# A Dynamic Model of Entrepreneurship with Borrowing Constraints: Theory and Evidence

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**Abstract**

Does wealth begets wealth and entrepreneurship, or is entrepreneurship mainly determined by individual's ability? A large literature studies the relationship between wealth and entry to entrepreneurship to inform this question. This paper shows that in a dynamic model, the existence of financial constraints to the creation of business implies a non-monotonic relationship between wealth and entry into entrepreneurship: the probability of becoming an entrepreneur as a function of wealth is increasing for low wealth levels – as predicted by standard static models - but it is decreasing for higher wealth levels. US data are used to study the qualitative and quantitative predictions of the dynamic model. The welfare cost of borrowing constraints are found to be significant, around 6% of lifetime consumption, and are mainly due to undercapitalized entrepreneurs (intensive margin), rather than to able people not starting businesses (extensive margin).

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# 1 Introduction

Does wealth begets wealth and entrepreneurship, or is entrepreneurship mainly determined by individual's ability? A large empirical literature have studied the importance of financial frictions to entrepreneurship. An important focus of this literature is the study and interpretation of the observed relationship between entry into entrepreneurship and wealth.[1] A positive relationship between entry into entrepreneurship and wealth is often seen as evidence in favor of borrowing constraints. Indeed, this is the basic implication of a simple static model (e.g., Evans and Jovanovic (1989)).

Nevertheless, as argued by many authors, it is important to understand the endogenous determination of wealth to be able to interpret this correlation. For example, people with no entrepreneurial ability tend to save less and are also less likely to start businesses. This paper studies the savings decision of an individual that face the choice to become an entrepreneur, but is financial constraint, to guide the interpretation of the data.

The analysis of the dynamic model provides three main predictions that are compared to data: (1) individuals who eventually become entrepreneurs have higher savings rates than individuals who expect to remain workers; (2) the growth rate of consumption of new entrepreneurs is higher than that of workers and old entrepreneurs; (3) the probability of becoming an entrepreneur as a function of wealth is increasing for low wealth levels – as predicted by standard static models - but it is decreasing for higher wealth levels.

The first prediction summarizes nicely the essential ingredient of the model: workers can save up to overcome borrowing constraints to become entrepreneurs. As discussed above, depending on their relative skills, some workers choose to never become entrepreneurs, either because it is not productively efficient or because the savings required to overcome the constraints require too large a sacrifice. Others find that becoming an entrepreneur will have a large enough surplus and consequently save more. The second prediction follows from the fact that new entrepreneurs are still constrained with respect to their capital level and consequently have a higher marginal product of capital than the market interest rate. The last prediction follows from the fact that wealth and entrepreneurship are jointly determined in the model. For low wealth levels,

[1] Of particular relevance for this paper is the work by Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz-Eakin et. al. (1994), Quadrini (1999), Glenn and Hubbard (2001), and Hurst and Lusardi (2004) for the US and Paulson and Townsend (2004) for Thailand.

entry into entrepreneurship increases with wealth because it relaxes the borrowing constraint, just as predicted by standard static models. For high wealth levels, however, entry into entrepreneurship and wealth become negatively related. This negative relationship reflects the fact that over time individuals with high entrepreneurial skills are selected out of the pool of workers and that this selection effect increases with wealth. Intuitively, if an individual is rich and still works for a wage, then it is unlikely that he has a high entrepreneurial skill. This prediction contrasts sharply with the conventional wisdom in the empirical literature on borrowing constraints and entrepreneurship.

The analysis of the model thus suggests that using data on saving rates, consumption growth, and the transition into entrepreneurship as a function of wealth would allow the estimation of the model. This is done by minimizing the distance between the statistics describing the behavior of saving rates of entrepreneurs and the dynamics of entrepreneurship in the U.S. data and the same statistics from the simulated model. In other words, the model is estimated by an Indirect Inference procedure as described by Gourieroux and Monfort (1996).

At the estimated parameter values, welfare costs of borrowing constraints are substantial. Consumption must be increase by 6% to make individuals indifferent between the economy with borrowing constraints and the one with perfect capital markets. At the same time, poverty traps tend not to be important for the US economy. This is because the entrepreneurial technology is estimated to have sharp decreasing returns to the variable factors that need to be financed.[2] Welfare costs arise mainly because it take long for extremely able entrepreneurs to operate their businesses at their unconstrained scale. Most profitable individuals eventually get to start businesses in the estimated model.

**Literature Review** A large empirical literature have studied the importance of financial frictions to entrepreneurship. An important focus of this literature is the study and interpretation of the observed relationship between entry into entrepreneurship and wealth.[3] A positive relationship between entry into entrepreneurship and wealth is often seen as evidence in favor of borrowing constraints. Indeed, this is the basic implication of a simple static model (e.g.,

[2] Note that this could be either because the span of control of (mainly small) businesses in the data is low or because they operate labor-intensive technologies.

[3] Of particular relevance for this paper is the work by Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz-Eakin et. al. (1994), Quadrini (1999), Glenn and Hubbard (2001), and Hurst and Lusardi (2004) for the US and Paulson and Townsend (2004) for Thailand.

Evans and Jovanovic (1989)). Nevertheless, as argued by many authors, it is important to understand the endogenous determination of wealth to be able to interpret this correlation. For example, people with no entrepreneurial ability tend to save less and are also less likely to start businesses. The observed correlation between entry and wealth partially reflects the endogenous determination of wealth as opposed to a "causal" effect of liquidity[4]. This paper directly addresses this question by modeling the endogenous determination of wealth and entrepreneurship. Interestingly enough, novel predictions arise from the analysis of a dynamic model of wealth determination and entrepreneurship.

Related, it is important to note that the model is able to fit the relationship between the transition into business ownership and wealth, specially the fact that "only for households at the top of the wealth distribution is there a strong and positive relationship between household wealth and business entry" (see Hurst and Lusardi (2004)). Through the lens of the dynamic model, the data is consistent with borrowing constraints playing a substantial role on the dynamics of entrepreneurship, although the effect of borrowing constraints on the creation of businesses appear to be transitory.[5]

This paper is also motivated by and complements a recent literature that studies the quantitative implications of models of occupational choice and borrowing constraints. Numerical solutions of related models are studied by Quadrini (2000) and Cagetti and De Nardi (2005). These authors have shown that models featuring entrepreneurship and financial frictions are important to explain the observed wealth distribution in the U.S economy. In a similar vein, Cagetti and De Nardi (2004), Li (2002), and Meh (forthcoming) quantify the effect of various policies in models featuring entrepreneurs and credit constraints for the US economy. This paper complements this literature by providing an analytical characterization of the joint dynamic of wealth and entrepreneurship in this type of models.

