CREDIT SUPPLY AND THE HOUSING BOOM

ALEJANDRO JUSTINIANO, GIORGIO E. PRIMICERI, AND ANDREA TAMBALOTTI

Abstract. An increase in credit supply driven by looser lending constraints in the mortgage market is the key force behind four empirical features of the housing boom before the Great Recession: the unprecedented rise in home prices, the surge in household debt, the stability of debt relative to house values, and the fall in mortgage rates. These facts are more difficult to reconcile with the popular view that attributes the housing boom only to looser borrowing constraints associated with lower collateral requirements, because they shift the demand for credit.

Key words and phrases: House prices, household debt, mortgage rates, leverage, down payments.

1. INTRODUCTION

The U.S. economy recently experienced a severe financial crisis that precipitated the worst recession since the Great Depression. The seeds for these events were sown during an unprecedented housing and mortgage boom, which was characterized by four main facts.

Fact 1: House prices rose dramatically. Between 2000 and 2006 real home prices increased roughly between 40 and 70 percent, depending on measurement, as shown in figure 1.1. This spectacular boom was followed by a bust after 2006.

Fact 2: Households’ mortgage debt surged. This is illustrated in figure 1.2 for both the aggregate household sector and for financially constrained households in the Survey of Consumer Finances—the group that is most informative for the parametrization of our model. Both measures of indebtedness were stable in the 1990s, but they increased by about

Date: First version: March 2014. This version: January 2017.
We thank Tobias Adrian, Larry Christiano, Sebastian Di Tella, Andreas Fuster, Simon Gilchrist, Bob Hall, Cosmin Ilut, Igor Livshits, Donato Masciandaro, Ander Perez, Monika Piazzesi, Vincenzo Quadrini, Giacomo Rondina, Martin Schneider, Amir Sufi, Harald Uhlig as well as seminar and conference participants for comments and suggestions. Primiceri thanks Bocconi University and EIEF for their hospitality while he conducted part of this research. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Banks of Chicago, New York or the Federal Reserve System.
Figure 1.1. Real house prices. FHFA (formerly OFHEO) all-transactions house price index for the United States and CoreLogic Home Price Index (HPI). Both indexes are deflated by the consumer price index, and normalized to 100 in 2000:Q1.

30 and 60 percentage points between 2000 and 2007, before falling during the financial crisis.

FACT 3: Mortgage debt and house prices increased in parallel. As a result, the ratio of home mortgages to the value of residential real estate remained roughly unchanged until 2006. This fact is documented in figure 1.3, which also shows that this aggregate measure of household leverage spiked when home values collapsed before the recession.

FACT 4: Real mortgage rates declined. Figure 1.4 plots the 30-year conventional mortgage rate minus various measures of inflation expectations from the Survey of Professional Forecasters. It shows that real mortgage rates fluctuated around 5% during the 1990s, but fell by 2 to 3 percentage points as the housing boom unfolded.

We study these events in a simple general equilibrium framework that draws a stark distinction between the supply and demand for credit. On the demand side, a collateral constraint limits households' ability to borrow against the value of real estate, as in the large literature spawned by Kiyotaki and Moore (1997). On the credit supply side, a lending constraint impedes the flow of savings to the mortgage market.

We show that the four facts described above are easy to explain as the consequence of a progressive relaxation of this lending constraint, which generates an expansion in the supply of mortgage credit available to borrowers. Reproducing the same stylized facts as the result of looser collateral requirements alone, as in much of the literature on the recent
credit cycle based on Kiyotaki and Moore (1997), is much harder. The reason is that, in these models, looser collateral requirements shift the demand for credit, putting upward pressure on interest rates. Our stylized model of credit demand and supply is designed to highlight these simple points, but the same basic economic forces would also be at play in more complex models.

Lending constraints are a simple modeling device to capture a combination of technological, institutional, and behavioral factors that restrain the flow of funds from savers to mortgage borrowers. Starting in the late 1990s, the explosion of securitization and of market-based financial intermediation, together with changes in the regulatory and economic environment, lowered many of these barriers. For example, the pooling and tranching
of mortgages into mortgage-backed securities (MBS) created highly rated assets out of risky mortgages. These assets could then be purchased by those institutional investors that are restricted by regulation to only hold fixed-income securities with high ratings. As a result, the boom in securitization contributed to channel into mortgages a large pool of savings that had previously been directed towards other assets, such as government bonds (Brunnermeier, 2009). Investing in those same senior MBS tranches also freed up intermediary capital, due to their lower regulatory charges. Combined with the rise of off-balance-sheet vehicles, this form of “regulatory arbitrage” allowed banks to increase leverage without raising new capital, expanding their ability to supply credit to mortgage markets (Acharya and Richardson, 2009, Acharya et al., 2013, Nadauld and Sherlund, 2009). Finally, international
factors also played an important role in increasing the supply of funds to U.S. mortgage borrowers—a phenomenon often referred to as the global saving glut (Bernanke, 2005).

We model this well-documented reduction in the frictions impeding the free flow of savings into mortgage finance as an exogenous relaxation of lending constraints, and analyze its macroeconomic effects through the lens of our general equilibrium model. An important assumption underlying this exercise is that the U.S. economy in the 1990s was constrained by a limited supply of funds to the mortgage market, rather than by a scarcity of housing collateral. Starting from this situation, we show that a progressive loosening of the lending constraint in the residential mortgage market increases household debt in equilibrium (fact 2). If the resulting shift in the supply of funds is large enough, the availability of collateral also becomes a binding constraint. Then, a further expansion of the lending limit boosts the collateral value of houses, increasing their price (fact 1), while the interest rate falls (fact 4). Moreover, higher real estate values endogenously relax the collateral constraint, boosting households’ borrowing capacity in tandem with the rise in house prices (fact 3). We show these results analytically, but a simple calibration of our model based on the Survey of Consumer Finances also approximates these facts quantitatively.
Our reconstruction of the boom based on an expansion in credit supply is consistent with a large body of microeconomic evidence (e.g. Ambrose and Thibodeau, 2004, Mian and Sufi, 2009 and 2011, Favara and Imbs, 2015, Di Maggio and Kermani, 2014). Our model, however, can also perfectly accommodate a contemporaneous loosening of collateral requirements of the kind documented by Duca et al. (2011), Favilukis et al. (2017), Geanakoplos (2010) and DeFusco and Paciorek (2016), among others. We illustrate this point with a simulation in which lending and borrowing constraints are relaxed simultaneously. This combined experiment also matches the stylized facts. However, looser lending constraints account for most of the house price boom. Most importantly, the associated shift in credit supply remains the only force pushing interest rates lower.

In this combined experiment, the shifts in collateral and lending limits happen independently, since our model neatly separates the two constraints. In reality, though, an increase in banks’ ability to lend would likely prompt them to accept lower down payments. This intuitive link between collateral requirements and lending limits, which is absent from the workhorse model of collateralized borrowing of Kiyotaki and Moore (1997), would connect the movements in the demand and supply of credit. Nevertheless, our results suggest that a satisfactory account of the credit boom requires a larger shift in credit supply than in loan demand, even if these shifts were driven by some common determinants.

The rest of the paper is organized as follows. Section 1.1 reviews the literature. Section 2 presents our simple model of lending and borrowing with houses as collateral and a lending constraint. Section 3 characterizes the model equilibrium analytically. Section 4 illustrates some simple quantitative experiments that compare the macroeconomic impact of looser lending and collateral constraints. Section 5 concludes.

