News, Noise, and Fluctuations: An Empirical Exploration

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Abstract

We explore empirically models of aggregate fluctuations with two basic ingredients: agents form anticipations about the future based on noisy sources of information and these anticipations affect spending and output in the short run. Our objective is to separate fluctuations due to actual changes in fundamentals (news) from those due to temporary errors in agents’ estimates of these fundamentals (noise). We use a simple forward-looking model of consumption to address some methodological issues: structural VARs cannot be used to identify news and noise shocks in the data, but identification is possible via a method of moments or maximum likelihood. Next, we estimate on U.S. data both our simple model and a richer DSGE model with the same information structure. Our estimates suggest that noise shocks play an important role in short-run consumption fluctuations.

Keywords: Aggregate shocks, business cycles, vector autoregression, invertibility.

JEL Codes: E32, C32, D83

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Introduction

A common view of the business cycle gives a central role to anticipations. Consumers and firms continuously receive information about the future, which sometimes is news, sometimes just noise. Based on this information, consumers and firms choose spending and, because of nominal rigidities, spending affects output in the short run. If ex post the information turns out to be news, the economy adjusts gradually to a new level of activity. If it turns out to be just noise, the economy returns to its initial state. Therefore, the dynamics of news and noise generate both short-run and long-run changes in aggregate activity. In this paper, we ask how aggregate time series can be used to shed light on this view of the business cycle.

We are interested in this view for two reasons. The first is that it appears to capture many of the aspects often ascribed to fluctuations: the role of animal spirits in affecting demand—spirits coming here from a rational reaction to information about the future—, the role of demand in affecting output in the short run, together with the notion that in the long run output follows a natural path determined by fundamentals.

The second is that it appears to offer an interpretation of structural VAR evidence based on the assumption of two major types of shocks: shocks with permanent effects and shocks with transitory effects on activity. As characterized by Blanchard and Quah (1989), Galí (1999), Beaudry and Portier (2006), among others, “permanent shocks” appear to lead to an increase in activity in the short run, building up to a larger effect in the long run, while “transitory shocks”—by construction—lead to a transitory effect on activity in the short run. It is tempting to associate shocks with permanent effects to news and shocks with transitory effects to noise.

In this paper, we focus on a simple model which provides a useful laboratory to address two issues: a methodological one and a substantive one. First, can structural VARs indeed be used to recover news and noise shocks? Second, what is the role of news and noise shocks in short-run fluctuations?

On the first question, we reach a strong negative conclusion—one which came as an unhappy surprise for one of the coauthors. In models of expectation-driven fluctuations in which consumers solve a signal extraction problem, structural VARs can typically recover neither the shocks nor their propagation mechanisms. The reason is straightforward: If agents face a signal extraction problem, and are unable to separate news from noise, then the econometrician, faced with either the same data as the agents or a subset of these data, cannot do it either.
To address the second question, we then turn to structural estimation, first using a simple method of moments and then by maximum likelihood. We find that our model fits the data well and gives a clear description of fluctuations as a result of three types of shocks: shocks with permanent effects on productivity, which build up slowly over time; shocks with temporary effects on productivity, which decay slowly; and shocks to consumers’ signals about future productivity. All three shocks affect agents’ expectations, and thus demand and output in the short run, and noise shocks are an important source of short-run volatility. In our baseline specification, noise shocks account for more than half of the forecast error variance at a yearly horizon, while permanent technology shocks account for less than one third. This result is somewhat surprising when compared with variance decompositions from structural VARs where transitory “demand shocks” often account for a smaller fraction of aggregate volatility at the same horizons and permanent technology shock capture a bigger share (e.g., Shapiro and Watson, 1989, and Galí, 1992). Our methodological analysis helps to explain the difference, showing why structural VARs may understate the contribution of noise/demand shocks to short-run volatility and overstate that of permanent productivity shocks.

Recent efforts to empirically estimate models of news-driven business cycles include Christiano, Ilut, Motto and Rostagno (2007) and Schmitt-Grohé and Uribe (2008). These papers follow the approach of Jaimovich and Rebelo (2006), modeling news as advanced, perfect information about shocks affecting future productivity. We share with those papers the emphasis on structural estimation. The main difference is that we model the private sector information as coming from a signal extraction problem and focus our attention on disentangling the separate effects of news and noise.

The problem with structural VARs emphasized in this paper is essentially an invertibility problem, also known as non-fundamentalness. There is a resurgence of interest in the methodological and practical implications of invertibility problems, see, e.g., Sims and Zha (2006) and Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007). Our paper shows that non-invertibility problem are endemic to models where the agents’ uncertainty is represented as a signal extraction problem. This idea has also recently surfaced in models that try to identify the effects of fiscal policy when the private sector receives information on future policy changes (see Leeper, Walker and Yang, 2009).

The paper is organized as follows. Sections 1 and 2 present and solve the model. Section 3 looks at the use of structural VARs. Section 4 presents the results of our structural estimation. Section 5 presents a explores a number of extensions and Section 6 concludes.
1 A simple model

We begin with the following model, which is both analytically convenient, and, as we shall see, provides a good starting point for looking at postwar U.S. data.

Productivity is driven by two shocks: a permanent shock and a transitory shock.\(^1\) Consumers do not observe the two shocks separately, but only the realized level of productivity. The permanent shock introduces uncertainty about the economy’s long run fundamentals. The presence of the transitory shock implies that consumers cannot back out the permanent shock from productivity observations, thus creating a signal extraction problem.

Consumers have access to an additional source of information, as they observe a noisy signal of the permanent component of productivity. This adds a third source of fluctuations, a shock to the error term in the signal, which we call “noise shock.”

Finally, we capture the idea that spending decisions are based on expected future productivity adopting a simple model of consumption. Consumers solve their signal extraction problem, form expectations about future productivity, and choose spending based on these expectations. Because of nominal rigidities, spending determines output in the short run.

Now to the specific assumptions.

Productivity \(a_t\) (in logs) is the sum of two components, the permanent component \(x_t\) and the transitory component \(z_t\),

\[ a_t = x_t + z_t. \]  \hspace{1cm} (1)

The permanent component follows the unit root process

\[ \Delta x_t = \rho \Delta x_{t-1} + \epsilon_t. \] \hspace{1cm} (2)

The transitory component follows the stationary process

\[ z_t = \rho z_{t-1} + \eta_t. \] \hspace{1cm} (3)

The coefficient \(\rho\) is in \([0, 1)\), and \(\epsilon_t\) and \(\eta_t\) are i.i.d. normal shocks with variances \(\sigma^2_\epsilon\) and \(\sigma^2_\eta\).\(^2\)

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\(^1\)Permanent shock is a slight (and common) misnomer, as it refers to a shock with permanent effects that build up gradually.

\(^2\)A similar process (with full information) was recently used by Aguiar and Gopinath (2007) in an open economy calibration exercise. Boz, Daude and Durdu (2008) explore the role of different informational
For most of the paper, we assume that the variances $\sigma^2_\epsilon$ and $\sigma^2_\eta$ satisfy the restriction

$$\rho \sigma^2_\epsilon = (1 - \rho)^2 \sigma^2_\eta. \quad (4)$$

This restriction implies that the univariate process for $a_t$ is a random walk, that is

$$E[a_{t+1}|a_t, a_{t-1}, ...] = a_t.$$

This assumption is analytically convenient and is also broadly in line with actual productivity data, as we shall see below. To see why this property holds, notice first that the implication is immediate when $\rho = \sigma_\eta = 0$. Consider next the case in which $\rho$ is positive and both variances are positive. An agent who observes a productivity increase at time $t$, can attribute it to an $\epsilon$ shock and forecast future productivity growth, or to an $\eta$ shock and forecast mean reversion. When (4) is satisfied, these two considerations exactly balance and expected future productivity is equal to current productivity. While assumption (4) is convenient and realistic, none of our central results depends on it.

On top of observing the realized productivity level $a_t$ each period, consumers receive a noisy signal about the permanent component $x_t$. The signal is given by

$$s_t = x_t + \nu_t, \quad (5)$$

where $\nu_t$ is i.i.d. normal with variance $\sigma^2_\nu$.

We drastically simplify the determination of output by considering an economy with a linear production function that employs only labor, in which consumption is the only component of demand and output is fully determined by the demand side. Consumers

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assumptions in that context.

3See Table 1.

4The formal proof is as follows. In general, (1)-(3) imply

$$Var[\Delta a_t] = \frac{1}{1-\rho^2} \sigma^2_\epsilon - \frac{2}{1 + \rho} \sigma^2_\eta,$$

and

$$Cov[\Delta a_t, \Delta a_{t-j}] = \rho^j \frac{1}{1-\rho^2} \sigma^2_\epsilon - \frac{1}{1 + \rho} \sigma^2_\eta$$

for all $j > 0$. If (4) holds, these yield $Var[\Delta a_t] > 0$ and $Cov[\Delta a_t, \Delta a_{t-j}] = 0$ for all $j > 0$. Quah (1990, 1991) offers general results on the decomposition of a univariate process in permanent and transitory components with orthogonal innovations.
set consumption (in logs) $c_t$ equal to their long run productivity expectations

$$c_t = \lim_{j \to \infty} E_t[a_{t+j}], \quad (6)$$

where $E_t$ is the expectation conditional on the consumers’ information at date $t$, i.e., conditional on current and past values of $a_t$ and $s_t$. Output (in logs) is equal to consumption

$$y_t = c_t,$$

and the labor input $n_t$ adjusts to produce $y_t$, given the current productivity $a_t$, so that,

$$n_t = y_t - a_t.$$

In the Online Appendix 6.2 we show that this model is the limit case of a standard new Keynesian model with Calvo pricing and a simple inflation targeting rule, when the frequency of price adjustment goes to zero. A useful property of this simple model is that consumption, by construction, is a random walk:

$$c_t = E_t[c_{t+1}].$$

### 1.1 Solving the model

Using equations (1)-(3), we can express the expected value of long-run productivity in (6) in terms of the expected values of $x_t$ and $x_{t-1}$ to get

$$c_t = E_t[x_t] + \rho \frac{1}{1-\rho} E_t[x_t - x_{t-1}]. \quad (7)$$

To complete the solution of the model, we need to solve the consumers’ signal extraction problem to express the expectations of $x_t$ and $x_{t-1}$ in terms of current and lagged values of the shocks $(\epsilon_t, \eta_t, \nu_t)$. This can be done using standard Kalman filtering. The formal steps are in Appendix 5.1.