The rest of the paper is organized as follows. Section 2 describes the individual's dynamic occupational choice problem. Section 3 presents the implications

[4]See Holtz-Eakin et. Al. (1994) for an early discussion of how the endogeneity of wealth may affect the interpretation of the entry and wealth relationship. In a recent paper, Hurst and Lusardi (2004) present evidence against wealth being a bad indicator of liquidity for individuals becoming entrepreneurs.

[5]Hurst and Lusardi (2004) also show that the average starting capital of small businesses is small. The estimated model is also consistent with this observation as a large fraction of entrants do so with low wealth levels, and therefore, at a small scale. Again, through the lens of the model this fact imply that a substantial fraction of the entrants have a large *relative* ability as entrepreneurs, i.e., large ability as entrepreneurs relative to their ability as workers.

of the model for the average relationship between wealth and the likelihood of a transition to entrepreneurship. Section 4 confronts the predictions of the model and measures the welfare cost of borrowing constraints by estimating the model using U.S. data.

# 2 The Model Economy

The model is set in continuous time. Households are endowed with entrepreneurial ability, $e$, and initial wealth, $a_0$. In each instant of their life, they have the option to work for a wage, $w$, and invest their wealth at a constant interest rate, $r$, or to work and invest in an individual specific technology with productivity $e$, i.e., to become entrepreneurs. If households decide to be entrepreneurs they must devote all their labor endowment to run their businesses, i.e. occupations are indivisible. This captures a fundamental non-convexity: households are more productive by specializing in one activity. Households are only allow to borrow up to a fraction of their wealth.

## 2.1 Preferences

Agents' preferences over consumption profiles are represented by the time separable utility function

$$U(c) = \int_0^\infty e^{-\rho t} u(c(t))\, dt \tag{1}$$

where $t$ is the age of the individual and $\rho$ is the rate of time preference. The utility function over consumption, $u(c)$, is strictly increasing and strictly concave.[6]

The infinite horizon is a convenient analytical assumption. The theory should be understood as describing the life-cycle of an individual. Under this interpretation, $\rho = \rho^* + p$, where $\rho^*$ is the rate of time preferences and $p$ is the constant rate at which agents die. When studying the quantitative implications of the theory, a discrete time version of the model with finite horizon will be simulated and estimated.

[6] In a life-cycle interpretation of the model $t$ is the age of the agents and $\rho = \rho^* + p$, where $\rho^*$ is the rate of time preferences and $p$ is the constant rate at which agents die. This interpretation of the model is used when studying the quantitative implications of the theory.

## 2.2 Resource Constraints and Technologies

Agents start their lives with wealth $a_0$. At any time $t \geq 0$, their wealth, $a(t)$, evolves according to the following law of motion

$$\dot{a}(t) = y(a(t)) - c(t) \quad t \geq 0, \tag{2}$$

where $y(a(t))$ is the income of the agent with wealth $a(t)$, and $\dot{a}$ refers to $\frac{\partial a(t)}{\partial t}$.[7] The shape of the income function depends on occupational choices as follows.

If agents choose to be wage earners, they will sell their labor endowment for a wage $w$ and invest their wealth at a rate of return $r$. In this case, their income $y(a)$ is

$$y^W(a) = w + ra, \tag{3}$$

where $ra$ is the return on their wealth $a$. I refer to $w$ as *the wage*, but it should be understood that wages are individual-specific. Formally, $w = \bar{w}l$, where $l$ are the efficiency units that an individual can supply and $\bar{w}$ is the price of an efficiency unit of labor.

If individuals run a business they must devote their entire labor endowment to operate the business. Their revenue is given by the function, $f(e,k)$, where $e$ is the agent-specific ability and $k$ is the amount of capital invested in the business. [8] $f(e,k)$ is assumed to be strictly increasing in both arguments, homogeneous of degree 1, and strictly concave in capital, $f_e(e,k) > 0$, $f_k(e,k) > 0$, $f_{kk}(e,k) < 0$. Inada conditions are assumed to hold, $\lim_{k \to 0} f_k(e,k) = \infty$ and $\lim_{k \to \infty} f_k(e,k) = 0$. A higher entrepreneurial ability is associated with a higher marginal product of capital, $f_{ek}(e,k) > 0$, also $f(0,k) = 0$ and $\lim_{e \to \infty} f(e,k) = \infty$.

The amount of capital that agents can invest in their businesses is constrained by their wealth. To focus the analysis on the interaction between individual savings and occupational choice, I choose a simple specification of borrowing constraints. In particular, I assume that the value of an individual's business assets, $k$, must be less than or equal to the value of their wealth, $k \leq a$.

[7] For simplicity of exposition, I drop the time as an argument of the different functions.

[8] The production function should be interpreted as the reduced form of a more general technology requiring capital and labor,

$$f(e,k) = \max_n \tilde{f}(e,k,n) - \bar{w}n,$$

where $n$ are the efficiency units of labor employed and $\bar{w}$ is the price of an efficiency unit of labor. When calibrating the model and when discussing the predictions of the model for technologies with different capital intensities, the more general notation will be used.

If wealth exceeds the value of desired business assets, the remaining wealth is invested at the rate $r$.[9]

Therefore, the income of an entrepreneur solves the following static profit maximization problem:

$$y^E(e,a) = \max_{k \leq a} \{f(e,k) + r(a-k)\}. \tag{4}$$

Note that the scale of the business equals the individual's wealth, $a$, as long as wealth is lower than the unconstrained scale of the business, $k_u(e)$. The unconstrained scale is the solution to the unconstrained profit maximization problem, i.e.,

$$k_u(e) = \arg\max_k \{f(e,k) - rk\}.$$

This function is strictly increasing. Inada conditions are necessary to guarantee that this function is well defined for all $e$.