1.1. Related Literature. This paper is related to the macroeconomic literature on the causes and consequences of the financial crisis. As in Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2012), Hall (2012), Midrigan and Philippon (2011), Favilukis et al. (2017), Boz and Mendoza (2014), Justiniano et al. (2015, 2014), and Huo and Rios-Rull (2014), we use a model of household borrowing to analyze the drivers of the boom and bust in credit and house prices that precipitated the Great Recession. Guerrieri and
Uhlig (2016) provide a comprehensive analysis of the interaction between credit and housing cycles, and an exhaustive overview of this literature.\textsuperscript{1}

We follow these studies by limiting borrowing through a collateral constraint à la Kiyotaki and Moore (1997), which is backed by houses as in Iacoviello (2005). What is new in our framework is the introduction of the lending constraint, as a device to model the financial liberalization of the 2000s and the resulting expansion in credit supply first documented by Mian and Sufi (2009). This is in contrast with the literature cited above, which tends to capture variation in the availability of credit in both phases of the cycle through changes in the tightness of the borrowing constraint.\textsuperscript{2} In these models, looser collateral requirements increase the demand for credit, putting upward pressure on interest rates, which is counterfactual.

Another novelty of our approach is the interaction of lending constraints with borrowing limits. This interplay generates rich patterns of debt and home values that improve the model’s ability to match the fundamental facts about the boom, even in an extremely simple economy. The interaction between constraints also sets our work apart from Kiyotaki et al. (2011), Adam et al. (2012), Garriga et al. (2012) and Kermani (2012). They study the effects of a reduction in the world interest rate on a small open economy with borrowing constraints. These effects are qualitatively similar to those of looser lending constraints in our framework, but they treat the decline in interest rates as exogenous. In our model, in contrast, lower interest rates result from a slacker lending constraint when the borrowing limit is binding. This mechanism connects the fall in mortgage rates to the liberalization of mortgage financing and with other well-documented domestic, rather than just international, developments.

This emphasis on the increased supply of domestic funds to the mortgage market distinguishes our work also from the literature on global imbalances (e.g. Caballero et al., 2008, Mendoza et al., 2009, Caballero and Krishnamurthy, 2009). These papers study the dynamics of international portfolios and net foreign assets when heterogeneous countries

\textsuperscript{1}Our paper is also broadly related to the work of Gerali et al. (2010) and Iacoviello (2015), who estimate large-scale dynamic stochastic general equilibrium models with several nominal and real frictions, including collateral constraints for households and entrepreneurs, and leverage restrictions for financial intermediaries. These papers, however, investigate the properties of business cycles, and do not focus on the recent boom-bust cycle.

\textsuperscript{2}This modeling device is also the foundation of many recent normative studies on macroprudential regulation, such as Bianchi et al. (2012), Mendicino (2012), Bianchi and Mendoza (2012 and 2013), Lambertini et al. (2013), Farhi and Werning (2016), Korinek and Simsek (2016).
become more financially integrated. Therefore, they attribute the decline in interest rates exclusively to the inflow of foreign funds into the domestic economy. Another difference from these studies is that we explicitly link the higher supply of funds, domestic or international, to the U.S. debt and housing boom. This connection between credit supply and the boom requires modeling houses as collateral, and would not emerge in our model as a simple consequence of lower interest rates, as we explain in section 4.³

Our study also builds on the vast literature that focuses on the microeconomic foundations of leverage restrictions on financial intermediaries, in environments with agency, informational or incomplete market frictions (e.g. Holmstrom and Tirole, 1997, Adrian and Shin, 2014, Geanakoplos, 2010, Gertler and Kiyotaki, 2010, Gertler and Karadi, 2011, Christiano and Ikeda, 2013, Bigio, 2013, Simsek, 2013). We take these leverage restrictions as given, as in Adrian and Shin (2010a), Gertler et al. (2012), Adrian and Boyarchenko (2012, 2013), Dewachter and Wouters (2014), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014). These papers focus on risk as the fundamental determinant of credit supply through its effects on asset prices and intermediaries’ leverage, on their fragility when leverage rises in tranquil times, and on the consequences of this fragility when tranquility gives way to turbulence. Instead, we abstract from risk entirely, to concentrate on the link between the availability of credit, household debt and home prices. The result is a very simple model of the forces behind the credit and housing boom.

Like us, Landvoigt (2014) also stresses the interaction between supply and demand of mortgage debt. He proposes a rich model of borrowing and lending with intermediation, mostly focused on the effects of securitization on mortgage finance over the past several decades. In his model, mortgages can default and securitization allows to transfer this risk from leverage-constrained intermediaries to savers with low risk aversion. The final section of his paper also studies the boom and bust of the 2000s. In this experiment, the credit cycle is driven by a slackening of collateral requirements, along with a perceived decline in the riskiness of mortgages, which turns out to be incorrect. This combination of shocks generates a boom and bust in debt and real estate values that is qualitatively plausible. However, the response of house prices is small, partly because the yield on MBS rises during the boom. This effect on mortgage rates is at odds with the data (fact 4),

³There is also a more descriptive literature on the relationship between global imbalances and the boom (e.g. Obstfeld and Rogoff, 2009, Bernanke et al., 2011). These papers are mainly empirical and they do not attempt to model this relationship.
and it is presumably due to the slackening of the collateral constraint, which puts upward pressure on interest rates, as suggested by our model.

Risk is also central to the analysis of Favilukis et al. (2017), who present a life cycle model with idiosyncratic income fluctuations and incomplete markets. In their framework, a loosening of borrowing constraints, together with lower transaction costs for housing, increases home prices by compressing their risk premium, since it improves the ability of households to insure against income risk. This effect accounts for a substantial share of the rise in house price-to-rent ratios during the boom. However, it is also accompanied by an increase in interest rates, since better risk sharing opportunities decrease precautionary saving and hence boost the demand for funds. To reverse this counterfactual increase in interest rates, the model needs an infusion of foreign capital to shift the supply of credit.

Even though we abstract from risk considerations, the price-to-rent ratio also rises in our framework. This is due to the increase in the collateral services provided by houses, as the amount of available credit increases and interest rates fall. Therefore, the relaxation of lending constraints provides a simple and parsimonious account of the major facts of the boom described above. Moreover, our focus on lending constraints captures an aspect of credit liberalization that is missing in models that only feature a borrowing constraint, providing a more direct channel through which the diffusion of securitization, shadow banking, and regulatory arbitrage, together with an increase in intermediaries’ leverage, can impact the macroeconomy in general equilibrium. Overall, we interpret our results as pointing to a relaxation of lending constraints as an important mechanism that can complement the reduction in housing risk premia emphasized by Favilukis et al. (2017) in accounting for the unprecedented housing boom of the 2000s.

2. The model

This section presents a simple model with heterogeneous households that borrow from each other, using houses as collateral. The model features constraints on both borrowing and lending, which act as shifters of credit demand and supply respectively. This sharp separation between the two sides of the credit market, together with some simplifying assumptions, delivers an analytical solution. Based on this solution, section 3 uses demand and supply logic to demonstrate that the key force behind the four facts discussed in the introduction must be a relaxation of lending constraints. The same basic logic implies that
an increase in the demand for credit driven by lower collateral requirements is much harder to reconcile with those facts.

2.1. *Objectives and constraints*. The economy is populated by two types of households, with different discount rates, as in Kiyotaki and Moore (1997), Iacoviello (2005), Campbell and Hercowitz (2009b) and our own previous work (Justiniano, Primiceri, and Tambalotti, 2014, 2015). Patient households are denoted by $l$, since in equilibrium they save and lend. Their discount factor is $\beta_l > \beta_b$, where $\beta_b$ is the discount factor of the impatient households, who borrow in equilibrium.