Figure 1 shows the responses of consumption and productivity to our three shocks, using estimated parameters from Section 2, below. The time unit is the quarter and the impulses are one standard deviation positive shocks. The persistence parameter is $\rho = 0.89$, implying slowly building permanent shocks and slowly decaying transitory shocks. The standard deviations of the two technology shocks are $\sigma_\epsilon = 0.07\%$ and $\sigma_\eta = 0.63\%$. 
Figure 1: Impulse responses

and that of the noise shock is $\sigma_v = 0.89\%$, implying a fairly noisy signal.

In response to a permanent shock $\epsilon_t$, productivity builds up slowly over time—the implication of a high $\rho_x$—and consumption also increases slowly. This reflects the fact that the volatilities of transitory and noise shocks are relatively large, so that it takes a while for consumers to recognize the permanent shock and adjust consumption. For our parameter values, however, consumption initially increases faster than productivity, generating a transitory increase in employment. A more volatile transitory shock or a less informative signal, can yield a slower consumption adjustment, generating an initial drop in employment.

In response to a transitory shock $\eta_t$, productivity initially increases, and then slowly declines over time. As agents put some weight on the productivity increase being due to a permanent shock, consumption initially increases. As agents learn that it was only a transitory shock, consumption returns back to normal. For our parameter values, consumption increases less than productivity, leading to an initial decrease in employment. Again, for different parameters, the outcome may be an increase or a decrease in employment.

Finally, in response to a noise shock $\nu_t$, consumption increases, and then returns to normal over time. The response of consumption need not be monotonic. In the simulation presented here, the response turns briefly negative, before returning to normal. By assumption, productivity does not change, so employment initially increases, to return to normal over time.

In the rest of the paper, we ask how we can recover the responses in Figure 1 from the data.
2 Identification and estimation

We now turn to issues of identification and estimation. We attack the problem from three sides. First, we derive the reduced form VAR representation of the process for consumption and productivity and show that, except in special cases, it is non-invertible. This means that it is not possible to use simple semi-structural identification assumptions to estimate the economy responses to our three shocks. Second, we show that, in our simple model, one can use three moments in the data to identify the model parameters. This exercise helps us understand what information in the data can be used to shed light on the role of news and noise shocks. Third, we show how to estimate the model using likelihood-based methods, which can be applied in general to models in which a representative consumer solves a signal extraction problem. We will apply the latter approach to a DSGE model in Section 3.

In terms of observables, we will consider both the case where the signal $s_t$ is observable, so the econometrician has access to time series for $a_t$, $c_t$, and $s_t$, and the case where only $a_t$ and $c_t$ are observable, as it will be the case in our empirical work.

2.1 Reduced form VAR

Given our assumptions, the reduced form VAR representation of $c_t$ and $a_t$ takes the following simple form:

$$c_t = c_{t-1} + u^c_t,$$

$$a_t = \rho a_{t-1} + (1 - \rho) c_{t-1} + u^a_t,$$

where $u^c_t$ and $u^a_t$ are innovations with respect to the econometrician information set. This reduced form representation applies both in the case in which $s_t$ is observable and in the case in which it is not observable. In other words, (8) and (9) are the reduced form representation of consumption and productivity both in a trivariate VAR in $(c_t, a_t, s_t)$ and in a bivariate VAR in $(c_t, a_t)$.

To prove this result, let us begin from (8) and notice that (7) implies

$$(1 - \rho)c_t = E_t x_t - \rho E_t x_{t-1},$$

so

$$(1 - \rho)E_{t-1}[c_t - c_{t-1}] = E_{t-1}[x_t - x_{t-1}] - \rho E_{t-1}[x_{t-1} - x_{t-2}] = 0,$$

where $u^c_t$ and $u^a_t$ are innovations with respect to the econometrician information set. This equation, in general, does not take a simple form like (8) and (9).
as it includes infinitely many lags of $a_t$ and $s_t$ with coefficients that are transforms of the Kalman filter coefficients.

The univariate representations of productivity is a random walk, by assumption. But when we move to a multivariate representation, past consumption helps to predict productivity as we can see from (9). The reason why consumption helps predict productivity is that it embeds the additional information on $x_t$ that the consumers obtain from observing $s_t$.\(^6\)

### 2.2 Structural VAR

Suppose we run a trivariate reduced form VAR in $(c_t, a_t, s_t)$ and obtain the reduced form innovations $(u^c_t, u^a_t, u^s_t)$. Can we recover the original shocks $(\epsilon_t, \eta_t, \nu_t)$ from the three reduced form innovations? Except in two special cases, the answer is no.

The first special case is that of a perfectly informative signal, with $\sigma_\nu = 0$. In this case, (8) and (9) simplify to:

\[
\begin{align*}
    c_t &= c_{t-1} + \frac{1}{1-\rho} \epsilon_t, \\
    a_t &= \rho a_{t-1} + (1-\rho) c_{t-1} + \epsilon_t + \eta_t.
\end{align*}
\]

Consumption responds only to the permanent shock, productivity to both. Imposing the long-run restriction that only one of the shocks has a permanent effect on consumption and productivity, we can recover $\epsilon_t$ and $\eta_t$, and their dynamic effects. So in this case a structural VAR approach works. The second special case is that of a completely uninformative signal, with $\sigma_\nu \to \infty$. In this case, consumers only observe $a_t$ and our random walk assumption implies that consumption and productivity are perfectly correlated with

\[
\begin{align*}
    c_t &= c_{t-1} + u_t, \\
    a_t &= a_{t-1} + u_t,
\end{align*}
\]

where the second equality follows from (2). This implies $E_t[u^c_t] = 0$. If the econometrician can observe $s_t$, the result follows immediately. If $s_t$ is unobservable, the result follows applying the law of iterated expectations to get $E^E_t[E_t[u^c_t]] = 0$, where $E^E_t$ denotes the expectation conditional on the econometrician’s information. Turning to (9) notice that (1) and (3) imply $a_t - \rho a_{t-1} = x_t - \rho x_{t-1} + \eta_t$. Taking expectations and using (10) then implies $E_{t-1}[a_t - \rho a_{t-1}] = (1 - \rho) E_{t-1} c_t = (1 - \rho) c_{t-1}$. This implies $E_t[u^a_t] = 0$. The argument is extended to the case of unobservable $s_t$ exactly as in the case of $u^c_t$.

\(^6\)The only special case in which consumption does not help to predict productivity is $\rho = 0$. As we shall see below, in this case $a_t$ and $c_t$ are perfectly collinear, so, given $a_{t-1}$, $c_{t-1}$ provides no additional information on $a_t$.\(^9\)
where $u_t$ denotes the common innovations in the two variables. In this case, it is not possible to recover $\epsilon_t$ and $\eta_t$ from the single innovation $u_t$. But the decomposition between temporary and permanent shocks is now irrelevant, given that no information is available to separate them. We can then take the random walk representation of productivity as our primitive and interpret the productivity innovation as the single, permanent shock. In terms of this alternative representation, a structural VAR approach works.

Once we move away from these special cases and have a partially informative signal, the reduced form VAR representation is non-invertible and a structural VAR approach cannot be applied. The reason for the non-invertibility is a singularity problem: even if we have three innovations $(u'_c, u'_a, u'_s)$, one them is a linear combination of the other two, so it is not possible to recover the three independent shocks $(\epsilon_t, \eta_t, \nu_t)$ from them.

The formal result is stated in Lemma 1 in the appendix. The lemma is formulated in the context of a general signal-extraction model, to show that singularity is endemic to this class of models. The crucial intuition is that agents’ decisions are functions of their expectations, so even if the econometrician observes the three variables $(c_t, a_t, s_t)$, the first variable is a function of the other two, which implies that there are only two innovations driving the system. But then it is impossible to recover three orthogonal shocks from two innovations.

Despite the singularity problem, one could still hope that long-run identification restrictions could be used to separate the effect of the permanent shock $\epsilon_t$ from the combined effect of the other two shocks, $\eta_t$ and $\nu_t$. Unfortunately also this partial identification fails, as we can see from a simple simulation exercise.

We generate data from our model—with the parameters used for Figure 1—and run a structural VAR with long-run restrictions à la Blanchard and Quah (1989) to identify a permanent and a temporary shock, labeled “BQ shocks.” Figure 2 shows the estimated impulse responses to the two BQ shocks (dashed lines) and the impulse responses to the three original shocks in the model (solid lines). The two panels on the left focus on shocks with permanent effects. For both productivity and consumption, the BQ shock has larger effects on impact and less of a gradual build up in later periods, relative to the original shock $\epsilon$. For consumption, this is especially pronounced, given that the response to the BQ shock is virtually flat. The two panels on the right show the responses to shocks with temporary effects. The productivity response to the BQ transitory shock is the only one

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7 All the responses are to one standard deviation shocks.
8 The thick line corresponds to the $\eta$ shock, the thin line to the $\nu$ shock, the dashed line to the BQ transitory shock.
close to that of the original $\eta$ shock. The identified response of consumption to the BQ transitory shock is zero, unlike those of $\eta$ and $\nu$.

The flat responses of consumption to the BQ shocks in Figure 2 are not special to our numerical example. In fact, a general result holds for our model: consumption displays a flat impulse response to any identified shock, whether identified à la Blanchard and Quah (1989) or by any other short-term or long-term restriction. This result is proved formally as Proposition 1 in the Online Appendix (Section 6.1).\footnote{The exact result holds asymptotically. In short samples, the impulse responses are only approximately flat, as in Figure 2.} The intuition is that since consumption is a random walk conditional on the consumer’s information, an econometrician with access to the same information, or less, cannot identify any shock with non-flat effects on consumption.

2.3 Matching moments

We now turn to structural estimation. Although we cannot recover the shocks exactly, we can still use moments in the data to recover the model parameters, in our case $\rho$ and the variances $\sigma_\epsilon$, $\sigma_\eta$, and $\sigma_\nu$. In particular, in our baseline model, three moments from the data are sufficient to identify all our parameters. First, $\rho$ is identified using the reduced form relation (9). Given $\rho$, it is then easy to recover the variances. This identification

Figure 2: Model and SVAR-identified impulse responses
exploits the model’s assumptions on consumers’ forward looking behavior and rational expectations.\footnote{The use of permanent income logic together with rational expectations to identify temporary and permanent shocks connects our approach to a large body of work on household income dynamics, e.g., Blundell and Preston (1998).}

Let us go through each step of our moment-based estimation, using U.S. quarterly data. This will also allow us to show that our reduced form benchmark model (8)-(9) fits the time series facts for productivity and consumption fairly well.

