

Adverse Selection and Unraveling in Common-Value Labor Markets*

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Abstract

We investigate a common-value labor setting in which firms interview a worker prior to hiring. When firms have private information about the worker's value and interview decisions are kept private, many firms may enter the market, interview, and hire with positive probability. When firms' interview decisions are revealed, severe adverse selection arises. As a result, all firms except for the highest-ranked firm are excluded from the hiring process. Sequential interviewing with revelation restores entry and improves upon the outcome with private interview decisions.

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1 Introduction

The hiring process in many firms includes several stages, at the end of which employment offers are made. This process often begins by conducting an initial evaluation of a potential worker's resumé and other credentials. If the evaluation proves favorable, the worker proceeds to the next stage, which may consist of an interview or fly-out, or a step of an administrative nature such as a "short list." At the end of the process, the firm may offer the worker a job. Many professionals, including academic economists, newly minted MBAs, law school graduates, and to some extent medical residents, are hired in this way.

We examine a common-value setting in which several privately-informed firms may be interested in hiring a worker, and ask how making firms' intermediate decisions known to other firms affects the hiring process and the resulting allocation. To be more concrete, consider the academic job market for economists. In recent years, an online resource called Econjobmarket¹ started listing universities' interview and fly-out decisions, nearly in real time. A natural question is how this information revelation affects which interviews, fly-outs, and job offers a candidate gets. Because universities' intermediate decisions (whether or not to interview or fly out a candidate) contain some information about the candidate, making them known has the potential to increase the amount of information available to universities in the hiring process. This, in turn, could lead to a better hiring outcome. A closer inspection, however, shows that this intuition is incomplete. Although each university would benefit from knowing other universities' intermediate decisions, a university may or may not benefit from having its intermediate decisions revealed to other universities. Which universities benefit? Which candidates benefit? Should such information revelation be facilitated or prohibited?

Understanding the overall effect of revealing firms' intermediate decisions in a multi-stage hiring setting is complicated, because the amount of information revealed by these decisions is determined endogenously. If firms anticipate that their intermediate decisions and those of other firms will be revealed, they may adjust their intermediate decisions for two reasons. First, a firm may advance a marginal candidate in the hiring process in the hopes of learning more from other firms' intermediate decisions. Second, a firm may "give up" on a seemingly good candidate because it expects that a positive intermediate decision will result in an offer from a firm that is more attractive to the candidate.²

To analyze these issues, we investigate a three-stage model in which firms may be interested in hiring a worker whose value is common to all firms. In the first stage, each

¹<http://bluwiki.com/go/Econjobmarket>.

²Thus, it is not clear whether seeing a low-ranked firm interviewing a candidate constitutes "good news" or "bad news" about the candidate from a high-ranked firm's point of view.

firm decides whether to pay a small cost to “enter” and participate in the hiring process. Each entering firm obtains a private signal about the value of the worker. Entry decisions are made simultaneously and are observable, and a firm that does not enter cannot later interview or hire the worker. In the second stage, all entering firms simultaneously choose whether to pay a small cost to “interview” the worker. This decision is based on each firm’s private signal. An interview is an indication that the firm is willing to proceed with the hiring process, and may be interpreted as a show of interest in the worker, as placing the worker on a “short list,” as a purely administrative step in the hiring process, as a fly-out, or as an actual interview. Interviews are a necessary step in the hiring process. A firm that does not interview the worker cannot later hire the worker. For expositional simplicity, we assume that interviews reveal no additional information about the worker; as long as all interviewing firms obtain the same information from the interview, the results of the paper hold without change. In the third stage, after all interviews have taken place, all firms simultaneously decide whether to make employment offers to the worker. The worker has a commonly-known strict ranking over firms, and accepts the highest-ranked offer among those he receives.³

Because the worker’s value is a function of all firms’ private information, each firm can make better hiring decisions if it has access to even coarse measures of other firms’ private information. This, in turn, is determined by whether firms’ interview decisions are revealed before employment offers are made. When interview decisions are not revealed (no revelation), no learning takes place between the interviewing and the hiring stages, so an interview is always followed by an offer. With no revelation, lower-ranked firms may be able to enter and make use of their private information. Theorem 1 describes the unique equilibrium in this case. With two firms and independent signals, for example, the strong firm will interview and hire the worker if its signal is sufficiently high. The weak firm will also interview the worker when it observes a sufficiently high signal. Because the weak firm is able to hire only when the strong firm does not make an offer, the signal it observes must be sufficiently high to offset the “bad news” that the strong firm did not interview. Section 4.1 describes a setting in which for any n all n firms enter and with positive probability interview and profitably hire the worker.

Next we analyze two effects of revealing firms’ interview decisions (revelation). Because revelation enables each firm to condition its hiring decision on the interview decisions of the other firms, the additional information improves the hiring decisions of all interviewing firms. This is the first effect. On the other hand, firms anticipate the impact of their interview decisions on subsequent offers, and this typically leads to changes in interviewing

³This seems to be the case in several entry-level labor markets, such as the market for MBAs or economics PhDs, where a more or less clear ranking of firms exists (overall or by field).

behavior, which could counteract the first effect. To isolate the first effect, we analyze a model in which interview decisions are unexpectedly revealed. Proposition 1 confirms that in this case revelation improves hiring outcomes from the firms' perspective relative to no revelation.

In contrast, the main result of the paper shows that revelation is quite detrimental to firms (and the worker) when firms make their interview decisions anticipating that these decisions and those of the other firms will be revealed before the hiring stage. Anticipated revelation creates increased adverse selection for all but the highest-ranked firm, and this erases all the benefits of revelation. Theorem 2 shows that in equilibrium only the highest-ranked firm enters the market and therefore no other firm interviews or hires. Compared to no revelation, all firms and the worker are weakly worse off. Any firm $2, \dots, n$ that enters with no revelation is strictly worse off. Firm 1 gets none of the benefits of revelation and is no better off than with no revelation. If the worker is hired by one of the firms $2, \dots, n$ with no revelation, he is unemployed and strictly worse off with revelation.

This result is surprising because interview decisions convey only coarse information about firms' private signals. Suppose, for example, that there are two firms and the value of the worker is the sum of the two firms' private signals. It may be that the low-ranked firm sees a very high signal and interviews. If the high-ranked firm sees a low signal, it may reason as follows: "I know that my signal is low, so if I interview and then see the low-ranked firm interviewing I will only be able to deduce that its signal is in some range whose expected value is not enough to offset my signal. Therefore, I will not interview given my low signal." Similarly, the low-ranked firm only sees that the high-ranked firm does not interview, and concludes that in expectation the high-ranked firm saw a higher signal than the high-ranked firm actually did. As a result, the low-ranked firm is willing to make the worker an offer while the high-ranked firm is not. Our result shows that this reasoning is inconsistent with equilibrium.⁴ Despite the fact that interviews only provide a coarse measure of a firm's private information, in equilibrium no firm can make use of its private information (except for the highest-ranked firm). Consequently, firms' entry choices are made as if they expect all their private information to be revealed. This is true even if a low-ranked firm has much better (or worse) private information about the worker's value than any other firm, and holds regardless of the number of firms.

The adverse effects of revelation can be mitigated by having firms interview the worker one after the other instead of simultaneously, with each firm's interview decision being

⁴In addition to considering the worker's expected value from the perspective of different firms, the proof also takes into account that firms may use mixed strategies and that different firms may attribute different probabilities to the same event, because they have different information. The proof applies to weakly affiliated signals and any number of firms.

revealed before the next firm makes its interview decision. Theorem 3 shows that each sequencing of firms induces a unique outcome. When firms are ordered from the lowest-ranked firm to the highest-ranked firm, the entry and hiring outcome is the same as in the setting with revelation (only the highest-ranked firm enters); when firms are ordered from the highest-ranked firm to the lowest-ranked firm, the entry and hiring outcome is the same as in the setting with no revelation. Firms' expenditures on interviews, however, are lower than with no revelation. This shows that the right sequencing of interviews, combined with revelation, improves upon no revelation.

When the common value assumption is relaxed, exclusion of weaker firms with revelation and simultaneous interviews will generally not occur. Indeed, suppose that for certain values of the worker a low-ranked firm is interested in hiring the worker but higher-ranked firms are not. Then, even if the value of the worker is commonly known, the low-ranked firm is not excluded.⁵ If, however, the value of the worker to every firm is weakly higher than the value to all lower-ranked firms, our exclusion result holds. The result is also robust to the hiring of multiple workers, provided there is sufficient separability across workers, and to informative interviews, provided that all interviewing firms gain the same information from interviewing.

There is an extensive literature on two-sided matching, beginning with the seminal work of Gale and Shapley (1962). The novelty of our paper is the focus on a specific hiring process and information revelation when firms have incomplete information about the value of the worker, which is common to all firms. Such a hiring process is often used in practice, and leads to new strategic considerations that influence firms' behavior. Such considerations do not arise in existing models of two-sided matching, both those that postulate complete information and those that postulate incomplete information of agents' preferences (see, for example, Roth and Sotomayor (1990), Sönmez (1999)). A more closely related framework is that of Chakraborty, Citanna, and Ostrovsky (2007), who consider a matching market with two-sided incomplete information, multiple firms, and multiple workers. They address the issue of stability and show that stable matchings fail to exist in general when firms observe the matching outcome, but do exist when workers' preferences are identical and firms observe only their own matches. Lee and Schwarz (2008) consider a two-sided market with incomplete information on both sides, in which both sides learn their preferences through costly interviews. In contrast to our model, agents do not base their interview decisions on private information. All agents on each side of the market are ex-ante

⁵Section 8.3.1 shows that with two firms and independent signals revelation makes the high-ranked firm better off and the low-ranked firm worse off compared to no revelation when the low-ranked firm has a lower hiring threshold than the high-ranked firm. Example 4 shows that this result does not generalize to more than two firms.

identical, values are private, and signals are independent and fully informative. The main issue is how coordination on which workers each firm should interview influences outcomes. Coles, Kushnir, and Niederle study the effect of signaling in a two-sided matching market with incomplete information. Masters (2009) studies hiring with interviews but does not consider revelation and the resulting interaction among firms. Josephson and Shapiro (2009) investigate a multistage hiring model with a different information structure, in which there are two rounds of interviewing and hiring. Interviews fully reveal a worker’s value, and interview decisions are unobservable within a round. They show that the employment outcome may be inefficient and suggest ways to overcome this.

The rest of the paper is organized as follows. Section 2 introduces the model and related notation. Section 3 conducts a preliminary analysis. Section 4 explores the setting with no revelation. Section 5 explores the setting in which firms anticipate no revelation but the interview decisions are nevertheless revealed. Section 6 explores the setting with revelation, and states and proves the main result. Section 7 explores the setting with sequential interviews. Section 8 discusses extensions to informative interviews, multiple workers, and heterogeneous worker value. Section 9 concludes. Appendix A contains the proof of Theorem 2 and related preliminaries. Appendix B contains the proof of the results of Section 7.

2 The Model and Notation

There are n risk-neutral firms and one worker. The set of firms $\{1, \dots, n\}$ is denoted by N . The worker is characterized by a vector of weakly affiliated signals, one for each firm. The set of possible signal realizations for firm i , denoted S_i , is finite⁶ and linearly ordered, with generic element s'_i . The vector of firms’ signals is drawn from a distribution F on $S = \times_i S_i$ with full support. We denote by s_i the random variable whose realization is an element in S_i , so s_1, \dots, s_n are weakly affiliated.

The worker can work for only one firm, and has a commonly-known strict ranking over firms. Firm 1 is the workers’ highest-ranked firm, firm 2 is the workers’ second highest-ranked firm, etc. The net value of employing the worker, in monetary units, is common to all firms. This value is given by a function v of all firms’ signals, and is strictly increasing in each firm’s signal. Together with the generality of the signal structure, which includes, for example, independent and conditionally independent signals, this specification of v accommodates heterogeneity in how well different firms are informed about the value of the worker. The function v is normalized so that firms’ outside option of not hiring the

⁶The assumption of finiteness simplifies the statement and proof of the results considerably.

worker is 0.

