

SUPPLEMENT TO “DECENTRALIZED TRADING WITH PRIVATE INFORMATION”: ONLINE APPENDIX  
(*Econometrica*, Vol. 82, No. 3, May 2014, 1055–1091)

BY MIKHAIL GOLOSOV, GUIDO LORENZONI, AND ALEH TSYVINSKI

S1. PROOF OF LEMMA 5

TO PROVE THE LEMMA, we will find two scalars  $\bar{x}$  and  $\underline{u}$  such that the set

$$X = \{x : x \in (0, \bar{x}]^2, U(x, \delta) \geq \underline{u} \text{ for some } \delta \in [0, 1]\}$$

satisfies the desired properties. The proof combines two ideas: use market clearing to put an upper bound on the holdings of the two assets, that is, to show that, with probability close to 1, agents have portfolios in  $(0, \bar{x}]^2$ ; use optimality to bound their holdings away from zero, by imposing the inequality  $U(x, \delta) \geq \underline{u}$ .

First, let us prove that  $X$  is a compact subset of  $R_{++}^2$ . The following two equalities follow from the fact that  $U(x, \delta)$  is continuous, non-decreasing in  $\delta$  if  $x^1 \geq x^2$ , and non-increasing if  $x^1 \leq x^2$ :

$$\begin{aligned} & \{x : U(x, \delta) \geq \underline{u} \text{ for some } \delta \in [0, 1], x^1 \geq x^2\} \\ &= \{x : U(x, 1) \geq \underline{u}, x^1 \geq x^2\}, \\ & \{x : U(x, \delta) \geq \underline{u} \text{ for some } \delta \in [0, 1], x^1 \leq x^2\} \\ &= \{x : U(x, 0) \geq \underline{u}, x^1 \leq x^2\}. \end{aligned}$$

The sets on the right-hand sides of these equalities are closed sets. Then  $X$  can be written as the union of two closed sets, intersected with a bounded set:

$$\begin{aligned} X &= (\{x : U(x, 1) \geq \underline{u}, x^1 \geq x^2\} \cup \{x : U(x, 0) \geq \underline{u}, x^1 \leq x^2\}) \\ &\quad \cap (0, \bar{x}]^2, \end{aligned}$$

and thus is compact. Notice that  $x \notin X$  if  $x^j = 0$  for some  $j$  because of Assumption 2 and  $\underline{u} > -\infty$ . Therefore,  $X$  is a compact subset of  $R_{++}^2$ .

Next, let us define  $\bar{x}$  and  $\underline{u}$  and the time period  $T$ . Given any  $\varepsilon > 0$ , set  $\bar{x} = 4/\varepsilon$ . Goods market clearing implies that

$$(48) \quad P(x_t^j > \bar{x} | s) \leq \varepsilon/4 \quad \text{for all } t, \text{ for } j = 1, 2.$$

To prove this, notice that

$$\begin{aligned} 1 &= \int x_t^j(\omega) dP(\omega | s) \geq \int_{x_t^j(\omega) > 4/\varepsilon} x_t^j(\omega) dP(\omega | s) \\ &\geq (4/\varepsilon)P(x_t^j > 4/\varepsilon | s), \end{aligned}$$

which gives the desired inequality. Let  $\bar{u}$  be an upper bound for the agents' utility function  $u(\cdot)$  (from Assumption 2). Choose a scalar  $u < \bar{u}$  such that

$$\frac{\bar{u} - U(x_0, \delta_0)}{\bar{u} - u} \leq \frac{\varepsilon}{8},$$

for all initial endowments  $x_0$  and initial beliefs  $\delta_0$ . Such a  $u$  exists because  $U(x_0, \delta_0) > -\infty$ , as initial endowments are strictly positive by Assumption 3, and there is a finite number of types. Then notice that  $U(x_0, \delta_0) \leq E[v_t | h^0]$  for all initial histories  $h^0$ , because an agent always has the option to refuse any trade. Moreover,

$$E[v_t | h^0] \leq P(v_t < u | h^0)u + P(v_t \geq u | h^0)\bar{u}.$$

Combining these inequalities and rearranging gives

$$P(v_t < u | h^0) \leq \frac{\bar{u} - U(x_0, \delta_0)}{\bar{u} - u} \leq \frac{\varepsilon}{8}.$$

Taking unconditional expectations shows that  $P(v_t < u) \leq \varepsilon/8$ . Since  $P(s) = 1/2$ , it follows that

$$(49) \quad P(v_t < u | s) \leq \varepsilon/4 \quad \text{for all } t, \text{ for all } s.$$

Choose  $T$  so that

$$(50) \quad P(|u_t - v_t| > u/2 | s) \leq \varepsilon/4 \quad \text{for all } t \geq T.$$

This can be done by Lemma 2, given that almost sure convergence implies convergence in probability. We can then set  $\underline{u} = u/2$ .

Finally, we check that  $P(x_t \in X | s) \geq 1 - \varepsilon$  for all  $t \geq T$ , using the following chain of inequalities:

$$\begin{aligned} P(x_t \in X | s) &\geq P(x_t \in (0, \bar{x}]^2, U(x_t, \delta_t) \geq \underline{u} | s) \\ &\geq P(x_t \in (0, \bar{x}]^2, v_t \geq u, |u_t - v_t| \leq u/2 | s) \\ &\geq 1 - \sum_j P(x_t^j > \bar{x} | s) - P(v_t < u | s) - P(|u_t - v_t| > u/2 | s) \\ &\geq 1 - \varepsilon. \end{aligned}$$

The first inequality follows because  $U(x_t(\omega), \delta_t(\omega)) \geq \underline{u}$  implies  $U(x_t(\omega), \delta) \geq \underline{u}$  for some  $\delta \in [0, 1]$ . The second follows because  $v_t(\omega) \geq u$  and  $|u_t(\omega) - v_t(\omega)| \leq u/2$  imply  $u_t(\omega) = U(x_t(\omega), \delta_t(\omega)) \geq u/2 = \underline{u}$ . The third follows from repeatedly applying Lemma 4. The fourth combines (48), (49), and (50).

## S2. PROOF OF LEMMA 6

The idea of the proof is as follows. We construct a Taylor expansion to compute the utility gains for any trade. Then we define the traded amount  $\zeta$  and the utility gain  $\Delta$  satisfying (9) and (10).

Choose any two portfolios  $x_A, x_B \in X$  and any two beliefs  $\delta_A, \delta_B \in [0, 1]$  such that  $\mathcal{M}(x_B, \delta_B) - \mathcal{M}(x_A, \delta_A) > \varepsilon$ . Pick a price  $p$  sufficiently close to the middle of the

interval between the marginal rates of substitution:

$$p \in [\mathcal{M}(x_A, \delta_A) + \varepsilon/2, \mathcal{M}(x_B, \delta_B) - \varepsilon/2].$$

This price is chosen so that both agents will make positive gains. Consider agent  $A$  and a traded amount  $\tilde{\zeta} \leq \bar{\zeta}$  (for some  $\bar{\zeta}$  which we will properly choose below). The current utility gain associated with the trade  $\tilde{z} = (\tilde{\zeta}, -p\tilde{\zeta})$  can be written as a Taylor expansion:

$$\begin{aligned} (51) \quad & U(x_A - \tilde{z}, \delta_A) - U(x_A, \delta_A) \\ &= -\pi(\delta_A)u'(x_A^1)\tilde{\zeta} + (1 - \pi(\delta_A))u'(x_A^2)p\tilde{\zeta} \\ &\quad + \frac{1}{2}(\pi(\delta_A)u''(y^1) + (1 - \pi(\delta_A))u''(y^2)p^2)\tilde{\zeta}^2 \\ &\geq (1 - \pi(\delta_A))u'(x_A^2)(\varepsilon/2)\tilde{\zeta} \\ &\quad + \frac{1}{2}[\pi(\delta_A)u''(y^1) + (1 - \pi(\delta_A))u''(y^2)p^2]\tilde{\zeta}^2, \end{aligned}$$

for some  $(y^1, y^2) \in [x_A^1, x_A^1 - \bar{\zeta}] \times [x_A^2 + p\bar{\zeta}, x_A^2]$ . The inequality above follows because  $p \geq \mathcal{M}(x_A, \delta_A) + \varepsilon/2$ . An analogous expansion can be done for agent  $B$ .

Now we want to bound the last line in (51). To do so, we first define the minimal and the maximal prices for agents with any belief in  $[0, 1]$  and any portfolio in  $X$ :

$$\begin{aligned} \underline{p} &= \min_{x \in X, \delta \in [0, 1]} \{\mathcal{M}(x, \delta) + \varepsilon/2\}, \\ \bar{p} &= \max_{x \in X, \delta \in [0, 1]} \{\mathcal{M}(x, \delta) - \varepsilon/2\}. \end{aligned}$$

These prices are well defined, as  $X$  is a compact subset of  $R_{++}^2$  and  $u(\cdot)$  has continuous first derivative on  $R_{++}^2$ . Then, choose  $\bar{\zeta} > 0$  such that, for all  $\tilde{\zeta} \leq \bar{\zeta}$  and all  $p \in [\underline{p}, \bar{p}]$ , the trade  $\tilde{z} = (\tilde{\zeta}, -p\tilde{\zeta})$  satisfies  $\|\tilde{z}\| < \theta$  and  $x + \tilde{z}$  and  $x - \tilde{z}$  are in  $R_+^2$  for all  $x \in X$ . This means that the trade is small enough. Next, we separately bound from below the two terms in the last line of the Taylor

expansion (51). Let

$$D'_A = \min_{x \in X, \delta \in [0,1]} (1 - \pi(\delta))u'(x^2)\varepsilon/2,$$

$$D''_A = \min_{\substack{x \in X, \delta \in [0,1], \bar{p} \in [\underline{p}, \bar{p}], \\ y \in [x^1, x^1 + \tilde{\zeta}] \times [x^2 - \bar{p}\tilde{\zeta}, x^2]}} \frac{1}{2} [\pi(\delta)u''(y^1) + (1 - \pi(\delta))u''(y^2)\bar{p}^2].$$

Note that  $D'_A$  is positive,  $D''_A$  is negative, but  $D''_A\tilde{\zeta}^2$  is of second order. Then, there exist some  $\zeta_A \in (0, \bar{\zeta})$  such that, for all  $\tilde{\zeta} \leq \zeta_A$ ,

$$D'_A\tilde{\zeta} + D''_A\tilde{\zeta}^2 > 0$$

and, by construction,

$$U(x_A - \tilde{z}, \delta_A) - U(x_A, \delta_A) \geq D'_A\tilde{\zeta} + D''_A\tilde{\zeta}^2.$$

Analogously, we can find  $D'_B, D''_B$ , and  $\zeta_B$  such that, for all  $\tilde{\zeta} \leq \zeta_B$ , the utility gain for agent  $B$  is bounded from below:

$$U(x_B + \tilde{z}, \delta_B) - U(x_B, \delta_B) \geq D'_B\tilde{\zeta} + D''_B\tilde{\zeta}^2 > 0.$$

We are finally ready to define  $\zeta$  and  $\Delta$ . Let  $\zeta = \min\{\zeta_A, \zeta_B\}$  and

$$\Delta = \min\{D'_A\zeta + D''_A\zeta^2, D'_B\zeta + D''_B\zeta^2\}.$$

By construction,  $\Delta$  and  $\zeta$  satisfy the inequalities (9) and (10).

To prove the last part of the lemma, let

$$\lambda = \frac{1}{2} \frac{\pi(1) \min_{x \in X} \{u'(x^1)\}}{\min\{D'_A, D'_B\}},$$

which, as stated in the lemma, only depends on  $X$  and  $\varepsilon$ . Using a second-order expansion similar to the one above, the utility gain associated to  $z = (\zeta, -p\zeta)$ , for an agent with portfolio  $x_A$  and any belief  $\delta \in [0, 1]$ , can be bounded below:

$$U(x_A - \tilde{z}, \delta) - U(x_A, \delta) \geq -\pi(1) \min_{x \in X} \{u'(x^1)\} \zeta + D'_A\zeta^2.$$

Therefore, to ensure that (11) is satisfied, we need to slightly modify the construction above, by choosing  $\zeta$  so that the following holds:

$$\frac{-\pi(1) \min_{x \in X} \{u'(x^1)\} \zeta + D'_A\zeta^2}{\Delta} > \lambda.$$

The definitions of  $\Delta$  and  $\lambda$  and a continuity argument show that this inequality holds for some positive  $\zeta \leq \min\{\zeta_A, \zeta_B\}$ , completing the proof.

## S3. PROOF OF LEMMA 7

For all  $x \in R^2$  and all  $\delta \in [0, 1]$ , define the measure  $G_t$  as follows:

$$G_t(x, \delta) \equiv \begin{cases} P(\omega : x_t(\omega) = x, \delta_t(\omega) = \delta | s_1) \\ \quad - P(\omega : x_t(\omega) = x, \delta_t(\omega) = \delta | s_2) & \text{if } \delta > 1/2, \\ 0 & \text{if } \delta \leq 1/2. \end{cases}$$

We first prove that  $G_t$  is a well-defined measure, and next, we prove properties (i)–(iii).