Representative household $j = \{b, l\}$ maximizes utility

$$
E_0 \sum_{t=0}^{\infty} \beta_j^t [u(c_{j,t}) + v_j(h_{j,t})],
$$

where $c_{j,t}$ denotes consumption of non-durable goods, and $v_j(h_{j,t})$ is the utility of the service flow derived from a stock of houses $h_{j,t}$ owned at the beginning of the period. The function $v(\cdot)$ is indexed by $j$ for reasons explained in section 2.2. Utility maximization is subject to the flow budget constraint

$$
c_{j,t} + p_t [h_{j,t+1} - (1 - \delta) h_{j,t}] + R_{t-1} D_{j,t-1} \leq y_{j,t} + D_{j,t},
$$

where $p_t$ is the price of houses in terms of the consumption good, $\delta$ is the depreciation rate of the housing stock, and $y_{j,t}$ is an exogenous endowment of consumption goods and new houses. $D_{j,t}$ is the amount of one-period debt accumulated by the end of period $t$, and carried into period $t+1$, with gross interest rate $R_t$. In equilibrium, debt is positive for the impatient borrowers and it is negative for the patient lenders, representing loans that the latter extend to the former. Borrowers can use their endowment, together with loans, to buy non-durable consumption goods and new houses, and to repay old loans with interest.

Households’ decisions are subject to two additional constraints.

2.1.1. *The collateral constraint*. On the liability side of their balance sheet, a collateral constraint limits debt to a fraction $\theta$ of the value of the borrowers’ housing stock, along the lines of Kiyotaki and Moore (1997). This constraint takes the form

$$(2.1) \quad D_{j,t} \leq \theta p_t h_{j,t+1},$$
where $\theta$ is the maximum allowed loan-to-value (LTV) ratio. Therefore, changes in $\theta$ affect households’ ability to borrow against a given value of their property.

In practice, higher values of $\theta$ capture looser collateral requirements, such as those associated with lower down payments, multiple mortgages on the same property (so-called piggyback loans), and more generous home equity lines of credit. A growing literature identifies changes in $\theta$, and in the credit conditions that they represent, as an important driver of the credit cycle of the 2000s (e.g. Eggertsson and Krugman, 2012, Guerrieri and Lorenzoni, 2012, Hall, 2012, Midrigan and Philippon, 2011, Favilukis et al., 2017, and Boz and Mendoza, 2014).

2.1.2. The lending constraint. The second constraint on households’ decisions applies to the asset side of their balance sheet, in the form of an upper bound on the total amount of mortgage lending that they can extend

\[(2.2) \quad -D_{j,t} \leq \bar{L}.\]

This lending constraint is meant to capture a variety of implicit and explicit regulatory, institutional and technological constraints on the economy’s ability to channel funds towards the mortgage market.\(^5\)

For simplicity, we impose this constraint directly on the ultimate lenders. However, appendix A shows that this formulation is equivalent to one in which financial intermediaries face a leverage (or capital) constraint and a cost of equity adjustment. When this cost becomes very large, the leverage constraint on intermediaries boils down to a lending constraint of the form (2.2). We focus on this extreme formulation of the lending limit for tractability, and symmetry with the more familiar collateral constraint imposed on the borrowers. As we show in an online appendix, this choice is immaterial to our findings, which only require an upward sloping credit supply schedule. This specification can be obtained

---

\(^4\)This type of constraint is often stated as a requirement that contracted debt repayments (i.e. principal plus interest) do not exceed the future expected value of the collateral. We focus on a contemporaneous constraint for simplicity. This choice is inconsequential for the results, which mostly pertain to steady state equilibria.

\(^5\)In this stylized economy, the lending constraint also limits households’ overall ability to save because mortgages are the only financial assets. However, this assumption is irrelevant for the results. If agents could save without restrictions using another asset, the equilibrium would be unaffected, as long as a limit remains on how much of these savings can be allocated to mortgage financing.
with any positive cost of equity adjustment. The version of the model with intermediaries provides a possible interpretation of slacker lending constraints as deriving from the relaxation of leverage restrictions on financial institutions.

2.1.3. Market clearing. The model is closed by imposing that borrowing is equal to lending

\[ D_{b,t} + D_{l,t} = 0, \]  

and that the housing market clears

\[ h_{b,t} + h_{l,t} = \bar{h}, \]  

where \( \bar{h} \) is a fixed supply of houses.

2.2. Functional forms. We make two convenient functional form assumptions to obtain an analytical solution, which we will characterize in the next section. First, we assume that the lenders’ utility function implies a rigid demand for houses at the level \( \bar{h}_l \). As a result, houses are priced by the borrowers, who are leveraged and face a fixed supply equal to \( \bar{h}_b \equiv \bar{h} - \bar{h}_l \). This assumption is appealing for two reasons. First, housing markets are highly segmented (e.g. Landvoigt et al., 2015), leading to little trading of houses between rich and poor agents, lenders and borrowers. Assuming a rigid demand by the lenders shuts down all trading between the two groups in the model’s equilibrium, thus approximating reality. Second, this simple modeling device captures the idea that houses are priced by the most leveraged individuals, as in Geanakoplos (2010), amplifying the potential effects of borrowing constraints on house prices.  

The second simplifying assumption is that utility is linear in non-durable consumption. As a result, the marginal rate of substitution between houses and non-durables does not depend on the latter. Furthermore, the level and distribution of income do not matter for

6The online appendix is available here: http://faculty.wcas.northwestern.edu/~gep575/css_OnlineAppendix6-3.pdf
7This is the reason why the utility from housing services \( v \) is indexed by \( j \).
8This simplifying assumption approximates what would happen to house prices in equilibrium if the housing markets for borrowers and lenders were highly segmented, even if we maintain the assumption of one homogenous house type, with one house price. Alternatively, one could assume directly that borrowers and lenders enjoy two different kinds of houses, which are traded in two separate markets. In such an environment, shifts in either the lending or the borrowing limit would only affect the price of the borrowers’ houses, through their impact on the multiplier. This result is qualitatively consistent with the evidence in Landvoigt et al. (2015), according to which cheaper houses (presumably those owned by borrowers) appreciated more than more expensive ones during the boom.
the equilibrium in the housing and debt markets, which makes the determination of house prices simple and transparent. Under these assumptions, the housing Euler equation of the borrowers yields 

\[ p_t = \frac{\beta_b}{(1 - \mu_t \theta)} \left[ mrs + (1 - \delta) E_t p_{t+1} \right], \tag{2.5} \]

where \( \mu_t \) is the Lagrange multiplier of the collateral constraint, \( mrs = v'(\bar{h}_b) \), and the constant marginal utility of consumption was normalized to one.

According to this expression, house prices are the discounted sum of two components. The first component is the marginal rate of substitution between houses and consumption. It represents the “dividend” from living in the house, and it is also equal to their shadow rent. The second component is the expected selling price of the undepreciated portion of the house. With a constant shadow rent, house prices can only vary due to fluctuations in the discount factor. This feature of the model is consistent with the fact that house prices are significantly more volatile than measured fundamentals, resulting in large fluctuations of price-rent ratios, as stressed for instance by Favilukis et al. (2017). Unlike in their framework, though, the discount factor in (2.5) does not depend on risk, but on the tightness of the collateral constraint, through both its shadow value \( \mu_t \) and the LTV ratio \( \theta \). Therefore, house prices depend crucially on the value of collateral services, as represented by \( \mu_t \).

3. Characterization of the Equilibrium

The model of the previous section features two balance sheet constraints, both limiting the equilibrium level of debt in the economy. The collateral constraint limits the amount of borrowing to a fraction of the value of housing \( (D_{b,t} \leq \theta p_t \bar{h}_b) \). The lending constraint, instead, puts an upper bound on the ability of savers to extend mortgage credit. But in our closed economy, where borrowing must be equal to lending in equilibrium, the lending limit also turns into a constraint on borrowing \( (D_{b,t} \leq \bar{L}) \).\(^9\) Which constraint binds at any given point in time depends on the parameters \( \theta \) and \( \bar{L} \), but also on house prices, which are endogenous. Moreover, both constraints bind when \( \theta p_t \bar{h}_b = \bar{L} \), a restriction that turns out to be far from knife-edge, due to the endogeneity of \( p_t \). In the region in which

---

\(^9\)This would continue to be true in an open economy, like the one in Justiniano et al. (2014). In this case, the market clearing condition for debt becomes \( D_{b,t} + D_{l,t} = L_{f,t} \), where \( L_{f,t} \) denotes the amount of foreign borrowing. The upper bound on domestic mortgage lending then implies \( D_{b,t} \leq \bar{L} + L_{f,t} \), with \( L_{f,t} \) shifting the constraint in the same way as \( \bar{L} \) does.
both constraints bind, increases in the supply of credit go hand in hand with increases in house prices, which endogenously slack the borrowing limit, making it possible for both constraints to continue binding together.