The timing of the market is as follows. First, before observing their signals, all firms simultaneously choose whether to enter the market. The cost of entry to firm i is $e_i > 0$. A firm that does not enter the market cannot participate in subsequent stages of the market. Entry decisions are publicly observed. After the entry stage, all entering firms observe their private signals and then simultaneously decide whether to interview the worker. The cost of an interview to firm i is $c_i > 0$. An interview is a necessary step in the hiring process, so the worker cannot be hired by a firm that did not interview him. We assume that interviews reveal no new information about the worker to the interviewing firm. All of our results hold if the realization of an additional signal s_0 is revealed during the interview, where s_0 is affiliated with the other signals, has full finite support, and v is increasing in s_0 . We analyze the model under three different information structures: no revelation of interview decisions, full revelation, and an intermediate model in which firms do not anticipate revelation. At the next stage, each firm decides whether to make an employment offer to the worker. The offers are made simultaneously.⁷ The worker accepts the offer made by the firm he prefers most among those that made him an offer.⁸ An entering firm i 's payoff from hiring a worker with signals s'_1, \dots, s'_n is $v(s'_1, \dots, s'_n) - c_i - e_i$. If the firm interviews but does not hire a worker, either because the firm does not make him an offer or because the worker does not accept the firm's offer, then the firm's payoff is $-c_i - e_i$. Firms' entry and interview costs are commonly known.

Positive entry costs and interview costs guarantee that entry and interview decisions are not "cheap talk." Because we are interested in the informational effects of signals and interviews, we will typically consider low entry and interview costs. By precluding "cheap talk," small entry and interview costs lead to significantly different predictions than do costs of 0. We maintain the assumption that there is some $M > 1$ such that the ratio of any two firms' interview costs is no more than M . The larger the bound M , the lower costs have to be for Theorems 2 and 3 to hold.

We analyze the game using the solution concept of sequential equilibrium (henceforth: equilibrium). Because the game is finite, a sequential equilibrium exists.

⁷If firms can make post-interview offers at any time up to a common time deadline, it is weakly dominant for all of them to make offers at the deadline.

⁸The assumption that the worker always accept the best employment offer he receives is without loss of generality. This is because if the workers prefers unemployment to working at a certain firm, then that firm will never enter the market, and can be ignored.

3 Preliminary Analysis

As a preliminary exercise, suppose that firms $2, \dots, n$ do not enter. Then, conditional on entering, firm 1 interviews the worker (and later makes an offer that will be accepted) if and only if its signal s'_1 satisfies

$$E[v|s_1 = s'_1] \geq c_1, \tag{1}$$

with possible mixing between interviewing and not interviewing if the inequality is an equality. Because the worker's value increases in every firm's signal and firms' signals are affiliated, a higher signal makes firm 1 more optimistic about the worker's expected value.⁹ This fact and the fact that the worker always accepts the firm's offer if it is made imply that the firm employs a threshold interviewing (and hiring) strategy. As the cost c_1 of interviewing decreases, the threshold decreases and the firm's expected profit increases. For the remainder of the paper, we make the following assumption.

A1 *(1) holds with a strict inequality for at least one signal when $c_1 = 0$.*

Assumption A1 guarantees that for low (but positive) interview costs, firm 1's post-entry expected profit is positive. Therefore, when the entry and interview costs are low (but positive), firm 1 will enter and make positive profits even if it is the only entering firm.

4 No Revelation

With no revelation, a firm will interview if and only if it plans to make an offer. Because the worker accepts the highest-ranked offer, we can solve for equilibrium by moving from firm 1 to firm n and identifying each firm's interviewing strategy given those of all higher-ranked firms. Firm 1 behaves as described in Section 3. Conditional on entering, it employs a threshold interviewing strategy, interviewing and hiring for every signal above the lowest signal s'_1 that satisfies (1) (if such signals exist), with possible mixing at the lowest signal if the inequality is an equality.

Given firm 1's interviewing strategy, its entry decision depends on whether its expected profits conditional on entering offset the entry costs.¹⁰ When the expected profits conditional on entering equal the entry cost, the firm may mix between entering and not

⁹To see this, apply Lemma 4 in Appendix A with $Z_{-i} = S_{-i} \times \Omega_{-i}$.

¹⁰If firm 1 mixes at the lowest signal for which it interviews with positive probability, then it makes 0 profits there, so its behavior there does not affect the profitability of entry.

entering. For low entry and interviewing costs e_1 and c_1 , however, firm 1 has a unique optimal strategy. To see this, denote by T_1 the lowest signal for which (1) holds with a strict inequality when $c_1 = 0$ (such a signal exists by Assumption A1). Then, for low e_1 and c_1 , firm 1's unique optimal strategy is to enter with probability 1 and interview and hire with probability 1 at all signals greater or equal to T_1 . The following result shows that for low entry and interviewing costs, there is in fact a unique equilibrium, in which every entering firm employs a threshold interviewing strategy.

Theorem 1 *For low $\max_{i \in N} e_i$ and $\max_{i \in N} c_i$ there is a unique equilibrium, which is in pure strategies. In this equilibrium, every entering firm i interviews for all signals greater or equal to some signal T_i . The equilibrium can be found by iterated elimination of strictly dominated strategies.*

Proof. We prove the following claim by induction: for any $i \in N$, for low $\max_{j \leq i} e_j$ and $\max_{j \leq i} c_j$, every firm $j \leq i$ has a strictly dominant (pure) strategy once the strictly dominated strategies of higher-ranked firms have been iteratively eliminated. As we have seen, the claim is true for $i = 1$, because for low e_1 and c_1 firm 1 has a strictly dominant threshold interviewing strategy with threshold T_1 . Now suppose that the claim is true for $i - 1 \geq 1$. Then, for low $\max_{j \leq i-1} e_j$ and $\max_{j \leq i-1} c_j$, in any equilibrium firms $1, \dots, i - 1$ play the strategies identified by the induction hypothesis. Given the strategies of firms $1, \dots, i - 1$, what should firm i do? Conditional on entering, firm i will succeed in hiring the worker when it makes an offer if and only if firms $1, \dots, i - 1$ do not make an offer or, equivalently, do not interview. Because firms $1, \dots, i - 1$ use pure strategies, the event that none of these firms interview the worker is the set $B = \times_{j \in N} B_j$, where

$$B_j = \begin{cases} \text{signals in } S_j \text{ for which firm } j \text{ does not interview} & j \leq i - 1 \\ S_j & j > i - 1 \end{cases}. \quad (2)$$

Note that $B_j = S_j$ if firm j does not enter. Conditional on entering, firm i 's net profit if it interviews and makes an offer to a worker at signal s'_i is

$$\Pr(B|s_i = s'_i) E[v|B, s_i = s'_i] - c_i. \quad (3)$$

Conditional on entering, for low c_i it is uniquely optimal for firm i to interview with probability 1 at precisely all signals s'_i for which the expression in (3) is strictly positive when c_i is replaced with 0. If there is at least one such signal s'_i , then for low e_i it is strictly optimal for firm i to enter. If there are no such signals, then it is strictly optimal for firm i not to enter. This shows that the induction hypothesis holds for i . Moreover, if the the expression in (3) is strictly positive for some signal s'_i when c_i is replaced with 0,

then the expression is also strictly positive for all signals $s_i'' > s_i'$. This is because (i) by affiliation and because v is strictly increasing $E[v|B, s_i = s_i']$ increases with s_i' (Lemma 4 in Appendix A) and (ii) $\Pr(B|s_i = s_i')$ is strictly positive for all signals s_i' if it is strictly positive for one signal s_i' (F has full support). Therefore, if firm i enters for low e_i , then for low c_i it interviews for all signals greater or equal to some signal T_i . ■

Costs and strategies of firms ranked lower than i do not appear in (2) and (3). Therefore, the unique equilibrium when entry and interview costs are low can be solved for by iteratively applying the process described in the proof of Theorem 1, proceeding from firm 1 to firm N . Note that the interview threshold of every firm $2, \dots, n$ is strictly higher than if would be if the firm was the only one in the market. Firms compensate for the adverse selection they experience from higher-ranked firms by increasing their interview threshold.

The requirement that interview costs be low is necessary to conclude that firms employ threshold interviewing strategies. To see this, suppose that e_1 and c_1 are low enough for firm 1 to use a threshold interviewing strategy with threshold T_1 . Consider firm 2 and (3). As s_2' increases, $\Pr(s_1 < T_1 | s_2 = s_2')$ decreases (affiliation) and $E[v|s_1 < T_1, s_2 = s_2']$ strictly increases (v is strictly increasing and affiliation - Lemma 4 in Appendix A). Therefore, the expression in (3) may be a non-monotonic function of s_2' . This means that the set of signals for which firm 2 interviews need not correspond to a threshold interviewing strategy when c_2 is not low.

Nevertheless, when firms' signals are independent, each firm employs a threshold interviewing strategy regardless of interviewing costs. This is because when firms' signals are independent, $\Pr(B|s_i = s_i')$ is independent of s_i' , so the expression in (3) strictly increases in s_i' .¹¹

4.1 No Revelation Example

Suppose each firm's signal is drawn uniformly and independently from the set $\{-\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon\} \cup \{-\frac{1}{2} + \frac{i}{2^k} - \varepsilon, -\frac{1}{2} + \frac{i}{2^k} + \varepsilon : i = 1, \dots, 2^k - 1\}$ for some $k \geq n$ and positive $\varepsilon < \frac{1}{2^{k+2}}$ (this approximates the uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$). Figure 1 illustrates the set of signals for $k = 2$ and small $\varepsilon > 0$ by depicting the signals as circles.

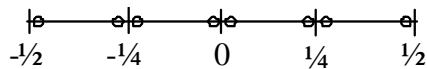


Figure 1: The set of signals for each firm in Example 1 for $k = 2$ and small $\varepsilon > 0$

¹¹If this expression equals 0 for some signal s_i' , then the firm may mix between interviewing and not interviewing at s_i' .

Suppose that $v = \sum_{i=1}^n s_i$. Then, if entry and interview costs are low, for a firm operating alone in the market it is uniquely optimal to interview and hire at any signal greater than or equal to ε , because the expected value of other firms' signals is 0. With no revelation and low entry and interview costs firm 1 enters and interviews and hires at any signal greater or equal to ε . Therefore, the expected value of firm 1's signal conditional on not interviewing is $-\frac{1}{4}$. As a result, for low entry and interview costs, firm 2 enters and interviews and hires at any signal greater or equal to $\frac{1}{4} + \varepsilon$. Proceeding in this way, we see that with no revelation and low entry and interview costs there is a unique equilibrium. In this equilibrium, all firms enter and every firm i interviews with probability 1 at all signals greater or equal to $T_i = \frac{1}{2} - \frac{1}{2^i} + \varepsilon$.

5 Unexpected Revelation

The intuition that revealing firms' interviewing decisions may make firms better off can perhaps be traced to the following result.

Proposition 1 *Suppose firms' interview decisions are made assuming no revelation. Then, relative to no revelation, revealing firms' interview decisions (a) weakly decreases the set of signals for which the worker is hired, (b) may shift the worker to lower-ranked firms but does not shift the worker to higher-ranked firms, and (c) weakly increases the utility of all firms.*

Proof. By assumption, revealing firms' interview decisions does not affect the set of signals for which each firm interviews. With no revelation, whenever the worker is interviewed, he is hired by the highest-ranked firm that interviews him. Therefore, no additional hiring results from unexpected revelation. Now consider the movement of the worker among firms. With no revelation, a firm hires the worker following an interview if and only if no higher-ranked firm interviews the worker, regardless of what lower-ranked firms do. This means that the worker cannot move up to a better firm because of unexpected revelation: if a firm hires the worker with unexpected revelation but does not hire him with no revelation, the firm must have interviewed with no revelation (since the firm interviews for the same signals with unexpected revelation), and the reason the worker is not hired by the firm with no revelation is that the worker is interviewed and hired by a higher-ranked firm, so the worker shifts down to be hired by the firm with unexpected revelation. To see that all firms are weakly better off compared to no revelation, note that since there is no movement of the worker to higher-ranked firms, with unexpected revelation each firm can obtain the same hiring outcome it obtains with no revelation by making an offer if and only if all higher-ranked firms do not interview. A firm may do better by not making an offer

for some signals that lead to the firm interviewing, and can also choose to make an offer when higher-ranked firms interview but choose not to hire based on other firms' interview decisions. ■

Proposition 1 suggests that the setting with no revelation is unstable in the following intuitive sense. Suppose that firms make their interview decisions expecting no revelation. After the interview stage and before the hiring stage, the firms can be made collectively better off by revealing their interview decisions. No firm would be worse off and some firms could be strictly better off compared to no revelation. Small transfers between firms could make all firms strictly better off.

5.1 Unexpected Revelation Example

Consider the setting of Section 4.1 with two firms, $k \geq 3$, and low entry and interview costs. Firm 1's interview threshold is ε , and firm 2's interview threshold is $\frac{1}{4} + \varepsilon$. If firms' interviewing decisions are revealed unexpectedly, then when firm 1 interviews and firm 2 does not, firm 1 makes offers and hires for all signals greater or equal to $\frac{1}{8} + \varepsilon$. This contrasts with the no-revelation setting, in which firm 1 makes an offer whenever it interviews. When both firms interview, firm 1 hires, and when only firm 2 interviews, firm 2 hires, just as with no revelation. A worker with $s_1 = \varepsilon$ and $s_2 < \frac{1}{4} + \varepsilon$ is hired by firm 1 with no revelation, but is not hired by any firm with unexpected revelation.