## 2.3 Consumer's Problem

Agents choose profiles for consumption, $c(t)$, wealth, $a(t)$, occupational choice, and business assets, $k(t)$, to solve

$$\begin{aligned}
&\max_{c(t),a(t),k(t)\geq 0} \int_0^\infty e^{-\rho t} u(c(t))\,dt \\
&s.t. \\
&\dot{a}(t) = y(a(t)) - c(t) \\
&y(e,a(t)) = \max\left\{y^E(e,a(t)), y^W(a(t))\right\}.
\end{aligned}$$

As is implicitly recognized in the statement of the problem, the occupational decision is a static one. That is, given current wealth, $a$, agents choose to be entrepreneurs if their income as entrepreneurs, $y^e(e,a)$, exceeds their income as wage earners, $y^w(a)$, i.e., $y^e(e,a) \geq y^w(a)$.

This can be expressed as a simple policy function. Define $\underline{e}$ to be the ability at which individuals are just indifferent between being wage earners and being entrepreneurs conditional on being able to borrow at the interest rate $r$.[10] Able

[9] When studying the quantitative implications of the theory, I allow for entrepreneurs to invest up to a fraction of their wealth, i.e., $k \leq \lambda a$ with $\lambda \geq 1$.

[10] $\underline{e}$ solves

$$\max_k f(\underline{e},k) - rk = w.$$

The left hand side of this equation is well define, increasing, continuous and take the value individuals (individuals with ability above $\underline{e}$) decide to be entrepreneurs if their current wealth is higher than the threshold wealth $\underline{a}(e)$, $a \geq \underline{a}(e)$,where $\underline{a}(e)$ solves

$$f(e, \underline{a}(e)) = w + r\underline{a}(e).$$

Intuitively, agents of a given ability choose to become entrepreneurs if they are wealthy enough to run their businesses at a profitable scale. Alternatively, agents of a given wealth $a$ choose to become entrepreneurs if their ability is high enough. Both ability and resources determine the occupational decision.

Given the optimal static decision, the dynamic program is equivalent to a standard capital-accumulation problem subject to a production function of the form

$$y(e,a) = \begin{cases} w + ra & \text{if } a \in [0, \underline{a}(e)) \\ f(e,a) & \text{if } a \in [\underline{a}(e), k_u(e)) \\ f(e, k_u(e)) + r(a - k_u(e)) & \text{if } a \in [k_u(e), \infty) \end{cases}.$$

This technology is given by the upper envelope of the "wage earner technology," $y^w(a)$, and the "entrepreneurial technology," $y^e(e,a)$. Figure 1 describes these technologies. Notice that this production function is not concave. The return to capital increases if individuals invest more than $\underline{a}(e)$.

Necessary conditions for the wealth accumulation problem are given by

1. the Euler equation,

$$\frac{u''(c)\,c}{u'(c)}\frac{\dot{c}}{c} = \begin{cases} r - \rho & \text{if } a \in [0, \underline{a}(e)) \\ f_k(e,a) - \rho & \text{if } a \in [\underline{a}(e), k_u(e)) \\ r - \rho & \text{if } a \in [k_u(e), \infty) \end{cases}, \tag{5}$$

   stating that the marginal rate of substitution should equal the marginal rate of transformation;

2. the law of motion for wealth,

$$\dot{a} = y(e,a) - c, \tag{6}$$

   describing the evolution of the individual wealth;

3. and the tranversality condition,

zero for $e = 0$ and goes to infinity as $e$ goes to infinity.

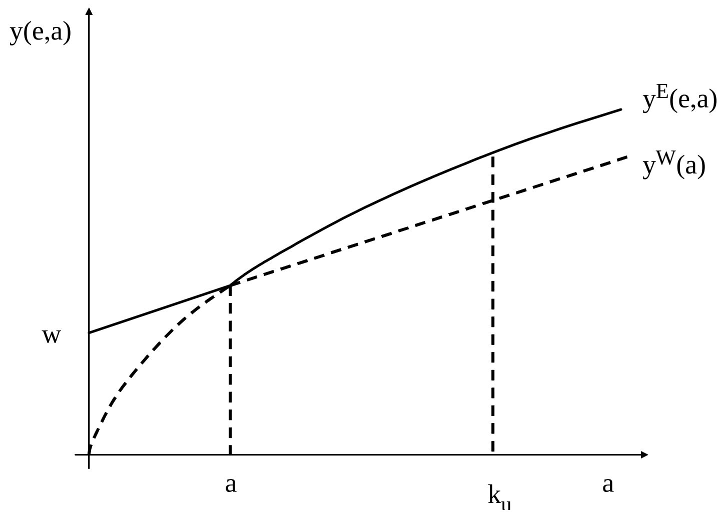


Figure 1: Technologies Available to Households

$$\lim_{t\to\infty} e^{-\rho t} u'\left(c\left(t\right)\right) a\left(t\right) = 0, \tag{7}$$

stating that value of wealth should converge to zero.

In the present case, these conditions are only necessary. As in any non-convex problem, there are solutions to the first order conditions that correspond to global minima or to local maxima that are not global maxima. Also, there can be multiple maxima. In section 4, I analyze the optimal accumulation path under this technology.

I conclude this section by noting that:

**Remark 1:** The model is homogeneous of degree 1 in $(a, w, e)$.

Exploiting this property, I normalize all the variables in the model by the wage. When studying the behavior of entrepreneurs in the data, this also suggests that wealth to wage ratios are the relevant measure of resources available to individuals and that the relevant notion of entrepreneurial ability to the model is relative ability, i.e., entrepreneurial ability relative to the ability as a worker $e/w$.

## 2.4 The Evolution of Individual Wealth

This section reviews a characterization of the evolution of individual wealth derived in Buera (2007). The main result are: (a) There exists a threshold wealth level, $a_s(e)$, such that individuals with initial wealth below the threshold, $a_0 < a_s(e)$, follow a path associated with decreasing wealth, converging to a zero-wealth steady state where they are wage-earners. Meanwhile, households with initial wealth above the threshold, $a_0 \geq a_s(e)$, save to become entrepreneurs and converge to a high-wealth entrepreneurial steady state. (b) The function $a_s(e)$ is strictly decreasing in entrepreneurial ability and there exists a minimum ability, $e_{high}$, such that individuals with ability above $e_{high}$ save to become entrepreneurs regardless of their initial wealth.

Proposition 1 contains the main result of this section: given an ability level $e$, households with low initial wealth will follow a path converging to a zero wealth worker steady state, and households with high initial wealth will follow a path converging to a high wealth entrepreneurial steady state.