Since  $P$  generates a discrete distribution over  $x$  and  $\delta$  for each  $t$ , to prove that  $G_t$  is a well-defined measure we only need to check that

$$P(x_t = x, \delta_t = \delta | s_2) \leq P(x_t = x, \delta_t = \delta | s_1),$$

so that  $G_t$  is nonnegative. Take any  $\delta > 1/2$ . Bayesian rationality implies that a consumer who knows his belief is  $\delta$  must assign probability  $\delta$  to  $s_1$ :

$$\delta = P(s_1 | x_t = x, \delta_t = \delta).$$

Moreover, Bayes's rule implies that

$$\frac{P(s_2 | x_t = x, \delta_t = \delta)}{P(s_1 | x_t = x, \delta_t = \delta)} = \frac{P(x_t = x, \delta_t = \delta | s_2)P(s_2)}{P(x_t = x, \delta_t = \delta | s_1)P(s_1)}.$$

Rearranging and using  $P(s_1) = P(s_2)$  and  $\delta > 1/2$  yields

$$\frac{P(x_t = x, \delta_t = \delta | s_2)}{P(x_t = x, \delta_t = \delta | s_1)} = \frac{1 - \delta}{\delta} < 1,$$

which gives the desired inequality.

Property (i) is immediately satisfied by construction. Property (ii) follows because  $P(x_t = x, \delta_t = 1 | s_2) = 0$  for all  $x$ , given that  $\delta_t = 1$  requires that we are at a history which arises with zero probability conditional on  $s_2$ . The proof of property (iii) is longer and involves the manipulation of market clearing relations and the use of our symmetry assumption. Using the assumption of uniform market clearing, find an  $M$  such that

$$(52) \quad \int_{x_t^2(\omega) \leq m} x_t^2(\omega) dP(\omega | s_1) \geq 1 - \varepsilon \quad \text{for all } m \geq M.$$

Notice that

$$\int_{x_t^2(\omega) \leq m} x_t^1(\omega) dP(\omega | s_1) \leq \int x_t^1(\omega) dP(\omega | s_1) = 1,$$

which, combined with (52), implies that

$$\int_{x_t^2(\omega) \leq m} (x_t^1(\omega) - x_t^2(\omega)) dP(\omega | s_1) \leq \varepsilon \quad \text{for all } m \geq M.$$

Decomposing the integral on the left-hand side gives

$$(53) \quad \int_{\substack{x_t^1 > x_t^2 \\ x_t \in [0, m]^2}} (x_t^1 - x_t^2) dP(\omega | s_1) + \int_{\substack{x_t^2 = x_t^1 \\ x_t \in [0, m]^2}} (x_t^1 - x_t^2) dP(\omega | s_1) \\ + \int_{\substack{x_t^1 < x_t^2 \\ x_t \in [0, m]^2}} (x_t^1 - x_t^2) dP(\omega | s_1) + \int_{\substack{x_t^1 > m \\ x_t^2 \leq m}} (x_t^1 - x_t^2) dP(\omega | s_1) \leq \varepsilon.$$

Let us first focus on the first three terms on the left-hand side of this expression. The second term is zero. Using symmetry to replace the third term, the sum of the first three terms can then be rewritten as

$$(54) \quad \int_{\substack{x_t^1 > x_t^2 \\ x_t \in [0, m]^2}} (x_t^1 - x_t^2) dP(\omega | s_1) + \int_{\substack{x_t^1 > x_t^2 \\ x_t \in [0, m]^2}} (x_t^2 - x_t^1) dP(\omega | s_2).$$

These two integrals are equal to the sums of a finite number of nonzero terms, one for each value of  $x$  and  $\delta$  with positive mass. Summing the corresponding terms in each integral, we have three cases: (a) terms with  $\delta_t = \delta > 1/2$  and  $P(x_t = x, \delta_t = \delta | s_1) > P(x_t = x, \delta_t = \delta | s_2)$  (by Bayes's rule), which can be written as

$$(x^1 - x^2)P(x_t = x, \delta_t = \delta | s_1) - (x^1 - x^2)P(x_t = x, \delta_t = \delta | s_2) \\ = (x^1 - x^2)G_t(x);$$

(b) terms with  $\delta_t = \delta = 1/2$  and  $P(x_t = x, \delta_t = \delta | s_1) = P(x_t = x, \delta_t = \delta | s_2)$  (by Bayes's rule), which are equal to zero,

$$(x^1 - x^2)P(x_t = x, \delta_t = \delta | s_1) - (x^1 - x^2)P(x_t = x, \delta_t = \delta | s_2) = 0;$$

(c) terms with  $\delta_t = \delta < 1/2$  and  $P(x_t = x, \delta_t = \delta | s_1) = P(x_t = x, \delta_t = \delta | s_2)$  (once more, by Bayes's rule), which can be rewritten as follows, exploiting symmetry:

$$(x^1 - x^2)P(x_t = (x^1, x^2), \delta_t = \delta | s_1) \\ - (x^1 - x^2)P(x_t = (x^1, x^2), \delta_t = \delta | s_2) \\ = (x^1 - x^2)[P(x_t = (x^2, x^1), \delta_t = 1 - \delta | s_2)$$

$$\begin{aligned}
 & - P(x_t = (x^2, x^1), \delta_t = 1 - \delta \mid s_t)] \\
 & = (x^2 - x^1)G_t((x^2, x^1), 1 - \delta).
 \end{aligned}$$

Combining all these terms, the integral (54) is equal to

$$\begin{aligned}
 & \int_{\substack{x^1 > x^2, \delta > 1/2 \\ x \in [0, m]^2}} (x^1 - x^2) dG_t(x, \delta) \\
 & \quad + \int_{\substack{x^1 > x^2, \delta < 1/2 \\ x \in [0, m]^2}} (x^2 - x^1) dG_t((x^2, x^1), 1 - \delta) \\
 & = \int_{\substack{x^1 > x^2, \delta > 1/2 \\ x \in [0, m]^2}} (x^1 - x^2) dG_t(x, \delta) + \int_{\substack{x^2 > x^1, \delta > 1/2 \\ x \in [0, m]^2}} (x^1 - x^2) dG_t(x, \delta) \\
 & = \int_{x \in [0, m]^2} (x^1 - x^2) dG_t(x, \delta),
 \end{aligned}$$

where the first equality follows from a change of variables and the second from the fact that  $G_t$  is zero for all  $\delta \leq 1/2$ . We can now go back to the integral on the right-hand side of (53), and notice that the integrand  $(x_t^1 - x_t^2)$  in the fourth term is positive, so replacing the measure  $P$  with the measure  $G_t$ , which is smaller than or equal to  $P$ , reduces the value of that term. Therefore, the inequality (53), in terms of the measure  $P$ , leads to the following inequality in terms of the measure  $G_t$ :

$$\int_{x^2 \leq m} (x^1 - x^2) dG_t \leq \varepsilon,$$

completing the proof of property (iii).

#### S4. PROOF OF LEMMA 3

We start with the usual convergence properties. Since the marginal rates of substitution of informed agents converge, by Proposition 1, and there is at least a mass  $\alpha$  of informed agents, using Lemmas 4 and 5 we can find a compact set  $X \subset R_{++}^2$  and a time  $T'$  such that there is a sufficiently large mass of informed agents with marginal rates of substitution sufficiently close to  $\kappa_t(s)$  (within  $\bar{\varepsilon}/2$ ) and portfolios in  $X$ :

$$P(|\mathcal{M}(x_t, \delta_t) - \kappa_t(s)| < \bar{\varepsilon}/2, \delta_t = \delta^l(s), x_t \in X \mid s) > (3/4)\alpha$$

for all  $t \geq T'$  and for all  $s$ , where  $\bar{\varepsilon}$  is defined as in Proposition 2.

Now we provide an important concept. We want to focus on the utility gains that can be achieved by small trades (of norm less than  $\theta$ ), by agents with marginal rates of substitution sufficiently different from each other (by at least  $\bar{\varepsilon}/2$ ). Formally, we proceed as follows. Take any  $\theta > 0$ . Using Lemma 6, we can then find a lower bound for the utility gain  $\Delta > 0$  from trade between two agents with marginal rates of substitution differing by at least  $\bar{\varepsilon}/2$  and with portfolios in  $X$ , making trades of norm less than  $\theta$ . It is important to notice that this is the gain achieved if the agents trade but do not change their beliefs. Therefore, it is also important to bound from below the gains that can be achieved by such trades if beliefs are updated in the most pessimistic way. This bound is also given by Lemma 6, which ensures that  $-\lambda\Delta$  is a lower bound for the gains of the agent offering  $z$  at any possible ex post belief (where  $\lambda$  is a positive scalar independent of  $\theta$ ).

Next, we want to restrict attention to agents who are close to their long-run expected utility. Per period utility  $u_t$  converges to the long-run value  $\hat{v}_t$ , by Lemma 2. We can then apply Lemma 4 and find a time period  $T \geq T'$  such that, for all  $t \geq T$  and for all  $s$ :

$$(55) \quad P(u_t \geq \hat{v}_t - \alpha\Delta/4, x_t \in X \mid s) > 1 - \bar{\varepsilon}/2,$$

and

$$(56) \quad P(|\mathcal{M}(x_t, \delta_t) - \kappa_t(s)| < \bar{\varepsilon}/4, u_t \geq \hat{v}_t - \alpha\Delta/8, \delta_t = \delta^t(s), x_t \in X \mid s) > \alpha/2.$$

Equation (55) states that there are enough agents, both informed and uninformed, close to their long-run utility. Equation (56) states that there are enough informed agents close to both their long-run utility and to their long-run marginal rates of substitution.

We are now done with the preliminary steps ensuring proper convergence and can proceed to the body of the argument.

Choose any  $t \geq T$ . By Proposition 2, two cases are possible: either (i) the informed agents' long-run marginal rates of substitution are far enough from each other,  $|\kappa_t(s_1) - \kappa_t(s_2)| \geq \bar{\varepsilon}$ ; or (ii) they are close to each other,  $|\kappa_t(s_1) - \kappa_t(s_2)| < \bar{\varepsilon}$ , but there is a large enough mass of uninformed agents with marginal rate of substitution far from that of the informed agents,  $P(|\mathcal{M}(x_t, \delta_t) - \kappa_t(s)| \geq 2\bar{\varepsilon} \mid s) \geq \bar{\varepsilon}$  for all  $s$ .

In the next two steps, we construct the desired trade  $z$  for each of these two cases, and then complete the argument in step 3.

*Step 1.* Consider the first case, in which  $|\kappa_t(s_1) - \kappa_t(s_2)| \geq \bar{\varepsilon}$ . In this case, an uninformed agent can exploit the difference between the informed agents' marginal rates of substitution in states  $s_1$  and  $s_2$ , making an offer at an intermediate price. This offer will be accepted with higher probability in the state



in which the informed agents' marginal rate of substitution is higher. In particular, suppose

$$\kappa_t(s_2) + \bar{\varepsilon} \leq \kappa_t(s_1)$$

(the opposite case is treated symmetrically). Lemma 6 and the definition of the utility gain  $\Delta$  imply that there is a trade  $z = (\zeta, -p\zeta)$ , with price  $p = (\kappa_t(s_1) + \kappa_t(s_2))/2$  and size  $\|z\| < \theta$ , that satisfies the following inequalities:

$$(57) \quad U(x_t + z, \delta_t) \geq u_t + \Delta \quad \text{if} \quad \mathcal{M}(x_t, \delta_t) > \kappa_t(s_1) - \bar{\varepsilon}/4 \text{ and } x_t \in X,$$

$$(58) \quad U(x_t - z, \delta_t) \geq u_t + \Delta \quad \text{if} \quad \mathcal{M}(x_t, \delta_t) < \kappa_t(s_2) + \bar{\varepsilon}/4 \text{ and } x_t \in X.$$

Equation (57) states that all (informed and uninformed) agents with marginal rate of substitution above  $(\kappa_t(s_1) - \bar{\varepsilon}/4)$  will receive a utility gain  $\Delta$  from the trade  $z$ , in terms of current utility. Equation (58) states that all (informed and uninformed) agents with marginal rate of substitution below  $(\kappa_t(s_1) + \bar{\varepsilon}/4)$  will receive a utility gain  $\Delta$  from the trade  $-z$ , in terms of current utility.

Combining conditions (56) and (57) shows that, in state  $s_1$ , there are at least  $\alpha/2$  informed agents with after-trade utility above the long-run utility,  $U(x_t + z, \delta_t) > \hat{v}_t$ . Since all these agents would accept the trade  $z$ , this implies that the probability of acceptance of the trade is  $\chi_t(z|s_1) > \alpha/2$ .