6 Revelation

With revelation, each firm can condition its hiring decision on all other firms' interview decisions. As a result, a firm's interview strategy may depend on its beliefs about all other firms' interview strategies, and not only on those of higher-ranked firms. This typically implies that the no-revelation equilibrium identified in Theorem 1 is not an equilibrium with revelation. To see why, consider the example of Section 4.1 with two firms. For low entry and interview costs, with no revelation the unique equilibrium is for both firms to employ threshold interviewing strategies. Firm 1's interview threshold is ε and that of firm 2 is $\frac{1}{4} + \varepsilon$. With revelation, this is no longer an equilibrium. Indeed, suppose that firm 2 maintained the same interviewing strategy. This implies that seeing firm 2 interviewing is good news about firm 2's signal. Firm 1 would then find it beneficial to interview for some signals lower than ε , and for those signals to make an offer only if it sees firm 2 interview. Firm 2's response to this behavior by firm 1 would be to change the set of signals for which it interviews. More generally, with revelation a firm may choose to interview at a signal because it expects to learn something about the worker's value from the other firms'

interview decisions. But the firm also knows that the other firms will learn something from its interview decision, which may affect its probability of hiring the worker and the value of the worker conditional on hiring.

More concretely, suppose that m firms enter in an equilibrium with revelation and assume for simplicity that firms use pure strategies, so that each firm has an “interview set” of signals for which it interviews. Consider the behavior of an entering firm. After interviewing, the firm can condition its hiring decision on each of the 2^{m-1} possible combinations of the other entering firms’ interview decisions. For each such combination, the firm determines a “hiring set” of signals for which it makes an offer after interviewing. These hiring sets, which depend on the other firms’ interview sets, determine the firm’s interview set. Because of this interdependence, all firms’ hiring sets for each combination of the other firms’ interview decisions and all interview sets are determined jointly. A sequential procedure like the one described in Theorem 1 can therefore not be used to solve for an equilibrium.

The analysis is further complicated by considering mixed strategies (which may depend on entry and interview costs) and non-threshold interview and hiring sets. An argument like the one used in Theorem 1 to show that firms use pure strategies and that entering firms use threshold interviewing strategies, all of which are independent of costs when costs are low, does not work with revelation. And if firms use non-threshold interview strategies, then seeing another firm interview is not necessarily “good news” about the worker’s value. Despite these difficulties, the following result fully characterizes equilibrium behavior with revelation.

Theorem 2 *For low entry and interview costs, in any equilibrium with revelation the only firm that enters is firm 1.*¹²

To illustrate the proof of Theorem 2, suppose that interview costs are 0, and that a firm interviews only if there is a positive probability that it can hire the worker and that conditional on hiring the worker the firm makes positive profits. By Condition A1, firm 1 enters and interviews for some signals, because it can always ignore the other firms. Suppose that firm $i > 1$ is the lowest-ranked firm that enters, and consider the lowest signal s'_i at which firm i interviews. When firm i observes this signal, there is a combination of the other firms’ interview decisions, or “interview schedule,” that arises with positive probability and at which firm i can profitably hire the worker with positive probability (otherwise firm i would not interview at s'_i).

¹²When there are only two firms ($n = 2$), the restriction on the ratio between firms’ interview costs assumed in Section 2 is not needed for the result.

The first step of the proof shows that this interview schedule consists of only firm i interviewing. To see why, suppose that such a schedule \mathcal{I} exists in which other firms interview, and denote by j the highest-ranked firm that interviews in \mathcal{I} . Because firm i successfully hires at \mathcal{I} , there must be signals at which firm j interviews but does not make an offer at \mathcal{I} . Suppose that firm i observes s'_i , firm j observes the highest signal s'_j for which it interviews but does not make an offer at \mathcal{I} , and the schedule \mathcal{I} arises. Then firm j 's expectation of the worker's value is higher than that of firm i : firm j observes s'_j and knows only that firm i 's signal is greater or equal to s'_i , whereas firm i observes s'_i and knows only that firm j 's signal is at most s'_j . Because firm i is willing to hire the worker when it observes s'_i and \mathcal{I} , firm j would be willing to hire the worker when it observes s'_j and \mathcal{I} . But then firm j should deviate and make an offer at s'_j . This is illustrated in Figure 2 below. The second step of the proof uses a similar argument to show that if firm i is willing to interview when it observes s'_i and make an offer when it is the only firm that interviews, then some other firm $l \neq i$ can profitably deviate and interview at some signal at which it is not supposed to interview, and then make an offer if it sees that the only other firm that interviews is firm i .

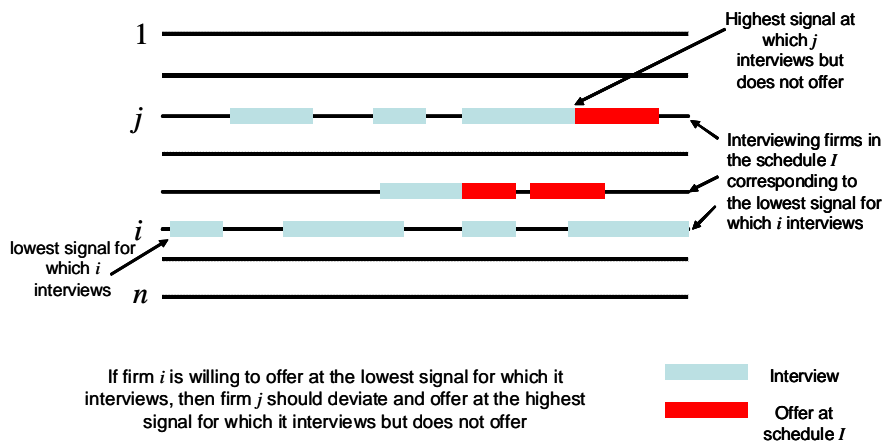


Figure 2: A profitable deviation for firm j

The proof of Theorem 2, which is in Appendix A, formalizes these steps and extends them to accommodate mixed strategies and positive interview costs (which imply that probabilities of certain events must be taken into account, and not only the worker's expected value). This makes the second step of the proof quite involved.

6.1 Discussion

Theorem 2 implies that when entry and interview costs are low, revelation leads to unraveling that excludes all but the highest-ranked firm from interviewing and hiring. The outcome is as if the set of firms included only firm 1. In this outcome, firms $2, \dots, n$ are just as disadvantaged as they would be if all of their private information were revealed, and firm 1 does not benefit.

Compared to the setting with no revelation, no firm is better off with revelation, and any firm $2, \dots, n$ that enters with no revelation is strictly worse off with revelation. In the setting of Section 4.1 above, in which all firms enter with no revelation, revelation makes firms $2, \dots, n$ strictly worse off because they are excluded. Theorem 2 also implies that the worker is no better off with revelation. Firm 1 hires the worker for the same set of signal profiles with revelation and with no revelation, and whenever the worker is hired by some firm $2, \dots, n$ with no revelation, he is unemployed with revelation. Thus, revelation lowers virtually any measure of welfare and efficiency when entry and interview costs are low.

We find this result surprising for several reasons. First, a firm's signal provides it with private information, and only a coarse measure of this information is made public when interviewing decisions are revealed. Second, with revelation, although each entering firm faces adverse selection from higher-ranked firms, it gains valuable information from the interview decisions of all other entering firms. Third, each firm may use non-interval interviewing strategies and employ mixed strategies. Fourth, because the function v is not assumed symmetric, the impact of one firm's signal on the worker's value may be high, while that of another firm is low. Thus, how informative a firm's signal is may vary across firms. In particular, when the number of firms is large, it may seem that at least some firms' interview decisions would not be so informative, which may allow these firms to participate in the market. Theorem 2 shows that the adverse selection is so strong that firms $2, \dots, n$ cannot make any use of their private information with revelation, regardless of n .

Because the game is finite, a sequential equilibrium is guaranteed to exist. Theorem 2 describes the equilibrium outcome, but does not specify firms' off-path behavior. To get an idea of off-path behavior that supports the equilibrium outcome, suppose first that firm 1 does not enter, and choose some set of entering firms. This is a proper subgame, and so has a sequential equilibrium. Any sequential equilibrium will do, because firm 1 will never find a deviation to not entering attractive, regardless of what other firms do. Now suppose that at least three firms enter, including firm 1. This is a proper subgame in which any sequential equilibrium will do, because no firm can reach this subgame by deviating unilaterally. Finally, suppose that two firms enter, firm 1 and firm $j \neq 1$, and consider a sequential equilibrium of this proper subgame. The proof of Theorem 2 applied to $n = 2$

shows that firm j cannot make strictly positive profits net of entry costs in the subgame. Therefore, firm j makes non-positive profits net of entry costs in this subgame, and will not deviate to entering (because entry is costly).¹³

7 Sequential Interviews

Suppose that firms make interview decisions sequentially instead of simultaneously, and that each interview decision is revealed after it is made.¹⁴ Each firm can condition its interview decision on those of the firms that preceded it, and all firms can condition their hiring decisions on the other firms' interview decisions.¹⁵ Then, when entry and interview costs are low, each ordering of firms leads to a unique equilibrium. This equilibrium is described by Theorem 3 in Appendix B. Underlying the result is an unraveling argument similar to the one used in the proof of Theorem 2. The argument shows that a firm that is followed by a higher-ranked entering firm can never profitably hire the worker in equilibrium, and therefore never enters. The set of entering firms therefore has the property that no entering firm is followed by a higher-ranked entering firm. To pin down the set of entering firms among the sets of firms that satisfy this property, we show that if an entering firm interviews and is followed by a lower-ranked entering firm, then the lower-ranked firm can never profitably hire the worker in equilibrium. Thus, an entering firm that follows a higher-ranked entering firm can only profitably hire, and therefore only interviews, if the higher-ranked firm does not interview. This pins down firms' behavior inductively conditional on entry, and leads to an iterative process of identifying firms that cannot make positive profits if they enter, even if they are not followed by higher-ranked firms. That these firms do not enter leads to a unique set of entering firms. The proof of Theorem 3 makes these arguments precise and describes the set of entering firms and their interviewing and hiring behavior.

Proposition 3 in Appendix B compares the hiring outcome with sequential interviews

¹³In the subgame, both firms will interview with a positive probability that is strictly less than 1. Firm j will interview with low probability, and will interview with probability strictly less than 1 at any signal. It will therefore have expected profits of 0 net of entry costs.

¹⁴If interview decisions are not revealed, then their order does not matter and we are in the setting with no revelation.

¹⁵We maintain the assumption that offers are made simultaneously. A similar analysis can be conducted when offers are made sequentially. In particular, if offers are made sequentially from the highest-ranked interviewing firm to the lowest-ranked interviewing firm, the outcome is as if the offers were made simultaneously; the reverse order of offers implies that only firm 1 enters.

to the outcome with simultaneous interviews when entry and interview costs are low. The proposition shows that if interviews are conducted sequentially according to an ordering π of firms and the set of entering firms in the unique equilibrium with sequential interviews is $K(\pi)$, then the hiring outcome in this equilibrium is the same as the hiring outcome that arises in the unique equilibrium described in Theorem 1 of a market with simultaneous interviews when the set of firms is $K(\pi)$. In particular, when firms are ordered from the highest-ranked firm to the lowest-ranked firm, the entry and hiring outcome is the same as in the setting with no revelation; when firms are ordered from the lowest-ranked firm to the highest-ranked firm, the entry and hiring outcome is the same as in the setting with revelation (only the highest-ranked firm enters). These observations are summarized by the following corollaries of Proposition 3, which are proved in Appendix B.