To illustrate the interaction between the two balance sheet constraints, we start from the standard case with only a borrowing limit, which is depicted in figure 3.1. The supply of funds is perfectly elastic at the interest rate represented by the (inverse of the) lenders’ discount factor. The demand for funds is also flat, at a higher interest rate determined by the borrowers’ discount factor. At the borrowing limit, however, credit demand becomes vertical. Therefore, the equilibrium is at the (gross) interest rate $1/\beta_b$, where demand meets supply and the borrowing constraint is binding, implying a positive multiplier on the collateral constraint ($\mu_t > 0$). In this equilibrium, the price of houses is determined by equation (2.5), pinning down the location of the kink in the demand for funds.

Figure 3.2 extends the analysis to a model with a lending constraint. Now the supply of funds also has a kink, at the value $\bar{L}$. Whether this constraint binds in equilibrium depends on the relative magnitude of $\bar{L}$ and $\theta p_t \bar{h}_b$. In figure 3.2, $\bar{L} > \theta p_t \bar{h}_b$, so that the lending constraint does not bind and the equilibrium is the same as in figure 3.1.\(^{10}\)

If instead $\bar{L} < \theta p_t \bar{h}_b$, the lending limit is binding, as shown in figure 3.3. The interest rate now settles at $1/\beta_b$, higher than before. At this rate of return, savers would be happy

\(^{10}\)For this to be an equilibrium, the resulting house price must of course satisfy $\bar{L} > \theta p_t \bar{h}_b$. 

Figure 3.1. Demand and supply of funds in a model with collateral constraints.
to expand their mortgage lending, but they cannot. At the same time, borrowers are not limited in their ability to bring consumption forward by the value of their collateral, but by the scarcity of funds that the savers can channel towards the mortgage market. Equation (2.5) again determines the price of houses. However, this price is below that in the scenarios illustrated in figures 3.1 and 3.2, since now the borrowing constraint does not bind (i.e. $\mu_t = 0$). In this equilibrium, house prices are lower because real estate is not valuable as collateral at the margin. An extra unit of housing does not allow any additional borrowing, since the binding constraint is on the supply side of the credit market.

Qualitatively, the transition from a steady state with a low $\bar{L}$, as in figure 3.3, to one with a higher $\bar{L}$, as in figure 3.2, causes interest rates to fall while household debt and house prices increase. These movements match the U.S. experience in the first half of the 2000s.

Last, we consider the case in which $\bar{L} = \theta p_t \bar{h}_b$, when the vertical arms of the supply and demand for funds exactly overlap. This is not an unimportant knife-edge case, as the equality might suggest, due to the endogeneity of house prices. In fact, there is a large and interesting region of the parameter space in which both constraints bind, so that $p_t = \frac{\bar{L}}{\bar{m}_b}$. Given $p_t$, equation (2.5) pins down the value of the multiplier $\mu_t$, which, in turn, determines the interest rate. This is an equilibrium as long as the implied value of $\mu_t$ is positive, and the interest rate lies in the interval $[1/\beta_t, 1/\beta_b]$. 

We formalize these intuitive arguments through the following proposition.
Proposition 1. There exist two thresholds for house prices, \( p \equiv \frac{\beta b \cdot mrs}{1 - \beta b (1 - \delta)} \) and \( \bar{p}(\theta) \equiv \frac{\beta(\theta) \cdot mrs}{1 - \beta(\theta)(1 - \delta)} \), such that:

(i) if \( \bar{L} < \theta p \bar{h}_b \), the lending constraint is binding and

\[
p_t = p, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b};
\]

(ii) if \( \bar{L} > \theta \bar{p}(\theta) \bar{h}_b \), the borrowing constraint is binding and

\[
p_t = \bar{p}(\theta), \quad D_{b,t} = \theta \bar{p}(\theta) \bar{h}_b \quad \text{and} \quad R_t = \frac{1}{\beta_t};
\]

(iii) if \( \theta p \bar{h}_b \leq \bar{L} \leq \theta \bar{p}(\theta) \bar{h}_b \), both constraints are binding and

\[
p_t = \frac{\bar{L}}{\theta \bar{h}_b}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1 - \beta_b (1 - \delta) - mrs \cdot \beta_b \theta \bar{h}_b / \bar{L}}{\theta} \right];
\]

where \( mrs \equiv v'(\bar{h}_b) \), \( \bar{\beta}(\theta) \equiv \frac{\beta \beta_b}{\theta \beta_b + (1 - \theta) \beta_t} \) and \( \bar{p}(\theta) \geq p \) for every \( \theta \geq 0 \).

Proof. See appendix B.

As a further illustration of Proposition 1, figure 3.4 plots the equilibrium value of house prices, debt and interest rates, as a function of the lending limit \( \bar{L} \), for a constant maximum LTV ratio \( \theta \). The equilibrium behavior of these variables features three regions. Starting from the left in the figure, the lending limit is binding while the borrowing limit is not (case
i). With a tight lending constraint, interest rates are high, while house prices and debt are low.

As $\bar{L}$ rises past $\theta p h_b$, both constraints are binding, so that $\theta p_t \tilde{h}_t = \bar{L}$ (case iii). In this middle region, interest rates fall and house prices increase, boosting households' ability to borrow. With more abundant credit, interest rates are lower, which makes borrowers even more eager to consume early, enhancing the value of the collateral that makes that borrowing possible.

However, the relationship between lending limits and house prices is not strictly monotonic. With further increases in $\bar{L}$, eventually only the borrowing constraint binds (case ii). In this region, the model becomes a standard one with only collateral constraints, in which lending limits are irrelevant for the equilibrium.
The transition towards looser lending constraints depicted in figure 3.4 reproduces qualitatively the stylized facts outlined in the introduction. In the next section, we illustrate this transition in a calibrated version of the model.

4. A Calibrated Example

This section presents some numerical experiments, to provide a more quantitative perspective on the simple model introduced above. We first compare the effects of looser lending constraints to those of slacker collateral requirements, and show that the former is much more successful than the latter at matching the stylized facts of the boom. Then, we perform an experiment in which lending and borrowing constraints are relaxed together. When the two are combined, lower collateral requirements magnify the effects of relaxing the lending limit. However, looser lending constraints still account for the majority of the increase in house prices and remain the only force that pushes interest rates lower.

The model is parametrized so that its steady state matches key statistics for the 1990s, a period of relative stability for the quantities we are interested in. We associate this steady state with a tight lending constraint, as in figure 3.3. This assumption seems appropriate for a period in which mortgage finance was less sophisticated, securitization was still developing, and as a result savers faced relatively high barriers to investing in the mortgage market.

More specifically, we pick the parameter values in table 1 to match five U.S. macro and micro targets: (i) a 0.3% quarterly depreciation rate of houses, based on the NIPA Fixed Asset Tables; (ii) a 5% real mortgage rate in the initial steady state in the 1990s; (iii) a 2.5% fall in real mortgage rates during the first half of the 2000s; (iv) a 43% average ratio of debt to the value of real estate for the agents that we identify as “borrowers” in the Surveys of Consumer Finances from the 1990s; and (v) an 80% average LTV on newly issued mortgages, also from the Surveys of Consumer Finances from the 1990s. To simultaneously match (iv) and (v) in our quantitative exercises, we follow Campbell and Hercowitz (2009b) and generalize the baseline model to allow mortgage borrowers to accumulate equity, as in reality. The speed of loan amortization is determined by the parameter \( \rho \) in table 1. Appendix B formally describes this generalization of the collateral constraint and presents additional details on the calibration.