Next, we want to show that the trade  $z$  is accepted with sufficiently low probability conditional on  $s_2$ . In particular, we want to show that  $\chi_t(z|s_2) < \alpha/4$ . The key step here is to make sure that the trade is rejected not only by informed but also by uninformed agents. The argument is that if this trade were to be accepted by uninformed agents, then informed agents should be offering  $z$  and gaining in utility. Formally, proceeding by contradiction, suppose that the probability of  $z$  being accepted in state  $s_2$  is large:  $\chi_t(z|s_2) \geq \alpha/4$ . Condition (56) implies that there is a positive mass of informed agents with  $\mathcal{M}(x_t, \delta_t) < \kappa_t(s_2) + \bar{\varepsilon}/2$ ,  $x_t \in X$ , and close enough to the long-run utility  $u_t \geq \hat{v}_t - \alpha\Delta/8$ . By (58), these agents would be strictly better off making the offer  $z$  and consuming  $x_t - z$  if the offer is accepted and consuming  $x_t$  if it is rejected, since

$$(1 - \chi_t(z|s_2))U(x_t, \delta_t) + \chi_t(z|s_2)U(x_t - z, \delta_t) > u_t + \alpha\Delta/4 > \hat{v}_t.$$

Since this strategy dominates the equilibrium payoff, this is a contradiction, proving that  $\chi_t(z|s_2) < \alpha/4$ .

*Step 2.* Consider the second case, in which the long-run marginal rates of substitution of the informed agents are close to each other and there is a large enough mass of uninformed agents with marginal rate of substitution far from that of the informed agents.

The argument is as follows: with positive probability, we can reach a point where it is possible to separate the marginal rates of substitution of a group of uninformed agents from the marginal rates of substitution of a group of informed agents. This means that the uninformed agents in the first group can make an offer  $z$  to the informed agents in the second group and they will accept the offer in *both* states  $s_1$  and  $s_2$ . If the probabilities of acceptance  $\chi_t(z|s_1)$  and  $\chi_t(z|s_2)$  are sufficiently close to each other, this would be a profitable deviation for the uninformed, since their ex post beliefs after the offer is accepted would be close to their ex ante beliefs. In other words, in contrast to the previous case, they would gain utility but not learn from the trade. It follows that the probabilities  $\chi_t(z|s_1)$  and  $\chi_t(z|s_2)$  must be sufficiently different in the two states, which leads to either (4) or to (5).

To formalize this argument, consider the expected utility of an uninformed agent with portfolio  $x_t$  and belief  $\delta_t$ , who offers a trade  $z$  and stops trading from then on:

$$u_t + \delta_t \chi_t(z|s_1)(U(x_t - z, 1) - U(x_t, 1)) \\ + (1 - \delta_t) \chi_t(z|s_2)(U(x_t - z, 0) - U(x_t, 0)),$$

where  $u_t$  is the expected utility if the offer is rejected and the following two terms are the expected gains if the offer is accepted, respectively, in states  $s_1$  and  $s_2$ . This expected utility can be rewritten as

$$(59) \quad u_t + \chi_t(z|s_1)(U(x_t - z, \delta_t) - U(x_t, \delta_t)) \\ + (1 - \delta_t)(\chi_t(z|s_2) - \chi_t(z|s_1))(U(x_t - z, 0) - U(x_t, 0)),$$

using the fact that  $U(x_t, \delta_t) = \delta_t U(x_t, 1) + (1 - \delta_t)U(x_t, 0)$  (by the definition of  $U$ ). To interpret (59), notice that, if the probability of acceptance was independent of the signal,  $\chi_t(z|s_1) = \chi_t(z|s_2)$ , then the expected gain from making offer  $z$  would be equal to the second term:  $\chi_t(z|s_1)(U(x_t - z, \delta_t) - U(x_t, \delta_t))$ . The third term takes into account that the probability of acceptance may be different in two states, that is,  $\chi_t(z|s_2) - \chi_t(z|s_1)$  may be different from zero. An alternative way of rearranging the same expression yields

$$(60) \quad u_t + \chi_t(z|s_2)(U(x_t - z, \delta_t) - U(x_t, \delta_t)) \\ + (1 - \delta_t)(\chi_t(z|s_1) - \chi_t(z|s_2))(U(x_t - z, 1) - U(x_t, 1)).$$

In the rest of the argument, we will use both (59) and (60).

Suppose that there exist a trade  $z$  and a period  $t$  which satisfy the following properties: (a) the probability that  $z$  is accepted in state 1 is large enough,

$$\chi_t(z|s_1) > \alpha/4,$$

and (b) there is a positive mass of uninformed agents with portfolios and beliefs that satisfy

$$(61) \quad u_t \geq \hat{v}_t - (\alpha/4)\Delta,$$

$$(62) \quad U(x_t - z, \delta_t) - U(x_t, \delta_t) \geq \Delta,$$

$$(63) \quad U(x_t - z, \delta) - U(x_t, \delta) \geq -\lambda\Delta \quad \text{for all } \delta \in [0, 1],$$

for some  $\Delta > 0$  and  $\lambda > 0$ . In words, the uninformed agents are sufficiently close to their long-run utility, their gains from trade at *fixed* beliefs have a positive lower bound  $\Delta$ , and their gains from trade at *arbitrary* beliefs have a lower bound  $-\lambda\Delta$ .

Now we distinguish two cases. Suppose first that  $\chi_t(z|s_2) \geq \chi_t(z|s_1)$ . Then, for the uninformed agents who satisfy (61)–(63), the expected utility (59) is greater than or equal to

$$\hat{v}_t - (\alpha/4)\Delta + \chi_t(z|s_1)\Delta - (\chi_t(z|s_2) - \chi_t(z|s_1))\lambda\Delta.$$

From individual optimality, this expression cannot be larger than  $\hat{v}_t$ , since  $\hat{v}_t$  is the maximum expected utility for a proposer in period  $t$ . We then obtain the following restriction on the acceptance probabilities  $\chi_t(z|s_1)$  and  $\chi_t(z|s_2)$ :

$$\chi_t(z|s_1)(1 + \lambda)\Delta \leq \alpha\Delta/4 + \chi_t(z|s_2)\lambda\Delta.$$

Since  $\chi_t(z|s_1) > \alpha/2$  and  $\chi_t(z|s_1) \geq \chi_t(z|s_2)$ , it follows that  $\alpha/4 < (1/2) \times \chi_t(z|s_2)$ , and we obtain

$$\chi_t(z|s_1)(1 + \lambda) \leq \chi_t(z|s_2)(1/2 + \lambda),$$

which is equivalent to

$$(64) \quad \chi_t(z|s_1) \geq \frac{1 + \lambda}{1/2 + \lambda} \chi_t(z|s_2).$$

This shows that the probability of acceptance in state  $s_1$  is larger than the probability of acceptance in state  $s_2$  by a factor  $(1 + \lambda)/(1/2 + \lambda)$  greater than 1.

Consider next the case  $\chi_t(z|s_2) < \chi_t(z|s_1)$ . Then, for the uninformed agents who satisfy (61)–(63), the expected utility (60) is greater than or equal to

$$\hat{v}_t - \alpha\Delta/4 + \chi_t(z|s_2)\Delta - (\chi_t(z|s_1) - \chi_t(z|s_2))\lambda\Delta.$$

An argument similar to the one above shows that optimality requires

$$\chi_t(z|s_2) \geq \frac{1 + \lambda}{1/2 + \lambda} \chi_t(z|s_1).$$

Some algebra shows that this inequality and  $\chi_t(z|s_1) > \alpha/2$  imply

$$(65) \quad 1 - \chi_t(z|s_1) > 1 - \frac{\alpha}{2} \frac{1/2 + \lambda}{1 + \lambda},$$

$$(66) \quad 1 - \chi_t(z|s_1) > \frac{(1 - \alpha/2)(1/2 + \lambda)}{(1 - \alpha/2)(1/2 + \lambda) - \alpha/4} (1 - \chi_t(z|s_2)),$$

giving us a positive lower bound for the probability of rejection  $1 - \chi_t(z|s_1)$  and showing that  $1 - \chi_t(z|s_1)$  exceeds  $1 - \chi_t(z|s_2)$  by a factor greater than 1.

To complete this step, we show that there exist a trade  $z$  and a period  $t$  which satisfy properties (a) and (b).

Notice that  $P(|\mathcal{M}(x_t, \delta_t) - \kappa_t(s_1)| \geq 2\bar{\varepsilon} \mid s_1) \geq \bar{\varepsilon}$  requires that either  $P(\mathcal{M}(x_t, \delta_t) \leq \kappa_t(s_1) - 2\bar{\varepsilon} \mid s_1) \geq \bar{\varepsilon}/2$  holds or  $P(\mathcal{M}(x_t, \delta_t) \geq \kappa_t(s_1) + 2\bar{\varepsilon} \mid s_1) \geq \bar{\varepsilon}/2$ . We concentrate on the first case, as the second is treated symmetrically. Set the trading price at  $p = \min\{\kappa_t(s_1), \kappa_t(s_2)\} - \bar{\varepsilon}/2$ . Lemma 6 implies that there are positive scalars  $\Delta$  and  $\lambda$  and a trade  $z = (\zeta, -p\zeta)$  with  $\|z\| < \theta$  that satisfy the following inequalities:

$$(67) \quad U(x_t - z, \delta_t) \geq u_t + \Delta, \quad U(x_t - z, \delta) \geq u_t - \lambda\Delta \quad \text{for all } \delta \in [0, 1],$$

if  $\mathcal{M}(x_t, \delta_t) < p - \bar{\varepsilon}/4$  and  $x_t \in X$ ,

and

$$(68) \quad U(x_t + z, \delta_t) \geq u_t + \Delta \quad \text{if } \mathcal{M}(x_t, \delta_t) > p + \bar{\varepsilon}/4 \text{ and } x_t \in X.$$

Since  $|\mathcal{M}(x_t, \delta_t) - \kappa_t(s_1)| < \bar{\varepsilon}/4$  implies  $\mathcal{M}(x_t, \delta_t) > \kappa_t(s_1) - \bar{\varepsilon}/4$  and  $\kappa_t(s_1) - \bar{\varepsilon}/4$  is larger than  $p + \bar{\varepsilon}/4$  by construction, conditions (56) and (68) guarantee that there is a positive mass of informed agents who accept  $z$ , ensuring that  $\chi_t(z|s_1) > \alpha/2$ , showing that  $z$  satisfies property (a).

Next, we want to prove that there is a positive mass of uninformed agents who gain from making offer  $z$ . To do so, notice that  $|\kappa_t(s_1) - \kappa_t(s_2)| < \bar{\varepsilon}$  implies

$$p - \bar{\varepsilon}/4 = \min\{\kappa_t(s_1), \kappa_t(s_2)\} - (3/4)\bar{\varepsilon}$$

$$\geq \kappa_t(s_1) - (7/4)\bar{\varepsilon} > \kappa_t(s_1) - 2\bar{\varepsilon},$$

which implies

$$P(\mathcal{M}(x_t, \delta_t) < p - \bar{\varepsilon}/4 \mid s_1) \geq P(\mathcal{M}(x_t, \delta_t) \leq \kappa_t(s_1) - 2\bar{\varepsilon} \mid s_1) \geq \bar{\varepsilon}/2.$$

This, using Lemma 4 and condition (55), implies

$$P(\mathcal{M}(x_t, \delta_t) < p - \bar{\varepsilon}/4, u_t \geq \hat{v}_t - \alpha\Delta/4, x_t \in X \mid s_1) > 0,$$

which, combined with (67), shows that the trade  $z$  satisfies property (b).

*Step 3.* Here we put together the bounds established above and define the scalars  $\beta$  and  $\rho$  in the lemma's statement. Consider the case treated in Step 1. In this case, we can find a trade  $z$  such that the probability of acceptance conditional on each signal satisfies  $\chi_t(z|s_1) > \alpha/2$  and  $\chi_t(z|s_2) < \alpha/4$ . Therefore, in this case, condition (4) is true as long as  $\beta$  and  $\rho$  satisfy

$$\beta \leq \alpha/2 \quad \text{and} \quad \rho \leq 2.$$

Consider the case treated in Step 2. In this case, we can find a trade  $z$  such that either  $\chi_t(z|s_1) > \alpha/2$  and (64) hold or (65) and (66) hold. This implies that either condition (4) or condition (5) holds, as long as  $\beta$  and  $\rho$  satisfy

$$\beta \leq 1 - \frac{\alpha}{2} \frac{1/2 + \lambda}{1 + \lambda}, \quad \rho \leq \frac{1 + \lambda}{1/2 + \lambda}, \quad \rho \leq \frac{(1 - \alpha/2)(1/2 + \lambda)}{(1 - \alpha/2)(1/2 + \lambda) - \alpha/4}.$$

Setting

$$\beta = \min \left\{ \alpha/2, 1 - \frac{\alpha}{2} \frac{1/2 + \lambda}{1 + \lambda} \right\} > 0,$$

$$\rho = \min \left\{ 2, \frac{1 + \lambda}{1/2 + \lambda}, \frac{(1 - \alpha/2)(1/2 + \lambda)}{(1 - \alpha/2)(1/2 + \lambda) - \alpha/4} \right\} > 1$$

ensures that all the conditions above are satisfied, completing the proof.