Productivity $a_t$ is the logarithm of the ratio of GDP to employment and consumption $c_t$ is the logarithm of the ratio of NIPA consumption to population. Our sample is from 1970:1 to 2008:1. An issue we have to confront is that, in contradiction to our model, and indeed to any balanced growth model, productivity and consumption have different growth rates over the sample (0.34% per quarter for productivity, versus 0.46% for consumption). This difference reflects factors left out of the model, from changes in participation, to changes in the saving rate, to changes in the capital-output ratio. For this reason, in what follows, we allow for a secular drift in the consumption-to-productivity ratio and remove it from the consumption series.\footnote{We are aware that, in the context of our approach, where we are trying to isolate potentially low frequency movements in productivity, this is an imperfect solution. But, given our purposes, it seems to be a reasonable first pass assumption. The reason why we concentrate on the sample 1970:1 to 2008:1 is precisely because, with longer samples, we are less confident that this approach does a satisfactory job at accounting for low frequency changes in the consumption-to-productivity ratio. In our DSGE exercise, we use a longer sample.}

Some basic features of the time series for productivity and consumption are presented in Table 1. Lines 1 and 2 show the results of estimated AR(1) for the first differences of the two variables. Recall that our model implies that both productivity and consumption should follow random walks, so the AR(1) term should be equal to zero. In both cases, the AR(1) term is indeed small, insignificant in the case of productivity, significant in the case of consumption.

The first step of our identification uses the reduced form equation (9) to recover $\rho$. Writing (9) as a cointegrating regression, we have

$$
\Delta a_t = (1 - \rho)(c_{t-1} - a_{t-1}) + \nu_t^a,
$$

which can be estimated by OLS. Our estimate is reported on Line 3 of Table 1. Line 4 allows for lagged rates of change of consumption and productivity, and shows the presence of richer dynamics than implied by our specification, with significant coefficients on the lagged rates of change of both variables.
Table 1: Consumption and Productivity Regressions.
Note: Sample: 1970:1 to 2008:1. $\Delta(j)a \equiv a(+j-1) - a(-1)$. In parenthesis: robust standard errors computed using Newey-West window with 10 lags.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta a$</th>
<th>$\Delta c$</th>
<th>$(c - a)(-1)$</th>
<th>implied $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.06 (0.09)</td>
<td>0.24 (0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.05 (0.03)</td>
<td>0.03 (0.02)</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.21 (0.10)</td>
<td>0.32 (0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.03 (0.15)</td>
<td>0.31 (0.30)</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.03 (0.30)</td>
<td>0.98 (0.43)</td>
<td>0.91</td>
<td></td>
</tr>
</tbody>
</table>

The model provides alternative ways of estimating $\rho$, exploiting the correlation of productivity growth and consumption at different horizons. Namely, the model implies

$$a_{t+j} - a_t = (1 - \rho^j)(c_{t-1} - a_{t-1}) + u_{t}^{a_j},$$

for any $j \geq 0$, where $u_{t}^{a_j}$ is a disturbance uncorrelated to the econometrician’s information at date $t$. Lines 5 to 7 explore this implication. The results are roughly consistent with the model predictions, and all point to relatively high values for $\rho$ (reported in the last column). The idea that the forecasting power of the consumption-to-productivity ratio tells us something about consumers’ information about future productivity is closely related to a similar observation made by Cochrane (1994) in terms of the consumption-to-output ratio. Indeed this observation was the motivating reason for Cochrane (1994) early suggestion to introduce news shocks in business cycle models.

The second step of our identification, is to estimate $\sigma_c$ and $\sigma_\eta$. For this, we exploit our univariate random walk assumption for $a_t$, that is, assumption (4), which implies the

\[ E_t[a_{t+j}] = (1 - \rho^j) c_t + \rho^j a_t. \]

Taking expectations at time $t-1$ on both sides yields

\[
E_{t-1}[a_{t+j}] = (1 - \rho^j) E_{t-1}[c_t] + \rho^j E_{t-1}[a_t]
= (1 - \rho^j) c_{t-1} + \rho^j((1 - \rho)c_{t-1} + \rho a_{t-1})
= (1 - \rho^{j+1}) c_{t-1} + \rho^{j+1} a_{t-1},
\]

the second equality follows from (8) and (9), the third from rearranging.

\[\text{13}\] The standard errors are corrected for the presence of autocorrelation due to overlapping intervals using the Newey-West estimator.
following relations between the two variances and the variance of $\Delta a_t$:

$$
\sigma^2_e = \frac{\text{Var}[\Delta a_t]}{(1 - \rho)^2} \\
\sigma^2_\eta = \frac{\text{Var}[\Delta a_t]}{\rho}
$$

Given a sample standard deviation of $\Delta a_t$ equal to 0.67% and given our estimate for $\rho$, we get estimates $\sigma_e = 0.03\%$ and $\sigma_\eta = 0.65\%$. These results imply a very smooth permanent component, in which small shocks steadily build up over time, and a large transitory component, which decays slowly over time.

The third and last step is to recover the variance of the noise shock $\sigma_\nu$. For this, we match the coefficient of correlation between the residual of regression (11) and consumption growth $\Delta c$. Numerical results show that, given the other parameters, this moment is an increasing function of $\sigma_\nu$. The coefficient of correlation between $\Delta c$ and the residual of the regression on line 3 (corresponding, respectively to $u^c_t$ and $u^a_t$) is equal to 0.52. If the signal was perfectly informative this correlation would be equal to 0.05, while if the signal was uninformative it would be 1. The observed correlation is in between these values and implies a standard deviation of the noise shock $\sigma_\nu = 2.1\%$.

### 2.4 Maximum likelihood

We now turn to estimation by maximum likelihood. Conditional on the model being correctly specified, a maximum likelihood approach dominates the moment matching approach of the last section, as it fully incorporates all the restrictions implied by the model. For example, in our simple model, a maximum likelihood approach exploits the correlation between $u^c_t$ and $u^a_t$ implied by the model.

A maximum likelihood approach has the advantage that it can easily be extended to richer models, like the DSGE model of Section 3. In Appendix 5.1, we show how to compute the likelihood function for a general representative-agent model with signal extraction. The main idea is first to solve the consumer’s Kalman filter to obtain the dynamics of consumer’s expectations, as discussed in Section 1.1, and next to build the econometrician’s Kalman filter including in the list of unobservable state variables the consumer’s expectations. This way of computing the likelihood function can also be used to apply Bayesian methods, as we shall do in Section 3.

\[14\] These bounds can be derived from the analysis in Section 2.1. To obtain the first, some algebra shows that under full information $\text{Cov}[u^c_t, u^a_t]/\sqrt{\text{Var}[u^c_t]\text{Var}[u^a_t]} = (1 - \rho)/\sqrt{(1 - \rho)^2 + \rho}$. The second bound is immediate.
Table 2 shows the results of estimation of the benchmark model presented as a grid over values of $\rho$ from 0 to 0.99. For each value of $\rho$, we find the values of the remaining parameters that maximize the likelihood function and in the last column we report the corresponding likelihood value. The table shows that the likelihood function has a well-behaved maximum at $\rho = 0.89$, yielding the parameters reported on line 6.

Recall that the maximum likelihood approach uses all the implicit restrictions imposed by the model. This explains the difference between the estimates obtained with the estimated obtained by ML and those obtained by moment matching in Section 2.3. In particular, the maximum likelihood approach favors smaller values of $\rho$ and $\sigma_\nu$. However, if we look at line 8 of Table 2, we see parameters closer to those in Section 2.3 and the likelihood gain from line 8 to line 6 is not very large. In other words, the data are consistent with a range of different combinations of $\rho$ and $\sigma_\nu$. When we look at the model’s implications in terms of variance decomposition, we will consider different values in this range.

It is useful to clarify that the random walk assumption for productivity is not necessary for identification of the model’s parameters. In particular, we can relax assumption (4), estimating independently $\sigma_\eta$ and $\sigma_\epsilon$ and we can allow for different coefficients $\rho_x$ and $\rho_z$ in equations (2) and (3). The estimation results are reported in Table 3 and are quite close to those obtained under the random walk assumption.

What do our results imply in terms of the dynamic effects of the shocks and of variance decomposition? If we use the estimated parameters from the benchmark model (line 6 in Table 2), the dynamic effects of each shock are given in Figure 1 and were dis-

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_\epsilon$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_\epsilon$</th>
<th>ML</th>
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<td>0.0067</td>
<td>0.0000</td>
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</tr>
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<td>0.0077</td>
<td>0.0023</td>
<td>0.0065</td>
<td>0.0026</td>
</tr>
<tr>
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<td>0.0071</td>
<td>0.0014</td>
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<td>0.0056</td>
</tr>
<tr>
<td>6</td>
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<td>0.0007</td>
<td>0.0063</td>
<td>0.0089</td>
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<td>7</td>
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<td>0.0007</td>
<td>0.0064</td>
<td>0.0099</td>
</tr>
<tr>
<td>8</td>
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<td>0.0068</td>
<td>0.0003</td>
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<td>0.0234</td>
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<td>9</td>
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<td>0.0063</td>
<td>0.0001</td>
<td>0.0063</td>
<td>0.0753</td>
</tr>
</tbody>
</table>

Table 2: Maximum likelihood estimation: benchmark model

---

For maximum likelihood estimation we used Dynare. Our observables are first differences of labor productivity and consumption, so we use a diffuse Kalman Filter to initialize the variance covariance matrix of the estimator (a variance-covariance matrix with a diagonal of 10).
Estimate | Standard error
--- | ---
\( \rho_x \) | 0.8879 | 0.0478
\( \rho_z \) | 0.8878 | 0.0474
\( \sigma_\eta \) | 0.0065 | 0.0004
\( \sigma_\epsilon \) | 0.0007 | 0.0003
\( \sigma_\nu \) | 0.0090 | 0.0052
ML | 1073.3

Table 3: Maximum likelihood estimation: unconstrained model

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Perm. tech.</th>
<th>Trans. Tech.</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.749</td>
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<tr>
<td>4</td>
<td>0.269</td>
<td>0.198</td>
<td>0.533</td>
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<tr>
<td>8</td>
<td>0.683</td>
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<td>12</td>
<td>0.832</td>
<td>0.046</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Table 4: Variance decomposition

cussed in Section 1.1: a slow and steady build up of permanent shocks on productivity and consumption, a slowly decreasing effect of transitory shocks on productivity and consumption, and a slowly decreasing effect of noise shocks on consumption.