**Proposition 1***: There exits a strictly positive ability level, $e_{low}$ and a finite ability level, $e_{high}$ such that:*

1. *For $e \leq e_{low}$ it is optimal for agents to follow the trajectory converging to the $(0, w)$ steady state for all levels of initial wealth ;*

2. *For $e \in (e_{low}, e_{high})$ there is a single initial wealth, $a_s(e)$, such that individuals starting with wealth level, $a_s(e)$, will be indifferent between following the trajectory converging to the $(0, w)$ steady state or the trajectory converging to the $(a_{ss}, c_{ss})$ steady state. Agents with initial wealth to the left of $a_s(e)$ prefer to follow the trajectory converging to the $(0, w)$ steady state. The converse holds for agents starting with wealth to the right of $a_s(e)$.*

3. *For $e \geq e_{high}$ it is optimal for agents to follow the trajectory converging to the $(a_{ss}, c_{ss})$ steady state for all levels of initial wealth .*

**Proof:** *See Buera (2007).*

Intuitively, households with low initial wealth require a larger investment in terms of forgone consumption to save up toward the efficient scale. Thus, they prefer to have a lower but smoother consumption profile as wage earners.

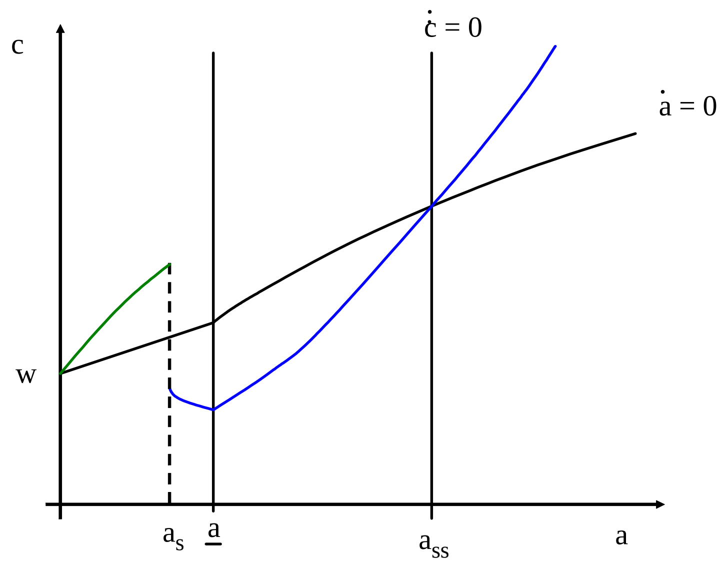


Figure 2: Optimal Trajectories (Intermediate Ability)

Figure 2 illustrates the optimal trajectories in the intermediate ability case[11].

This proposition tells us that the typical policy function for consumption will be discontinuous. For agents with low initial wealth, it is optimal to start with relatively high, but decreasing, consumption. For agents with high initial wealth it is optimal to start with a relatively low, but increasing, consumption. Moreover, there is a unique threshold on initial wealth that divides individuals into these two groups. I refer to this threshold as the poverty trap threshold. The poverty trap threshold is a function of entrepreneurial ability.

This characterization implies the following corollary.

**Corollary to Proposition 1:** (a) *The saving rate of individuals who eventually become entrepreneurs is higher than the saving rate of individuals who remain wage earners. (b) The growth rate of consumption increases after individuals become entrepreneurs.*

This suggests two obvious tests for the model that are performed in section 4.

The next result states that the threshold $a_s(e)$ is decreasing in the agent's entrepreneurial ability. It also tells us that there is a minimum ability $e_{low}$ and

[11] For low enough ability $e$ it will the case that $a_s > \underline{a}$, implying that there are individuals that start as entrepreneurs, but choose to eat their wealth an eventually become workers.

a maximum ability $e_{high}$ such that nobody with ability lower than $e_{low}$ is an entrepreneur in the long run and everybody with ability higher than $e_{high}$ is an entrepreneur in the long run.

**Proposition 2**: *If* $a_s(e) \leq \underline{a}(e)$, *the "poverty trap",* $a_s(e)$, *is strictly decreasing in the agent's ability, i.e.,*

$$\frac{\partial a_s(e)}{\partial e} < 0.$$

**Proof:** *See Buera (2007).*

The intuition of this result is straightforward. For individuals that are still workers, entrepreneurial ability only affects those who plan to follow the entrepreneurial path. In other words, the more profitable it is to be an entrepreneur, the less likely an individual will be stuck in a poverty trap. The ability to save gives an upper bound on the importance of borrowing constraints.

# 3 Entry into Entrepreneurship and Wealth

Most of the empirical literature on entrepreneurship and borrowing constraints has focused on measuring the effect of wealth on the likelihood that an individual becomes an entrepreneur. A positive relationship is often seen as evidence in favor of the direct effect of borrowing constraints on entrepreneurship. Understandably, many researchers have expressed the concern that wealth and unobservable ability may be positively correlated, making such interpretation problematic. In this section I address these issues.

The likelihood that an individual becomes an entrepreneur as a function of current wealth, conditional on age, $P(transition|a,t)$, is a non-monotonic function of wealth. The fraction of individuals that enter entrepreneurship is an increasing function of wealth for low wealth levels, as in a static model (e.g. Evans and Jovanovic (1989)), and a decreasing function of wealth for high wealth levels. Moreover, with time the transition probability is compressed from below and above. The intuition for this result is extremely simple. When looking at transitions, we are considering the agents that are working today. But this is a selected sample. In particular, these are individuals that did not find it profitable to start a business by the current period, i.e. they are relatively low ability individuals. Moreover, this selection increases with the wealth of the agents. If somebody is rich and hasn't started a business yet, then he must be a bad entrepreneur. Next, I give a formal statement of this result (see Figure 3 for an illustration this result).