## S5. PROOF OF LEMMA 8

Since  $\delta_t(\omega)$  are equilibrium beliefs, Bayesian rationality requires  $P(s_1 | \delta_t < \varepsilon/(1 + \varepsilon)) < \varepsilon/(1 + \varepsilon)$  for all  $\varepsilon > 0$ . The latter condition implies  $P(s_2 | \delta_t < \varepsilon/(1 + \varepsilon)) > 1 - \varepsilon/(1 + \varepsilon)$  and thus

$$\frac{P(s_1 | \delta_t < \varepsilon/(1 + \varepsilon))}{P(s_2 | \delta_t < \varepsilon/(1 + \varepsilon))} < \varepsilon,$$

for all  $\varepsilon > 0$ . Bayes's rule implies that

$$\frac{P(s_1 | \delta_t < \varepsilon/(1 + \varepsilon))}{P(s_2 | \delta_t < \varepsilon/(1 + \varepsilon))} = \frac{P(\delta_t < \varepsilon/(1 + \varepsilon) | s_1)P(s_1)}{P(\delta_t < \varepsilon/(1 + \varepsilon) | s_2)P(s_2)}.$$

Combining the last two equations and using  $P(s_1) = P(s_2) = 1/2$  yields

$$P(\delta_t < \varepsilon/(1 + \varepsilon) | s_1) < \varepsilon P(\delta_t < \varepsilon/(1 + \varepsilon) | s_2) \leq \varepsilon,$$

which gives the desired inequality.

## S6. PROOF OF LEMMA 9

Let us begin from the first part of the lemma. Suppose, by contradiction, that

$$|\kappa_{t+J}(s_1) - \kappa_t(s_1)| > \varepsilon$$

for some  $\varepsilon > 0$  for infinitely many periods. Then, at some date  $t$ , an informed agent with marginal rate of substitution close to  $\kappa_t(s)$  can find a profitable deviation by holding on to his portfolio  $x_t$  for  $J$  periods and then trade with other informed agents at  $t + J$ . Let us formalize this argument. Suppose, without loss of generality, that

$$\kappa_{t+J}(s_1) > \kappa_t(s_1) + \varepsilon$$

for infinitely many periods (the other case is treated in a symmetric way). Next, using our usual steps and Proposition 1, it is possible to find a compact set  $X$ , a time  $T$ , and a utility gain  $\Delta > 0$  such that the following two properties are satisfied: (i) in all periods  $t \geq T$ , there is at least a measure  $\alpha/2$  of informed agents with marginal rate of substitution sufficiently close to  $\kappa_t(s)$ , utility close to its long-run level, and portfolio  $x_t$  in  $X$ , that is,

$$(69) \quad P(|\mathcal{M}(x_t, \delta_t) - \kappa_t(s)| < \varepsilon/3, u_t \geq \hat{v}_t - \gamma^J \alpha \Delta/2, x_t \in X, \delta_t = 1 | s) > \alpha/2,$$

and (ii) in all periods  $t \geq T$  in which  $\kappa_{t+J}(s) > \kappa_t(s) + \varepsilon$ , there is a trade  $z$  such that

$$(70) \quad U(x - z, 1) > U(x, 1) + \Delta \quad \text{if} \quad \mathcal{M}(x, 1) < \kappa_t(s) + \varepsilon/3 \quad \text{and} \quad x \in X,$$

and

$$(71) \quad U(x + z, 1) > U(x, 1) + \Delta \quad \text{if} \quad \mathcal{M}(x, 1) > \kappa_{t+J}(s) - \varepsilon/3 \quad \text{and} \quad x \in X.$$

Pick a time  $t \geq T$  in which  $\kappa_{t+J}(s) > \kappa_t(s) + \varepsilon$  and consider the following deviation. Whenever an informed agent reaches time  $t$  and his portfolio  $x_t$  satisfies  $\mathcal{M}(x_t, 1) < \kappa_t(s) + \varepsilon/3$  and  $x_t \in X$ , he stops trading for  $J$  periods and then makes an offer  $z$  that satisfies (70) and (71). If the offer is rejected, he stops trading from then on. The probability that this offer is accepted at time  $t + J$  must satisfy  $\chi_{t+J}(z|s_1) > \alpha/2$ , because of conditions (69) and (71). Therefore, the expected utility from this strategy, from the point of view of time  $t$ , is

$$u_t + \gamma^J \chi_{t+J}(z|s_1)(U(x_t - z, 1) - u_t) > u_t + \gamma^J \alpha \Delta/2 \geq \hat{v}_t,$$

so this strategy is a profitable deviation and we have a contradiction.

The second part of the lemma follows from the first part, using Proposition 1 and the triangle inequality.

## S7. EXAMPLES

In this section, we present two examples in which the equilibrium can be analyzed analytically. The first objective of these examples is to show existence of equilibrium with symmetry across states and uniform market clearing in some special cases. The second objective is to study how information acquisition takes place in equilibrium. The third objective is to study how the equilibrium is affected by changing the parameter  $\gamma$ , which controls the probability that the game ends. In particular, we are interested in what happens when  $\gamma$  goes to 1.

Higher values of  $\gamma$  correspond to economies in which agents have the chance to do more rounds of trading before the game ends. We can interpret the limit  $\gamma \rightarrow 1$  as a frictionless trading limit, in which agents have the chance to make infinite rounds of trading before the game ends. In a full information economy in which agents can trade forever (i.e., with no discounting), Gale (1986a) showed that bilateral bargaining yields a Walrasian outcome, in which agents making the first offer do not have any monopoly power, due to the fact that their partners have unlimited chances to make further trades in the future. We can then ask whether a similar result applies in our model with asymmetric information when  $\gamma$  goes to 1. Our first example shows that, in general, the result does not extend. In particular, in that example, agents remain uninformed even as  $\gamma$  goes to 1. This bounds the gains from trade that can be reaped by responders who refuse to trade in the first round. Therefore, agents who are selected as proposers in the first period keep some monopoly power. In our second example, on the other hand, uninformed agents have the opportunity to acquire perfect information in equilibrium and payoffs converge to those of perfect competition.

In all the examples, there are two types, with initial portfolios  $x_{1,0} = (\omega, 1 - \omega)$  and  $x_{2,0} = (1 - \omega, \omega)$ , for some  $\omega \in (1/2, 1)$ . Let  $\phi(s_1) = \varphi > 1/2$  and recall that, by symmetry,  $\phi(s_2) = 1 - \varphi$ . Informed agents with endowment  $x_{1,0}$  are called “rich informed agents” in state  $s_1$  and “poor informed agents” in state  $s_2$ , as their endowment’s present value is greater in the first case. The opposite labels apply to informed agents with  $x_{2,0}$ .

For analytical tractability, we modify the setup of our model in the first round of trading and make the following assumption. In period  $t = 1$ , the matching process is such that agents meet other agents with complementary endowments with probability 1, that is, type 1 agents only get matched with type 2 agents. In all following periods, agents meet randomly as in the setup of Section 1. All our results from previous sections hold in this modified environment, as they only rely on the long-run properties of the game. The purpose of this assumption is to construct equilibria in which almost all trades take place in the first round.

We consider two examples. In the first example, uninformed agents do not learn anything about the state  $s$  and keep their initial beliefs at  $\delta = 1/2$ . In the second example, all uninformed agents learn the state  $s$  exactly in the first round of trading. For ease of exposition, we present the main results for the two

examples in Sections S7.1 and S7.2, and present some more technical derivations behind the examples in Sections S7.3 and S7.4.

### S7.1. Example 1: An Equilibrium With No Learning

For this example, we introduce an additional modification to our baseline model, assuming that, in period  $t = 1$ , all informed agents get to be proposers with probability 1. In particular, in  $t = 1$ , informed agents are only matched with uninformed agents and the informed agent is always selected as the proposer.<sup>11</sup> If two uninformed agents meet at  $t = 1$ , each is selected as the proposer with probability  $1/2$ . From period  $t = 2$  onward, the matching and the selection of the proposer are as in the baseline model. That is, each agent has the same probability of meeting an informed or uninformed partner and each agent has probability  $1/2$  of being selected as the proposer. As pointed out above, the changes made in period  $t = 1$  do not affect the long-run properties of the game and the general results of the previous sections still hold.

For a given scalar  $\eta \in (0, 1)$ , to be defined below, our aim is to construct an equilibrium in which strategies and beliefs satisfy:

S1. In  $t = 1$ , all proposers of type  $i$  offer  $z_{i,E} = (\eta, \eta) - x_{-i,0}$ . All responders accept.

S2. In  $t = 2, 3, \dots$ , all proposers offer zero trade.

S3. In  $t = 2, 3, \dots$ , all uninformed responders reject any offer  $z$  that satisfies

$$\min\{\varphi z^1 + (1 - \varphi)z^2, (1 - \varphi)z^1 + \varphi z^2\} < 0;$$

all informed responders reject any offer  $z$  that satisfies

$$\begin{aligned} \varphi z^1 + (1 - \varphi)z^2 < 0 & \quad \text{if } s = s_1, \quad \text{or} \\ (1 - \varphi)z^1 + \varphi z^2 < 0 & \quad \text{if } s = s_2. \end{aligned}$$

B1. Uninformed responders keep their beliefs unchanged after offer  $z_{i,E}$  in period  $t = 1$  and after offer 0 in period  $t \geq 2$ .

B2. In  $t = 1$ , after an offer  $z \neq z_{i,E}$ , uninformed responders adjust their belief to  $\delta = 1$  if they are of type 1 and to  $\delta = 0$  if they are of type 2.

B3. In  $t = 2, 3, \dots$ , after an offer  $z \neq 0$ , uninformed responders adjust their belief to  $\delta = 1$  if  $\varphi z^1 + (1 - \varphi)z^2 < (1 - \varphi)z^1 + \varphi z^2$  and to  $\delta = 0$  if  $\varphi z^1 + (1 - \varphi)z^2 > (1 - \varphi)z^1 + \varphi z^2$ .

Notice that, in equilibrium, all agents reach endowments on the 45 degree line after one round of trading and remain there from then on. S1–S3 and B1–B3 describe strategies and beliefs in equilibrium and along a subset of off-the-equilibrium-path histories. This is sufficient to show that we have an equilibrium, since we can prove that, if other agents' strategies satisfy S1–S3,

<sup>11</sup>This requires assuming  $\alpha < 1/2$ .



the payoff from any deviating strategy is bounded above by the equilibrium payoff. An important element of our construction is that following off-the-equilibrium-path offers at  $t \geq 2$ , uninformed agents hold “pessimistic” beliefs, meaning that they expect the state  $s$  to be the one for which the present value of the offer received is smaller. This, together with the fact that all agents are on the 45 degree line starting at date 2, implies that deviating agents have limited opportunities to trade after period 1.

The argument to prove that an equilibrium with these properties exists is in two steps. First, we show that zero trade from period  $t = 2$  on is a continuation equilibrium. Second, we go back to period  $t = 1$  and show that making and accepting the offers  $z_{i,E}$  is optimal at  $t = 1$ .

To show that no trade is an equilibrium after  $t = 2$ , we use property S3. Let  $V(x, \delta)$  denote the continuation utility of an agent with endowment  $x$  and belief  $\delta \in [0, 1]$  at any time  $t \geq 2$ .<sup>12</sup> This value function is independent of  $t$  since the environment is stationary after  $t = 2$ . Since other agents’ strategies satisfy S3, the endowment process of a deviating agent who starts at  $(x, \delta)$  satisfies the following property: if the state is  $s_1$ , any endowment  $\tilde{x}$  reached with positive probability at future dates must satisfy

$$\varphi \tilde{x}^1 + (1 - \varphi) \tilde{x}^2 \leq \varphi x^1 + (1 - \varphi) x^2.$$

This property holds because, in  $s_1$ , neither informed nor uninformed agents will accept trades that increase the expected value of the proposer’s endowment computed using the probabilities  $\varphi$  and  $1 - \varphi$ . A similar property holds in  $s_2$ , reversing the roles of the probabilities  $\varphi$  and  $1 - \varphi$ . These properties, together with concavity of the utility function, imply that the continuation utility  $V(x, \delta)$  is bounded as follows:

$$(72) \quad V(x, \delta) \leq \delta u(\varphi x^1 + (1 - \varphi) x^2) + (1 - \delta) u((1 - \varphi) x^1 + \varphi x^2).$$

In the continuation equilibrium, all agents start from a perfectly diversified endowment with  $x^1 = x^2$ . So an agent can achieve the upper bound in (72) by not trading. This rules out any deviation by proposers on the equilibrium path. A similar argument shows that the off-the-equilibrium-path responses in S3 are optimal.<sup>13</sup> So we have an equilibrium for  $t \geq 2$ . The argument for no trade in periods  $t \geq 2$  is closely related to the no-trade theorem of Milgrom and Stokey (1982).

Turning to period  $t = 1$ , consider a rich informed proposer with endowment  $x_{1,0}$  in state  $s_1$ . If he deviates and offers  $z \neq z_{1,E}$ , the uninformed responder’s belief goes to  $\delta = 0$ . Then the offer will only be accepted if  $V(x_{1,0} + z, 0) \geq V(x_{1,0}, 0)$  and the payoff of the proposer, if the offer is accepted, would be

<sup>12</sup>The value function is defined at the beginning of the period, before knowing whether the game ends or there is another round of trading.