Table 4 presents the implications of the estimated parameters for variance decomposition, showing the contribution of the three shocks to forecast error variance at different horizons. Noise shocks are an important source of short run volatility here, accounting for more than 70% of consumption volatility at a 1-quarter horizon and more than 50% at a one year horizon, while permanent technology shocks play a smaller role, having almost no effect on quarterly volatility and explaining less than 30% at a 4-quarter horizon. Clearly, the baseline model allows for a limited number of shocks, so it will be interesting to explore the role of noise shocks in richer specifications in the next section. However, it is useful to compare the exercise here to traditional SVAR exercises, such as Shapiro and Watson (1989) and Gali (1992), that also use a small number of shocks and follow Blanchard and Quah (1988) to identify supply shocks. In those papers, transitory demand shocks typically explain a smaller fraction of aggregate volatility than our noise shocks and permanent technology shocks play a bigger role. The analysis in Section 2.2 helps to explain the difference with our results, by showing that, asymptotically, a SVAR is biased towards assigning 100% of consumption volatility to the permanent shock.
2.5 Recovering states and shocks

So far we have focused on using structural estimation to estimate the model’s parameters. Now we turn to the question: what information on the unobservable states and shocks can be recovered from structural estimation?

Here the idea is to exploit the fact that the econometrician has access to the whole sample. Looking at what happened to productivity after consumers chose consumption, we may be able to get a better sense of what the states and shocks were. In other words, using data from times 1 to T, we can form our best estimates of states and shocks at any time \( t \leq T \). This is precisely the job of the Kalman smoother.

The top panel of Figure 3 plots estimates for the permanent component of productivity \( x_t \) obtained from our benchmark model. The solid line correspond to \( x_t \), the dashed line to the consumers’ real time estimate of the same variable, denoted by \( x_{t|t} \). Notice that both \( x_t \) and \( x_{t|t} \) are unobservable states for the econometrician, so the two lines correspond to the Bayesian estimates of the respective state.\(^\text{16}\)

Looking at medium-run movements, the model identifies a gradual adjustment of consumers’ expectations to the productivity slowdown in the 70s and a symmetric gradual adjustment in the opposite direction during the faster productivity growth after the mid 90s. Around these medium-run trends, temporary fluctuations in consumers’ expectations produce short-run volatility.

To gauge the short-run effects of expectational errors, the consumers’ expectations of \( x_t \) are not sufficient, given that consumers project future growth based on their expectations of both \( x_t \) and \( x_{t-1} \). For this reason, in the bottom panel of Figure 3, we plot the smoothed series for the consumers’ real time expectations regarding long-run productivity, \( x_{t+\infty|t} = (x_{t|t} - \rho x_{t-1|t})/(1 - \rho) \), and compare it to the same expression computed using the smoothed series \( x_{t|T} \) and \( x_{t-1|T} \). The model generates large short-run consumption volatility out of temporary changes in consumers’ expectations. Sometimes these changes occur when consumers’ overstate current \( x_t \) (e.g., at the end of the 80s), other times when consumers slowly catch up to an underlying productivity acceleration and understate \( x_{t-1} \) (e.g., at the end of the 90s). Obviously, the model is too stylized to give a credible account of all cyclical episodes. For example, given the absence of monetary policy shocks the recession of 1981-82 is fully attributed to animal spirits. In the next section, we repeat the exercise in a richer model that allows for monetary shocks.

The Kalman smoother also tell us what is the root-mean-square error (RMSE) of the

\(^{16}\)Appendix 5.1 shows that \( x_{t|t} \) is, in general, an unobservable state for the econometrician.
estimates of $x_t$ made both by the econometrician and by the consumer. It turns out that in steady state these two estimates coincide and the RMSE is 0.44% for estimates using data up to date $t$. If we can use all possible future data the RMSE halves, to 0.28%, but remains positive.\footnote{That is, this is the RMSE of the estimate of $x_t$ based on data up to time $T > t$ when we let $T \to \infty$.} Appendix 6.4 contains more details.

Turning to the shocks, we know from our discussion of structural VARs that the information in current and past values of $c_t$ and $a_t$ is not sufficient to derive the values of the current shocks. However, this does not mean that the data contain no information on the shocks. In particular, the Kalman smoother gives estimates of $\epsilon_t$, $\eta_t$, and $\nu_t$ using the entire time series available. Figure 4 plots these estimates for our benchmark model.

Notice the apparent high degree of autocorrelation of the estimated permanent shocks in the top panel of Figure 4. The smoothed estimates of $\epsilon_t$ in consecutive quarters tend to be highly correlated, as the econometrician does not know to which quarter to attribute...
an observed permanent change in productivity. Notice that the autocorrelation of the estimated shocks is not a rejection of the assumption of i.i.d. shocks, but purely a reflection of the econometrician’s information. In fact, performing the same estimation exercise on simulated data delivers a similar degree of autocorrelation.

3 A DSGE exercise

In this section, we start from the same productivity process and information structure of Section 1, but we embed them in a small scale DSGE model. The model includes investment and capital accumulation, an explicit treatment of nominal rigidities in prices and wages, a monetary policy rule à la Taylor, and allows for various adjustment costs that have been proposed in the literature to capture the observed dynamics of aggregate quantities.

Our objective here is twofold. First, we want to explore the robustness of our findings

\[\text{In Appendix 6.4 we show that the RMSE for the } \epsilon \text{ shock is very high, about } 94\% \text{ of the prior standard deviation } \sigma_{\epsilon}.\]
to the use of a richer model, with a larger number of shocks. Second, we want to show that it is easy to embed a signal-extraction information structure in DSGE models.

The model is estimated using Bayesian methods, as is now common for DSGE models with a relatively large number of parameters. The approach to compute the likelihood function is the one outlined in Section 2.4. However, a useful result makes the estimation even easier. Namely, we exploit the fact that the model’s information structure is observationally equivalent to the information structure of a model with full information and correlated shocks. One can then estimate the full-information model subject to a restriction on the correlation of the shocks and, at the end, recover the parameters of the original signal-extraction model. The observational equivalence is established formally in Appendix 5.3.

Since the model is standard, we describe here its main ingredients and leave the details and the log-linearization to the Online Appendix (Section 6.2). The model is similar to those in Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005). The preferences of the representative household are given by the utility function

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \log \left( C_t - hC_{t-1} \right) - \frac{1}{1+\xi} \int_{0}^{1} N_{jt}^{1+\xi} \, dj \right) \right], \quad (12)$$

where $C_t$ is consumption, the term $hC_{t-1}$ captures internal habit formation, and $N_{jt}$ is the supply of specialized labor of type $j$. The presence of differentiated labor introduces monopolistic competition in wage setting as in Erceg, Henderson and Levin (2000). The capital stock $\bar{K}_t$ is owned and rented by the representative household and the capital accumulation equation is

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + D_t \left[ 1 - \mathcal{G} \left( I_t / I_{t-1} \right) \right] I_t, \quad (13)$$

where $\delta$ is the depreciation rate, $D_t$ is a stochastic investment specific technology parameter, and $\mathcal{G}$ is a quadratic adjustment cost in investment

$$\mathcal{G}(I_t/I_{t-1}) = \chi(I_t/I_{t-1}) - \Gamma)^2 / 2,$$

where $\Gamma$ is the long run gross growth rate of TFP. The model features variable capacity

---

19Therefore, a signal-extraction model can be seen as a way of imposing restrictions on a class of models with correlated shocks.
utilization: the capital services supplied by the capital stock $\bar{K}_t - 1$ are

$$K_t = U_t \bar{K}_t - 1,$$

(14)

where $U_t$ is the degree of capital utilization and the cost of capacity utilization, in terms of current production, is $C(U_t)\bar{K}_t - 1$, where $C(U_t) = U_t^{1+\xi} / (1 + \xi)$.

The final good is a Dixit-Stiglitz aggregate of a continuum of intermediate goods, produced by monopolistic competitive firms, with staggered price setting à la Calvo (1983). Similarly, specialized labor services are supplied under monopolistic competition, with staggered nominal wages. The monetary authority sets the nominal interest rate following a standard inertial Taylor rule.

The model is estimated on U.S. time series for GDP, consumption, investment, employment, the federal funds rate, inflation, and wages for the period 1954:3-2011:1. More details on the data are in the Online Appendix (Section 6.3).

The parameter estimates are reported in Table 5. Figures 5 and 6 show the impulse responses for our seven observed variables following a permanent technology shock, a temporary technology shock and a noise shock.

The responses of consumption to the three shocks are qualitatively similar to those in the simple model of Section 1, plotted in Figure 1: in the short-run consumption responds mostly to the noise shock and to the transitory shock, with responses of similar magnitudes to the two shocks. In the long-run the permanent shock dominates. The main quantitative differences are that the DSGE favors a larger coefficient of autocorrelation for growth shocks—$\rho = 0.94$ vs $\rho = 0.89$ in Section 2.4—and that it attributes larger volatility to both fundamental and noise shocks.

Unlike the simple model of Section 1, the DSGE model also has implications for investment. The second column of Figure 5 shows that investment increases gradually and permanently after a permanent shock and has a hump-shaped response to a transitory shock. Following a noise shock the investment response is first positive and hump-shaped and later turns negative. What is happening is that, at some point, agents realize the shock was just noise, the consumption response goes back to zero and the investment response turns negative as the economy reverts to its original capital stock.