4.1. The role of credit supply. This section studies the effects of a progressive relaxation of the lending constraint in the calibrated model and it compares them to those of looser
collateral requirements. The first exercise assumes that at the end of the 1990s the U.S. economy was constrained by a limited supply of credit, as in figure 3.3 above. Starting in 2000, the lending constraint is gradually lifted, following a linear path. Each movement in $\bar{L}$ is unanticipated by agents and the experiment is timed so that the lending constraint no longer binds in 2006, thereby becoming irrelevant for the equilibrium. In the bare bones model presented above, an increase in $\bar{L}$ affects house prices and interest rates only in the region in which both the lending and borrowing constraints bind, as demonstrated in proposition 1. Therefore, the movements in $\bar{L}$ are calibrated to make this region coincide with the period between 2000 and 2006, when the macroeconomic developments highlighted by the four stylized facts were most pronounced.

The solid lines in figure 4.1 represent the response of the key variables to the loosening of $\bar{L}$ described above. The expansion in credit supply lowers mortgage rates by 2.5 percentage points. This decline reflects the gradual transition from a credit-supply-constrained economy, where the interest rate equals $\frac{1}{\beta_b}$, to an economy that is constrained on the demand side of credit, with a lower interest rate $\frac{1}{\beta_l}$. This permanent fall in mortgage rates is a distinctive feature of our environment with lending constraints. It cannot be replicated in standard models with only a borrowing limit, since their steady state interest rate is always pinned down by the discount factor of the lenders. Quantitatively, the decline in rates matches the evidence presented in the introduction by construction, through the choice of the discount factors of the two sets of households.

Turning to the other key variables, as lending constraints become looser and mortgage rates fall below $\frac{1}{\beta_b}$, impatient households increase their demand for credit up to the limit allowed by the collateral constraint, which becomes binding. With lower interest rates, borrowing to bring consumption forward becomes more desirable, increasing the value of the collateral that makes that borrowing possible. Formally, this effect is captured by a higher shadow value of the collateral constraint ($\mu_t$), as shown in equation (2.5). This collateral effect is not connected to the behavior of interest rates through the mechanical discounting mechanism of representative-agent models. Instead, interest rates affect the

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\beta_b$</th>
<th>$\beta_l$</th>
<th>$\theta$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>0.9879</td>
<td>0.9938</td>
<td>0.80</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

Table 1. Model calibration.
Figure 4.1. Response of macro variables to a change in lending limits, compared to the response to a change in collateral requirements.

Eagerness to borrow of the constrained households, therefore influencing the value of houses as collateral. This point can be illustrated by comparing the baseline to a simpler version of the model, in which the borrowing constraint does not depend on the value of real estate. In this case, house prices are unrelated to interest rates because this collateral effect is absent, given the maintained assumption of linear preferences.

In the calibration, house prices increase by almost 40 percent in real terms following the shift in credit supply, close to the U.S. experience depicted in figure 1.1. This substantial increase in house prices relaxes the collateral constraint in equilibrium, allowing households to borrow more against the higher value of their homes, with an unchanged debt-to-real estate ratio. In summary, a progressive loosening of the lending constraint generates a large increase in household debt that is associated with an equally large increase in house prices, a stable debt-to-collateral ratio, and a fall in mortgage rates, as in the four stylized facts of the boom.
To help put the quantitative effects of a shift in the lending limit in perspective, figure 4.1 compares them to those of a relaxation in collateral requirements in an economy without lending constraints. In the simulation, the rise in $\theta$ from the baseline value of 0.8 to 1.02 is calibrated to match the same increase in debt as in the previous experiment.\(^{11}\)

The variables of interest respond to the increase in $\theta$ in ways that are not in line with the stylized facts. First, interest rates do not change, since lenders are unconstrained and their discount factor pins down the interest rate. In a model with short-run dynamics, such as one with concave utility, interest rates would even increase in the short-run to convince savers to lend additional funds to the now less constrained borrowers. This is a robust feature of models with a borrowing constraint, since shifts in this constraint affect credit demand (e.g. Favilukis et al., 2017 and Justiniano et al., 2015, as well as Eggertsson and Krugman, 2012 and Guerrieri and Lorenzoni, 2012, who consider a tightening of the constraint).

Second, house prices move little in response to an increase in the maximum LTV, a finding that is also common in the literature (e.g. Kiyotaki et al., 2011, Garriga et al., 2012 and Sommer et al., 2013). In fact, house prices would rise even less in a model with short-run dynamics, due to the temporary increase in interest rates that reduces the collateral value of real estate (e.g. Iacoviello and Neri, 2010 and Justiniano et al., 2015).\(^{12}\) Third, the increase in household debt is accompanied by a rise in the debt-to-real estate ratio, as shown in the lower-right panel. We return to this aspect of the simulation in the next section.

4.2. Adding looser collateral requirements. The experiments described above suggest that looser lending constraints are a more plausible driver of the housing and credit boom than changes in required down payments. Only the former can possibly have caused the notable fall in interest rates during this period, while remaining compatible with the other stylized facts highlighted in the introduction. This conclusion might seem at odds with an established line of empirical research that has documented less stringent down payment

---

\(^{11}\)Since in this model the interest rate is constant at $1/\beta_l$, we set $\beta_l$ at 0.9879 to match the same real mortgage rate target of 5 percent in the 1990s. For $\beta_b$ we choose the value 0.9820 to maintain the same gap from the discount factor of the lenders as in the previous experiment.

\(^{12}\)One notable exception to this general finding is Favilukis et al. (2017), in which home prices rise more significantly because the better risk-sharing opportunities afforded by higher LTVs compress housing risk premia. This channel is absent in our model, which abstracts from risk entirely. However, the increase in credit supply also boosts house prices in our model through a reduction in their discount factor, as discussed in reference to equation 2.5.
requirements during the early 2000s, especially for certain borrowers such as first-time home
buyers (e.g. Duca et al., 2011, Faviilikis et al., 2017 and Geanakoplos, 2010).\footnote{Along similar
lines, DeFusco and Paciorek (2016) document greater reliance on second-lien loans during the
boom to circumvent the conforming limit for first mortgages.}

This subsection reconciles our narrative with this evidence. It shows that increases
in maximum LTVs of the kind identified in that literature can be layered on top of an
expansion in credit supply in our model, with only marginal effects on the results. This
finding confirms our main conclusion that a shift in credit supply was the main engine of the
boom, even if down payment requirements did indeed fall at the same time. One possible
interpretation of this combined experiment is as a simple way of capturing the connection
between laxer lending standards and the surge in the flow of funds towards the mortgage
market associated with a financial liberalization (e.g. Dell’Ariccia et al., 2012, Keys et al.,

The effects of a contemporaneous slackening of collateral and lending limits in the cali-
brated model are presented in figure 4.2. This simulation combines: (i) a gradual rise in \( \theta \)
from 0.8 to 1.02, and (ii) an increase in \( \bar{L} \) sufficient to produce a decline in mortgage rates
of 2.5 percentage points by 2006, as in section 4.1. In response to these changes, house
prices rise substantially, more than in the baseline experiment of section 4.1, bringing them
even closer to the data. However, the quantitative contribution of \( \theta \) is small, as we can see
by comparing the outcomes of the combined experiment to the simulations in figure 4.1.
Similarly, the increase in \( \theta \) has no impact on interest rates.