<sup>13</sup>See Proposition 6 in Section S7.3.

$V(x_{1,0} - z, 1)$ . In Section S7.3, we derive an upper bound on this payoff. We can then find parametric examples and choose  $\eta$  so that offering  $z_{1,E}$  yields a higher payoff. The reason why this is possible is that, if the proposer offers  $z_{1,E}$ , the uninformed responder's belief remains at  $\delta = 1/2$ , so the informed proposer is able to trade at better terms in period 1.

Consider next a poor informed proposer with endowment  $x_{2,0}$  in state  $s_1$ . If he deviates, the uninformed responder adjusts his belief to  $\delta = 1$ . Since the poor informed proposer also holds belief  $\delta = 1$ , we can show that the best deviation by a poor informed proposer is to make an offer that reaches the 45 degree line for both. However, the responder's outside option is higher at  $\delta = 1$  than at  $\delta = 1/2$ , because he holds a larger endowment of asset 1. This makes the participation constraint of the responder tighter than at the equilibrium offer  $z_{2,E}$  and allows us to construct parametric examples in which the poor informed proposer prefers not to deviate.

Having shown that both the rich informed proposer and the poor informed proposer prefer not to deviate, we can then show that an uninformed proposer prefers not to deviate either, using the argument that the payoff of an uninformed agent under a deviation is weakly dominated by the average of the payoffs of an informed agent with the same endowment. In Section S7.3, we derive sufficient conditions that rule out deviations in period  $t = 1$  and show how to construct parametric examples that satisfy these conditions. The following proposition contains such an example.

**PROPOSITION 4:** *Suppose the utility function is  $u(c) = c^{1-\sigma}/(1-\sigma)$  and the parameters  $(\sigma, \omega, \varphi)$  are in a neighborhood of  $(4, 9/10, 9^4/(9^4+1))$ . There is an  $\eta \in (0, 1)$  and a cutoff  $\bar{\alpha} > 0$  for the fraction of informed agents in the game, such that S1–S3 and B1–B3 form an equilibrium if  $0 \leq \alpha < \bar{\alpha}$ .*

An important ingredient in the construction of the example in the proposition is to assume that the fraction of informed agents  $\alpha$  is sufficiently small. This puts a bound on the utility from trading in periods  $t \geq 2$ , because it implies that an agent only gets a chance to trade with informed agents with a small probability. In particular, property S3 means that an agent who starts at  $x$  at  $t = 2$  and only trades with uninformed agents before the end of the game can only reach endowments  $\tilde{x}$  that satisfy both

$$\varphi \tilde{x}^1 + (1 - \varphi) \tilde{x}^2 \leq \varphi x^1 + (1 - \varphi) x^2$$

and

$$(1 - \varphi) \tilde{x}^1 + \varphi \tilde{x}^2 \leq (1 - \varphi) x^1 + \varphi x^2.$$

This restriction is crucial in constructing upper bounds on the continuation utility of deviating agents at date  $t = 1$ . The intuition is that the trades  $z_{i,E}$  at date  $t = 1$  are proposed and accepted in equilibrium because the outside

option is to trade with fully diversified, mostly uninformed agents in periods  $t = 2, 3, \dots$ . In period  $t = 1$ , there are large gains from trade coming from the fact that agents are not diversified, but all these gains from trade are exhausted in the first round of trading. From then on, the presence of asymmetric information limits the gains from trade for a deviating agent who is still undiversified at the end of  $t = 1$ .

Somewhat surprisingly, under the assumptions in Proposition 4 an equilibrium can be constructed for any value of  $\gamma$ . This is because what bounds the continuation utility in period  $t = 2$  is the small probability of trading with informed agents in future periods. So for any value of  $\gamma$ , we can choose the cutoff  $\bar{\alpha}$  sufficiently small to make the probability of trading with informed agents approach zero and sustain our equilibrium. Larger values of  $\gamma$  correspond to economies in which agents trade more frequently before the game ends. Then the observation above can be interpreted as follows. There can be economies close to the frictionless limit—that is, with  $\gamma$  close to 1—in which no information is revealed in equilibrium. This happens because more frequent trade implies that the diversification motive for trade is exhausted more quickly. However, once the diversification motive is exhausted, the no-trade theorem implies that no further trade occurs and so no further information is revealed.

When the mass of informed agents  $\alpha$  is zero, the equilibrium holds for all  $\gamma$ , so we can take  $\gamma \rightarrow 1$ . We then have an economy in which the limit equilibrium allocation is an allocation in which uninformed agents with the *same* endowments get different consumption levels depending on whether they were selected as proposers or responders in the very first period of the game. So we have an example of an economy in which the presence of more frequent rounds of trading does not lead, in the limit, to a perfectly competitive outcome, unlike in the economies with perfect information analyzed by Gale (1986a). This shows that the presence of asymmetric information can have powerful effects in decentralized economies. Again, the underlying idea is that the no-trade theorem limits the agents' ability to trade in the long run, and this induces agents to accept trades in the early stages of the game, when diversification motives are stronger. This undermines the ability of future rounds of trading to act as a check on the monopoly power of proposers in the early stages of the game.

### S7.2. Example 2: An Equilibrium With Learning

We now turn to an example in which uninformed agents acquire perfect information in the first round of trading. As in Example 1, we assume that, in the first round of trading, each agent is matched with an agent with complementary endowments. Moreover, we assume that almost all agents are informed, so there is only a zero mass of uninformed agents. We also assume that preferences display constant absolute risk aversion:

$$u(c) = -e^{-\rho c}.$$

This assumption allows us to characterize analytically the value function  $V(x, \delta)$  for  $\delta = 0$  and  $\delta = 1$ . Notice that CARA preferences do not satisfy the property  $\lim_{c \rightarrow 0} u(c) = -\infty$ , which was assumed in Section 2 (Assumption 2). However, the only purpose of that property was to ensure that endowments stay in a compact set with probability close to 1 in equilibrium. Here, we can check directly that endowments remain in a compact set in equilibrium. So all our general results still apply.

The analysis of this example proceeds in two steps. First, we characterize the equilibrium focusing on the behavior of informed agents—which can be done, given that uninformed agents are in zero mass. Second, we look at the uninformed agent's problem at date  $t = 1$  and derive conditions that ensure that his optimal strategy is to experiment, making an offer that perfectly reveals the state  $s$ .

For the first step, we need to derive four equilibrium offers  $z_{i,E}(s)$  which depend on the proposer type  $i$  and on the state  $s$ . In equilibrium, all proposers of type  $i$  make offer  $z_{i,E}(s)$  in state  $s$ , all responders accept, and both proposer and responder reach a point on the 45 degree line. The offers  $z_{i,E}(s)$  are found maximizing  $V(x_{i,0} - z, \delta)$  subject to

$$V(x_{-i,0} + z, \delta) \geq V(x_{-i,0}, \delta),$$

with  $\delta = 1$  if  $s_1$  and  $\delta = 0$  if  $s_2$ . Since the two agents share the same beliefs, it is not difficult to show that the solution to this problem yields an allocation on the 45 degree line for both agents and that they stop trading from period  $t = 2$  onward. For this argument, it is sufficient to use the upper bound (72), which was used for our Example 1 and also holds here. We then obtain the following proposition.<sup>14</sup>

**PROPOSITION 5:** *If all agents are informed, there is an equilibrium in which all agents reach the 45 degree line in the first round of trading and stop trading from then on.*

Before turning to our second step, however, we need to derive explicitly the form of the  $V$  function and the offers  $z_{i,E}(s)$ . These steps are more technical and are presented in Section S7.4. The assumption of CARA utility helps greatly in these derivations, as it allows us to show that the value function takes the form  $V(x) = -\exp\{-\rho x^1\}f(x^2 - x^1)$  for some decreasing function  $f$  and that the function  $f$  can be obtained as the solution of an appropriate functional equation.

We can then turn to our second step and consider uninformed agents in period  $t = 1$ . The case of uninformed responders is easy. Since they meet informed proposers with probability 1 and these proposers make different offers

<sup>14</sup>A detailed proof is in Section S7.4.

in the two states, they acquire perfect information on  $s$  and respond like informed agents, accepting the offer and reaching the 45 degree line.

The case of uninformed proposers is harder. We want to show that an uninformed proposer with endowment  $i$  makes offer  $z_{i,E}(s_1)$  if  $i = 1$  and offer  $z_{i,E}(s_2)$  if  $i = 2$ , where  $z_{i,E}(s)$  are the offers derived above for informed agents. In other words, uninformed proposers mimic the behavior of *rich* informed proposers. By making these offers, uninformed proposers get to learn exactly the state  $s$ , because their offer is accepted with probability 1 in one state and rejected with probability 1 in the other. While this implies that they become informed from period  $t = 2$  onward, it also implies that, with probability  $1/2$ , they do not reach the 45 degree line in the first round of trading. Our characterization of the  $V$  function in Section S7.4 allows us to compute their payoff in this case and to characterize their trading in periods  $t = 2, 3, \dots$ . In particular, the optimal behavior of uninformed agents who fail to trade in period  $t = 1$  is to make an infinite sequence of trades in all periods  $t \geq 2$  in which they are selected as proposers. The difference  $x^1 - x^2$  is reduced by a factor of  $1/2$  each time they get to trade, so that they asymptotically reach the 45 degree line.

Let us show that the offers described above are optimal for the uninformed proposer. We focus on type  $i = 1$ , as the case of  $i = 2$  is symmetric. To prove that offering  $z_{1,E}(s_1)$  is optimal at time  $t = 1$ , we need to check that:

1. offer  $z_{1,E}(s_1)$  is rejected by informed agents in  $s_2$ ;
2. offer  $z_{1,E}(s_1)$  dominates any other offer accepted by informed agents only in  $s_1$ ;
3. offer  $z_{1,E}(s_1)$  dominates any offer accepted by informed agents only in state  $s_2$ ;
4. offer  $z_{1,E}(s_1)$  dominates any offer accepted by informed agents in both states.

Conditions 1 to 3 are proved in Section S7.4 and hold for any choice of parameters. The most interesting condition is the last one, which ensures that the uninformed agent prefers to learn even though it entails not trading with probability  $1/2$  in the first period. In Section S7.4, we derive an upper bound for the expected utility from any offer accepted by informed responders in both  $s_1$  and  $s_2$ . Making the following assumptions on parameters:

$$\rho = 1, \quad \varphi = 2/3, \quad \omega = 1,$$

we can compute the expected utility from offering  $z_{1,E}(s_1)$  and the upper bound just discussed. The values we obtain are plotted in Figure S1 for different values of  $\gamma$ .

As the figure shows, there is a range of  $\gamma$  for which experimenting dominates non-experimenting, so uninformed agents offer  $z_{1,E}(s_1)$ . Notice that as  $\gamma$  goes to 1, the expected utility of the uninformed agent converges to the expected utility of the informed agent and both converge to the expected utility in a Walrasian rational expectations equilibrium. So unlike in Example 1, in this

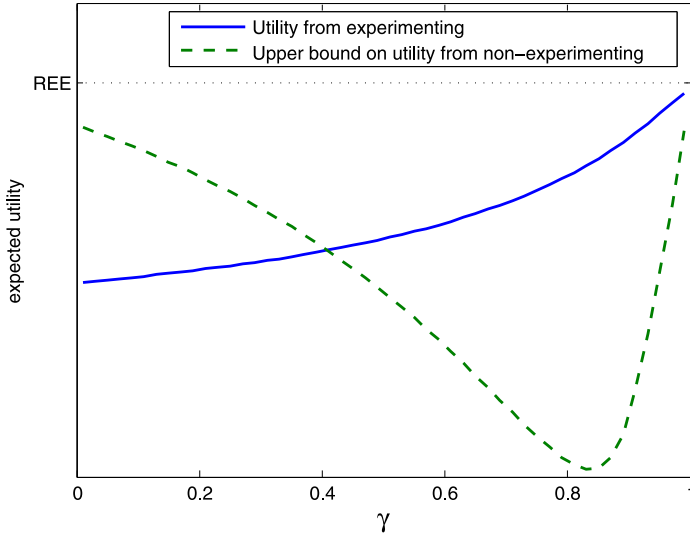


FIGURE S1.—Example 2.

case, as the frequency of trading increases, the equilibrium payoffs converge to those of a perfectly competitive rational expectations equilibrium.

As a final remark, notice that, in this example, some agents—the uninformed proposers whose offer is rejected in the first round—only reach an efficient allocation asymptotically. However, we can characterize the speed at which this convergence occurs. To measure distance from efficiency, let us use the distance  $d_t \equiv |x_t^1 - x_t^2|$ . Since this distance is reduced by  $1/2$  every time the agent gets to make an offer and trade, after  $n$  trades in which the agent is selected as the proposer, the distance is reduced to  $2^{-n}d_0$ . It is then possible to show that, for every  $\epsilon$ , there is a  $\gamma$  large enough that the probability of  $d_t < \epsilon$  is larger than  $1 - \epsilon$ . That is, with  $\gamma$  close to 1, the allocation approaches efficiency also for uninformed first-round proposers.