Notice that, in the first quarters following a noise shock, consumption and investment move in the same direction. There is now a growing literature on whether expectational shocks can generate this type of comovement in business cycle models.20 The crucial

20See Lorenzoni (2011) for a review.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
<th>Conf. bands</th>
<th>Distribution</th>
<th>Prior st. dev.</th>
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<td>0.5262</td>
<td>0.4894</td>
<td>0.5787</td>
</tr>
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</tr>
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<td>2.8912</td>
<td>4.3021</td>
</tr>
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<td>$\chi$</td>
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<td>4.3311</td>
<td>3.6751</td>
<td>5.5079</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Calvo prices</td>
<td>0.66</td>
<td>0.8770</td>
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<td>0.8998</td>
</tr>
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<td>$\theta_w$</td>
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</tr>
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<td>$\gamma_\pi$</td>
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</tr>
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<td>$\gamma_y$</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.015</td>
<td>0.024</td>
</tr>
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</table>

**Shock processes**

Neutral technology and noise

| $\rho$ | 0.6 | 0.9426 | 0.9230 | 0.9618 | Beta | 0.2 |
| $\sigma_u$ | 0.5 | 1.1977 | 1.0960 | 1.2975 | Inv. Gamma | 1 |
| $\sigma_v$ | 1 | 1.4738 | 0.7908 | 2.3176 | Inv. Gamma | 1 |

Investment specific

| $\rho_d$ | 0.6 | 0.4641 | 0.3263 | 0.5743 | Beta | 0.2 |
| $\sigma_d$ | 0.15 | 11.098 | 8.4323 | 14.910 | Inv. Gamma | 1.5 |

Mark ups

| $\rho_p$ | 0.6 | 0.7722 | 0.6991 | 0.8461 | Beta | 0.2 |
| $\phi_p$ | 0.5 | 0.4953 | 0.3749 | 0.6557 | Beta | 0.2 |
| $\sigma_p$ | 0.15 | 0.1778 | 0.1508 | 0.2027 | Inv. Gamma | 1 |
| $\rho_w$ | 0.6 | 0.9530 | 0.9534 | 0.9650 | Beta | 0.2 |
| $\phi_w$ | 0.5 | 0.5583 | 0.5125 | 0.6224 | Beta | 0.2 |
| $\sigma_w$ | 0.15 | 0.3057 | 0.2847 | 0.3264 | Inv. Gamma | 1 |

Policy

| $\rho_q$ | 0.4 | 0.0413 | 0.0024 | 0.0807 | Beta | 0.2 |
| $\sigma_q$ | 0.15 | 0.3500 | 0.3148 | 0.3782 | Inv. Gamma | 1 |
| $\rho_g$ | 0.6 | 0.9972 | 0.9938 | 0.9998 | Beta | 0.2 |
| $\sigma_g$ | 0.5 | 0.2877 | 0.2680 | 0.3078 | Inv. Gamma | 1 |
mechanism here is based on nominal rigidities. The increase in consumption drives up demand in the short run, while the real wage increases slowly due to stickiness (bottom right panel of Figure 6). This increases the marginal profitability of capital and thus investment. Given that productivity in unaffected, output increases above its natural level, generating inflation (bottom middle panel of Figure 6). The central bank responds by raising interest rates (bottom left panel of Figure 6). A sufficient degree of nominal rigidity and a sufficiently low responsiveness of the Taylor rule imply that the interest rate increase is not enough to undo the increase in investment.

The role of nominal rigidities in the model is illustrated in Figure 7. The first row of plots compares the impulse responses of quantities to a noise shock with our estimated parameters and with the same parameters except for flexible prices and wages \((\theta = \theta_w = 0)\). Absent nominal rigidities the effects of a noise shock on consumption are muted and the effects on investment are reversed. The estimated values of the Calvo parameters \(\theta\) and \(\theta_w\) in the baseline estimation are fairly high. Therefore, in the second row of plots, we experiment with smaller values \((\theta = \theta_w = 0.75)\) and find responses close to the baseline. In the third and fourth row of Figure 7 we explore the role of habit formation and adjustment costs, plotting responses when \(h = 0\) and when \(\chi = 0\). Habit formation does not seem to play a major role. Adjustment costs, on the other hand, dampen the response of investment to noise shocks. Absent adjustment costs, the investment response is larger by an order of magnitude.

It is useful to compare our results to Christiano, Ilut, Motto and Rostagno (2008), who use a similar model to study the effects of a simpler form of “news shock.” Namely, they consider a shock that leads agents to perfectly anticipate a standard technology shock which will occur four quarters later. Our informational setup offers two advantages over this approach: first, changing the parameter \(\rho\) we have a smoother way of controlling the horizon at which the technology shock is expected to kick in; second, by introducing a noisy signal, we can account explicitly for “mistakes,” i.e., for changes in expectations about the future that are later disappointed. Christiano, Ilut, Motto and Rostagno (2008) obtain two results seemingly in contrast to ours: comovement can arise also under flexible prices and adjustment costs tend to magnify the response of investment. The crucial difference is the horizon at which agents expect technology to improve. Under our estimated parameters, a change in the signal \(s_t\) today anticipates technology improvements.

\[21\] The experiments in Figure 7 are comparative dynamics exercises in which we try different parameter values without re-estimating the model, to understand the mechanics of the model. Table 7 below considers experiments in which some parameters are kept fixed and others are re-estimated.

23
that occur relatively far in the future: with $\rho = 0.94$ only about 22% of the long-run productivity increase is expected to occur in a year. In Christiano, Ilut, Motto and Rostagno (2008) investment increases in anticipation of productivity increases that will take place relatively soon, while in our model investment mostly responds to higher consumer demand. Our effect can only arise under sticky prices and is stronger if adjustment costs are weaker.\textsuperscript{22} This discussion emphasizes that the horizon at which productivity improvements are expected to occur is an important parameter to determine the model response and its sensitivity to other parameter choices.

Turning to variance decomposition, our results are reported in Table 3. The noise shock is the main short-run driver of consumption, accounting for more than half of consumption volatility in the very short term and about 1/4th of it at a two year horizon. On the other hand, the noise shock only accounts for a very small fraction of investment volatility. Virtually all short-run investment volatility is due to the investment specific shock. This can be reconciled with the results above on comovement by noticing that conditional on noise shocks consumption and investment have a similar volatility (as seen from Figure 5). However, unconditional investment volatility is larger than consumption volatility and the investment specific shock plays the crucial role in accounting for it. Finally, the three most important drivers of output are the investment specific shock, the transitory technology shock and the noise shock, with the latter explaining about 20% of volatility at a one year horizon.

It is useful to compare our results to those of Justiniano, Primiceri and Tambalotti (2010), who recently emphasized the role of investment specific technological shocks. Our results on the role of the investment specific shock are similar, while the noise shock in our setup plays a role similar to their shock to intertemporal preferences. Allowing for both noise shocks and intertemporal preference shocks poses serious identification problems as, in the short run, they have very similar effects on consumption. Basically, they both act as shocks to the right-hand side of the Euler equation, one based on temporary errors in expectations about future consumption, the other based simply on changes in time preferences.\textsuperscript{23} To us, noise shocks offer a more appealing explanation for consumption fluctuations than high-frequency changes in intertemporal preferences. However the two explanations, while similar on the positive side, have different implications in terms of

\textsuperscript{22}In unreported results, we look at the effects of a noise shock in a setup with $\rho = 0.2$ and all the other parameters unchanged. In that case, the horizon of our noise shock is shorter and we do obtain comovement of consumption, investment, and employment even under flexible prices.

\textsuperscript{23}Consistently with this observation, some estimation exercises we have tried in this direction seem very sensitive to the prior.
Table 6: Variance decomposition

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<th>Quarter</th>
<th>Perm. tech.</th>
<th>Trans. Tech.</th>
<th>Noise</th>
<th>Inv. specific</th>
<th>Price markup</th>
<th>Wage markup</th>
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<td></td>
</tr>
<tr>
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<td>0.005</td>
<td>0.011</td>
<td>0.971</td>
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<td>0.006</td>
<td>0.000</td>
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</tr>
<tr>
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<td>0.021</td>
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<td>0.008</td>
<td>0.016</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>8</td>
<td>0.031</td>
<td>0.036</td>
<td>0.027</td>
<td>0.869</td>
<td>0.009</td>
<td>0.027</td>
<td>0.000</td>
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<tr>
<td>12</td>
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<td>0.046</td>
<td>0.025</td>
<td>0.769</td>
<td>0.009</td>
<td>0.029</td>
<td>0.000</td>
<td>0.003</td>
</tr>
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<td>Output</td>
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<td>0.003</td>
<td>0.249</td>
<td>0.200</td>
<td>0.372</td>
<td>0.083</td>
<td>0.026</td>
<td>0.001</td>
<td>0.066</td>
</tr>
<tr>
<td>4</td>
<td>0.040</td>
<td>0.272</td>
<td>0.198</td>
<td>0.363</td>
<td>0.057</td>
<td>0.039</td>
<td>0.003</td>
<td>0.028</td>
</tr>
<tr>
<td>8</td>
<td>0.228</td>
<td>0.270</td>
<td>0.134</td>
<td>0.267</td>
<td>0.036</td>
<td>0.035</td>
<td>0.006</td>
<td>0.024</td>
</tr>
<tr>
<td>12</td>
<td>0.477</td>
<td>0.200</td>
<td>0.083</td>
<td>0.167</td>
<td>0.023</td>
<td>0.023</td>
<td>0.008</td>
<td>0.020</td>
</tr>
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</table>
policy design. So further work to disentangle their relative importance is warranted.

We conclude this section with some additional robustness checks, presented in Table 7. Line 1 reports our baseline results, for comparison. The exercise on line 2 explores the monetary rule. The parameter estimates in Table 5 give an interest rate rule that is very insensitive to inflation: $\gamma_\pi$ is close to 1, while the literature typically finds larger numbers. Therefore, in our first exercise we redo the estimation using a tighter prior for $\gamma_\pi$, namely a prior centered at 1.5 with a standard deviation of 0.05. Line 2 of Table 7 shows that short run consumption volatility is still mostly driven by noise shocks. However, the estimation requires higher values for the Calvo parameters $\theta$ and $\theta_w$. Therefore, in line 3, we ask what happens if we restrict at the same time the Taylor coefficient at 1.5 and both Calvo parameters at 0.85. The effect of noise shocks on consumption is still relevant but substantially reduced relative to our baseline. The marginal likelihood suggests that these restrictions are strongly rejected by the data. An unappealing feature of the estimates in line 3 is also that they imply a bigger role of the wage markup shock in consumption volatility, in particular, that shock now explains 17% of consumption volatility at a 1 year horizon.
The experiments on line 2 and 3 of Table 7 show that the model requires a combination of high levels of nominal rigidity and sluggish monetary policy response to get noise shocks to match consumption volatility. The reason for this is that those two ingredients mute the response of the real interest rate. In the Euler equation, current consumption is only driven by expected future consumption and by the real rate. If the real rate responds less, consumption is more responsive to movements in expected future consumption and thus to noise shocks. This suggests that a drawback of the model used here is the assumption of a unit elasticity of intertemporal substitution (log utility), which makes consumption too sensitive to the real interest rate.\footnote{Our model needs preferences consistent with balanced growth, as it includes a non-stationary technology process. Therefore, moving away from unit elasticity will require to introduce non-separable preferences à la King, Plosser, and Rebelo (1988).}
Table 7: Variance of consumption due to noise shocks in restricted models

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q4</th>
<th>Marginal Likelihood</th>
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<tbody>
<tr>
<td>1. Baseline</td>
<td>0.51</td>
<td>0.43</td>
<td>-1539.5</td>
</tr>
<tr>
<td>2. Tighter prior on $\gamma_\pi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_\pi = 1.34$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.87$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_w = 0.91$</td>
<td>0.54</td>
<td>0.36</td>
<td>-1548.5</td>
</tr>
<tr>
<td>3. Fixed monetary rule and fixed Calvo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_\pi = 1.5$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.85$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_w = 0.85$</td>
<td>0.47</td>
<td>0.19</td>
<td>-1554.1</td>
</tr>
<tr>
<td>4. Tighter prior on low variance of investment shocks and low adjustment costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_d = 0.5503$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_d = 6.1204$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi = 2.5$</td>
<td>0.55</td>
<td>0.47</td>
<td>-1541.5</td>
</tr>
</tbody>
</table>
4 Conclusions

On the methodological side, we have explored the problem of estimating models with news and noise, which we think provide an appealing description of the cycle. We have shown the limits of SVAR estimation and shown how these models can be estimated with structural methods. This implies that to identify the role of news and noise in fluctuations one must rely more heavily on the model’s structure. Our simple model shows that a central role for identification is played by the consumer’s Euler equation, that is, by the assumption that current movements in consumption are primarily driven by changes in the consumers’ expectations on the economy’s long run potential.