On the contrary, the increase in \( \theta \) boosts household debt, but this effect reflects a higher
debt-to-collateral ratio, as in the \( \theta \)-only experiment. This is shown in the lower-right
panel of figure 4.2. The reason for this counterfactual behavior is that, in the model,
all borrowers take immediate advantage of lower down payments by borrowing more, all
the way up to the new higher limit. This assumption is standard in the literature and
it helps to make the model more transparent and tractable, but it is extreme. In reality,
many existing homeowners amortize their mortgages without re-leveraging the collateral.
For these homeowners, the debt-to-collateral ratio tends to fall over time, the more so
the faster house prices rise. Unlike in our simple model, therefore, the behavior of the
aggregate debt-to-real estate ratio in the data depends on the relative weight of the two
groups of homeowners: those that repay their existing mortgage, and those with new or
recently refinanced mortgages, whose initial LTVs reflect current credit and housing market conditions.\footnote{During the boom, this latter group grew significantly, since refinancing activity was at historically high levels, and an unusually large fraction of that activity was accompanied by equity withdrawals.}

To address this shortcoming of the baseline model, we experimented with an extended version that includes this type of heterogeneity among borrowers.\footnote{Details on this model are available in the online appendix.} For a reasonable calibration of the fraction of households belonging to the two groups, the extended model generates a flat debt-to-real estate ratio in response to a combined increase in $\bar{L}$ and $\theta$, as in fact 3. Under this calibration, the decline in leverage among those that repay tends to balance the increase among those that refinance at the new higher $\theta$, producing little change in aggregate leverage. These simple calculations can therefore be interpreted as further evidence in favor of a narrative of the boom in which the shift in credit supply was accompanied by looser collateral requirements. As we have stressed in this section, however,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig42.png}
\caption{Response of macro variables to a combined relaxation of lending and collateral constraints.}
\end{figure}
looser lending constraints are the essential, and so far under-emphasized, ingredient of this
narrative, which is necessary to match all four stylized facts, particularly the large increase
in house prices and the fall in mortgage rates.

4.3. Possible causes of the bust. This paper focuses on the housing and credit boom,
because this is where the stark distinction between credit demand and supply factors high-
lighted by our simple model is more likely to advance our understanding of the fundamental
forces at play. Compared to the boom, the financial crisis and recession that followed are
arguably even more complicated phenomena, which our simple framework is not designed
to address. Nevertheless, it is useful to ask what our model has to say about the possible
triggers of the housing bust.

The simplest answer to this question is that the bust was the result of a reversal of the
same forces that had fueled the upswing. This is usually the line adopted in the literature.\textsuperscript{16}
In our model, such a reversal would come in the form of a tighter supply of credit, resulting
in a decline of the collateral value of real estate (represented by $\mu_t$ in equation 2.5) and
hence of house prices and debt. This effect is the opposite of the one behind the boom,
when more plentiful credit drives collateral values higher because real estate is the ticket
to accessing that credit. This reconstruction, however, treats the boom and the bust as
unrelated. We can gain some insights into the possible connections between the two phases
of the cycle by focusing instead on changes in $\theta$.

Movements in $\theta$ have two counteracting effects on house prices. On the one hand, an
increase in maximum LTVs allows agents to borrow more against their house, which pushes
house prices higher. In equation (2.5), this first effect is captured by the positive impact
of $\theta$ on $p_t$ for given $\mu_t$. However, lower down payments also put upward pressure on
credit demand and interest rates. Given that credit becomes more expensive, borrowers
feel less constrained by the scarcity of their collateral, which tends to depress house prices.
Mathematically, this second effect of $\theta$ on $p_t$ comes through the impact of $\theta$ on $\mu_t$. The
strength of this dampening effect depends on the local elasticity of credit supply. When the
elasticity is low, the upward pressure on interest rates from higher LTVs can induce a large

\textsuperscript{16}Burnside et al. (2016) present a model with houses, but no credit, in which the boom can sow the seeds
of the bust due to heterogeneous expectations and social dynamics. Guerrieri and Uhlig (2016) attribute
credit cycles to private information and adverse selection frictions. These papers model some potential
endogenous triggers of the bust. In contrast, most of the literature assumes an exogenous reversal of the
process that gave rise to the boom, be it financial liberalization, or some other factor.
enough decline in $\mu_t$ to actually drive house prices lower. In summary, house prices could drop as a consequence of lower LTVs when credit supply is highly elastic, but also due to higher LTVs if credit supply is very inelastic.

The latter possibility is interesting because it suggests that the turnaround in house prices might have been a consequence of the mature phase of the credit liberalization, when collateral requirements continued to be relaxed, even as the expansion in the supply of credit had run its course. According to our model, this is the point when the expansion in credit demand ran against a rigid supply. Real estate is no longer so valuable in this phase of the boom because it is now plentiful in relation to the amount of available credit that it can collateralize. This hypothesis can explain the leveling off of house prices and their initial decline at a time when credit standards were still loose, as in 2006, without invoking a sudden reversal of the ongoing credit liberalization.

5. Concluding Remarks

The unprecedented boom and bust in house prices and household debt have been among the defining features of the U.S. macroeconomic landscape since the turn of the millennium. Common accounts of this credit cycle, in the economics literature and beyond, have pointed to changes in the tightness of borrowing constraints, and to the consequent shifts in credit demand, as its key driver. In this paper, we argued that the focus of this discussion should shift from constraints on borrowing to obstacles to lending, or equivalently from factors affecting credit demand to those behind its supply, when it comes to understanding the boom phase of the cycle.

Using a stylized model of borrowing and lending between patient and impatient households, we showed that the progressive erosion of these barriers to lending is the key force behind four important empirical facts characterizing the boom: the large increase in house prices and mortgage debt, a stable ratio between mortgages and the value of the real estate that collateralizes them, and the fall in mortgage interest rates. The model’s ability to reproduce these facts depends on the interaction between borrowing and lending constraints, and it cannot be reproduced with either of the two constraints in isolation.

To maximize our model’s tractability, and the transparency of its insights, we abstracted from risk entirely. According to Favilukis et al. (2017), this is an important ingredient to understand the evolution of house prices in response to a credit liberalization. Enriching
our framework along these lines represents an interesting, although challenging, avenue for future research.

**Appendix A. A Simple Model with Financial Intermediaries and Capital Requirements**

This appendix shows that our simple baseline model with a parametric lending limit $\bar{L}$ is equivalent to the limiting case of a more realistic model with financial intermediation. In this model, intermediaries face a capital requirement that their equity be above a certain fraction of their assets, as in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). Intermediaries finance mortgages by collecting savings from the patient households in the form of either debt (i.e., deposits) or equity, where the latter can only be adjusted by paying a convex cost, similar to Jermann and Quadrini (2012). In the limit in which the marginal cost of adjustment tends to infinity, so that equity is fixed in equilibrium, the capital requirement becomes a hard constraint on the funds supplied to the borrowers, exactly as in the baseline model.

Although this case with infinite adjustment costs is extreme, it is qualitatively consistent with the evidence on the stickiness of intermediaries’ equity first uncovered by Adrian and Shin (2010b). If the marginal cost of adjusting the intermediaries’ capital were not prohibitively large, as assumed here, the resulting supply of funds would be differentiable, rather than having a kink, but it would still be upward sloping. The positive slope of the supply of funds is what matters for our results, which will hence hold also under less extreme assumptions regarding the costs of equity adjustment, as shown in the online appendix.

In the model with intermediaries, competitive “banks” finance mortgages with a mix of equity and deposits collected from the savers. Although the borrowers receive funds from the intermediaries, rather than directly from the savers, their optimization problem is identical to the one in section 2. The lenders, in contrast, maximize the same utility function as in section 2, but subject to the flow budget constraint

$$c_{t,t} + p_t [h_{t,t+1} - (1 - \delta) h_{t,t}] - D_{t,t} + E_t \leq y_{t,t} - R_{t-1} D_{t,t-1} + R_{t-1} E_{t-1},$$

where $-D_{t,t}$ represents “deposits”, which pay a gross interest rate $R_{t}^D$, and $E_t$ represents equity capital, with rate of return $R_{t}^E$. These interest rates can differ from the borrowing rate $R_t$. 
With linear utility in consumption, the first order conditions of the problem of the lenders become

\[(A.1)\quad R_t^D = R_t^E = \frac{1}{\beta},\]

together with the condition \(h_{t,t} = \tilde{h}_t\) following from the maintained assumption that the lenders’ demand for houses is rigid.