### S7.3. Example 1: Proofs

**PROPOSITION 6:** *In the economy of Example 1, individual strategies are optimal for  $t \geq 2$ .*

**PROOF:** Consider first a proposer, informed or uninformed, who has not deviated up to time  $t$ , so he holds an endowment  $x$  on the 45 degree line, either  $(\eta, \eta)$  or  $(1 - \eta, 1 - \eta)$ , and believes  $\delta \in [0, 1]$ . From inequality (72), the expected payoff from any deviating strategy is bounded above by

$$\delta u(\varphi x^1 + (1 - \varphi)x^2) + (1 - \delta)u((1 - \varphi)x^1 + \varphi x^2) = u(x^1),$$

given that  $x^1 = x^2$ . Making a zero offer today and not trading in all future periods achieves the upper bound  $u(x^1)$ , so the agent cannot gain by deviating. Turning to a responder, consider first an uninformed responder. Suppose he receives an off-the-equilibrium-path offer  $z \neq 0$  at  $t \geq 2$ , with

$$\min\{\varphi z^1 + (1 - \varphi)z^2, (1 - \varphi)z^1 + \varphi z^2\} < 0.$$

Suppose, without loss of generality, that  $\varphi z^1 + (1 - \varphi)z^2$  is smaller than  $(1 - \varphi)z^1 + \varphi z^2$ , and thus smaller than zero. From B3, his belief after receiving offer  $z$  is  $\delta = 1$ . Using (72), his expected utility after accepting the offer is bounded above by

$$u(\varphi(x^1 + z^1) + (1 - \varphi)(x^2 + z^2)) < u(\varphi x^1 + (1 - \varphi)x^2) = u(x^1).$$

Rejecting the offer yields the payoff  $u(x^1)$  and strictly dominates accepting the offer. A similar argument shows optimality for informed responders. *Q.E.D.*

The following lemma provides an upper bound on the continuation utility which will be used below. Define the function

$$(73) \quad W(x, \delta) \equiv \max_z U(x - z, \delta)$$

$$\text{s.t. } \varphi z^1 + (1 - \varphi)z^2 \geq 0,$$

$$(1 - \varphi)z^1 + \varphi z^2 \geq 0,$$

and the quantity

$$\Xi(\alpha) \equiv \frac{1 - \gamma}{1 - \gamma + \alpha\gamma/2},$$

which is the probability of trading only with uninformed agents between any period  $t \geq 2$  and the end of the game.

LEMMA 10: *The following is an upper bound on the value function  $V(x, \delta)$ :*

$$V(x, \delta) \leq \Xi(\alpha)[\delta W(x, 1) + (1 - \delta)W(x, 0)]$$

$$+ (1 - \Xi(\alpha))\bar{u} \quad \text{for } \delta \in \{0, 1\}.$$

PROOF: Consider first an informed agent with  $\delta = 1$ . Consider a deviating strategy starting at  $(x, 1)$  at time  $t$ , and consider any history  $h^{t+j}$  along which the agent only meets uninformed agents. Let  $x(h^{t+j}) = x + \sum z_n$  be his endowment at that history, where  $z_n$  are all the successful trades made along the history. Each trade  $z_n$  must satisfy  $\varphi z_n^1 + (1 - \varphi)z_n^2 \geq 0$  and  $(1 - \varphi)z_n^1 + \varphi z_n^2 \geq 0$ ,

and so  $z = \sum z_n$  must satisfy the same inequalities. By the definition of  $W$ , the expected utility at history  $h^{t+j}$  then satisfies

$$U(x(h^{t+j}), 1) \leq W(x, 1).$$

Following any history in which the agent meets informed agent, the utility is bounded by the upper bound  $\bar{u}$ . Taking expectation over all future histories yields the bound  $\Xi W(x, 1) + (1 - \Xi)\bar{u}$ . An uninformed agent cannot do better than receiving perfect information on the state  $s$  and then re-optimize, which yields the bound in the lemma. *Q.E.D.*

For the following result, we define

$$(74) \quad w_R^* \equiv \max_z W(x_{1,0} - z, 1)$$

$$\text{s.t. } W(x_{2,0} + z, 0) \geq U(x_{2,0}, 0).$$

PROPOSITION 7: *If  $\eta$  satisfies*

$$(75) \quad u(1 - \eta) > w_R^*,$$

$$(76) \quad u(\eta) < U(x_{2,0}, 0),$$

$$(77) \quad u(\eta) > \frac{1}{2}W(x_{2,0}, 0) + \frac{1}{2}W(x_{2,0}, 1),$$

*there is an  $\hat{\alpha} \in (0, 1)$  such that, if  $\alpha < \hat{\alpha}$ , the strategies in S1–S3 are individually optimal.*

PROOF: Proposition 6 shows optimality in all periods  $t \geq 2$ , so we can restrict attention to time  $t = 1$ . Consider first the behavior of responders. We focus on responders with  $x_{2,0}$ ; the case of responders with  $x_{1,0}$  is symmetric. All responders are uninformed and they accept the equilibrium offer if

$$(78) \quad u(\eta) \geq \frac{1}{2}V(x_{2,0}, 1) + \frac{1}{2}V(x_{2,0}, 0).$$

Using Lemma 10, a sufficient condition for (78) is

$$u(\eta) \leq \Xi(\alpha) \left[ \frac{1}{2}W(x_{2,0}, 0) + \frac{1}{2}W(x_{2,0}, 1) \right] + (1 - \Xi(\alpha))\bar{u}.$$

Assumption (77), together with  $\lim_{\alpha \rightarrow 0} \Xi(\alpha) = 1$ , ensures that this condition holds for  $\alpha \rightarrow 0$ .



Consider next informed proposers. We focus on proposers with endowment  $x_{1,0}$ ; the case of proposers with  $x_{2,0}$  is symmetric. Suppose the proposer deviates by offering  $z \neq z_{1,E}$  and the responder's belief goes to  $\delta = 0$ . The  $z$  offer will be rejected if

$$\Xi(\alpha)W(x_{2,0} + z, 0) + (1 - \Xi(\alpha))\bar{u} < U(x_{2,0}, 0),$$

given that the left-hand side is an upper bound on the continuation utility after accepting the offer and the right-hand side is a lower bound on the continuation utility after rejecting the offer. If the state is  $s_1$ , the proposer is a rich informed agent and his continuation utility is bounded above by  $W(x_{1,0} - z, 1)$ . The utility from deviating is then bounded above by

$$\begin{aligned} w_R(\alpha) &= \max_z \Xi(\alpha)W(x_{1,0} - z, 1) + (1 - \Xi(\alpha))\bar{u} \\ \text{s.t. } &\Xi(\alpha)W(x_{2,0} + z, 0) + (1 - \Xi(\alpha))\bar{u} \geq U(x_{2,0}, 0). \end{aligned}$$

The function  $w_R(\alpha)$  is continuous in  $\alpha$  and  $w_R(0) = w_R^*$ , so assumption (75) ensures that  $u(1 - \eta) > w_R(\alpha)$  as  $\alpha \rightarrow 0$ . If the state is  $s_2$ , the proposer is a poor informed agent and the utility from deviating is bounded above by

$$\begin{aligned} w_P(\alpha) &= \max_z \Xi(\alpha)W(x_{1,0} - z, 0) + (1 - \Xi(\alpha))\bar{u}, \\ \text{s.t. } &\Xi(\alpha)W(x_{2,0} + z, 0) + (1 - \Xi(\alpha))\bar{u} \geq U(x_{2,0}, 0). \end{aligned}$$

When  $\alpha = 0$ , the solution to this problem is given by perfect risk sharing with  $x_{2,0}^1 + z^1 = x_{2,0}^2 + z^2 = u^{-1}(U(x_{2,0}, 0))$ . The proposer's payoff is then

$$w_P(0) = u(1 - u^{-1}(U(x_{2,0}, 0))).$$

This payoff is strictly dominated by offering  $z_{1,E}$  if

$$1 - \eta > 1 - u^{-1}(U(x_{2,0}, 0)),$$

which is equivalent to assumption (76). A continuity argument ensures that  $u(1 - \eta) > w_P(\alpha)$  for  $\alpha \rightarrow 0$ .

Finally, consider uninformed proposers. Since informed proposers can condition their strategy on the realization of  $s$ , an uninformed proposer cannot do better than the expected gain of the informed proposer's deviations. Since this gain is negative under both values of  $s$ , the expected gain is negative and the uninformed proposer strictly prefers not to deviate. *Q.E.D.*

### S7.3.1. Proof of Proposition 4

Given Proposition 7, we need to find parameters that satisfy conditions (75)–(77). The following lemma simplifies this construction.

LEMMA 11: *The function  $W$  satisfies the following properties:*

1.  $W(x_{2,0}, 1) = u(\varphi(1 - \omega) + (1 - \varphi)\omega)$ .
2. *If*

$$(79) \quad \frac{\varphi u'(\omega)}{(1 - \varphi)u'(1 - \omega)} = 1,$$

then

$$(80) \quad w_R^* = W(x_{1,0}, 1) = \varphi u(\omega) + (1 - \varphi)u(1 - \omega).$$

PROOF: Consider problem (73), which defines  $W$ . The problem is concave, so the following first-order conditions, together with the constraints, are sufficient for an optimum:

$$\begin{aligned} \pi(\delta)u'(x^1 - z^1) &= \varphi\lambda + (1 - \varphi)\mu, \\ (1 - \pi(\delta))u'(x^2 - z^2) &= (1 - \varphi)\lambda + \varphi\mu. \end{aligned}$$

At  $(x, \delta) = (x_{2,0}, 1)$ , we have a solution with  $\lambda > 0$ ,  $\mu = 0$ , and  $z^2 > 0 > z^1$ , which gives us property 1.

At  $(x, \delta) = (x_{2,0}, 0)$ , we have a solution at  $z = 0$  with

$$(81) \quad \lambda = \frac{u'(1 - \omega) - u'(\omega)}{\frac{\varphi}{1 - \varphi} - \frac{1 - \varphi}{\varphi}} > 0,$$

$$\mu = \frac{\frac{\varphi}{1 - \varphi}u'(\omega) - \frac{1 - \varphi}{\varphi}u'(1 - \omega)}{\frac{\varphi}{1 - \varphi} - \frac{1 - \varphi}{\varphi}} > 0,$$

where the second inequality follows from (79) and  $\varphi > 1/2$ . This implies  $W(x_{2,0}, 0) = U(x_{2,0}, 0)$  and the envelope theorem implies

$$(82) \quad \frac{\partial W(x_{2,0}, 0)}{\partial x^1} = (1 - \varphi)u'(1 - \omega),$$

$$(83) \quad \frac{\partial W(x_{2,0}, 0)}{\partial x^2} = \varphi u'(\omega).$$

A symmetric argument applies to the case  $(x, \delta) = (x_{1,0}, 1)$ , leading to  $W(x_{1,0}, 1) = U(x_{1,0}, 1)$  and

$$\begin{aligned} \frac{\partial W(x_{1,0}, 1)}{\partial x^1} &= \varphi u'(\omega), \\ \frac{\partial W(x_{1,0}, 1)}{\partial x^2} &= (1 - \varphi)u'(1 - \omega). \end{aligned}$$

Consider problem (74), which defines  $w_R^*$ . The first-order conditions for this problem are

$$\begin{aligned}\frac{\partial W(x_{1,0} - z, 1)}{\partial x^1} &= \lambda \frac{\partial W(x_{2,0} + z, 0)}{\partial x^1}, \\ \frac{\partial W(x_{1,0} - z, 1)}{\partial x^2} &= \lambda \frac{\partial W(x_{2,0} + z, 0)}{\partial x^2}.\end{aligned}$$

These conditions are satisfied by setting  $z = 0$  and  $\lambda = 1$ , so we have  $w_R^* = W(x_{1,0}, 1) = U(x_{1,0}, 1)$ . *Q.E.D.*

Given Proposition 7 and Lemma 11, to construct an example it is sufficient to find a utility function, probabilities, and endowments that satisfy the four conditions:

$$\begin{aligned}\varphi u'(\omega) &= (1 - \varphi)u'(1 - \omega), \\ u(1 - \eta) &> \varphi u(\omega) + (1 - \varphi)u(1 - \omega), \\ u(\eta) &< \varphi u(\omega) + (1 - \varphi)u(1 - \omega), \\ u(\eta) &> \frac{1}{2}u(\varphi(1 - \omega) + (1 - \varphi)\omega) + \frac{1}{2}(\varphi u(\omega) + (1 - \varphi)u(1 - \omega)).\end{aligned}$$

With CRRA utility, the first condition boils down to

$$\frac{\varphi}{1 - \varphi} = \left( \frac{\omega}{1 - \omega} \right)^\sigma,$$

and the remaining conditions are satisfied for  $\sigma = 4$ ,  $\omega = 0.9$ ,  $\varphi = 0.9^4/(1 + 0.9^4)$ , and  $\eta = 0.1265$ .