Corollary 1 *For low entry and interview costs, the hiring outcome corresponding to the equilibrium of Theorem 3 (sequential interviews) with the ordering $\pi = (n, n - 1, \dots, 1)$ is the same as the hiring outcome corresponding to the equilibrium of Theorem 2 (revelation). The expenditures on interviews and firms' profits are the same in both equilibria.*

Corollary 2 *For low entry and interview costs, the hiring outcome corresponding to the equilibrium of Theorem 3 (sequential interviews) with the ordering $\pi = (1, 2, \dots, n)$ is the same as the hiring outcome corresponding to the equilibrium of Theorem 1 (no-revelation). The expenditures on interviews are lower and firms' profits are higher with sequential interviews.¹⁶*

Corollary 2 shows that when interviews are conducted sequentially from the highest-ranked firm to the lowest-ranked firm the hiring outcome is the same as in the no-revelation setting. This outcome is achieved, however, with lower interview expenditures and higher firm profits. The reason is that with sequential interviews each firm interviews and makes an offer only when all higher-ranked firms do not interview, so an offer is always accepted. With simultaneous interviews an offer is accepted under the same conditions, but a firm cannot condition its interview decision on those of higher-ranked firms and therefore interviews and makes an offer even when higher-ranked firms make offers. Corollary 2 also suggests a way to overcome the bad outcome associated with revelation and simultaneous interviews. By interviewing in sequence, from the highest-ranked firm to the lowest-ranked firm, firms are better off and the worker is no worse off with revelation than with no revelation.

¹⁶The logic behind this result is somewhat similar to the one underlying the existence result of Chakraborty et al. (2007), in that higher-ranked schools do not condition their actions on those of lower-ranked schools.

8 Extensions

8.1 Informative Interviews

Suppose that an interview conveys additional information about the worker to the interviewing firm. As long as all interviewing firms obtain the same information from the interview, the results of the paper hold without change. Formally, this information is an additional signal s_0 that is affiliated with the other signals and has full finite support, such that v is increasing in s_0 . The realization of s_0 is revealed during the interview.

8.2 Multiple workers

The analysis applies to multi-worker markets in which all interviews are conducted simultaneously before all offers are made as long as there is enough separability across workers. For this we require that the vectors of signals for each worker be independent across workers, that each firm can hire any number of workers, and that the value of the workers hired by a firm be additively separable across workers. These assumptions imply that with no revelation we can analyze firms' interviewing (and hiring) decisions for each worker separately, so Theorem 1 holds.¹⁷

With revelation things are more delicate, because a firm's decision whether to interview a worker could depend on the signals it observes for other workers. To see why, suppose that the other firms believe that firm i decides whether to interview worker k based on the signals firm i observes for other workers. Then, the other firms will infer something about the value of workers other than k from firm i 's interview decision regarding worker k . If firm i were to then decide whether to interview worker k based only on the signal it observes for worker k , then the other firms' hiring decisions regarding the other workers may be affected to the detriment of firm i . Thus, firm i may optimally condition its interview decision regarding worker k on the signals it observes for other workers.¹⁸

We would like to rule out such behavior, because the signal a firm observes for one worker contains no information about the value of other workers. We therefore consider separable equilibria, which are defined as follows.

¹⁷In principle, a firm could choose whether to interview a worker based on the signals it observes for other workers. These signals, however, do not tell the firm anything about the worker's value, so the firm could optimally use them only as a randomization device. For low enough entry and interview costs, Proposition 1 rules out such behavior because it shows that firms use pure strategies.

¹⁸Such behavior does not arise with no revelation, because no firm observes the other firms' interview decisions when it makes its hiring decisions.

Definition 1 *A separable equilibrium is a sequential equilibrium in which each firm's decision whether to interview a worker does not depend the firm's signals for other workers.*

In a separable equilibrium, a firm's interviewing decision regarding worker k does not contain any information about the value of other workers. Therefore, when analyzing separable equilibria we need only consider strategies in which a firm's hiring decision regarding worker k does not depend on the other firms' interviewing decisions regarding the other workers.¹⁹ Because firms' signals are independent across workers and the value of the workers to a firm is additively separable across workers, the continuation of any separable equilibrium conditional on entry is the conjunction of sequential equilibria conditional on entry in markets with one worker, one market for each of the workers in the original market. In particular, for low entry costs a firm finds it optimal to enter in a sequential equilibrium of the original market if and only if it finds it profitable to enter in the sequential equilibrium of at least one of the markets corresponding to a single worker. Therefore, Theorem 2 characterizes all separable equilibria when entry and interview costs are low.

8.3 Heterogeneous Worker Value

The assumption that the worker's value is common to all firms is key to the analysis. If lower-ranked firms have a positive probability of hiring the worker when all firms' signals are made public, we would not expect revelation to exclude these firms (see Section 8.3.2). With this in mind, the common value assumption can be replaced with the assumption that the value of the worker to firm j is weakly higher than the value to firm i whenever $j < i$.²⁰ This maintains the property that if all firms' signals are public, a higher-ranked firm wants to hire the worker if a lower-ranked firm does.

8.3.1 Two Firms and Independent Signals

With only two firms and independent signals, we can compare the setting with revelation to that with no revelation even if the value of the worker to firm 2 is higher than his value to firm 1. Specifically, let the value of the worker be $v(s_1, s_2)$ to firm 1 and $v(s_1, s_2) + w_2$ to firm 2, for some $w_2 > 0$. If v can take negative values lower than w_2 , then it may be that firm 2 wants to hire the worker and firm 1 does not. With no revelation, Theorem 1 and its proof hold without change.

¹⁹If a firm is indifferent between hiring and not hiring a worker, it may use the other firms' interviewing decisions regarding the other workers as a randomization mechanism. This does not change the statement or proof of Theorem 2.

²⁰One example is adding a firm-specific constant to v with higher-ranked firms having a higher constant.

Now consider the setting with revelation, and suppose both firms enter. For expositional simplicity, suppose that each firm interviews whenever it is indifferent between interviewing and not interviewing, and makes an offer whenever it is indifferent between making an offer and not making an offer. Because signals are independent, each firm employs a threshold interviewing strategy. Therefore, seeing the worker interviewed by firm 2 is “good news” and seeing the worker not interviewed by firm 2 is “bad news” for firm 1 about the worker’s value, regardless of firm 2’s interviewing threshold. This means that for any signal s'_1 , firm 1’s estimation of the worker’s value is higher when it sees firm 2 interview than what it would be with no revelation, which in turn is higher than its estimation when it sees firm 2 not interview. That is,

$$E[v|s_1 = s'_1, s_2 \geq T_2^0] \geq E[v|s_1 = s'_1] \geq E[v|s_1 = s'_1, s_2 < T_2^0],$$

where T_2^0 is firm 2’s interview threshold with revelation (the lowest signal for which it interviews). In particular,

$$E[v|s_1 = T_1, s_2 \geq T_2^0] \geq E[v|s_1 = T_1] \geq E[v|s_1 = T_1, s_2 < T_2^0],$$

where T_1 is firm 1’s interview threshold with no revelation. So the lowest signal at which firm 1 is willing to interview and make an offer if firm 2 interviews is weakly lower than T_1 . Denote this signal by T_1^1 . The lowest signal at which firm 1 is willing to interview and make the worker an offer if firm 2 does not interview is weakly higher than T_1 . Denote this signal by T_1^0 . Because $T_1^0 \geq T_1^1$, firm 1’s interview threshold with revelation conditional on entry is T_1^1 . If $s'_1 \in [T_1^1, T_1^0)$, then firm 1 makes the worker an offer only if firm 2 interviews. In this interval of signals, firm 1 interviews the worker for the “option value” of hiring him.

How does T_2^0 , firm 2’s interview threshold with revelation, compare to T_2 , firm 2’s interview threshold conditional on entry with no revelation? Suppose firm 2 interviews the worker. Because $T_1^1 \leq T_1^0$, firm 2 can only hire the worker when firm 1 doesn’t interview. And because $T_1^1 \leq T_1$, firm 2 faces greater adverse selection with revelation than with no revelation. That is, for any signal s'_2 , the expected value of the worker to firm 2 conditional on firm 2 being able to hire the worker is lower with revelation than with no revelation, so

$$E[v|s_1 \leq T_1^1, s_2 = s'_2] \leq E[v|s_1 \leq T_1, s_2 = s'_2].$$

In particular,

$$E[v|s_1 \leq T_1^1, s_2 = T_2] \leq E[v|s_1 \leq T_1, s_2 = T_2].$$

Therefore, with revelation firm 2’s interviewing (and hiring) threshold T_2^0 is weakly higher than T_2 . The following figure summarizes these results.

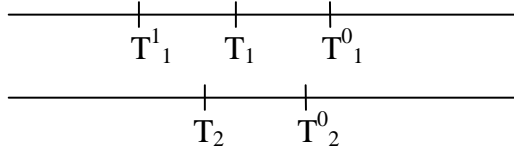


Figure 3: Interview and hiring thresholds with and without revelation for two firms with independent signals

The unambiguous location of the interviewing and hiring thresholds with revelation relative to those with no revelation leads to the following observations. First, if the worker is hired with revelation, then he is also hired with no revelation. This is because with revelation the worker is hired by firm 1 if $s_1 \geq T_1^1$ and $s_2 \geq T_2^0 \geq T_2$ (in which case with no revelation the worker is hired by firm 2 if not by firm 1) or if $s_1 \geq T_1^0 \geq T_1$ (in which case with no revelation the worker is hired by firm 1), and by firm 2 if $s_1 \leq T_1^1 \leq T_1$ and $s_2 \geq T_2^0 \geq T_1$ (in which case with no revelation the worker is hired by firm 2). This also shows that if the worker is hired by firm 2 with revelation, then he is also hired by firm 2 with no revelation, so the only possible movement of the worker as a result of revelation is from firm 2 to firm 1 (when $s_1 \in [T_1^1, T_1)$ and $s_2 \geq T_2^0$), and from both firms to unemployment (when $s_1 \in [T_1, T_1^0)$ and $s_2 \leq T_2^0$, or $s_1 \leq T_1^0$ and $s_2 \in [T_2, T_2^0)$). As a result, firm 1 is weakly better off and firm 2 is weakly worse off with revelation. The worker may be better off or worse off. The following proposition summarizes these results.²¹

Proposition 2 *Suppose that $n = 2$ and the two firms have independent signals. When entry and interview costs are low so that Theorem 1 holds, the following statements are true for any equilibrium with revelation.*

1. *Revelation makes firm 1 weakly better off. Firm 1's interview threshold is weakly lower with revelation than with no revelation.*
2. *Revelation makes firm 2 weakly worse off. Firm 2's interview threshold is weakly higher with revelation than with no revelation.*
3. *If the worker is hired by firm 2 with no revelation, he may be hired by firm 1 with revelation.*

²¹Although we explicitly considered equilibria in which each firm interviews whenever it is indifferent between interviewing and not interviewing, and makes an offer whenever it is indifferent between making an offer and not making an offer, the same analysis holds for all equilibria, as does Proposition 2.

4. *If the worker is hired by firm 2 with revelation, then he is hired by firm 2 with no revelation.*
5. *If the worker is hired with no revelation, he may be unemployed with revelation.*
6. *If the worker is hired with revelation, he is hired with no revelation.*

Note that the possible movement of the worker, from firm 2 to firm 1, is the opposite of what happens with unexpected revelation. There, movement is only possible from higher-ranked firms to lower-ranked firms.

The proposition holds even without assuming that both firms enter. Indeed, if firm 1 enters, it is weakly better off if firm 2 enters, so by Assumption A1 firm 1 always enters and is therefore at least as well off with revelation as it is with no revelation. If firm 2 does not enter, then it is weakly worse off than with no revelation. The following example shows that both firms may enter with revelation.

8.3.2 An Example in which the Value of the Worker to Firm 2 is Higher than to Firm 1

Suppose that there are two firms, and each firm's signal is drawn uniformly and independently from the set $\{-\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon\} \cup \{-\frac{1}{2} + \frac{i}{12k} - \varepsilon, -\frac{1}{2} + \frac{i}{12k} + \varepsilon : i = 1, \dots, 12k - 1\}$ for some $k \geq 1$ and positive $\varepsilon < \frac{1}{48k}$ (this approximates the uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$). Suppose that the value of the worker is $s_1 + s_2$ to firm 1 and $s_1 + s_2 + \frac{1}{2}$ to firm 2. Without revelation, if entry and interview costs are low, there is a unique equilibrium. In this equilibrium, both firms enter and interview with probability 1 at all signals greater or equal to their interview thresholds, which are $T_1 = \varepsilon$ and $T_2 = -\frac{1}{4} + \varepsilon$. With revelation, if entry and interview costs are low, there is a unique equilibrium in which both firms enter. This equilibrium has thresholds $T_1^1 = T_2^0 = -\frac{1}{6} + \varepsilon$ and $T_1^0 = \frac{1}{3} + \varepsilon$. In particular, $T_1^1 < T_1 < T_1^0$ and $T_2 < T_2^0$, as in Figure 3. A worker with s_i in $[-\frac{1}{6} + \varepsilon, \varepsilon)$ and $s_2 \geq -\frac{1}{6} + \varepsilon$ is hired by firm 2 with no revelation and by firm 1 with revelation. A worker with s_1 in $[\varepsilon, \frac{1}{3} + \varepsilon)$ and $s_2 \leq -\frac{1}{6} + \varepsilon$, or with $s_1 \leq \frac{1}{3} + \varepsilon$ and s_2 in $[-\frac{1}{4} + \varepsilon, -\frac{1}{6} + \varepsilon)$, is employed with no revelation but unemployed with revelation.