The competitive financial intermediaries maximize profits

\[(A.2)\quad R_t D_{b,t} + R_t^D D_{l,t} - R_t^E [1 + f(E_t)] E_t\]

subject to the constraint that assets must equal liabilities,

\[(A.3)\quad D_{b,t} + D_{l,t} = E_t,\]

and to a “capital requirement” that limits lending to a multiple of equity,

\[(A.4)\quad D_{b,t} \leq \chi E_t.\]

The function \(f(E_t)\) represents a convex cost of issuing equity. As in Jermann and Quadrini (2012), this cost is positive, creating a pecking order of liabilities whereby debt is preferred to equity. We parametrize it as

\[f\left(\frac{E_t}{E}\right) = \tau \left(\frac{E_t}{E}\right)^\gamma\]

so that the bank’s first order conditions become

\[(A.5)\quad R_t - R_t^D = \phi_t\]

\[(A.6)\quad R_t^E \left[1 + \tau (1 + \gamma) \left(\frac{E_t}{E}\right)^\gamma\right] - R_t^D = \chi \phi_t,\]

where \(\phi_t\) is the Lagrange multiplier on the capital requirement.

Combining these two conditions with the fact that \(R_t^D = R_t^E\), we find that the interest rate on loans is a weighted average of the cost of funding these loans with equity and deposits

\[(A.7)\quad R_t = \frac{1}{\chi} R_t^D \left[1 + \tau (1 + \gamma) \left(\frac{E_t}{E}\right)^\gamma\right] + \frac{\chi - 1}{\chi} R_t^D.\]
In this expression, $1/\chi$ is the share of bank liabilities held as equity when the capital requirement is binding. Its cost is a markup over the interest rate on deposits $R_t^D$, which reflects the marginal cost of issuing equity.

Since this marginal cost is everywhere positive, debt is always preferable to equity, making the capital requirement constraint always binding for the financial intermediary. Therefore, we can turn equation (A.7) into the supply of funds by substituting $E_t = D_{b,t}/\chi$ to obtain

$$R_t = \frac{1}{\beta_t} \left[ 1 + \frac{\tau (1 + \gamma)}{\chi} \left( \frac{D_{b,t}}{\chi E_t} \right)^\gamma \right].$$

This supply function is increasing and convex for $\gamma > 1$. When $\gamma \to \infty$, the function exhibits a kink at $D_{b,t} = \chi E$, thus establishing the equivalence between this model with intermediation and the simple model with a lending constraint, if we set $\bar{L} = \chi E$. This equivalence furthermore provides an interpretation for changes in the lending limit $\bar{L}$, as stemming from changes in the leverage ratio of intermediaries $\chi$, or in their cost of issuing equity.

**APPENDIX B: THE GENERAL MODEL WITH HOME EQUITY ACCUMULATION**

This appendix presents the model with home equity accumulation used to generate the quantitative results of section 4, as well as some details on its calibration. Following Campbell and Hercowitz (2009b), generalizing the baseline model of section 2 to account for home equity accumulation consists of replacing the simple collateral constraint (2.1) with

$$D_{b,t} \leq \theta p_t H_{b,t+1}$$

(B.1)

$$H_{b,t+1} = \sum_{j=0}^{\infty} (1 - \rho)^j [h_{t+1-j} - (1 - \delta) h_{t-j}],$$

where the last expression can be written recursively as

$$H_{b,t+1} = (1 - \rho) H_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}].$$

The variable $H_{b,t+1}$ denotes the amount of housing stock that can be used as collateral at any point in time, which does not necessarily coincide with the physical stock of houses, $H_{b,t+1}$. Equation (B.1) describes the evolution and composition of $H_{b,t+1}$. The houses built today $(h_{t+1} - (1 - \delta) h_t)$ can all be pledged as collateral. Hence, they can “sustain”
an amount of borrowing equal to a fraction $\theta$ of their market value. Over time, though, these houses lose their collateral “power” at a rate $\rho$. Only a fraction $(1 - \rho)^j$ of the houses purchased in $t - j$ can be collateralized, with the remaining share representing amortization of the loan and the associated accumulation of home equity. If $\rho = \delta$, amortization and depreciation coincide, so that the entire housing stock can always be pledged. In this case $H_{b,t+1}$ is equal to $H_{b,t+1}$ and the collateral constraint is identical to (2.1). If $\rho > \delta$, however, contractual amortization is faster than depreciation, leading to accumulation of equity, as in reality.

B.1. Characterization of the equilibrium. With this generalized version of the collateral constraint, the optimality conditions of the problem of the borrowers are

\[(B.2)\]
\[(1 - \mu_t) u'(c_{b,t}) = \beta_t R_t E_t u'(c_{b,t+1})\]

\[(B.3)\]
\[(1 - \zeta_t) u'(c_{b,t}) p_t = \beta_b v'_b(h_{b,t+1}) + \beta_b (1 - \delta) E_t [(1 - \zeta_{t+1}) u'(c_{b,t+1}) p_{t+1}]\]

\[(B.4)\]
\[(\zeta_t - \theta \mu_t) u'(c_{b,t}) p_t = \beta_b E_t [(1 - \rho) \zeta_{t+1} u'(c_{b,t+1}) p_{t+1}]\]

\[(B.5)\]
\[c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}\]

\[(B.6)\]
\[\mu_t (D_{b,t} - \theta p_t H_{b,t+1}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \theta p_t H_{b,t+1},\]

\[(B.7)\]
\[H_{b,t+1} = (1 - \rho) H_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}]\]

where $u'(c_{b,t}) \mu_t$ and $u'(c_{b,t}) p_t \zeta_t$ are the Lagrange multipliers on the constraint $D_{b,t} \leq \theta p_t H_{b,t+1}$ and on the evolution of $H_{b,t+1}$ respectively. The optimality conditions of the problem of the lenders are

\[(B.8)\]
\[(1 + \xi_t) u'(c_{l,t}) = \beta_t R_t E_t u'(c_{l,t+1})\]

\[(B.9)\]
\[u'(c_{l,t}) p_t = \beta_t v'_l(h_{l,t+1}) + \beta_l (1 - \delta) E_t [u'(c_{l,t+1}) p_{t+1}]\]

\[(B.10)\]
\[c_{l,t} + p_t [h_{l,t+1} - (1 - \delta) h_{l,t}] + R_{t-1} D_{l,t-1} = y_{l,t} + D_{l,t}\]
(B.11) \[ \xi_t (-D_{t,t} - L) = 0, \quad \xi_t \geq 0, \quad -D_{t,t} \leq L, \]

and the market clearing conditions are given by equations (2.3) and (2.4).

To solve this model, first note that

\[ \mathcal{H}_{b,t+1} = \frac{\delta}{\rho} h_b. \]

Suppose now that the lending constraint is binding and the collateral constraint is not, so that \( D_{b,t} = L < \theta p_t \frac{\delta}{\rho} h_b, \xi_t > 0 \) and \( \mu_t = 0 \). With linear utility in consumption, \( R_t = 1/\beta_b \) follows from equation (B.2), and equations (B.3) and (B.4) imply \( p_t = \frac{\beta_b mrs}{1 - \beta_b(1-\delta)} \equiv \bar{p} \). For this to be an equilibrium, the collateral constraint must actually not be binding, as assumed above. This requires \( \bar{L} < \theta \bar{p} \frac{\delta}{\rho} h_b \).

Suppose now to be in the opposite situation in which the collateral constraint is binding, while the lending constraint is not. It follows that \( D_{b,t} = \theta p_t \frac{\delta}{\rho} h_b < \bar{L}, \xi_t = 0 \) and \( \mu_t > 0 \). We can now derive \( R_t = 1/\beta_l \) from equation (B.8), while equation (B.2) implies \( \mu_t = 1 - \beta_b/\beta_l \). Substituting the expression for \( \mu_t \) into equation (B.4) and combining it with (B.3) yields

\[ p_t = \frac{\beta_b mrs}{1 - \beta_b(1-\delta)} \cdot \frac{1 - \beta_b(1-\rho)}{1 - \beta_b(1-\rho) - \theta (1 - \beta_b/\beta_l)} \equiv \bar{p} (\theta, \rho) > \bar{p}. \]

This is an equilibrium, provided that \( \bar{L} > \theta \bar{p} (\theta) \frac{\delta}{\rho} h_b \).