On the empirical side, the data appear consistent with a view of fluctuations where the pattern of technological change is smooth, subject to random shocks which only build up slowly, while a sizable fraction of short-run volatility in consumption and output comes from noisy information on these long-run trends.

A useful extension for future work is to add variables to the empirical exercise, to better capture consumers’ expectations about the future. For example, one could include financial market prices, following Beaudry and Portier (2006), or survey measures of consumer confidence, as Barsky and Sims (2008). However, the analysis in Section 2.2, where we allow the econometrician to directly observe all the signals observed by the consumers, shows that adding these variables will not solve the identification problems of SVARs.

Finally, it is useful to notice that the applicability of SVAR methods depends crucially on the way in which one models the information structure. In models where the consumer exactly observes shocks which will affect productivity in the future, invertibility problems may be less damning. However, we believe that signal extraction models provide a more realistic and flexible way of introducing expectational shocks.

5 Appendix

In this appendix, we formulate a general representative agent dynamic linear model with signal extraction. Both the simple model of Section 1 and the full DSGE model of Section 3 are special cases of this formulation. We use this model for various purposes: (i) to set up the agents’ Kalman filter used in the model solution (Section 1.1) and the econometrician’s Kalman filter used to construct the likelihood function in Section 2.4; (ii) to derive the
general singularity result for signal-extraction models discussed in Section 2.2; (iii) to derive the equivalent full information model which simplifies estimation in Section 3.

Uncertainty is captured by the exogenous state vector $X_t$ that follows the process

$$X_t = AX_{t-1} + BV_t,$$

where $V_t$ is an $n$-dimensional vector of mutually independent i.i.d. shocks, with positive variance. The representative agent observes the $m$-dimensional vector

$$S_t = CX_t + DV_t.$$  

In Sections 1 and 3, the states vector is $X_t = (x_t, x_{t-1}, z_t)'$, the shocks vector is $V_t = (\epsilon_t, \eta_t, \nu_t)'$ and the vector of consumer observations is $S_t = (a_t, s_t)$. So the matrices $A, B, C, D$ are

$$A = \begin{bmatrix} 1 + \rho & -\rho & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

Let $Y_t$ denote a vector of endogenous state variables controlled by the agent. Suppose the economic model can be described in terms of the difference equation

$$FE_t [Y_{t+1}] + GY_t + HY_{t-1} + MS_t + NE_t [S_{t+1}] = 0,$$

where $F, G, H, M, N$ are matrices of parameters. Notice that the unobservable exogenous state $X_t$ only enters the equilibrium through the observable vector $S_t$, reflecting the assumption that the information set of the representative agent is given only by past and current values of $S_t$ and of the endogenous state $Y_t$. Suppose there is a unique stable solution of the model:

$$Y_t = PY_{t-1} + QS_t + RX_t | t,$$

where we use the notation $X_t | t$ for the agents’ expectation $E [X_t | S_t, S_{t-1}, ...]$. The matrices $P, Q, R$ can be found solving the three matrix equations

$$FP^2 + GP + H = 0, \quad (FP + G) Q + M = 0, \quad (FP + G) R + [F (QC + R) + NC] A = 0.$$  

See Uhlig (1995) for techniques to solve the first equation in $P$. The solution of the other two is straightforward as they are linear in $Q$ and $R$.

The economic model of Section 1 is given directly in the form (18), by equation (7). The economic model of Section 3 is presented in the Online Appendix.
5.1 Kalman filters

We can use the Kalman filter to express the agents’ expectations $X_t|t$ in recursive form as

$$
X_t|t = AX_{t-1}|t-1 + K(S_t - S_t|t-1)
= A(I - KC)X_{t-1}|t-1 + KS_t,
$$

(19)

where the matrix of Kalman gains $K$ depends on the parameters of the productivity process. We assume that (19) is stable, i.e., all eigenvalues of $A(I - KC)$ are smaller than one in absolute value. Notice that stability of the filter does not require $X_t$ to be stationary, e.g., the model used in Sections 1 and 3 is non-stationary and yet the filter is stable.

The vector of states for the econometrician is given by $(X_t, X_t|t, Y_t)$. The dynamics of $X_t$ are given by (15). The dynamics of $X_t|t$ are given by

$$
X_t|t = A(I - KC)X_{t-1}|t-1 + KCAX_{t-1} + (KCB + KD)V_t,
$$

which follows from (15) and (19). The dynamics of $Y_t$ are given by (18). To set up the econometrician’s Kalman filter we use the dynamic system just described for $(X_t, X_t|t, Y_t)$ and the observation equation

$$
S_t^E = T\begin{bmatrix} Y_t & S_t \end{bmatrix}.
$$

(20)

5.2 Singularity

Solving (18) backward and substituting in (20), we can express the econometrician’s observables $S_t^E$ in terms of distributed lags of the agents’ observables $S_t$ and of the agents’ expectations $X_{t|t}$:

$$
S_t^E = \Xi(L)(S_t\ X_{t|t})',
$$

(21)

Define the vector of innovations for the econometrician as

$$
U_t = S_t^E - E[S_t^E|S_{t-1}^E, S_{t-2}^E, ...].
$$

(22)

We say that the VAR in $Y_t$ is invertible if $V_t$ can be expressed as a linear combination of current and past values of $U_t$.

**Lemma 1** If the dimension of the agent’s observation vector is smaller than the dimension of the shock vector, $m < n$, then the VAR in $S_t^E$ is not invertible.

**Proof.** The agents’ Kalman filter can be solved backward to express $X_{t|t}$ as a function of current and past values of $S_t$. This, combined with (21) and (22), implies that $S_t^E$ and
thus $U_t$ can be expressed as a function of current and past values of $S_t$. This implies that $\text{Var}[V_t|U_t, U_{t-1}, ...] \geq \text{Var}[V_t|S_t, S_{t-1}, ...]$. Standard derivations allow us to express the innovations in $S_t$, as $S_t - E[S_t|S_{t-1}, S_{t-2}, ...] = \Psi(L) V_t$ where

$$\Psi(L) = CA [I - (I - KC) AL]^{-1} [I - K (C + D)] + C + D,$$

and since $\Psi$ is $m \times n$ $V_t$ cannot be expressed in terms of the agent’s innovations, so $\text{Var}[V_t|S_t, S_{t-1}, ...] > 0$. Combining this with the inequality above yields $\text{Var}[V_t|U_t, U_{t-1}, ...] > 0$.

The main point of the lemma is that what matters is not the number of variables observed by the econometrician, but the number of variables observed by the agent. If the agent has not enough information to back up the shocks $V_t$, an econometrician cannot generate additional information on these shocks by observing the agent’s behavior.

### 5.3 Equivalent full information model

Write the joint dynamics of $X_{t|t}$ and $S_t$ as follows:

$$X_{t|t} = AX_{t-1|t-1} + K(S_t - CAX_{t-1|t-1}),$$

$$S_t = CAX_{t-1|t-1} + S_t - CAX_{t-1|t-1},$$

where the first equation follows from (19). Let $\Sigma_S$ denote the variance-covariance matrix $\text{Var}_{t-1} [S_t]$, which is obtained from the Kalman filter. Suppose this matrix can be factorized as $\Sigma_S = GG'$ for some matrix $G$. Consider the model

$$\hat{X}_t = A\hat{X}_{t-1} + KG\hat{V}_t,$$

$$S_t = CAX_{t-1|t-1} + G\hat{V}_t,$$

where $\hat{V}_t$ is an $m$-dimensional vector of mutually independent, i.i.d. standard normal shocks. Identifying $\hat{X}_t$ with $X_{t|t}$ and $\hat{V}_t$ with $S_t - CAX_{t-1|t-1}$ we obtain the following result.

**Lemma 2** For any matrix $G$ that satisfies $GG' = \Sigma_S$ the original signal extraction model is observationally equivalent to (23)-(24) with the assumption that the agent perfectly observes the state $\hat{X}_t$ and the shock $\hat{V}_t$. 

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References


6 Online appendix

6.1 More on non invertibility

This is the result discussed at the end of Section 2.2. The result is derived for the theoretical impulse responses, ignoring sampling error.

**Proposition 1** Suppose the econometrician observes \((c_t, a_t, s_t)\) or \((c_t, a_t)\). The impulse response of \(c_t\) to any identified shock from a structural VAR are either permanent and flat or zero.

**Proof.** Let \(w_t\) be an identified shock, corresponding to a linear combination of current and past observables. Denote by \(I_t\) the consumer’s information and by \(I_e t\) the econometrician’s information. Applying the law of iterated expectations we get

\[
E\left[c_{t+k}|w_t, I_{t-1}\right] = E\left[\lim_{j \to \infty} E\left[a_{t+k+j}|I_{t+k}\right] |w_t, I_{t-1}\right] = \lim_{j \to \infty} E\left[a_{t+j}|w_t, I_{t-1}\right],
\]

for all \(k \geq 0\) and, similarly,

\[
E\left[c_{t+k}|I_{t-1}\right] = \lim_{j \to \infty} E\left[a_{t+j}|I_{t-1}\right].
\]

It follows that the response of consumption to \(w_t\) is constant and equal to

\[
E\left[c_{t+k}|w_t, I_{t-1}\right] - E\left[c_t|I_{t-1}\right] = \lim_{j \to \infty} E\left[a_{t+k+j}|w_t, I_{t-1}\right] - \lim_{j \to \infty} E\left[a_{t+j}|I_{t-1}\right],
\]

for all \(k \geq 0\). 