8.3.3 More than Two Firms

Even when there are more than two firms, firm 1 is always made weakly better off by revelation compared to no revelation. This is because firm 1 benefits from more information and faces no adverse selection. In contrast to the two-firms case, however, the effect of revelation on firms other than firm 1 when a lower-ranked firm may have a higher value

for the worker is no longer unambiguous. In particular, revelation may make a low-ranked firm better off. To see this, consider three firms, with firm 3's signal so uninformative that firms 1 and 2 ignore firm 3's interviewing decision when they make their hiring decisions. Our results from the two-firm setting suggest that sometimes the worker hired by firms 1 and 2 with no revelation may be unemployed with revelation. If the value of the worker to firm 3 is then high enough, then firm 3 would like to hire the worker when it is not hired by firms 1 and 2, so revelation may make firm 3 better off. This is what happens in the following example.

8.3.4 An Example in which a Low-Ranked Firm is Better Off with Revelation

Modify the setting of Section 8.3.2 above by adding firm 3 with the same signal structure as firms 1 and 2, and set the value of the worker to firm 1 at $s_1 + s_2 + \delta s_3$, to firm 2 at $s_1 + s_2 + \delta s_3 + \frac{1}{2}$, and to firm 3 at $s_1 + s_2 + \delta s_3 + w_3$, for $\delta < \frac{\varepsilon}{2}$ and $w_3 > 0$ to be specified below. For low entry and interview costs, we still have that with no revelation firms 1 and 2 enter, $T_1 = \varepsilon$, and $T_2 = -\frac{1}{4} + \varepsilon$. Because with no revelation firm 3 can hire only when the other firms do not interview, the expected net value of a worker that firm 3 can hire with no revelation at signal s_3 is $-\frac{1}{4} - \frac{3}{8} + \delta s_3 + w_3 = -\frac{5}{8} + \delta s_3 + w_3$. With revelation, for low entry and interview costs, firms 1 and 2 ignore firm 3's interviewing decision (because δ is small), so $T_1^1 = T_2^0 = -\frac{1}{6} + \varepsilon$ and $T_1^0 = \frac{1}{3} + \varepsilon$ as in Section 8.3.2. In this case, firm 3 can hire when firm 1's signal is below T_1^0 and firm 2 doesn't interview. The expected value of the worker is then $-\frac{1}{12} - \frac{1}{3} + \delta s_3 + w_3 = -\frac{5}{12} + \delta s_3 + w_3 > -\frac{5}{8} + \delta s_3 + w_3$. Moreover, the probability that firm 3 is able to hire with revelation is higher than with no revelation. If $w_3 \geq \frac{5}{12} + \varepsilon$, then for low entry and interview costs, with revelation firm always 3 interviews the worker and is strictly better off than with no revelation: it succeeds in hiring with higher probability and obtains a worker that has a higher expected value. In addition, if w_3 is in $[\frac{5}{12} + \varepsilon, \frac{5}{8} - \varepsilon)$, then firm 3 does not enter with no revelation, whereas with revelation, for low entry and interview costs, it enters and makes strictly positive profits.

9 Conclusion

This paper has investigated a model in which privately-informed firms interview a worker before making their hiring decisions, and the value of the worker is common to all firms. When firms' interview decisions are kept private, each firm can make use of its private information, even though all but the highest-ranked firm face adverse selection akin to a "winner's curse." When firms' interview decisions are revealed, the adverse selection

becomes so strong that regardless of the number of firms only the top firm can make use of its private information - all other firms stay out of the market. Revelation of firms' interview decisions, which has the potential to improve market outcomes through the sharing of private information, leads to complete unraveling and less usage of information than with no revelation. The outcome with revelation is worse than with no revelation according to virtually any efficiency or social welfare criterion. This effect may be less pronounced when complementarity/substitution among workers, private value components, and other real-world features of labor markets are introduced.²² We view our result as indicative of the potential for adverse selection in common-value markets with intermediate, coarse information disclosure, even when there are many firms and the information structure is fairly general.

A possible solution is having firms interview the worker in sequence, with each firm deciding whether to interview the worker based on preceding firms' interview decisions. Each ordering of firms' interviews leads to a unique hiring outcome. The two "extreme" orderings, from the highest-ranked firm to the lowest-ranked firm and vice-versa, lead to the hiring outcomes associated with no revelation and revelation. The high-to-low ordering not only overcomes the unraveling associated with revelation, it also improves upon the no-revelation setting because it achieves the no-revelation hiring outcome with lower interview expenditures.

Two modeling assumptions suggest avenues for future research. The first is that firms' hiring decisions are binary. This assumption may be suitable for studying certain entry-level labor markets or university admission processes, in which competition through wages does not seem to be a dominant factor. A model with wage competition would fit other settings but presents significant technical challenges. A preliminary investigation suggests that if firms are ranked in their valuation for the worker, a logic similar to the one underlying Theorem 2 may lead to unraveling even with wage competition. A second modeling assumption is that whether firms' interview decisions are revealed is determined exogenously. An interesting question is under what circumstances we would expect revelation to arise endogenously. Suppose that the only way an interview is revealed is if the worker or firm that participated in the interview reveal the fact of the interview (no lying is possible). Then it is an equilibrium for all workers and firms to reveal all interviews, because a unilateral deviation by a worker or a firm does not affect revelation. As discussed in Section 5, Proposition 1 suggests that the setting with no revelation is unstable. A full characterization of all equilibria when revelation is endogenized is beyond the scope of this paper.

²²Of course, such features would make the model much less tractable.

A Theorem 2

A.1 Preliminaries and Notation

To model mixed strategies, we assume that each firm i observes the outcome of a uniform lottery over $\Omega_i = [0, 1]$, and denote by ω_i the realization of this lottery. The lotteries of different firms are statistically independent, and are also independent of all firms' signals.

We use the following notation for post-entry interviewing and hiring mixed strategies parameterized by k , that is, strategies that take the set of entering firms as given. Firm i chooses a measurable set $\tilde{I}_i^k \subset S_i \times \Omega_i$ following whose elements it interviews the worker. We define $\sigma_i^k(s_i) = \text{Prob}\{\omega_i : (s_i, \omega_i) \in \tilde{I}_i^k\}$ as the probability that firm i interviews after observing the signal s_i . For each subset $\mathcal{I} \subset \{1, \dots, n\}$ such that $i \in \mathcal{I}$, firm i chooses a measurable set $\tilde{O}_{i,\mathcal{I}}^k \subset S_i \times \Omega_i$ following whose elements it makes an offer if it interviewed and observed interview schedule \mathcal{I} (that is, if it observed precisely the firms in \mathcal{I} interviewing). For every interview schedule \mathcal{I} such that $i \in \mathcal{I}$, we define $\tau_i^k(s_i; \mathcal{I}) = \text{Prob}\{\omega_i : (s_i, \omega_i) \in \tilde{O}_{i,\mathcal{I}}^k\}$ as the probability that firm i makes an offer if it both (i) interviewed after observing signal s_i and (ii) observed interview schedule \mathcal{I} . We denote by $\underline{s}_i^k = \min\{s_i : \sigma_i^k(s_i) > 0\}$ the lowest signal for which firm i interviews with positive probability, by $\bar{s}_i^k = \max\{s_i : \sigma_i^k(s_i) < 1\}$ the highest signal for which firm i interviews with probability less than one, and by $\bar{s}_{i,\mathcal{I}}^k = \max\{s_i : \tau_i^k(s_i; \mathcal{I}) < 1, \sigma_i^k(s_i) > 0\}$ the highest signal for which firm i interviews with positive probability and makes an offer with probability less than one after interviewing and observing interview schedule \mathcal{I} .

Let $I_i^k = \tilde{I}_i^k \times \prod_{j \neq i} (S_j \times \Omega_j)$ and $O_{i,\mathcal{I}}^k = \tilde{O}_{i,\mathcal{I}}^k \times \prod_{j \neq i} (S_j \times \Omega_j)$. For a set of firms \mathcal{I} , we denote by $\hat{\mathcal{I}} = \bigcap_{j \in \mathcal{I}} I_j^k \cap \bigcap_{j \notin \mathcal{I}} \neg I_j^k$ the event that exactly this set of firms interviews. The set $\Phi_{i,\mathcal{I}}^k = \bigcap_{j \in \mathcal{I}, j < i} \neg O_{j,\mathcal{I}}^k$ is the event at which firm i could possibly have its offer accepted if precisely the firms in \mathcal{I} interview (because all stronger interviewing firms do not make offers).

A.2 Technical Lemmas

Denote by G_i the uniform CDF on $\Omega_i = [0, 1]$. Endow $\Omega = \times_{i \in N} \Omega_i$ with the product CDF $G = \times G_i$. Denote by μ^G the probability measure on Ω induced by G , by μ_i^G the probability measure on Ω_i induced by G_i , and by μ_{-i}^G the probability measure on Ω_{-i} induced by G_{-i} , where $-i$ is the set of indices other than i . Consider the probability space defined by $S \times \Omega$ and the probability measure $\mu^{F \times G}$ induced by $F \times G$. Denote by $\mu_i^{F \times G}$ and $\mu_{-i}^{F \times G}$ the induced probability measures on the measurable spaces $S_i \times \Omega_i$ and $S_{-i} \times \Omega_{-i}$.

Lemma 1 *Every measurable set $Z_i \subseteq S_i \times \Omega_i$ can be represented uniquely as $\cup_{s'_i \in S_i} (\{s'_i\} \times A(s'_i))$, where $A(s'_i)$ are measurable subsets of Ω_i .*

Proof. The set $\Delta_i = \{\cup_{s'_i \in S_i} (\{s'_i\} \times A(s'_i)) : A(s'_i) \text{ are measurable subsets of } \Omega_i\}$ is a σ -algebra: $S_i \times \Omega_i$ is an element of Δ_i , the complement of an element of Δ_i is in Δ_i , and a countable union of elements in Δ_i are in Δ_i . Moreover, Δ_i is the smallest σ -algebra of $S_i \times \Omega_i$

with respect to which the projection mappings $\pi_1 : S_i \times \Omega_i \rightarrow S_i$ and $\pi_2 : S_i \times \Omega_i \rightarrow \Omega_i$ are continuous. To see that the projection mappings are continuous, note that for any $B \subseteq S_i$,

$$\pi_1^{-1}(B) = \cup_{s'_i \in B} (\{s'_i\} \times \Omega_i) \cup_{s'_i \notin B} (\{s'_i\} \times \phi),$$

and for any $C \subseteq \Omega_i$,

$$\pi_2^{-1}(C) = \cup_{s'_i \in S_i} (\{s'_i\} \times C).$$

Now consider some σ -algebra $\tilde{\Delta}_i$ of $S_i \times \Omega_i$ with respect to which the projection mappings are continuous. By continuity, for any $s'_i \in S_i$ and measurable $B \subseteq \Omega_i$, the sets $\pi_1^{-1}(\{s'_i\}) = \{s'_i\} \times \Omega_i$ and $\pi_2^{-1}(B) = \cup_{s'_i \in S_i} (\{s'_i\} \times B)$ are elements of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under finite intersections, $(\{s'_i\} \times \Omega_i) \cap \cup_{s'_i \in S_i} (\{s'_i\} \times B) = \{s'_i\} \times B$ is an element of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under countable unions, $\Delta_i \subseteq \tilde{\Delta}_i$. By definition as the smallest σ -algebra with respect to which the projection mappings are continuous, the product σ -algebra on $S_i \times \Omega_i$ is therefore Δ_i , so every measurable subset of $S_i \times \Omega_i$ is an element of Δ_i . Uniqueness of the representation follows from the fact that every $s'_i \in S_i$ appears only once in the representation. ■

Consider sets Z_1, \dots, Z_n such that for every $i \in N$, Z_i is a positive-measure subset of $S_i \times \Omega_i$. Let

$$\tilde{S}_i = \{s'_i \in S_i : \mu_i^G(A(s'_i)) > 0\},$$

where $A(s'_i)$ is such that $\{s'_i\} \times A(s'_i)$ appears in the unique representation of Z_i from Lemma 1. The set \tilde{S}_i is comprised of the signals in S_i that appear in Z_i with positive probability. Let $\tilde{S} = \times_{i \in N} \tilde{S}_i$, and for every $s' = (s'_1, \dots, s'_n) \in \tilde{S}$, let

$$\delta(s') = f(s') \prod_{i \in N} \mu_i^G(A(s'_i)) > 0.$$

For every $s' \in \tilde{S}$, let $h(s') = \frac{\delta(s')}{\sum_{s'' \in \tilde{S}} \delta(s'')}$. Then h induces a probability distribution on \tilde{S} . Denote the CDF of this probability distribution by H . For every $i \in N$, let \tilde{s}_i be the random variable induced by H on \tilde{S}_i .