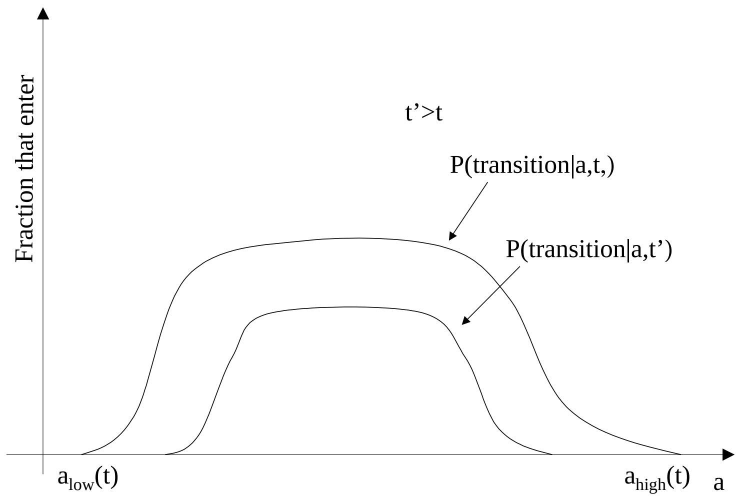


Figure 3: Transition into Entrepreneurship as a Function of Wealth and Age

**Proposition 4:** *There exists an age $\bar{t}$, an increasing function $a_{low}(t)$ and a decreasing function $a_{high}(t)$, satisfying $a_{low}(t) \leq a_{high}(t)$, a positive constant $\xi > 0$, and neighborhoods $\mathcal{N}(a_{high}(t), \xi)$* and *$\mathcal{N}(a_{low}(t), \xi)$ such that:*

1. *for all $t < \bar{t}$*

$$P(transition|a,t) = 0 \quad \forall a \in [0, a_{low}(t)] \cup [a_{high}(t), \infty);$$

and

$$P(transition|a,t) > 0 \quad \forall a \in \mathcal{N}(a_{high}(t), \xi) \text{ and } a \leq a_{high}(t)$$
$$P(transition|a,t) > 0 \quad \forall a \in \mathcal{N}(a_{low}(t), \xi) \text{ and } a \geq a_{low}(t)$$

2. *For all $t \geq \bar{t}$*

$$P(transition|a,t) = 0 \quad \text{all } a.$$

This proposition suggests a way to obtain information about the importance of borrowing constraints. In particular, it suggests a way to obtain a lower bound on the unconstrained scale of businesses, i.e., the unconstrained scale of the least efficient business for a fixed wage and borrowing constraints coefficient, $\lambda$. Indeed, if we were to observe that the decreasing portion of the transition into entrepreneurship occurs at very high wealth levels we would infer that the entrepreneurial technology is close to linear, large $\phi$, or that borrowing constraints are very tight, low $\lambda$.

**Remark 2:** *The minimum wealth level such that nobody with wealth higher than this makes the transition to entrepreneurship is a lower bound on the unconstrained scale of the least efficient business in operation, i.e.* $a_{high}(t) \leq k_u(\underline{e}) = \underline{a}(\underline{e})$.[12]

Next, I consider a straightforward implication of this result.

An important concern of the empirical literature on entrepreneurship and borrowing constraints is to measure the causal effect of wealth on the likelihood that an individual starts a business. It is often believed that the actual correlation between entry into entrepreneurship and wealth overestimates the causal effect (e.g. Holtz-Eakin et. al. (1994)). It turns out that in a dynamic model, the opposite is true. From proposition 5 we know that, for high wealth levels, wealth close to $a_{high}$, the transition to entrepreneurship as a function of wealth is decreasing. But the effect of a wealth transfer is still strictly positive. Thus, the observed effect of wealth on entrepreneurship underestimates the effect of a wealth transfer. This is the content of the next remark.

**Remark 3:** *For* $0 < t < \bar{t}$

$$\frac{\partial P(transition|a,t,\tilde{a})}{\partial a} < \frac{\partial P(transition|a,t,\tilde{a})}{\partial \tilde{a}}$$
$$\forall a \in N(a_{high}(t), \xi) \text{ and } a \leq a_{high}(t),$$

*where* $\tilde{a}$ *corresponds to an exogenous component of wealth at time* $t$ *that individuals expect to be zero with certainty.*

We are now ready to study the full set of implications of the dynamic model.

# 4 Empirical Evidence and Structural Estimation

In this section, U.S. data on savings rates, consumption growth of entrepreneurs, and the relationship between entry and wealth are used to evaluate the predictions of the dynamic model and to estimate it using Indirect Inference as describe in Gourieroux and Monfort (2002). The idea is to choose the parameters of the dynamic model to minimize the distance between a set of statistics

[12] In the case $\lambda > 1$ (i.e., we allow borrowing), this inequality becomes $\lambda a_{high}(t) \leq k_u(\underline{e})$. A large $a_{high}(t)$ can be rationalized by a large scale of businesses (large $\phi$) or by a tight borrowing constraint (low $\lambda$).

describing the behavior of savings rates and entrepreneurial dynamics in the U.S. data and those same statistics calculated from the simulated model. I begin by discussing the main features of the savings rates of entrepreneurs and the dynamics of entrepreneurship in two U.S. datasets.

## 4.1 Empirical Evidence

I use a yearly panel for the period 1984–1995 from the Panel Study of Income Dynamics (PSID) with rich information on occupational choice, ownership of businesses, and the wealth of U.S. households; and a quarterly rotating panel (1984–1999) from the Consumer Expenditure Survey (CEX) providing consumption data and information on occupational choice. Since the model provides a theory of the initial transition into entrepreneurship, I estimate the model with data on the savings behavior and the dynamics of entrepreneurship for young households (those that are up to 31 years old). Therefore, unless otherwise notice, all statistics are calculated for household that are up to 31 years old in the initial period. The data used is described in the data appendix.

Following the recent literature (see Gentry and Hubbard (2001) and Hurst and Lusardi (2004)), an entrepreneur in the data is identified as someone who reports to own a business. Unfortunately, this information is not available for the CEX. In the case of the CEX, an entrepreneur is identified as someone who reports to be self-employed. Whenever it is possible, results are shown for both definitions.

The first and second column of Table 1 report the main facts regarding the behavior of savings rates and consumption growth of entrepreneurs as measured by the ownership of a business and the self-employed status of the head of the household[13]. Here I sketch a summary of the data.

- **Individuals save more prior to starting a business.** Among the young households (age$\leq 31$), those that became business owners between periods $t$ and $t+1$ save 7% more in the previous 5 years (between $t-5$ and $t$) than households that neither owned a business in $t$ nor in $t+1$ (see first row of Table 1). Related, individuals becoming business owners between $t-5$ and $t$ save 26% more between these years than those that

[13] All the moments are calculated using individuals characteristics as controls (sex, marital status and education of the head). The value of the different moments should therefore be interpreted as the one for a single white male with college education.

do not own businesses in $t-5$ and $t$. This is referred to as the savings rate differential during entry in Table 2[14].