6.2 DSGE Model

6.2.1 Setup

**Households.** The household preferences are given by (12) in the text. The household budget constraint is

\[
P_t C_t + P_t I_t + T_t + B_t + P_t C(U_t)K_t - R_{t-1}B_{t-1} + Y_t + \int_0^1 W_{jt} N_{jt} dj + R^k_t K_t = (25)
\]

where \(P_t\) is the price level, \(T_t\) is a lump sum tax, \(B_t\) are holdings of one period bonds, \(R_t\) is the one period nominal interest rate, \(Y_t\) are aggregate profits, \(W_{jt}\) is the wage of specialized labor of type \(j\), \(R^k_t\) is the capital rental rate. The investment-specific technology parameter \(d_t = \log D_t\) follows the stochastic process

\[
d_t = \rho d_{t-1} + \epsilon_d t.
\]

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\[ \varepsilon_{dt} \text{ and all the variables denoted } \varepsilon \text{ from now on are i.i.d. shocks.} \]

Households choose consumption, bond holdings, capital utilization, and investment each period so as to maximize their expected utility subject to (25) and a standard no-Ponzi condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires \( B_t = 0 \).

**Production and labor services.** Consumption and investment are in terms of a final good which is produced by competitive final good producers using the CES production function

\[
Y_t = \left( \int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}} d\tilde{j} \right)^{1+\mu_{pt}},
\]

which employs a continuum of intermediate inputs. \( Y_{jt} \) is the quantity of input \( j \) employed and \( \mu_{pt} \) captures a time-varying elasticity of substitution across goods, where 

\[
\log(1 + \mu_{pt}) = \log (1 + \mu_p) + m_{pt} \text{ and } m_{pt} \text{ follows the process}
\]

\[
m_{pt} = \rho_p m_{pt-1} + \varepsilon_{pt} - \psi_p \varepsilon_{pt-1}.
\]

The production function for intermediate good \( j \) is

\[
Y_{jt} = (K_{jt})^\alpha (A_t L_{jt})^{1-\alpha},
\]

where \( K_{jt} \) and \( L_{jt} \) are, respectively, capital and labor services employed. As in our baseline model, the technology parameter \( a_t \) follows the process (1)-(3) and the representative consumer does not observe \( x_t \) and \( z_t \) separately, but observes \( a_t \) and the signal \( s_t \) given by (5). However, here we treat explicitly the constant term in TFP growth by letting

\[
A_t = \Gamma^t e^{a_t}.
\]

Intermediate good prices are sticky with price adjustment as in Calvo (1983). Each period intermediate good firm \( j \) can freely set the nominal price \( P_{jt} \) with probability \( 1 - \theta_p \) and with probability \( \theta_p \) is forced to keep it equal to \( P_{jt-1} \). These events are purely idiosyncratic, so \( \theta_p \) is also the fraction of firms adjusting prices each period.

Labor services are supplied to intermediate good producers by competitive labor agencies that combines specialized labor of types in \([0, 1]\) using the technology

\[
N_t = \left[ \int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} d\tilde{j} \right]^{1+\mu_{wt}},
\]

where \( \log(1 + \mu_{wt}) = \log (1 + \mu_w) + m_{wt} \text{ and } m_{wt} \text{ follows the process}
\]

\[
m_{wt} = \rho_w m_{wt-1} + \varepsilon_{wt} - \psi_w \varepsilon_{wt-1}.
\]
Specialized labor wages are also sticky and set by the household. For each type of labor $j$, the household can freely set the price $W_{jt}$ with probability $1 - \theta_w$ and has to keep it equal to $W_{jt-1}$ with probability $\theta_w$.

Market clearing in the final good market requires
\[ C_t + I_t + C (U_t) \bar{K}_{t-1} + G_t = Y_t, \quad (27) \]
market clearing in the market for labor services requires $\int L_{jt} dj = N_t$.

**Government spending and monetary policy.** Government spending is set as a fraction of output and the ratio of government spending to output is $G_t / Y_t = \psi + g_t$, where $g_t$ follows the stochastic process
\[ g_t = \rho g_{t-1} + \epsilon_{gt}. \]
Monetary policy follows the interest rate rule
\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) (\gamma_\pi \pi_t + \gamma_y \hat{y}_t) + q_t \]
where $r_t = \log R_t - \log R$ and $\pi_t = \log P_t - \log P_{t-1} - \pi$, $\pi$ is the inflation target, $\hat{y}_t$ is defined below and $q_t$ follows the process
\[ q_t = \rho_q q_{t-1} + \epsilon_{qt}. \]

### 6.2.2 Optimality conditions

**Households.** Define the marginal utility of consumption
\[ \Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left[ \frac{1}{C_{t+1} - hC_t} \right]. \quad (29) \]
The consumers Euler equation is then
\[ \Lambda_t = \beta R_t E_t \left[ \frac{\Lambda_{t+1}}{P_t} \right]. \quad (30) \]
The optimality conditions for $\bar{K}_{t-1}$ and $I_t$ are
\[ \Phi_t = \beta E_t \left[ \Lambda_{t+1} \left( R_{t+1}^k U_{t+1} - P_t C (U_{t+1}) \right) \right] + (1 - \delta) \beta E_t \Phi_{t+1}, \quad (31) \]
\[ P_t \Lambda_t = \Phi_t D_t \left[ 1 - G_t - \frac{I_t}{I_{t-1} G'_t} \right] + \beta E_t \left[ \Phi_{t+1} D_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 G'_{t+1} \right], \quad (32) \]
where $\Phi_t$ is the Lagrange multiplier on the capital accumulation constraint. The optimality condition for capacity utilization $U_t$ is
\[ R_t^k = C' (U_t). \quad (33) \]
**Firms.** The optimality condition for the final good producer yields the demand for intermediate good $j$,

$$Y_{jt} = Y_t \left( P_{jt} / P_t \right)^{-(1+\mu_{pt})/\mu_{pt}},$$

while the zero profit condition gives the final good price $P_t = (\int P_{jt}^{1/\mu_{pt}} dj)^{\mu_{pt}}$.

Cost minimization by the intermediate good producers and constant returns to scale imply that the nominal cost of producing $Y_{jt}$ is $M_t Y_{jt}$, where $M_t$ is the marginal cost

$$M_t = a^{-\alpha}(1-\alpha)^{-(1-\alpha)} \left( R_t^k \right)^{\alpha} (W_t / A_t)^{1-\alpha}. \quad (34)$$

Cost minimization also implies that all intermediate good firms choose the same capital-labor ratio

$$\frac{K_t}{N_t} = \frac{a}{1-\alpha} \frac{W_t}{R_t^k}. \quad (35)$$

Calvo pricing implies that the firms changing their price in period $t$ choose the price $P_t^*$ that maximizes

$$E_t \left[ \sum_{\tau=0}^{\infty} \theta^\tau P_t^* Y_{jt+\tau} \right] = 0. \quad (36)$$

subject to $Y_{jt+\tau} = Y_{jt} \left( P_t^* / P_{t+\tau} \right)^{-(1+\mu_{pt+\tau})/\mu_{pt+\tau}}$. Their optimality condition is thus

$$E_t \left[ \sum_{\tau=0}^{\infty} \theta^\tau P_t^* \frac{\Lambda_t+\tau}{P_{t+\tau}} \left( \frac{1}{\mu_{pt+\tau}} - \frac{1+\mu_{pt+\tau}M_{t+\tau}}{\mu_{pt+\tau}P_t^*} \right) Y_{jt+\tau} \right] = 0. \quad (37)$$

The problem of the labor agency is similar to that of the final good producer, so the demand for specialized labor of type $j$ is

$$N_{jt} = N_t \left( W_{jt} / W_t \right)^{-(1+\mu_{wt})/\mu_{wt}},$$

and the price of labor services is $W_t = (\int W_{jt}^{1/\mu_{wt}} dj)^{\mu_{wt}}$. Calvo pricing for wage setters implies that the workers that can adjust their wage maximize

$$E_t \left[ \sum_{\tau=0}^{\infty} \theta^\tau W_t^* N_{jt+\tau} \right] = 0. \quad (37)$$

and their optimality condition is

$$E_t \left[ \sum_{\tau=0}^{\infty} \theta^\tau W_t^* \left( \frac{\Lambda_t+\tau}{P_{t+\tau}} W_t^* N_{jt+\tau} \right) \right] = 0. \quad (38)$$
6.2.3 Log-linear approximation

First, we need to normalize some variables to ensure their stationarity. In particular, we define

$$\hat{c}_t = \log(C_t / A_t) - \log(C / A)$$

where $C / A$ denotes the value of $C / A_t$ in the deterministic version of the model in which $A_t$ grows at the constant growth rate $\Gamma$. Analogous definitions apply to the quantities $\hat{y}_t, \hat{k}_t, \hat{i}_t$. The quantities $N_t$ and $U_t$ are already stationary, so we have

$$n_t = \log N_t - \log N,$$

and a similar expression for $u_t$. For nominal variables, we also need to take care of non-stationarity in the price level, so we define

$$\hat{w}_t = \log \left( W_t / (A_t P_t) \right) - \log \left( W / (A P) \right), \quad r_t^k = \log \left( R_t^k / P_t \right) - \log \left( R^k / P \right),$$

$$m_t = \log \left( M_t / P_t \right) - \log (M / P), \quad r_t = \log R_t - \log R, \quad \pi_t = \log \left( P_t / P_{t-1} \right) - \pi.$$

Finally, for the Lagrange multipliers we define

$$\hat{\lambda}_t = \log(\Lambda_t A_t) - \log(\Lambda A), \quad \hat{\phi}_t = \log(\Phi_t A_t / P_t) - \log(\Phi A / P).$$

Notice that the hat is only used for variables normalized by $A_t$.