Finally, we must find the equilibrium of the model in the region of the parameter space in which \( \theta \bar{p} (\theta) \frac{\delta}{\rho} h_b \leq \bar{L} \leq \theta \bar{p} (\theta) \frac{\delta}{\rho} h_b \). Combining equations (B.2) and (B.8) implies that at least one of the two constraints must be binding, and the results above show that the value of the parameters in this region is inconsistent with only one of them being binding. It follows that both constraints must bind at the same time, implying \( D_{b,t} = \bar{L} = \theta p_t \frac{\delta}{\rho} h_b \) and \( p_t = \frac{\theta \bar{L}}{\delta \partial h_b} \). Substituting the expression for \( p_t \) into equations (B.3) and (B.4), we can compute the equilibrium value of \( \mu_t = \frac{1 - \beta_b(1-\delta)-mrs-\beta_b\delta\theta h_b/(\rho L)}{\theta \beta_b(1-\delta)} \cdot \frac{1 - \beta_b(1-\rho)}{1 - \beta_b(1-\delta)} \), and verify that it is positive if \( \theta \bar{p} (\theta) \frac{\delta}{\rho} h_b \leq \bar{L} \leq \theta \bar{p} (\theta) \frac{\delta}{\rho} h_b \). We can then obtain \( R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1 - \beta_b(1-\delta)-mrs-\beta_b\delta\theta h_b/(\rho L)}{\theta} \cdot \frac{1 - \beta_b(1-\rho)}{1 - \beta_b(1-\delta)} \right] \) using (B.2).

These results can be summarized in the following proposition.

**Proposition 2.** In the model of section 4 there exist two threshold house prices, \( \bar{p} \equiv \frac{\beta_b mrs}{1 - \beta_b(1-\delta)} \) and \( \bar{p} (\theta, \rho) \equiv \frac{\beta_b mrs}{1 - \beta_b(1-\delta)} \cdot \frac{1 - \beta_b(1-\rho)}{1 - \beta_b(1-\delta) - \theta(1 - \beta_b/\beta_l)} \), such that:
(i) if $L < \theta p \frac{\delta}{\rho} h_b$, the lending constraint is binding and $$p_t = \bar{p}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b};$$

(ii) if $L > \theta \bar{p} (\theta, \rho) \frac{\delta}{\rho} h_b$, the borrowing constraint is binding and $$p_t = \bar{p} (\theta, \rho), \quad D_{b,t} = \theta \bar{p} (\theta, \rho) \frac{\delta}{\rho} h_b \quad \text{and} \quad R_t = \frac{1}{\beta_l};$$

(iii) if $\theta p \frac{\delta}{\rho} h_b \leq L \leq \theta \bar{p} (\theta, \rho) \frac{\delta}{\rho} h_b$, both constraints are binding and

$$p_t = \frac{\rho}{\delta} \frac{\bar{L}}{\theta h_b}; \quad D_{b,t} = \bar{L} \quad \text{and}$$

$$R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1 - \beta_b (1 - \delta) - mrs \cdot \beta_b \delta \theta h_b / (\rho \bar{L})}{\theta} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \delta)} \right];$$

where $mrs \equiv v'(\bar{h}_b)$ and $\bar{p} (\theta) \geq p$ for every $0 \leq \theta \leq 1$.

It is easy to verify that proposition 1 in the main text is a special case of proposition 2, when $\rho = \delta$.

B.2. Calibration. The calibration of this model, summarized in table 1, is based on U.S. macro and micro targets. Time is in quarters. We set the depreciation rate of houses ($\delta$) equal to 0.003, based on the NIPA Fixed Asset Tables. To pick the discount factors of the borrowers and lenders, we look at the evolution of real mortgage rates, as shown in figure 1.4. They hovered around 5% in the 1990s and fell by about 2.5 percentage points between 2000 and 2005. This decline in mortgage rates is the appropriate target for our calibration because it mirrors not only the effect of domestic factors shifting the supply of credit, like the explosion of securitization, but also the impact of foreign capital inflows directed towards U.S. mortgage products. Since these foreign funds also targeted other U.S. safe assets, such as Government bonds, their impact is not fully reflected in the behavior of the spread between mortgage and Treasury rates.\textsuperscript{17} In light of these considerations, we set the discount factor of the borrowers to match a 5% real rate in the initial steady state, implying $\beta_b$ equal to 0.9879. Given this value, we calibrate the lenders’ discount factor to generate a fall in interest rates of 2.5 percentage points following the relaxation of the

\textsuperscript{17}The spread between the 30-year mortgage and the 5- or 7-year (the peak years for mortgage termination according to Calhoun and Deng, 2002) Treasury rates declined by 147 and 109 basis points, albeit mostly after 2002.
lending constraint, yielding $\beta_l = 0.9938$. The resulting gap in discount factors between patient and impatient households is in line with that chosen by Krusell and Smith (1998) or Carroll et al. (2013) to match the wealth distribution in the U.S.

The borrowing constraint with amortization features two parameters, $\theta$ and $\rho$, which allow the model to match information on maximum LTVs at origination, as well as on the average ratio of mortgages to the value of real estate among borrowers. To measure these objects, we first need to identify households in the data that resemble the borrowers in the model.

One straightforward option would be to call borrowers all households with mortgage debt, since only borrowers are indebted in the model. The problem with this strategy is that in the real world many mortgage borrowers also own a substantial amount of financial assets, which arguably makes them less severely constrained than the impatient borrowers in the model, who only own the equity in their house. In some cases, however, the assets held by these rich borrowers are illiquid, or otherwise unavailable to smooth consumption, which makes them behave as “hand-to-mouth” consumers, as discussed by Kaplan et al. (2014), Kaplan and Violante (2014), Campbell and Hercowitz (2009a), and Iacoviello and Pavan (2013).

In light of this evidence, we follow the more conservative strategy of calling “borrowers” the mortgage holders with limited liquid assets. We carry out this exercise in the Survey of Consumer Finances (SCF), which is a triennial survey of the assets and liabilities of U.S. households. Following Iacoviello and Pavan (2013) and Hall (2011), we set the limit on liquid assets at two months of total income, where liquid assets are the sum of money market, checking, savings and call accounts, directly held mutual funds, stocks, bonds, and T-Bills, net of credit card debt, as in Kaplan and Violante (2014).

Given this definition of borrowers, we calibrate the initial loan-to-value ratio, $\theta$, as the average LTV on “new” mortgages, which are those taken out by the borrowers in the year immediately preceding each survey. These new mortgages include both purchases and refinancings, but only if the amount borrowed is at least half the value of the house, since mortgages with lower initial LTVs are unlikely to be informative on the credit conditions experienced by marginal buyers (Campbell and Hercowitz, 2009b). A time-series average of these ratios computed over the three surveys of 1992, 1995 and 1998 yields a value for $\theta$ of 0.8. This is a pretty standard initial LTV for typical mortgages and also broadly in line
with the cumulative loan-to-value ratio of first-time home buyers estimated by Duca et al. (2011) for the 1990s.

For $\rho$, the parameter that governs the amortization speed on loans, we pick a value of 0.0056 to match the average ratio of debt to real estate for the borrowers in the three SCFs from the 1990s, which is equal to 0.43.

References


Federal Reserve Bank of Chicago

*E-mail address: ajustiniano@frbchi.org*

Northwestern University, CEPR, and NBER

*E-mail address: g-primiceri@northwestern.edu*

Federal Reserve Bank of New York

*E-mail address: a.tambalotti@gmail.com*