Lemma 2 *If the random variables s_1, \dots, s_n are affiliated (under F), then so are $\tilde{s}_1, \dots, \tilde{s}_n$ (under H).*

Proof. Choose s', s'' in $\tilde{S} \subseteq S$. Because $\tilde{S} = \times_{i \in N} \tilde{S}_i$, $(s' \vee s'') \in \tilde{S}$ and $(s' \wedge s'') \in \tilde{S}$, where \vee is the component-wise maximum and \wedge is the component-wise minimum. It remains to show that $h(s' \vee s'') h(s' \wedge s'') \geq h(s') h(s'')$. We have

$$\begin{aligned} h(s' \vee s'') h(s' \wedge s'') &= \frac{\delta(s' \vee s'') \delta(s' \wedge s'')}{(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s}))^2} \\ &= \frac{1}{(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s}))^2} f(s' \vee s'') \prod_{i \in N} \mu_i^G(A(\max(s'_i, s''_i))) f(s' \wedge s'') \prod_{i \in N} \mu_i^G(A(\min(s'_i, s''_i))) \\ &= \frac{f(s' \vee s'') f(s' \wedge s'')}{(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s}))^2} \prod_{i \in N} \mu_i^G(A(s'_i)) \mu_i^G(A(s''_i)) \end{aligned}$$

$$\geq \frac{f(s') f(s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} \prod_{i \in N} \mu_i^G(A(s'_i)) \mu_i^G(A(s''_i)) = h(s') h(s''),$$

where the inequality follows from affiliation under F . ■

In what follows, we will use the following well-known property of affiliation.

Lemma 3 *If s_1, \dots, s_n are affiliated and $v(s_1, \dots, s_n)$ is non-decreasing in each of its arguments, then $E(v(s_1, \dots, s_n) | s_1 = s'_1)$ is non-decreasing in s'_1 .*

Proof. Milgrom and Weber (1982), Theorem 5 (page 1100). ■

Corollary 3 *If s_1, \dots, s_n are affiliated and $v(s_1, \dots, s_n)$ is strictly increasing in each of its arguments, then $E(v(s_1, \dots, s_n) | s_1 = s'_1)$ is strictly increasing in s'_1 .*

Proof. Let $s''_1 \geq s'_1$. We have

$$\begin{aligned} E(v(s_1 = s'_1, \dots, s_n) | s_1 = s'_1) &\leq E(v(s_1 = s'_1, \dots, s_n) | s_1 = s''_1) \\ &< E(v(s_1 = s''_1, \dots, s_n) | s_1 = s''_1), \end{aligned}$$

where the first inequality is an application of the lemma, and the second inequality follows because it holds for every realization of s_2, \dots, s_n . ■

Suppose that a firm has some conjecture about the realization of other firms' signals. The next lemma shows that regardless of this conjecture, seeing a higher signal makes the firm more optimistic about the value of the worker.

Lemma 4 *Suppose that s_1, \dots, s_n are affiliated, and $v(s_1, \dots, s_n)$ is strictly increasing in each of its arguments. Let $i \in N$, and for every $j \neq i$ let Z_j be a positive-measure subset of $S_j \times \Omega_j$. If s'_i and s''_i are elements of S_i such that $s''_i \geq s'_i$ and A and B are positive-measure subsets of Ω_i , then $E(v|\{s''_i\} \times A, Z_{-i}) \geq E(v|\{s'_i\} \times B, Z_{-i})$. If the first inequality is strict, then so is the second.*

Proof. Because G_i is statistically independent of F and G_{-i} , and v is not a function of Ω_i , we have $E(v|\{s''_i\} \times A, Z_{-i}) = E(v|s''_i, Z_{-i})$ and $E(v|\{s'_i\} \times A, Z_{-i}) = E(v|s'_i, Z_{-i})$. Let $Z_i = \{s'_i, s''_i\} \times \Omega_i$, and define \tilde{S} from (Z_i, Z_{-i}) as described above. By Lemma 2 and Corollary 3, $E_H(v|\tilde{s}_i = s''_i, \tilde{s}_{-i}) \geq E_H(v|\tilde{s}_i = s'_i, \tilde{s}_{-i})$, with a strict inequality if $s''_i > s'_i$. Therefore, it suffices to show that $E(v|s'_i, Z_{-i}) = E_H(v|\tilde{s}_i = s'_i, \tilde{s}_{-i})$ and $E(v|s''_i, Z_{-i}) = E_H(v|\tilde{s}_i = s''_i, \tilde{s}_{-i})$. We will show the first equality; the second follows by replacing s'_i with s''_i . Using the notation introduced above, we have

$$\begin{aligned} E_H(v|\tilde{s}_i = s'_i, \tilde{s}_{-i}) &= \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-i}} h(s'_i, s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} h(s'_i, s'_{-i}) v(s'_i, s'_{-i}) \\ &= \frac{\sum_{s' \in \tilde{S}} \delta(s')}{\sum_{s'_{-i} \in \tilde{S}_{-i}} \delta(s'_i, s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \frac{\delta(s'_i, s'_{-i})}{\sum_{s' \in \tilde{S}} \delta(s')} v(s'_i, s'_{-i}) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-1}} \delta(s'_i, s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \delta(s'_i, s'_{-i}) v(s'_i, s'_{-i}) \\
&= \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-1}} f(s'_i, s'_{-i}) \underbrace{\mu_i^G(A(s'_i)) \prod_{j \neq i} \mu_j^G(A(s'_j))}_{\Omega_i}} \\
&\quad \sum_{s'_{-i} \in \tilde{S}_{-i}} f(s'_i, s'_{-i}) \underbrace{\mu_i^G(A(s'_i)) \prod_{j \neq i} \mu_j^G(A(s'_j))}_{\Omega_i} v(s'_i, s'_{-i}) \\
&= \frac{1}{\mu^{F \times G}(\{s'_i\} \times \Omega_i \times Z_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \mu^{F \times G}(\{s'_i, s'_{-i}\} \times \Omega_i \times_{j \neq i} A(s'_j)) v(s'_i, s'_{-i}) \\
&= E(v|s'_i, Z_{-i}).
\end{aligned}$$

■

For any $i \in N$ and $s'_i \in S_i$, denote by $f_i(s'_i) = \sum_{s'_{-i} \in S_{-i}} f(s'_i, s'_{-i})$ the marginal probability of s'_i . For any $s' \in S$, let $\tilde{f}(s') = \prod_{i \in N} f_i(s'_i) > 0$, and denote by \tilde{F} the CDF on S corresponding to \tilde{f} . Denote by $\mu^{\tilde{F} \times G}$ the measure on $S \times \Omega$ induced by $\tilde{F} \times G$. By definition, under $\mu^{\tilde{F} \times G}$ the measurable events in $S_i \times \Omega_i$ are statistically independent of those in $S_j \times \Omega_j$, for any $i \neq j$. Clearly, a set $X \subseteq S \times \Omega$ is $\mu^{F \times G}$ -measurable if and only if it is $\mu^{\tilde{F} \times G}$ -measurable. By definition, for any measurable subset $Z_i \subseteq S_i \times \Omega_i$ we have $\mu^{F \times G}(Z_i \times S_{-i} \times \Omega_{-i}) = \mu^{\tilde{F} \times G}(Z_i \times S_{-i} \times \Omega_{-i})$. For any $s' \in S$, let $\phi(s) = \frac{f(s)}{\tilde{f}(s')} > 0$. Let $\phi_{\min} = \min_{s' \in S} \phi(s')$ and $\phi_{\max} = \max_{s' \in S} \phi(s')$.

Lemma 5 *If X is a measurable subset of $S \times \Omega$, then*

$$\phi_{\min} \mu^{\tilde{F} \times G}(X) \leq \mu^{F \times G}(X) \leq \phi_{\max} \mu^{\tilde{F} \times G}(X).$$

Proof. For any $s' \in S$ and every measurable set $A \subseteq \Omega$, we have

$$\mu^{F \times G}(\{s'\} \times A) = f(s') \mu^G(A) = \phi(s') \tilde{f}(s') \mu^G(A) = \phi(s') \mu^{\tilde{F} \times G}(\{s'\} \times A). \quad (4)$$

A proof similar to that of Lemma 1 shows that every measurable subset of $S \times \Omega$ can be represented uniquely as $\cup_{s' \in S} \{s'\} \times A(s')$, where $A(s')$ are measurable subsets of Ω . This observation, together with equation (4) implies the result. ■

Corollary 4 *Suppose X_1, X_2, \dots is sequence of measurable subsets of $S \times \Omega$. Then $\mu^{F \times G}(X_k) \xrightarrow[k \rightarrow \infty]{} 0$ if and only if $\mu^{\tilde{F} \times G}(X_k) \xrightarrow[k \rightarrow \infty]{} 0$.*

Proof. Immediate from Lemma 5. ■

Corollary 5 *A measurable subset X of $S \times \Omega$ has positive measure under $\mu^{F \times G}$ if and only if it has positive measure under $\mu^{\tilde{F} \times G}$. For such a positive-measure set,*

$$\phi_{\min} \leq \frac{\mu^{F \times G}(X)}{\mu^{\tilde{F} \times G}(X)} \leq \phi_{\max}.$$

In particular, if $X = \times_{i \in N} Z_i$ for positive-measure sets $Z_i \subseteq S_i \times \Omega_i$, then

$$\phi_{\min} \leq \frac{\mu^{F \times G}(X)}{\prod_{i \in N} \mu^{F \times G}(Z_i \times S_{-i} \times \Omega_{-i})} \leq \phi_{\max}.$$

Proof. The first two claims are immediate from Lemma 5 and Corollary 4. The last claim follows from the definition of $\mu^{\tilde{F} \times G}$. ■

Corollary 6 *Suppose X_1, X_2, \dots and Y_1, Y_2, \dots are sequences of measurable subsets of $S \times \Omega$, and $\mu^{F \times G}(Y_k)$ is bounded away from 0 for all k . Then (i) $\mu^{\tilde{F} \times G}(Y_k)$ is bounded away from 0 for all k , and (ii) $\mu^{F \times G}(X_k|Y_k) \xrightarrow{k \rightarrow \infty} 0$ if and only if $\mu^{\tilde{F} \times G}(X_k|Y_k) \xrightarrow{k \rightarrow \infty} 0$.*

Proof. Part (i) is immediate from Lemma 5. For part (ii), let $C_k = X_k \cap Y_k$. Let $\mu^{F \times G}(X_k|Y_k) = \frac{\mu^{F \times G}(C_k)}{\mu^{F \times G}(Y_k)}$ and $\mu^{\tilde{F} \times G}(X_k|Y_k) = \frac{\mu^{\tilde{F} \times G}(C_k)}{\mu^{\tilde{F} \times G}(Y_k)}$. Because both $\mu^{F \times G}(Y_k)$ and $\mu^{\tilde{F} \times G}(Y_k)$ are at most 1 and are bounded away from 0, $\mu^{F \times G}(X_k|Y_k) \xrightarrow{k \rightarrow \infty} 0$ if and only if $\mu^{F \times G}(C_k) \xrightarrow{k \rightarrow \infty} 0$, and $\mu^{\tilde{F} \times G}(X_k|Y_k) \xrightarrow{k \rightarrow \infty} 0$ if and only if $\mu^{\tilde{F} \times G}(C_k) \xrightarrow{k \rightarrow \infty} 0$. Now apply the Corollary 4 to the sequence C_1, C_2, \dots . ■

A.3 Proof of Theorem 2

Recall that the ratio between any two firms' interview costs is at most some $M > 1$. Choose some $M > 1$, and consider a sequence of strictly positive interviewing costs $c^k = (c_1^k, \dots, c_n^k)$ whose maximal element approaches 0 and which satisfy $\max_{i,j \in N} \frac{c_i^k}{c_j^k} < M$. Choose a sequence of strictly positive entry fees $e^k = (e_1^k, \dots, e_n^k)$ (that need not approach 0). Choose the entry fees and interviewing costs low enough so that firm 1 enters in any equilibrium with revelation.