**Optimality conditions.** Conditions (29)-(35) can be log-linearized to yield the following seven conditions

$$\hat{\lambda}_t = \frac{h \beta \Gamma}{(\Gamma - h \beta) (\Gamma - h)} E_t \hat{c}_{t+1} - \frac{\Gamma^2 + h^2 \beta}{(\Gamma - h \beta) (\Gamma - h)} \hat{c}_t + \frac{h \Gamma}{(\Gamma - h \beta) (\Gamma - h)} \hat{c}_{t-1} +$$

$$+ \frac{h \beta \Gamma}{(\Gamma - h \beta) (\Gamma - h)} E_t [\Delta a_{t+1}] - \frac{h \Gamma}{(\Gamma - h \beta) (\Gamma - h)} \Delta a_t, \quad (38)$$

$$\hat{\lambda}_t = r_t + E_t \left[ \hat{\lambda}_{t+1} - \Delta a_{t+1} - \pi_{t+1} \right], \quad (39)$$

$$\hat{\phi}_t = (1 - \delta) \beta \Gamma^{-1} E_t \left[ \hat{\phi}_{t+1} - \Delta a_{t+1} \right]$$

$$+ \left( 1 - (1 - \delta) \beta \Gamma^{-1} \right) E_t \left[ \hat{\lambda}_{t+1} - \Delta a_{t+1} + r_{t+1}^k \right], \quad (40)$$

$$\hat{\lambda}_t = \hat{\phi}_t + d_t - \chi \Gamma^2 \left( \hat{i}_{t} - \hat{i}_{t-1} + \Delta a_t \right) + \beta \chi \Gamma^2 E_t \left[ \hat{i}_{t+1} - \hat{i}_t + \Delta a_{t+1} \right], \quad (42)$$

$$r_t^k = \xi u_t, \quad (43)$$

$$m_t = \alpha r_t^k + (1 - \alpha) \hat{w}_t, \quad (44)$$

$$r_t^k = \hat{w}_t - \hat{k}_t + n_t, \quad (45)$$

39
Resource constraints and inflation. Log-linearizing conditions (14) and (13) yields

$$
\hat{k}_t = u_t + \hat{k}_{t-1} - \Delta a_t, \tag{46}
$$

$$
\hat{k}_t = (1 - \delta) \Gamma^{-1} (\hat{k}_{t-1} - \Delta a_t) + \left(1 - (1 - \delta) \Gamma^{-1}\right) \left(d_t + \hat{i}_t\right). \tag{47}
$$

Approximating and aggregating (26) over intermediate good producers and using the final good production function yields

$$
\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) n_t. \tag{48}
$$

Market clearing in the final good market (27) yields

$$
(1 - \psi) \hat{y}_t = C/Y \hat{c}_t + I/Y \hat{i}_t + R^{k} K/\mu Y u_t + g_t. \tag{49}
$$

$C/Y, I/Y$ and $R^{k} K/\mu Y$ are all equilibrium ratios in the deterministic version of the model in which $A_t$ grows at the constant rate $\Gamma$.

Aggregating individual optimality conditions for price setters yields

$$
\pi_t = \beta E_t \pi_{t+1} + \kappa m_t + \kappa m_{pt}, \tag{50}
$$

where $\kappa = (1 - \theta \beta)(1 - \theta)/\theta$. Aggregating individual optimality conditions for wage setters yields

$$
\bar{w}_t = \frac{1}{1 + \beta} \bar{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \bar{w}_{t+1} - \frac{1}{1 + \beta} (\pi_t + \Delta a_t) + \frac{\beta}{1 + \beta} E_t (\pi_{t+1} + \Delta a_{t+1})
$$

$$
- \kappa_w (\bar{w}_t - \zeta n_t + \hat{\lambda}_t) + \kappa_w m_{wt}, \tag{51}
$$

where

$$
\kappa_w = \frac{(1 - \theta_w \beta)(1 - \theta_w)}{\theta_w (1 + \beta) \left(1 + \zeta \left(1 + \frac{1}{\mu_w}\right)\right).}
$$

Summing up, we have 23 variables: the 9 exogenous variables

$$
d_t, a_t, x_t, z_t, s_t, m_{pt}, m_{wt}, g_t, q_t,
$$

and the 14 endogenous variables

$$
\hat{c}_t, \hat{i}_t, \hat{y}_t, n_t, \hat{k}_t, u_t, \hat{\lambda}_t, \hat{\phi}_t, r^k, \bar{w}_t, \pi_t, m_t, r_t.
$$

The model dynamics are given by the exogenous processes for the exogenous variables above and by the 14 equations (28) and (38)-(51).

The observables that are matched to the data are $\Delta y_t, \Delta c_t, \Delta i_t, \Delta w_t, \Delta n_t, r_t, \pi_t$, where the first four variables are obtained by adding $\Delta \hat{a}_t$ to the first differences of the corre-
sponding variables, e.g.,
\[ \Delta y_t = \dot{y}_t - \dot{y}_{t-1} + \Delta a_t. \]

### 6.2.4 New Keynesian model

The standard new Keynesian model is a special case of the DSGE model above when:
(i) the capital stock and capacity utilization are fixed (and hence the investment specific technology shock is absent),
(ii) the habit parameter in the preferences \( h \) is set to zero,
(iii) labor is homogeneous and wages are flexible. Furthermore, we consider a simple new Keynesian model with no fiscal policy \( (G_t = 0) \) and a simplified monetary rule with \( \rho_r = \gamma_y = 0 \) and no monetary shocks \( (q_t = 0) \).

In this case, the log-linearized model boils down to two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium:

\[ y_t = E_t [y_{t+1}] - \gamma\pi_t + E_t [\pi_{t+1}], \]
\[ \pi_t = \tilde{\kappa}(y_t - a_t) + \beta E_t [\pi_{t+1}], \]

where \( y_t = \dot{y}_t + a_t \). The first comes from (28), (38), (39), (49). The second comes from (38), (44), (50) and the fact that (51) requires \( \tilde{\omega}_t = \zeta n_t - \hat{\lambda}_t \) when \( \theta_w \to 0 \). The coefficient \( \tilde{\kappa} \) is given by \( (1 + \zeta / (1 - \alpha)) \kappa \). As long as \( \gamma\pi > 1 \) this system has a unique locally stable solution where \( y_t \) and \( \pi_t \) are linear functions of the four exogenous state variables \( a_t, x_{t|t}, x_{t-1|t}, z_{t|t} \),

\[
\begin{pmatrix}
  y_t \\
  \pi_t
\end{pmatrix} = D
\begin{pmatrix}
  a_t \\
  x_{t|t} \\
  x_{t-1|t} \\
  z_{t|t}
\end{pmatrix},
\]

The matrix \( D \) can be found using the method of undetermined coefficient as the solution to

\[
\begin{bmatrix}
  1 & \gamma\pi \\
  -\tilde{\kappa} & 1
\end{bmatrix} D = \begin{bmatrix}
  0 & 0 & 0 & 0 \\
  -\tilde{\kappa} & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
  1 & 1 \\
  0 & \beta
\end{bmatrix} D \begin{bmatrix}
  0 & 1 + \rho_x & -\rho_x & \rho_z \\
  0 & 1 + \rho_x & -\rho_x & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 0 & \rho_z
\end{bmatrix}.
\]

The elements of \( D \) are a continuous non-linear function of \( \tilde{\kappa} \) and a bit of algebra shows that

\[
\lim_{\tilde{\kappa} \to 0} D = \frac{1}{1 - \rho_x} \begin{bmatrix}
  0 & 1 - \rho_x & 0 \\
  0 & 0 & 0 \\
  0 & 0 & 0
\end{bmatrix}.
\]
Since $\bar{\kappa} \to 0$ when $\theta \to 1$, this shows that in the limit with very infrequent price adjustments we get (7).

6.3 Data

Our dataset spans the period 1954:3 to 2011:1. The start date is chosen because the effective Federal Funds Rate quarterly series, published by the Federal Reserve Board, is not available before that date.

The series for Real GDP, Real Personal Consumption Expenditures, Real Personal Durable Consumption Expenditures, Real Gross Private Domestic Investment, Wages and the GDP Implicit Price Deflator were obtained from the Saint Louis Fed online database. Population and Employment series were obtained from the Bureau of Labor Statistics online database (series IDs LNS10000000Q and LNS12000000Q respectively). The Federal Funds Rate series was obtained from the Federal Reserve Board online database (series ID H15/H15/RIFSPFF N.M).

The GDP series was constructed by dividing Real GDP by population and taking logs. The consumption series was constructed by subtracting Real Personal Durable Consumption from Real Personal Consumption, dividing by population, and taking logs. The investment series was constructed by dividing the sum of Real Gross Investment and Real Personal Durable Consumption by population, and taking logs. The labor input series was constructed by dividing Employment by Population and taking logs. Inflation was constructed by computing the quarterly log difference of the Price Deflator. The real wage was constructed by dividing Real Wages by the Price Deflator, and taking logs. The nominal interest rate is the effective Federal Funds Rate.

6.4 More on recovering states and shocks

This section presents more results on the Kalman smoother for the baseline model, following up on Section 2.5. Figure 8 plots the RMSE of the smoothed estimates of $x_t$ and $z_t$, when data up to $t + j$ are available, for $j = 0, 1, 2, \ldots$. The RMSE is the square root of $E_t[(x_t - E_t[x_t])^2]$, and can be computed using two different information sets: the econometrician’s, which only includes observations of $c_t$ and $a_t$, and the consumer’s, which also includes $s_t$. For simplicity, we compute the RMSE at the steady state of the Kalman filter, that is, assuming the forecaster has access to data from $-\infty$ to $t + j$. In this case, the econometrician’s information set coincides with the consumer’s, that is, the
The econometrician can back up the current value of $s_t$ perfectly from current and past observations of $c_t$ and $a_t$. Although we have not established this result analytically, it holds numerically in all our examples: the computed RMSE of the econometrician’s estimate of $s_t$ is zero at $j = 0$. This implies that, in our model, with a sufficiently long data set, the direct observation of $s_t$ does not add much to the econometrician’s ability to recover the unobservable states or the shocks.

Figure 8 illustrates the results discussed in the text. Notice that most of the relevant information arrives in the first six quarters after $t$; there are minimal gains after that.

Figure 9 we report the RMSE of the estimates of the shocks $\epsilon, \eta$ and $\nu$. To help the interpretation, each RMSE is normalized dividing it by the ex ante standard deviation of the respective shock (e.g., by $\sigma_\epsilon$ for $\epsilon$).

Notice that if the model was invertible, all RMSE would be zero at $j = 0$. The fact that all RMSE are bounded away from zero at all horizons shows that even an infinite data set would not allow us to recover the shocks exactly.

The transitory shock $\eta_t$ is estimated with considerable precision already on impact and the precision of its estimate almost doubles in the long run. The noise shock $\nu_t$ is less precisely estimated, but the data still tell us a lot about it, giving us an RMSE which is about $1/3$ of the prior uncertainty in the long run. The shock that is least precisely estimated is the permanent shock $\epsilon_t$. Even with an infinite series of future data, the residual variance is about 94% of the prior uncertainty on the shock.
Figure 9: Normalized RMSE of the estimated shocks at time $t$ using data up to $t + j$

How do we reconcile the imprecision of the estimate of $\epsilon_t$ with the fact that we have relatively precise estimates of the state $x_t$, as seen in Figure 8? The explanation is that the econometrician can estimate the cumulated effect of permanent productivity changes by looking at productivity growth over longer horizons, but cannot pinpoint the precise quarter in which the change occurred. Therefore, it is possible to have imprecise estimates of past $\epsilon_t$'s, while having a relatively precise estimate of their cumulated effect on $x_t$. 