- **Business owners have higher saving rates than non-business owners, and their saving rates decrease sharply with age**. Household that own a business in $t-5$ and $t$ and are up to 31 years old in period $t-5$ save 26% more than households that neither own a business in $t-5$ nor in $t$. Among mature households (those that are between 32 and 41 years old in $t-5$), business owners save just 10% more than non-business owners.

- **Individuals becoming self-employed have higher consumption growth, both relative to workers and to individuals that are already self-employed.** The growth in consumption between $t-1$ and $t$ of individuals becoming self-employed between $t-1$ and $t$ is 7% higher than that of those who are workers in both periods. The consumption growth of households that are already self-employed is not particularly high.

The following fact is illustrated in Figure 4.

- **Poor individuals and extremely rich individuals are less likely to start businesses.** Among the young, the probability that a household that is not a business owner in period $t-5$ starts a business in period $t$ is mostly constant (around 10%) for the first 3 wealth to wage ratio quartiles, increases sharply to 20% for wealth to wage ratios in the 90th to 95th percentiles, and then decreases for wealth to wage ratios in the top 5 percentiles.

Related datasets and facts have been studied by other authors. Quadrini (1999, 2000), also using the PSID, present evidence that entrepreneurs, and workers becoming entrepreneurs, are more likely to increases their wealth to income ratios. Using the 1983-1989 panel from the Survey of Consumer Finances, Gentry and Hubbard (2001) report business owners having larger savings rates than non-business owners, and this effect being stronger for younger households as oppose to middle-aged households. They also find that households save more while becoming business owners. In their sample, the median household that becomes a business owner between $t-6$ and $t$ save 30% more between $t-5$

[14] Although the first is a more appropiate measure of savings in anticipation of entry, it cannot always be calculated in the simulated model since for some parameter values all entrepreneurs enter in less than 5 years.

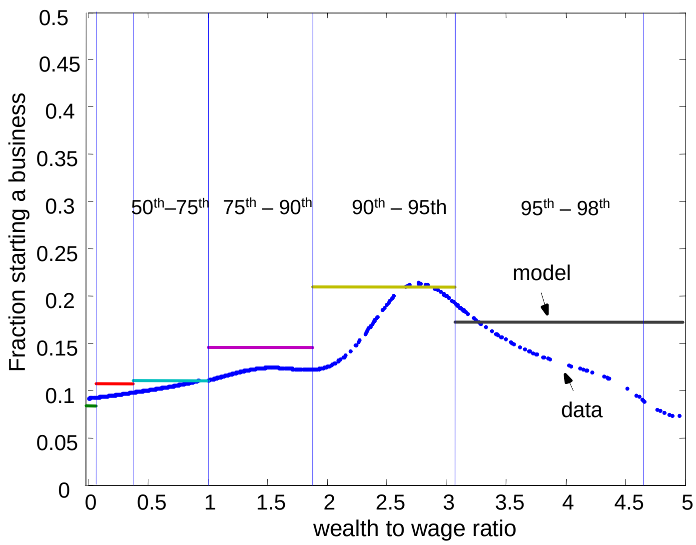


Figure 4: Wealth to wage ratios and the Transition into Business Ownership, PSID and Estimated Model. The relationship in the data corresponds to a semiparametric regression using the method described in Robinson (1988).

| Moments | Data | | Model | |
|---|---|---|---|---|
| | Business | Self-employed | $\phi^* = 0.36$ | $\phi = 0.70$ |
| Savings rate differentials of entrepreneurs vis-a-vis workers | | | | |
| Prior to entry | 0.07 | -0.06 | 0.30 | 0.30 |
| | (0.05) | (0.05) | | |
| During entry * | 0.26 | 0.19 | 0.34 | 0.43 |
| | (0.04) | (0.04) | | |
| After entry | | | | |
| young * | 0.26 | 0.30 | 0.25 | 0.32 |
| | (0.08) | (0.05) | | |
| mature * | 0.10 | 0.11 | 0.11 | 0.13 |
| | (0.05) | (0.06) | | |
| Consumption growth differential entrepreneurs vis-a-vis workers | | | | |
| During entry | – | 0.09 | 0.24 | 0.33 |
| | | (0.04) | | |
| After entry | – | 0.01 | 0.09 | 0.16 |
| | | (0.03) | | |

Table 1: Data and Simulated Moments (see Data Appendix for details). The moments that are signaled with "*" are the ones targeted when estimating the model.

and $t$ than households that are neither business owners in $t-6$ nor in $t$. Unfortunately, given the short SCF panel they cannot study the savings behavior in anticipation of the entry decision as is done in the current paper. More in line with my findings on relatively low savings rates in anticipation of entry, Hurst and Lusardi (2004), also using the PSID, find that previous wealth changes are weakly (and even negatively) correlated with future entry decision. They also study the relationship between wealth and the likelihood of becoming a business owner and find that the positive effect of wealth is stronger for the top percentiles of the wealth distribution. They do not report evidence on the negative relationship between wealth and entry into entrepreneurship for extremely wealthy individuals, as measured by their wealth to wage ratios.

## 4.2 Structural Estimation

The structure of the model is characterized by two preferences parameters, $\sigma$ and $\rho$, one technological parameter, $\phi$, the borrowing constraint coefficient, $\lambda$, and the distribution of ability, $g(e)$. To close the model, the interest rate, $r$, and initial distribution of wealth, $h(a_0)$, also need to be specified. The parameters are chosen using a combination of calibration and structural estimation. As in Section 5, I choose values for the preferences parameters, the interest rate and the depreciation rate following the standard practices: $\sigma = 1.5$, $\rho = r = 0.02$ and $\delta = 0.08$. The distribution of initial wealth is chosen to correspond to the distribution of wealth to income ratios of non-business owners that are up to 26 years old in the PSID. This leaves the technology parameter, $\phi$, the borrowing constraint coefficient, $\lambda$, and the distribution of ability to be jointly estimated by minimizing the distance between the moments in the data and the moments in the model. The details of the algorithm are described in the appendix. An heuristic discussion of the identification of the models follows.