#### S7.4. Example 2: Proofs

##### S7.4.1. Characterization of the Value Function $V$

Throughout this section, we fix the value of  $\delta$  at either 0 or 1. Our objective is to prove Proposition 8 below, which shows that the maximum continuation utility  $V(x, \delta)$  is well defined for all  $x \in R^2$  and shows how to compute it. We exploit the fact that, with exponential utility, the per-period utility  $U$  takes the form

$$(84) \quad U(x, \delta) = -e^{-\rho x_1} f_0(x_2 - x_1),$$

where

$$f_0(\xi) \equiv \pi(\delta) + (1 - \pi(\delta))e^{-\rho\xi}.$$

PROPOSITION 8: *If the utility function is exponential and all agents are on the 45 degree line at  $t = 2$ , then there exists a continuation equilibrium in which the maximum continuation utility  $V(x, \delta)$  is well defined for any  $x \in \mathbb{R}^2$ , agents accept any offer that satisfies  $V(z, \delta) \geq V(0, \delta)$ , and  $V$  takes the form*

$$(85) \quad V(x, \delta) = -e^{-\rho x_1} f(x_2 - x_1),$$

where  $f$  solves the functional equation

$$f(\xi) = (1 - \gamma)f_0(\xi) + \gamma \left( \frac{1}{2}f(\xi) + \frac{1}{2} \left[ f\left(\frac{\xi}{2}\right) \right]^2 \right).$$

The proof of this proposition is split in a number of lemmas.

LEMMA 12: *Suppose the continuation utility  $V(x, \delta)$  is well defined for all  $x \in \mathbb{R}^2$ . Suppose all agents are on the 45 degree line and an agent with endowment  $x$  accepts offer  $z$  if it satisfies  $V(x + z, \delta) \geq V(x, \delta)$ . Then  $V$  satisfies two properties:*

- (i) *it takes the form (85) for some function  $f$ ;*
- (ii) *it satisfies the Bellman equation*

$$(86) \quad V(x, \delta) = (1 - \gamma)U(x, \delta) + \gamma \left( \frac{1}{2}V(x, \delta) + \frac{1}{2} \max_{z: V(z, \delta) \geq V(0, \delta)} V(x - z, \delta) \right).$$

PROOF: Consider two agents with endowments  $x$  and  $x + a$ , where  $a$  is any scalar. The second agent can follow the trading strategy of the first agent and get, in every period, the same utility level scaled by a factor  $e^{-\rho a}$ . This implies  $V(x + a, \delta) \geq e^{-\rho a}V(x, \delta)$ . A symmetric argument for the first agent implies  $V(x, \delta) \geq e^{\rho a}V(x + a, \delta)$ . Combining these two inequalities, we have

$$(87) \quad V(x + a, \delta) = e^{-\rho a}V(x, \delta) \quad \text{for all } a.$$

Defining  $f(\xi) = -V((0, \xi), \delta)$ , this implies (85). If  $x$  is on the 45 degree line, (87) implies that  $V(x + z, \delta) \geq V(x, \delta)$  is equivalent to  $V(z, \delta) \geq V(0, \delta)$ . So if all agents are on the 45 degree line, an agent with endowment  $x$  who gets selected to make an offer, chooses  $z$  to maximize  $V(x - z, \delta)$  subject to  $V(z, \delta) \geq V(0, \delta)$ . A standard dynamic programming argument implies that the value function  $V(x, \delta)$  satisfies the functional equation (86). *Q.E.D.*

Our next step is to show that there is a function  $V$  that solves the functional equation (86). Let  $T$  denote the mapping that, given a function  $v: \mathbb{R}^2 \rightarrow \mathbb{R}$ , yields

$$Tv(x) = (1 - \gamma)U(x, \delta) + \gamma \left( \frac{1}{2}v(x) + \frac{1}{2} \max_{z: v(z) \geq v(0)} v(x + z) \right).$$

A fixed point of  $T$  is a solution to the functional equation (86). To establish existence of a fixed point, our strategy is the following: restrict attention to  $v$  functions that are generated by functions  $f$  in some set  $\mathcal{B}$ ; define a self-map  $\hat{T}$  on the space  $\mathcal{B}$ ; find a fixed point of  $\hat{T}$  in  $\mathcal{B}$  and use it to construct a function  $v$  that is a fixed point of the original operator  $T$ .

DEFINITION 4:  $\mathcal{A}$  is the set of continuous, non-increasing functions  $f : R \rightarrow R_+$  that satisfy  $f(0) = 1$  and

$$(88) \quad f(\lambda\xi' + (1-\lambda)\xi'') \leq [f(\xi')]^\lambda [f(\xi'')]^{1-\lambda}$$

for all  $\xi' \neq \xi''$  and all  $\lambda \in (0, 1)$ .

Condition (88) is essentially a property of log-convexity of  $f$ . The next lemma shows that (88) is equivalent to concavity of  $v$ .

LEMMA 13: Let  $v(x) = -e^{-\rho x_1} f(x_2 - x_1)$  for some continuous, non-increasing function  $f : R \rightarrow R_+$ . The function  $v$  is concave iff  $f$  satisfies (88).

PROOF:  $v$  is concave iff the set  $\{(x_1, \xi) : v(x_1, x_1 + \xi) \geq -\kappa\}$  is convex for any  $\kappa > 0$  (for  $\kappa \leq 0$  the set is empty). But  $v(x_1, x_1 + \xi) \geq -\kappa$  is equivalent to  $e^{-\rho x_1} f(\xi) \leq \kappa$ . Take two values  $\xi'$  and  $\xi''$  and choose  $x'_1$  and  $x''_1$  so that

$$e^{-\rho x'_1} f(\xi') = e^{-\rho x''_1} f(\xi'') = 1.$$

The convexity of  $\{(x_1, \xi) : e^{-\rho x_1} f(\xi) \leq 1\}$  implies that

$$e^{-\rho[\lambda x'_1 + (1-\lambda)x''_1]} f(\lambda\xi' + (1-\lambda)\xi'') \leq 1 = [e^{-\rho x'_1} f(\xi')]^\lambda [e^{-\rho x''_1} f(\xi'')]^{(1-\lambda)},$$

which yields property (88). The converse is easy and is omitted. Q.E.D.

LEMMA 14: Take any function  $v(x) = -e^{-\rho x_1} f(x_2 - x_1)$  for some  $f \in \mathcal{A}$ . Then, the following offer solves the maximization problem in (86):

$$z_1 = \frac{1}{\rho} \log f\left(\frac{x_2 - x_1}{2}\right),$$

$$z_2 = z_1 + \frac{x_2 - x_1}{2},$$

and

$$Tv(x) = -e^{\rho x_1} h(x_2 - x_1),$$

where  $h$  is in  $\mathcal{A}$  and satisfies

$$(89) \quad h(\xi) = (1-\gamma)f_0(\xi) + \gamma\left(\frac{1}{2}f(\xi) + \frac{1}{2}\left[f\left(\frac{\xi}{2}\right)\right]^2\right).$$

PROOF: Using  $\xi = x_2 - z_2 - (x_1 - z_1)$ , we can rewrite the optimization problem as

$$(90) \quad \max_{z_1, z_2} -e^{-\rho(x_1 - z_1)} f(\xi),$$

$$\text{s.t.} \quad -e^{-\rho z_1} f(x_2 - x_1 - \xi) \geq -1.$$

Substituting for  $z_1$  in the constraint, we have

$$(91) \quad \max_{\xi} -e^{-\rho x_1} f(\xi) f(x_2 - x_1 - \xi).$$

From (88) and

$$\frac{1}{2}\xi + \frac{1}{2}(x_2 - x_1 - \xi) = \frac{x_2 - x_1}{2},$$

we have

$$(92) \quad f(\xi) f(x_2 - x_1 - \xi) \geq \left[ f\left(\frac{x_2 - x_1}{2}\right) \right]^2.$$

The last inequality implies that setting  $\xi = (x_2 - x_1)/2$  is optimal. This gives us the optimal value for  $z_2 - z_1$ . To get the optimal level of  $z_1$ , we use the constraint (90). The expression for  $h$  follows from substituting the optimal choices of  $z_1$  and  $\xi$  in the objective function (91), substituting in (86), and using (84). Continuity and monotonicity of  $h$  and the fact that  $h(0) = 1$  follow immediately from (89) and the definition of  $f_0$ . It remains to establish the log-convexity of  $h$ . The maximization problem in (86) yields a concave function of  $x$ , because  $v$  is concave. Then  $Tv$  is a convex combination of concave functions and so is concave. Lemma 13 implies that  $h$  satisfies (88). *Q.E.D.*

Lemma 14 suggests that, to prove existence of a fixed point for  $T$ , we define the mapping  $\hat{T}$  as

$$\hat{T}f(\xi) = (1 - \gamma)f_0(\xi) + \gamma\left(\frac{1}{2}f(\xi) + \frac{1}{2}\left[f\left(\frac{\xi}{2}\right)\right]^2\right),$$

and look for a fixed point of this mapping. The advantage is that we can choose any positive scalar  $M$  and take as the domain of  $\hat{T}$  the space of continuous functions on the interval  $[0, M]$ , since  $\xi \in [0, M]$  implies  $\xi/2 \in [0, M]$ .

DEFINITION 5:  $\mathcal{B}_M$  is the set of continuous, non-increasing functions  $f: [0, M] \rightarrow [0, 1]$  that satisfy  $f(0) = 1$ , (88), and

$$(93) \quad f(\xi') - f(\xi) \geq -\rho(\xi' - \xi) \quad \text{if} \quad \xi' \geq \xi.$$

The additional property (93) is useful, as it ensures the equicontinuity of the functions in  $\mathcal{B}_M$ . We can show that  $\hat{T}$  is a self-map on  $\mathcal{B}_M$ .

LEMMA 15:  $\hat{T}$  is a mapping from  $\mathcal{B}_M$  to  $\mathcal{B}_M$ .

PROOF: That  $\hat{T}$  preserves continuity, monotonicity,  $f(\xi) \geq 0$ ,  $f(0) = 1$ , and (88) is an immediate corollary of Proposition 14. It remains to prove that  $\hat{T}$  preserves the bound  $f(\xi) \leq 1$  and that it preserves (93). Property (88) implies  $f(\xi/2)^2 \leq f(\xi)$ . Substituting in the definition of  $\hat{T}f$ , we then have

$$\hat{T}f(\xi) \leq (1 - \gamma)f_0(\xi) + \gamma f(\xi) \leq (1 - \gamma)f_0(0) + \gamma f(0) = 1,$$

since both  $f_0$  and  $f$  are non-increasing. To prove (93), take two values  $\xi' \geq \xi$ . By convexity,  $f_0$  satisfies

$$\begin{aligned} f_0(\xi') - f_0(\xi) &\geq f_0'(\xi)(\xi' - \xi) = -\rho(1 - \pi)e^{-\rho\xi}(\xi' - \xi) \\ &\geq -\rho(\xi' - \xi). \end{aligned}$$

Moreover, since  $f$  satisfies

$$f(\xi') - f(\xi) \geq -\rho(\xi' - \xi)$$

and is bounded above by 1, we have

$$\begin{aligned} &\left[ f\left(\frac{1}{2}\xi'\right) \right]^2 - \left[ f\left(\frac{1}{2}\xi\right) \right]^2 \\ &= \left[ f\left(\frac{1}{2}\xi'\right) + f\left(\frac{1}{2}\xi\right) \right] \left[ f\left(\frac{1}{2}\xi'\right) - f\left(\frac{1}{2}\xi\right) \right] \geq -2\rho\left(\frac{1}{2}\xi' - \frac{1}{2}\xi\right). \end{aligned}$$

Combining the last three inequalities, we have

$$\begin{aligned} &\hat{T}f(\xi') - \hat{T}f(\xi) \\ &= (1 - \gamma)[f_0(\xi') - f_0(\xi)] + \frac{\gamma}{2}[f(\xi') - f(\xi)] \\ &\quad + \frac{\gamma}{2} \left\{ \left[ f\left(\frac{1}{2}\xi'\right) \right]^2 - \left[ f\left(\frac{1}{2}\xi\right) \right]^2 \right\} \\ &\geq -\rho(\xi' - \xi), \end{aligned}$$

which completes the argument.

We can now state our existence result for  $f$ .

*Q.E.D.*

LEMMA 16: The mapping  $\hat{T}$  has a fixed point  $f$  in  $\mathcal{B}_M$ .