Lemma 6 *For low entry and interviewing costs e_1 and c_1 , with revelation it is strictly optimal for firm 1 to enter with probability 1, regardless of other firms' strategies.*

Proof. With no revelation, Assumption A1 guarantees that for low interviewing costs firm 1 enters with probability 1. With revelation, firm 1 is weakly better off conditional on entering than with no revelation, regardless of other firms' strategies (because its offer is always accepted, it can mimic its no-revelation outcome by ignoring other firms' interviewing decisions). Therefore, for low interviewing costs firm 1 enters with probability 1 with revelation. ■

Because signals are affiliated and v is increasing, a higher signal is good news about a worker's value for any interview schedule of the other firms. This implies the following result.

Lemma 7 For any $s''_i > s'_i$ such that $\sigma_i^k(s'_i) > 0$ and $\sigma_i^k(s''_i) > 0$, If $\tau_i^k(s'_i; \mathcal{I}) > 0$, then $\tau_i^k(s''_i; \mathcal{I}) = 1$.

Proof. Because $\sigma_i^k(s'_i) > 0$ and $\tau_i^k(s'_i; \mathcal{I}) > 0$, conditional on observing s'_i , interviewing, and observing interview schedule \mathcal{I} , firm i weakly prefers making an offer to not making an offer. Therefore, $E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s'_i) \geq 0$. By Lemma 4, $s''_i > s'_i$ implies that $E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s''_i) > 0$, so conditional on observing s''_i , interviewing, and observing interview schedule \mathcal{I} , firm i is strictly better off making an offer than not making an offer. ■

We will show by reverse induction on $i \in \{2, \dots, n\}$ that for low maximal interviewing costs (large enough k) firm i enters with probability 0 in any equilibrium with revelation, entry costs e^k , and interviewing costs c^k . This will prove Theorem 2. Choose $i \in \{2, \dots, n\}$, and suppose that for large enough k all firms $j > i$ enter with probability 0 in any equilibrium with revelation, entry costs e^k , and interviewing costs c^k . It suffices to show that for large enough k firm i enters with probability 0. Suppose in contradiction that there exists a subsequence of interviewing costs, without loss of generality the sequence itself, such that for any e^k and c^k in the sequence there exists a corresponding equilibrium q^k with revelation in which firm i enters with positive probability. Because entry is costly, for every k and equilibrium q^k in the sequence, firm i must make strictly positive expected profits conditional on entering.

Consider the following preliminary observation: If, given a set of entering firms, a firm interviews with sufficiently small probability, which depends only on the distribution F of the signals, then there is no signal conditional on which the firm interviews with probability 1. In particular, interviewing is not a strict best reply for any signal, so conditional on interviewing the firm expects a profit of 0. This observation is true because F has finite, full support. Because firm i makes strictly positive expected profits conditional on entering in q^k , the preliminary observation means that for every k there is some set J^k of firms that enter in q^k with positive probability, with $i \in J^k$, such that when the set of firms that enter is precisely J^k , firm i interviews with a probability that is uniformly bounded away from 0 for all k .

Consider firm i 's strategy in the equilibrium q^k when the set of entering firms is the set J^k specified above. By Lemma 6, $1 \in J^k$. Because firm i interviews with positive probability at signal \underline{s}_i^k , there is an interview schedule \mathcal{I} with $i \in \mathcal{I}$ such that conditional on \underline{s}_i^k (i) with positive probability precisely the firms in \mathcal{I} interview and all firms in \mathcal{I} ranked higher than i do not hire and (ii) conditional on this event, firm i 's expected value of the worker is positive. Formally, $\Pr(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k | s_i = \underline{s}_i^k) > 0$ and

$$E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = \underline{s}_i^k) > 0. \quad (5)$$

If not, then conditional on interviewing with \underline{s}_i^k , firm i could not cover its interviewing costs. We now show that this \mathcal{I} can only be the singleton $\{i\}$. Let $j = \min \mathcal{I}$ be the highest-ranked firm in \mathcal{I} , and suppose $j \neq i$. Because $\Pr(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k) > 0$, the signal $\bar{s}_{j,\mathcal{I}}^k$ is well defined (there is at least one signal for which firm j interviews with positive probability

and hires with a probability less than 1 when precisely the firms in \mathcal{I} interview). Because firms' signals are affiliated and v is increasing,

$$0 < E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = \underline{s}_i^k) \quad (6)$$

$$\leq E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k) \quad (7)$$

$$\leq E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_j = \bar{s}_{j,\mathcal{I}}^k) \quad (8)$$

$$\leq E(v|\hat{\mathcal{I}}, s_j = \bar{s}_{j,\mathcal{I}}^k). \quad (9)$$

The first inequality between conditional expectations follows from the definition of \underline{s}_i^k as i 's lowest signal consistent with $\hat{\mathcal{I}}$,²³ the second from the definition of $\bar{s}_{j,\mathcal{I}}^k$ as the highest signal of j consistent with $\Phi_{i,\mathcal{I}}^k$,²⁴ and the third from the fact that $\Phi_{i,\mathcal{I}}^k$ is bad news about the worker's value (Lemma 7).²⁵

The inequality $0 < E(v|\hat{\mathcal{I}}, s_j = \bar{s}_{j,\mathcal{I}}^k)$ implies that in the positive-probability event in which firm j sees signal $\bar{s}_{j,\mathcal{I}}^k$ and interview schedule \mathcal{I} (at which firm j interviews, because $j \in \mathcal{I}$), firm j would profit from hiring the worker. Because j is the strongest firm in \mathcal{I} , it would hire the worker if it made him an offer. Thus, j strictly prefers to make an offer at $\bar{s}_{j,\mathcal{I}}^k$, whereas by definition it makes an offer at $\bar{s}_{j,\mathcal{I}}^k$ with a probability less than 1, a contradiction. This shows that $j = i$, so $\mathcal{I} = \{i\}$ and $\Pr(\Phi_{i,\mathcal{I}}^k) = 1$.

Because $\mathcal{I} = \{i\}$ is the only schedule satisfying (5), this schedule arises with positive probability conditional on firm i seeing the signal \underline{s}_i^k , as discussed above. This means that every entering firm $j \in J^k \setminus \{i\}$ interviews with probability less than 1. Recall that \bar{s}_j^k is the highest signal for which firm j interviews with probability less than 1. From (5) and because $\Pr(\Phi_{i,\mathcal{I}}^k) = 1$, for any $j \in J^k \setminus \{i\}$ we have

$$0 < E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \quad (10)$$

$$\leq E(v|\hat{\mathcal{I}})$$

$$\leq E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k).$$

These inequalities follow, as above, from the assumption that firms' signals are affiliated and v is increasing.

Lemma 8 *There exists some $\delta > 0$ and a subsequence, without loss of generality the sequence itself, such that for all large enough k ,*

$$E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) > \delta \quad (11)$$

for some $j \in J^k \setminus \{i\}$.

²³For every s_i^k with $\sigma_i^k(s_i^k) > 0$, apply Lemma 4 with $Z_{-i} = \hat{\mathcal{I}}_{-i} \cap \Phi_{i,\mathcal{I}}^k$, $s'_i = \underline{s}_i^k$, $A = \sigma_i^k(\underline{s}_i^k)$, $s''_i = s_i^k$, and $B = \sigma_i^k(s_i^k)$.

²⁴For every s_j^k with $\sigma_j^k(s_j^k) > 0$ and $\tau_j^k(s_j^k; \mathcal{I}) < 1$, apply Lemma 4 with $Z_{-j} = \hat{\mathcal{I}}_{-j} \cap \Phi_{i,\mathcal{I},-j}^k$, $s'_i = s_i^k$, $A = \{\omega_j : (s_j^k, \omega_j) \notin \tilde{O}_{j,\mathcal{I}}^k\}$, $s''_i = \bar{s}_{j,\mathcal{I}}^k$, and $B = \{\omega_j : (s_j^k, \omega_j) \notin \tilde{O}_{j,\mathcal{I}}^k\}$.

²⁵Apply Lemma 4 iteratively, for every $l \in \mathcal{I} \setminus \{i\}$, as in the previous footnote.

Proof. By (10), the claim is clearly true if there exists some $\delta > 0$ and a subsequence such that for all large enough k either $E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) \geq \delta$ for some $j \in J^k$, or $E(v|\hat{\mathcal{I}}) - E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \geq \delta$. Suppose to the contrary that for every $\delta > 0$ there exists some $R(\delta)$ such that for all $k > R(\delta)$ and every firm $j \in J^k \setminus \{i\}$ we have (i) $E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) < \delta$ and (ii) $E(v|\hat{\mathcal{I}}) - E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) < \delta$. The inequality (i) implies that for every firm $j \in J^k$

$$\frac{\Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)}{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k)} \xrightarrow{k \rightarrow \infty} 0,$$

because whenever $\Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k) \neq 0$

$$E(v|\hat{\mathcal{I}}) = \frac{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k)}{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k) + \Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)} E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) \quad (12)$$

$$+ \frac{\Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)}{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k) + \Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)} E(v|\hat{\mathcal{I}}, s_j \neq \bar{s}_j^k) \quad (13)$$

so

$$E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) = \frac{\Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)}{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k) + \Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)} \left(E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}, s_j \neq \bar{s}_j^k) \right)$$

and v is strictly increasing. Moving on, by Corollary 5

$$\frac{\Pr(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k)}{\Pr(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k)} = R \frac{\Pr(\neg \tilde{I}_j^k \setminus (\bar{s}_j^k \times \Omega_j))}{\Pr(\neg \tilde{I}_j^k \cap (\bar{s}_j^k \times \Omega_j))}$$

for some constant $R > 0$ that depends only on the distribution F . Therefore,

$$\frac{\Pr(\neg \tilde{I}_j^k \setminus (\bar{s}_j^k \times \Omega_j))}{\Pr(\neg \tilde{I}_j^k \cap (\bar{s}_j^k \times \Omega_j))} \xrightarrow{k \rightarrow \infty} 0. \quad (14)$$

Similarly, (ii) implies that

$$\frac{\Pr(\tilde{I}_i^k \setminus (\underline{s}_i^k \times \Omega_i))}{\Pr(\tilde{I}_i^k \cap (\underline{s}_i^k \times \Omega_i))} \xrightarrow{k \rightarrow \infty} 0. \quad (15)$$

For every $l \in J^k \setminus \{i\}$, by (repeatedly) decomposing $E(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k)$ as we did $E(v|\hat{\mathcal{I}})$ in (12) and applying Corollary 5 using (14) for all $j \in J^k \setminus \{i, l\}$ and (15), we obtain

$$E(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k) \xrightarrow{k \rightarrow \infty} E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k), \quad (16)$$

where $-i$ is the set of indices $J^k \setminus \{i\}$. Now consider two possibilities. The first is that for some subsequence, without loss of generality the sequence itself,

$$E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k) \xrightarrow{k \rightarrow \infty} x, x \leq 0.$$

Then, because the number of signals is finite, $E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k) \leq 0$ holds for all large enough k . But then for large enough k we have

$$E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \leq E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k) \leq 0,$$

a contradiction to (10). The second possibility is that for some subsequence, without loss of generality the sequence itself,

$$E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k) \rightarrow x, x > 0.$$

Then, because the number of signals is finite, $E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k) > 2\delta$ holds for some fixed $\delta > 0$ for all large enough k . This, together with (16), implies that (11) holds for large enough k and any $j \in J^k \setminus \{i\}$. ■

Now suppose that for the $j \in J^k$ specified in Lemma 8, $\Pr(I_i^k \cap_{m \in J^k \setminus \{i,j\}} \neg I_m^k | s_j = \bar{s}_j^k)$ is bounded away from 0 along some subsequence, without loss of generality the sequence itself. Lemma 8 above shows that for some $\alpha > 0$ and all large enough k we would have

$$\Pr(I_i^k \cap_{m \in J^k \setminus \{i,j\}} \neg I_m^k | s_j = \bar{s}_j^k) E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) \geq \alpha.$$

But for large enough k , $c_j^k < \alpha$, so it is strictly optimal for firm j to interview with probability 1 at \bar{s}_j^k (and make an offer, which will be accepted, when firm i interviews and all other firms do not interview). This contradicts the definition of \bar{s}_j^k as the highest signal for which firm j interviews with probability less than 1.