There are eight parameters ($\phi$, $\lambda$, and six probabilities describing the ability distribution) that are estimated by matching nine moments (three moments related to the savings behavior of entrepreneurs vis-a-vis workers and the entry rates for six wealth to income groups[15]). Given values for $\phi$ and $\lambda$, the distribution of ability is identified from the fraction of entrants at each wealth group. To do this mapping, we need to use the information on the initial distribution of wealth and the dynamic of wealth implied by the model. Obviously, the shape of the distribution of ability for ability levels such that individuals never transition into entrepreneurship cannot be identified, i.e., for $e$ such that $\underline{a}(e) < a_s(e)$. The parameters $\phi$ and $\lambda$ are then identified by matching the relationship between wealth to wage ratios and entry for large values of the wealth to wage ratios as suggested by Remark 2, and the saving rate differential. In particular, I use the information on how fast business owners converge to the steady state scale of their businesses as measure by the decrease in their saving rate differential with tenure.

The first column of Table 2 reports the parameter estimates and their standard errors. An economy where entrepreneurs face strong decreasing returns to scale, $\phi = 0.36$, and tight borrowing constraints, $\lambda = 1.01$, best fits the data in the first column of Table 1. The distribution of ability turned out to be

[15] The wealth to income groups are: the bottom 25th percentile, 25th to 50th percentiles, 50th to 75th percentiles, 75th to 90th percentile, 90th to 95th percentile, and the top 5 percentile

bimodal with 40% of the profitable entrepreneurs (those with relative ability $\frac{e}{w} > 1$) having low returns (they could earn 5% more income as unconstrained entrepreneurs) and 30% facing large returns to entrepreneurship (they could earn three times as much income as unconstrained entrepreneurs)[16]. The estimate for $\phi$ is in line with the previous estimates by Evans and Jovanovic (1989) also using micro data for the U.S., while the estimate for $\lambda$ is relatively low. The second column of Table 2 reports the estimate for the borrowing constraint and the distribution of talent when the curvature parameter is constrained to take a larger value, $\phi = 0.7$. To rationalized the data, a less tight borrowing constraint is needed, $\lambda = 4.9$.

| Parameters | Estimate | |
|---|---|---|
| | Unconstrained | Constrained |
| Technology, $\phi$ | 0.36 | 0.70 |
| | (0.01) | – |
| Borrowing Constraint, $\lambda$ | 1.01 | 4.90 |
| | (0.03) | (0.04) |
| Probability (e \| e>1) | | |
| e = 1.05 | 0.39 | 0.39 |
| 1.20 | 0.02 | 0.00 |
| 1.50 | 0.23 | 0.05 |
| 3.00 | 0.32 | 0.37 |
| 5.00 | 0.05 | 0.00 |
| 10.00 | 0.00 | 0.20 |
| Average Welfare Costs | 6.24 | 8.84 |

Table 2: Parameter Estimates

Welfare cost of borrowing constraints are large (see Table 3). For the median among the productive entrepreneurs (individuals with $e > 1$) welfare costs amounts to 22% of lifetime consumption. The average welfare cost for the economy are 6% of lifetime consumption[17]. But, because there are strong decreasing

[16] For the estimated economy, since all transitions intro entrepreneurship are permanent, 46% of individuals are entrepreneurs and the probability of $e = 1$ is 0.24 . This is an artifact of not modelling exit and reentry of businesses.

[17] To calculate the average welfare cost, I force the distribution of ability to match the fraction of entrepreneurs in the data. This is done by shifting the mass of the ability distribution

returns to scale, individuals can undo most of the welfare cost of borrowing constraints by internally financing their businesses. This is specially true for very able individuals. In the same direction, the size and economic significance of poverty traps are low. Only individuals that would increase their income by just 5% decide not to save to start a business (12% of the population fall in this category).

| Ability | Wealth | | | |
|---|---|---|---|---|
| | zero wealth | 50th perc. wealth/ wage | 75th perc. wealth/ wage | 90th perc. wealth/ wage |
| 1.05 | 5.00 | 4.94 | 4.86 | 4.71 |
| 1.20 | 16.12 | 14.99 | 13.35 | 9.81 |
| 1.50 | 24.48 | 22.07 | 18.62 | 12.11 |
| 3.00 | 40.96 | 34.42 | 26.20 | 18.62 |
| 5.00 | 52.08 | 39.10 | 31.02 | 23.52 |

Table 3: Welfare Costs by Ability (rows) and Initial Wealth (columns) Measured as a Fraction of Lifetime Consumption (%)

The third column of Table 1 and Figure 4 reports the moments from the simulated model at the estimated parameters. The model does a good job of fitting the relationship between the transition into business ownership and wealth, specially the fact that "only for households at the top of the wealth distribution is there a strong and positive relationship between household wealth and business entry" (see Hurst and Lusardi (2004)). Savings rates of entrepreneurs, both young and mature, are matched but the model substantially overpredicts the savings rates prior to entry. The same is true for the growth rate of consumption of entrepreneurs. One the one hand, the behavior of savings rates and consumption growth suggest that borrowing constraints are not very important. But the relationship between wealth and the transition into entrepreneurship suggests the opposite conclusion. The estimation is the result of a compromise between these sets of moments.[18]

from ability values with $e > 1$ to values with $e = 1$, in a proportional way so that the distribution of ability conditional on $e > 1$ is not affected (and therefore not affecting the shape of the transition function (see Figure 4)).

[18] If the curvature parameter were to be constrained to take a larger value, $\phi = 0.7$, the model would imply a substantially larger saving rates differential for entrepreneurs vis-a-vis workers. This would be particularly true for younger businesses (see column 4 in Table 1). The fit of the model in terms of the relationship between entry and wealth, Figure 4, is almost

# A Data

I use a yearly panel for the period 1984–1995 from the Panel Study of Income Dynamics (PSID) with rich information on occupational choices, ownership of businesses and the wealth of US households; and a quarterly rotating panel (1984–1999) from the Consumer Expenditure Survey (CEX) providing consumption data and information on occupational choices.

In the case of the PSID, I create a 7 years panel pooling the households in the 1984-1990 and 1989-1995 samples. This gives a panel with two observations for wealth (1984 and 1989 in the 1984-1990 subsample, 1989 and 1994 in the 1989-1995 subsample), and yearly observations on the ownership of businesses and income. Using the pooled panel, I construct savings rates between the first and the fifth years, $savings_{1-5} = \frac{wealth_5 - wealth_1}{\sum_{t=1}^{5} income_t}$, wealth to income ratios $= \frac{wealth_t}{income_t}$ (the relevant measure of wealth in the model) and business ownership histories. "Wealth" corresponds the sum of net equity in a main home, other real estate, vehicles, farm/business, stocks, savings accounts and other assets, less debt; "income" equals total family money income plus food stamps minus federal income taxes paid; and business ownership status is determined by the question "Do you (or anyone in your family living there) own part or all of a farm or business?" For comparison purposes, I also use information about the self-employ status of the head of the household as an alternative proxy for entrepreneurship.