PROOF: Define the sequence of functions  $\{f_n\}_{n=0}^\infty$  starting at  $f_0$  and letting  $f_n = \hat{T}f_{n-1}$  for  $n = 1, 2, 3, \dots$ . First, we want to show that the sequence  $\{f_n\}$  is monotone. We prove it by induction. Notice that

$$(94) \quad f_1(\xi) = \left(1 - \frac{\gamma}{2}\right)f_0(\xi) + \frac{\gamma}{2}\left[f_0\left(\frac{1}{2}\xi\right)\right]^2 \leq f_0(\xi),$$

since the log-convexity of  $f_0$  implies  $f_0(\xi/2)^2 \leq f_0(\xi)$ . The definition of the sequence means that

$$f_n(\xi) = (1 - \gamma)f_0(\xi) + \frac{\gamma}{2}f_{n-1}(\xi) + \frac{\gamma}{2}f_{n-1}\left(\frac{1}{2}\xi\right).$$

Writing the same equation at  $n + 1$  and taking differences side by side, we have

$$f_{n+1}(\xi) - f_n(\xi) = \frac{\gamma}{2}[f_n(\xi) - f_{n-1}(\xi)] + \frac{\gamma}{2}\left[f_n\left(\frac{1}{2}\xi\right) - f_{n-1}\left(\frac{1}{2}\xi\right)\right].$$

This means that  $f_n \leq f_{n-1}$  implies  $f_{n+1} \leq f_n$ . Since  $f_1 \leq f_0$  from (94), by induction we have  $f_n \leq f_{n-1}$  for all  $n$ :

$$f_1(\xi) - f_0(\xi) = \frac{\gamma}{2}[f_n(\xi) - f_{n-1}(\xi)] + \frac{\gamma}{2}\left[f_n\left(\frac{1}{2}\xi\right) - f_{n-1}\left(\frac{1}{2}\xi\right)\right].$$

Since  $f_0$  is in  $\mathcal{B}_M$ , all the functions in the sequence are in  $\mathcal{B}_M$  and so they all satisfy

$$-\rho(\xi' - \xi) \leq f_n(\xi') - f_n(\xi) \leq 0$$

for any pair  $\xi' \geq \xi$ . This implies that the sequence  $\{f_n\}_{n=0}^\infty$  is uniformly bounded and equicontinuous. Notice that  $\mathcal{B}_M$  is closed in the sup-norm topology. Then by the Arzelà–Ascoli theorem, the sequence  $\{f_n\}_{n=0}^\infty$  admits a subsequence  $\{f_{n_k}\}_{k=0}^\infty$  that converges uniformly to a function  $f$  in  $\mathcal{B}_M$ . Moreover, the fact that the original sequence  $\{f_n\}_{n=0}^\infty$  is monotone implies that it also converges uniformly to  $f$ . It is easy to show that the mapping  $\hat{T}$  is continuous on  $\mathcal{B}_M$ . Therefore,

$$f = \lim_{n \rightarrow \infty} f_n = \lim_{n \rightarrow \infty} \hat{T}f_{n-1} = \hat{T} \lim_{n \rightarrow \infty} f_{n-1} = \hat{T}f,$$

which completes the argument. *Q.E.D.*

To complete the proof of Proposition 8, we need to go back to the function  $V(x, \delta)$ . If  $x_2 \geq x_1$ , we can choose any  $M > x_2 - x_1$ , find the function  $f$  that is a fixed point of  $\hat{T}$  on  $\mathcal{B}_M$ , and set  $V(x, \delta) = -e^{-\rho x_1} f(x_2 - x_1)$ . If  $x_2 < x_1$ , we can



proceed in a symmetric fashion, and prove the existence of a function  $g$  such that  $V(x, \delta) = -e^{-\rho x_2} g(x_1 - x_2)$ . The function  $f$  is then found setting

$$f(\xi) = e^{-\rho \xi} g(-\xi),$$

completing the proof.

#### S7.4.2. Equilibrium in Periods $t \geq 2$

We need to show that if an agent is already on the 45 degree line, his optimal strategy is never to trade. That is, we need to show that the trade  $z = (0, 0)$  is a solution to the maximization problem

$$\max_z V(x - z, \delta), \quad \text{s.t.} \quad V(z, \delta) \geq V(0, \delta),$$

if  $x_1 = x_2$ . To do so, we argue that  $V$  satisfies the property

$$(95) \quad V(x, \delta) \leq u(\pi(\delta)x_1 + (1 - \pi(\delta))x_2)$$

with equality if  $x_1 = x_2$ .

PROPOSITION 9: *The function  $V$  satisfies (95).*

PROOF: First, we prove that the mapping  $T$  preserves this property. If  $v$  satisfies (95), then we want to prove that the function  $J$ , defined as

$$J(x) = \max_z v(x - z), \quad \text{s.t.} \quad v(z) \geq v(0),$$

also satisfies (95). Suppose, by contradiction, that  $J(x) > u(\pi x_1 + (1 - \pi)x_2)$ . Then there exists a  $z$  such that  $v(x - z) > u(\pi x_1 + (1 - \pi)x_2)$ . Since  $v$  satisfies (95), we have

$$u(\pi x_1 + (1 - \pi)x_2) < u(\pi(x_1 - z_1) + (1 - \pi)(x_2 - z_2)),$$

which implies

$$\pi x_1 + (1 - \pi)x_2 < \pi(x_1 - z_1) + (1 - \pi)(x_2 - z_2).$$

Similarly,  $v(z) \geq v(0)$  and (95) imply

$$0 \leq \pi z_1 + (1 - \pi)z_2.$$

Summing side by side the last two inequalities yields a contradiction. The function  $Tv$  is equal to a convex combination of  $U$  and  $J$ . It is easy to show that  $U$  satisfies (95) and the property is preserved under convex combination. So  $Tv$  satisfies (95). It remains to prove that the condition holds as an equality if

$x_1 = x_2$ , and this follows from the fact that  $z = 0$  is always feasible. Since the function  $V(x, \delta)$  can be derived as the limit of sequence of functions  $v_n$ , starting at  $v_0 = U$  (see Lemma 16),  $U$  satisfies (95), and the set of functions that satisfy (95) is closed, the result follows. *Q.E.D.*

The following is an immediate corollary.

**COROLLARY 1:** *For  $t \geq 2$ , zero trade is optimal for all agents with endowment on the 45 degree line.*

**PROOF:** In the proof of Proposition 9, we show that

$$\max_z \{V(x - z, \delta), \text{ s.t. } V(z, \delta) \geq V(0, \delta)\} \leq u(\pi x_1 + (1 - \pi)x_2).$$

If  $x_1 = x_2$ , setting  $z = 0$  achieves the upper bound  $u(\pi x_1 + (1 - \pi)x_2)$ , hence zero trade is optimal. *Q.E.D.*

### S7.4.3. Strategies at $t = 1$

Let us first consider equilibrium offers of informed agents. We focus on rich informed agents in  $s_1$ , but analogous results hold for the other cases.

**PROPOSITION 10:** *In period  $t = 1$ , if informed agent of type 1 meets type 2 and  $s_1$ , the proposer offers a trade  $z_{1,E}(s_1)$  such that*

$$x_{1,0} - z_{1,E}(s_1) = (1 - \eta, 1 - \eta),$$

$$x_{2,0} + z_{1,E}(s_1) = (\eta, \eta),$$

where

$$\eta = 1 - \omega - [\log f(2\omega - 1)]/\rho.$$

**PROOF:** Given the value function  $V$  derived above, it is easy to check that  $\eta$  satisfies  $V((\eta, \eta), 1) = V((1 - \omega, \omega), 1)$ , since  $V((\eta, \eta), 1) = -e^{-\rho\eta}$  and  $V((1 - \omega, \omega), 1) = -e^{-\rho(1-\omega)}f(2\omega - 1)$ . We need to show that  $z_{1,E}(s_1)$  is optimal for the proposer. To do so, notice that, given the definition of  $\eta$ , the inequality  $V((1 - \omega, \omega) + z) \geq V((1 - \omega, \omega))$  can be rewritten as  $V((\eta, \eta) + z - z_{1,E}(s_1)) \geq V((\eta, \eta))$  or, given the properties of  $V$ , as  $V(z - z_{1,E}(s_1), \delta) \geq V(0, \delta)$ . So the maximization problem of the proposer can be rewritten as

$$\max_{\tilde{z}} \{V((1 - \eta, 1 - \eta) - \tilde{z}, \delta) \text{ s.t. } V(\tilde{z}, \delta) \geq V(0, \delta)\}.$$

The argument for Corollary 1 shows that it is optimal to choose  $\tilde{z} = 0$ , that is,  $z = z_{1,E}(s_1)$ . *Q.E.D.*

#### S7.4.4. *Uninformed Agents Experiment*

Without loss of generality, consider an uninformed proposer of type 1, with endowment  $(\omega, 1 - \omega)$ . We want to show that, at  $t = 1$ , he finds it optimal to experiment, by making an offer that is only accepted by the informed agents in one state of the world. In particular, we want to show that it is optimal for him to offer  $z_{1,E}(s_1)$ . If the offer is accepted, he stops trading; if it is rejected, he trades in all following periods, whenever he is the proposer, offering the trades described in Lemma 14. Given that the agent learns the state  $s$  from the fact that his offer is rejected, and given that from  $t = 2$  onward he will meet, with probability 1, informed agents with endowments on the 45 degree line, his behavior for  $t \geq 2$  is optimal by the results of the previous subsections.

To prove optimality at  $t = 1$ , we need to check that:

1. offer  $z_{1,E}(s_1)$  is rejected by informed agents in  $s_2$ ;
2. offering  $z_{1,E}(s_1)$  is better than any other offer accepted by the informed agent only in  $s_1$ ;
3. offering  $z_{1,E}(s_1)$  is better than any offer accepted by the informed agent only in state  $s_2$ ;
4. offering  $z_{1,E}(s_1)$  is better than any offer accepted by the informed agent in both states.

To check part 1, we need to show that

$$(96) \quad V((1 - \omega, \omega) + z_{1,E}(s_1), 0) < V((1 - \omega, \omega), 0).$$

But since  $(1 - \omega, \omega) + z_{1,E}(s_1) = (\eta, \eta)$  (where  $\eta$  is defined in Proposition 10) and  $V((\eta, \eta), 0) = V((\eta, \eta), 1) = V((1 - \omega, \omega), 1)$ , this condition boils down to

$$V((1 - \omega, \omega), 1) < V((1 - \omega, \omega), 0).$$

This inequality follows from the fact that, given that  $\omega > 1 - \omega$ ,  $V((1 - \omega, \omega), \delta)$  is monotone decreasing in  $\delta$ .

Part 2 can be proved as follows. To ensure that the offer is accepted by the informed in state  $s_1$ , the offer must satisfy

$$(97) \quad V((1 - \omega, \omega) + z, 1) \geq V((1 - \omega, \omega), 1),$$

$$(98) \quad V((1 - \omega, \omega) + z, 0) < V((1 - \omega, \omega), 0).$$

The payoff of the uninformed agent if he makes an offer accepted only in state 1 is

$$(99) \quad \frac{1}{2}V((\omega, 1 - \omega) - z, 1) + \frac{1}{2}V((\omega, 1 - \omega), 0).$$

Consider the problem of maximizing (99) subject to (97) and (98). If we relax the problem by omitting constraint (98), the optimal offer is  $z_{1,E}(s_1)$ , because it

maximizes the first term of (99) and the second term is a constant independent of  $z$ . But since  $z_{1,E}(s_1)$  satisfies (96), the second constraint is also satisfied and so  $z_{1,E}(s_1)$  solves the original problem. The payoff from this offer is

$$V_E = \frac{1}{2}V((\omega, 1 - \omega) - z_{1,E}(s_1), 1) + \frac{1}{2}V((\omega, 1 - \omega), 0).$$

To check part 3, we need to check that there is an upper bound on the utility the proposer can get by making an offer accepted only in state 2 and that this upper bound corresponds to the payoff from no trade  $(1/2)V((\omega, 1 - \omega), 1) + (1/2)V((\omega, 1 - \omega), 0)$ . In particular, inspecting indifference curves for our numerical examples, we see that all the offers that induce acceptance only in state 2 involve that the proposer buys asset 1 in exchange for asset 2 and that these offers are dominated by no trade.

To check part 4, notice that the payoff of the uninformed agent if he makes an offer accepted in both states is bounded above by the solution to the following problem:

$$(100) \quad \bar{V}_{NE} = \max_z \frac{1}{2}V((\omega, 1 - \omega) - z, 1) + \frac{1}{2}V((\omega, 1 - \omega) - z, 0)$$

subject to

$$V((1 - \omega, \omega) + z, \delta) \geq V((1 - \omega, \omega), \delta)$$

for  $\delta = 0, 1$ . This is because the continuation utility of the uninformed is lower than or equal to the objective function in (100). So we need to check the inequality

$$\bar{V}_{NE} < V_E.$$

This is the condition discussed in the main text and represented graphically in Figure S1.

*Dept. of Economics, Princeton University, 111 Fisher Hall, Princeton, NJ 08544, U.S.A.; [golosoov@princeton.edu](mailto:golosov@princeton.edu),*

*Dept. of Economics, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208, U.S.A.; [guido.lorenzoni@northwestern.edu](mailto:guido.lorenzoni@northwestern.edu),*

*and*

*Dept. of Economics, Yale University, 28 Hillhouse Avenue, New Haven, CT 06511, U.S.A.; [a.tsyvinski@yale.edu](mailto:a.tsyvinski@yale.edu).*

*Manuscript received October, 2009; final revision received February, 2014.*