I restrict the sample to the households that are at least 22 and at most 61 years old in the first period, that are working in the 1, 2, 6 and 7th periods (these are the periods for which business ownership information is used) and I drop the observations with savings rates below and above the 1st and 99th percentiles respectively. These restrictions leaves 5354 observations that are used to calculate the moments reported in Table 1.

In the case of the CEX I use the quarterly Interview component for the 1984-1999 period. From this dataset I use information on non-durable consumption, self-employed status of the head of the household and demographic characteristics. After applying similar restrictions to the PSID sample, 5545 observations of household that are up to 31 years old are left to calculate the moments reported in Table 1.

identical.

# B Algorithm

Given values for the preferences and technology parameters $(\sigma,\ \rho,\ \phi, \delta)$, the borrowing constraint coefficient $(\lambda)$, the interest rate $(r)$, the individual decision problem is solved for each ability level in the ability grid,
$e = \{1, 1.05, 1.1, 1.2, 1.5, 2, 3, 5, 7, 10\}$, by backward induction from the last period, $T = 40$. Given the policy functions and 10000 values for the initial wealth drawn from the distribution of wealth to income ratio of non-business owners that are up to 26 years old in the PSID, 10000 histories are generated for each value of ability, $\{x_t(e)\}_{t=1}^{T}$. Where the initial wealth to income ratio takes values on the grid $a_0 = \{0, 0.17, 0.49, 1, 1.84, 2.87, 8.03\}$ with probabilities $h(a_0) = \{0.25, 0.25, 0.25, 0.15, 0.05, 0.03, 0.02\}$. Then, by stacking the data for individuals at different ages I obtained a simulated (unbalanced) panel of 400000 observations (10000 forty years observations, $\{x_t(e)\}_{t=1}^{T}$, 10000 thirty-nine years observations, $\{x_t(e)\}_{t=2}^{T}$, ...) that is used to calculate the statistics for each value of ability. Given the distribution of ability, a vector of dimension 10, the statistics from the model are calculated. The algorithm then search over values of the returns to scale parameter, $\phi$, the borrowing constraint coefficient, $\lambda$, and the distribution of ability to minimized the weighted distance between the statistics calculated using the PSID data and the data from the simulated model. The weights are given by the inverse of the covariance matrix of the moments from the PSID data.

A important simplification is introduced by the fact that ability is fixed over the life of an individual, implying that the individual decision problem is independent of the ability distribution. For given values of $\phi$ and $\lambda$, a gradient based method (matlab routine fmincon.m) is used to minimize over the distribution of ability subject to the constraint that at most six ability values get to be assigned a positive probability. Then, grid search is used to minimized over $\phi$ and $\lambda$.

# C Proof of the Results in the Paper

**Assumption A.1:** The policy function, $a(a_0, t, e)$, is strictly increasing and continuous in the entrepreneurial ability, $e$, for $a_0 \geq a_s(e)$.

This is a natural assumption. It requires that present consumption does not increase too much when ability increases. In terms of assumptions about preferences, it needs that the preferences between consumption today and tomorrow are not too bias toward present consumption. For example, it will true in a two period model if preferences are homothetic.

**Proof of Proposition 4.** The transition probability conditional on wealth, $a$, and age, $t$, is given by the integral over all agents with ability high enough such that they find it profitable to start a business by $t+\Delta$, $e > \underline{\underline{a}}^{-1}(a+\tilde{a},\Delta)$, but not so high for them to be already entrepreneurs, $e < \underline{a}^{-1}(a)$. Where $\underline{\underline{a}}(e,\Delta)$ solves the equation $a\left(\underline{\underline{a}}(e,\Delta),\Delta,e\right) = \underline{a}(e)$. Formally

$$P(transition|a,t,\tilde{a},\Delta) = \frac{\int\limits_{e\in\mathcal{E}^w(a,t)\,,\,e>\underline{\underline{a}}^{-1}(a+\tilde{a},\Delta)} g(e)\,h(a_0(a,t,e))\,de}{1-P(entrepreneur|a,t)} \tag{8}$$

where $\mathcal{E}^w(a,t)$ is the support of ability conditional on wealth $a$, age $t$ and currently being a worker of those individuals that will eventually become entrepreneurs, $\mathcal{E}^w(a,t) = \{e \in R_+ : \exists a_0$ for which $a(a_0,t,e) = a$, $e < \underline{a}^{-1}(a)$ and $e \geq a_s^{-1}(a)\}$. Then, to proof this result we need to characterize the lowest and highest wealth levels, $a_{low}(t)$ and $a_{high}(t)$, such that this set is non-empty.

That there exist an increasing function $a_{low}(t)$ such that for $a < a_{low}(t)$ the set $\mathcal{E}^w(a,t)$ is empty follows trivially from the fact that the wealth of individuals that will eventually become entrepreneurs increases overtime and that $a_0 \geq 0$.

The function $a_{high}(t) = \underline{a}(e_{\min}(t))$ where $e_{\min}(t) = \inf\{e \in [e_{low},\infty] : a(a_s(e),t,e) = \underline{a}(e)\}$. Note that $e_{\min}(t)$ is a strictly increasing function of $t$ since for $e$ close to $e_{low}$ $(a_s(e_{low}) = \underline{a}(e_{low}))$ we know that $a(a_s(e),t,e) > \underline{a}(e)$ and, by continuity, we know that for the minimum root of the equation $a(a_s(e),t,e) = \underline{a}(e)$ the function $a(a_s(e),t,e)$ is decreasing and crosses the function $a_s(e)$ from below. Thus, the function $a_{high}(t)$ is then decreasing since it is the composition of a strictly decreasing function with an increasing function.

The maximum age such that the set $\mathcal{E}^w(a,t)$ is non-empty for some wealth level is define by

$$\bar{t} = \inf\{t \in R_{++} : a(a_s(e),t,e) \geq \underline{a}(e) \quad \forall e \geq e_{\min}\}$$

Age $\bar{t}$ is finite since $\underline{a}(e) < a_{ss}(e)\ \forall e \geq e_{\min}$. ■

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