Therefore,

$$\Pr(I_i^k \cap_{m \in J^k \setminus \{i,j\}} \neg I_m^k | s_j = \bar{s}_j^k) \xrightarrow{k \rightarrow \infty} 0 \quad (17)$$

for some $j \in J^k$. The fact that $\Pr(I_i^k)$ is bounded away from 0 and (17) imply that $\Pr(\neg I_l^k) \rightarrow 0$ for some firm $l \in J^k \setminus \{j, i\}$.²⁶ (if $J^k = \{i, j\}$, which happens for example if $n = 2$, we have a contradiction and we are done.) For this firm $l \in J^k \setminus \{j, i\}$, therefore, $\frac{\Pr(I_i^k)}{\Pr(\neg I_l^k)} \xrightarrow{k \rightarrow \infty} \infty$. By definition of conditional expectation and Corollary 5,

$$\begin{aligned} \frac{\Pr(I_i^k \cap_{m \in J^k \setminus \{i,l\}} \neg I_m^k | s_l = \bar{s}_l^k)}{\Pr(\cap_{m \in J^k \setminus \{i\}} \neg I_m^k | s_i = \underline{s}_i^k)} &= \frac{\Pr(s_i = \underline{s}_i^k)}{\Pr(s_l = \bar{s}_l^k)} \frac{\Pr(I_i^k \cap_{m \in J^k \setminus \{i,l\}} \neg I_m^k \cap s_l = \bar{s}_l^k)}{\Pr(\cap_{m \in J^k \setminus \{i\}} \neg I_m^k \cap s_i = \underline{s}_i^k)} \\ &\geq \frac{\Pr(s_i = \underline{s}_i^k)}{\Pr(s_l = \bar{s}_l^k)} \frac{\Pr(I_i^k) \prod_{m \in J^k \setminus \{i,l\}} \Pr(\neg I_m^k) \Pr(s_l = \bar{s}_l^k)}{\Pr(\neg I_l^k) \prod_{m \in J^k \setminus \{i,l\}} \Pr(\neg I_m^k) \Pr(s_i = \underline{s}_i^k)} R = \frac{\Pr(I_i^k)}{\Pr(\neg I_l^k)} R \end{aligned}$$

for some constant $R > 0$ that depends only on the distribution F . We conclude that

$$\frac{\Pr(I_i^k \cap_{m \in J^k \setminus \{i,l\}} \neg I_m^k | s_l = \bar{s}_l^k)}{\Pr(\cap_{m \in J^k \setminus \{i\}} \neg I_m^k | s_i = \underline{s}_i^k)} \xrightarrow{k \rightarrow \infty} \infty. \quad (18)$$

²⁶To see why, apply Corollary 6 to Equation (17), and then use Corollary 4.

Because firm i interviews at \underline{s}_i^k with positive probability,

$$\Pr\left(\bigcap_{m \in J^k \setminus \{i\}} \neg I_m^k \mid s_i = \underline{s}_i^k\right) \mathbf{E}(v \mid \hat{\mathcal{I}}, s_i = \underline{s}_i^k) \geq c_i^k. \quad (19)$$

Together with (10) for $j = l$ and the fact that $\frac{c_l^k}{c_i^k} < M$, (19) implies that

$$\Pr\left(\bigcap_{m \in J^k \setminus \{i\}} \neg I_m^k \mid s_i = \underline{s}_i^k\right) E(v \mid \hat{\mathcal{I}}, s_l = \bar{s}_l^k) > \frac{c_l^k}{M}.$$

For large enough k , (18) gives us

$$\Pr\left(I_i^k \cap_{m \in J^k \setminus \{i, l\}} \neg I_m^k \mid s_l = \bar{s}_l^k\right) E(v \mid \hat{\mathcal{I}}, s_l = \bar{s}_l^k) > c_l^k.$$

But then, for large enough k , it is strictly optimal for firm l to interview with probability 1 at \bar{s}_l^k . This contradicts the definition of \bar{s}_l^k , and therefore shows that for large enough k there is no equilibrium with revelation and costs c^k in which firm i enters with positive probability.

B The Results of Section 7

For expositional simplicity, we assume that interview costs are the same for all firms.²⁷

B.1 Preliminaries

To describe the unique equilibrium given an ordering of firms we require the following preliminaries. We denote the ordering by $\pi = (i_1, \dots, i_n)$, with the interpretation that if two different firms i_j and i_k enter, then $j < k$ means that firm i_j makes its interview decision before firm i_k . In particular, if all firms enter and interview then firm i_k is the k^{th} firm that interviews. Given a non-empty subset of firms N^q that includes firm 1, denote by K^q the subsequence of N^q according to the ordering π , such that a firm is in K^q if and only if it is ranked higher than all the firms in N^q that follow it in π . Note that firm 1 and the last firm in N^q according to π are in K^q , and that the firms in K^q are ordered in π from the highest-ranked firm to the lowest-ranked firm. The procedure we just described defines the set of firms $K^q \subseteq N^q$ from the set N^q . For example, suppose that $n = 6$ and $\pi = (2, 1, 3, 5, 6, 4)$. If $N^q = N$, then $K^q = \{1, 3, 4\}$. If $N^q = N \setminus \{4\}$, then $K^q = \{1, 3, 5, 6\}$.

We now define a set of firms N^{q+1} from N^q and K^q . Suppose the firms in K^q make their interview decisions in sequence, from the highest-ranked firm to the lowest ranked firm, using the following decision rule. Firm i in K^q interviews if and only if all higher-ranked firms in K^q did not interview and the expected value of the worker conditional on

²⁷When interview costs differ across firms, the same use of M that appears in the proof of Theorem 2 is needed in Theorem 3. The proof is then also more technically involved, because Lemma 10 below requires more work, but is conceptually similar to the proof presented here for the case of identical interview costs.

firm i 's signal and on all higher-ranked firms in K^q not interviewing is strictly positive (ignoring interview costs). For each firm i in K^q , this defines a set of signals L_i^q for which it interviews conditional on all higher-ranked firms in K^q not interviewing. If this set of signals is non-empty for all firms in K^q , then set $N^{q+1} = N^q$ (which implies $K^{q+1} = K^q$). Otherwise, denote by p^q the highest-ranked firm i in K^q for which L_i^q is empty, and set $N^{q+1} = N^q \setminus \{p^q\}$. By assumption A1, $p^q \neq 1$ so firm 1 is in N^{q+1} (recall that firm 1 is assumed in N^q). Now, let $N^1 = N$ and iteratively define N^q and K^q for all $q \geq 1$. Because there are n firms in N , for all $q > n - 1$ we have $N^q = N^{n-1}$ and $K^q = K^{n-1}$. Let $K(\pi) = K^{n-1}$.

B.2 Statement and Proof of Theorem 3

Theorem 3 *For low entry and interview costs, when firms make interview decisions sequentially according to an ordering π there is a unique equilibrium outcome. In this outcome, the firms in $K(\pi)$ enter and the firms in $N \setminus K(\pi)$ do not enter. The firms in $K(\pi)$ make their interview decisions in sequence, from the highest-ranked firm to the lowest-ranked firm. Firm i in $K(\pi)$ interviews if and only if all higher-ranked firms in $K(\pi)$ did not interview and the expected value of the worker conditional on firm i 's signal and on all higher-ranked firms in $K(\pi)$ not interviewing is strictly positive. An interviewing firm makes the worker an offer.*

The proof of Theorem 3 requires several lemmas. The first Lemma shows that if a firm interviews, then all lower-ranked firms that appear later in the ordering do not interview.

Lemma 9 *Suppose that firm j appears before firm i in π and that $j < i$. If both firms enter and firm j interviews, then firm i does not interview.*

Proof. Suppose to the contrary that both firms enter and firm i interviews with positive probability after firm j interviews. Consider the lowest signal \underline{s}_i for which firm i interviews with positive probability after firm j interviews. Because interviewing is costly, there is an interview schedule for which firm i hopes to profitably hire the worker at signal \underline{s}_i after seeing firm j interview. Because firm j interviews in this interview schedule, it must be that firm j does not make an offer with certainty when this interview schedule arises. But a sequence of inequalities similar to Inequalities (6)-(8) shows that when this interview schedule arises firm j could profitably deviate by making an offer with certainty at the highest signal for which it interviews and does not make an offer with certainty. ■

The next Lemma shows that a firm will not interview if a higher-ranked firm that appears after it in π enters.

Lemma 10 *Suppose that firm i appears before firm j in π and $j < i$. For low interview costs, if both firms enter then firm i does not interview.*

Proof. Suppose that both firms enter and firm i interviews, and consider the lowest signal \underline{s}_i for which firm i interviews with positive probability. By Lemma 9, when firm i interviews, all lower-ranked entering firms that appear after firm i in the ordering do not interview. By the same argument used in the proof of Lemma 9, in any interview schedule for which firm i hopes to hire the worker at signal \underline{s}_i all entering firms that rank higher than firm i and appear after firm i in the ordering do not interview. Therefore, in such a schedule all entering firms that appear after firm i in the ordering do not interview. Consider such an interview schedule \mathcal{I} that arises with positive probability, and denote by k the last entering firm in the ordering ranked higher than i . Because \mathcal{I} arises with positive probability, firm k does not always interview when the firms preceding it in the ordering make interview decisions in accordance with \mathcal{I} . But when the firms preceding firm k in the ordering make interview decisions in accordance with \mathcal{I} , at the highest signal for which firm k does not interview with certainty, for low interview costs firm k would be strictly better off if it interviewed with certainty and made an offer.²⁸ That it would be weakly better off is immediate because interview costs are the same for firms i and k . Strict improvement is not difficult to show.²⁹ ■

The following corollary is an immediate implication of Lemmas 9 and 10.

Corollary 7 *For low interview costs, in any equilibrium an entering firm i that is not followed in π by a higher-ranked entering firm interviews (and makes an offer) if and only if all higher-ranked entering firms that precede firm i in π do not interview and the expected value of the worker conditional on this event and the firm's signal is strictly positive.*

Recall that the firms in $K(\pi)$ are ordered in π from the highest-ranked firm to the lowest-ranked firm. Corollary 7 shows that to complete the proof of Theorem 3 it remains to show that for low entry and interview costs the set of entering firms is $K(\pi)$. This is the content of Lemma 12, which relies on the following Lemma.

Lemma 11 *Suppose that $N^q \neq N^{q+1}$. For any $r \geq q$ the firms in K^q that are ranked higher than firm p^q (and therefore precede p^q in π) are in K^r , and any firm in $K^r \setminus K^q$ is ranked lower than firm p^q .*

Proof. The proof is by induction on r . For $r = q$ the claim holds trivially. Suppose the claim holds for $r - 1 \geq q$. To show that the first part of the claim holds for r , suppose in contradiction that a firm i in K^q ranked higher than firm p^q is not in K^r . Because firm i is in K^{r-1} , it is higher-ranked than all the firms in N^{r-1} that follow it in π . Firm i is therefore

²⁸Because firm k is the last entering firm in the ordering ranked higher than firm i , firm k 's interview decision does not affect any of the following firms' behavior - they do not interview since firm i interviewed and they are ranked lower than firm i .

²⁹If the probability that all firms between i and k do not interview is smaller than 1, then the fact that it is weakly profitable for firm i to interview means that it is strictly profitable for firm k to deviate. If the probability that all firms between i and k do not interview is 1, then an argument similar to the one used in Lemma 8 shows that it is strictly profitable for firm k to deviate when interview costs are small.

higher-ranked than all the firms in any subset of N^{r-1} that follow it in π . Consequently, if firm i were in $N^r \subseteq N^{r-1}$, it would also be in K^r . This means that $p^{r-1} = i$, so L_i^{r-1} is empty. By definition, whether L_i^{r-1} is empty depends only on which firms in K^{r-1} rank higher than firm i . By the induction hypothesis and because firm i is ranked higher than firm p^q , the firms in K^{r-1} that rank higher than firm i are the firms in K^q that rank higher than firm i . But because p^q is the highest-ranked firm j in N^q for which L_j^q is empty and firm i is ranked higher than firm p^q , L_i^q is non-empty. Therefore, L_i^{r-1} is non-empty, a contradiction. To show that the second part of the claim holds for r , it suffices to show that any firm in $K^r \setminus K^{r-1}$ is ranked lower than firm p^q . If $N^r = N^{r-1}$, this is true because $K^r = K^{r-1}$. If $N^r \neq N^{r-1}$ and $K^r \neq K^{r-1}$, consider a firm i in $K^r \setminus K^{r-1}$. That firm i is not in K^{r-1} means that a higher-ranked firm in N^{r-1} follows it in π ; that firm i is in K^r means that no higher-ranked firm in N^r follows it in π . Therefore, firm p^{r-1} is ranked higher than firm i . To show that firm i is ranked lower than firm p^q , it suffices to show that firm p^{r-1} is ranked no higher than firm p^q . But p^{r-1} is in K^{r-1} by definition, and the first part of the claim, which we proved for r , shows that any firm in K^{r-1} ranked higher than firm p^q is in $K^r \subseteq N^r$, whereas p^{r-1} is not in N^r . ■

Lemma 12 *For low entry and interview costs the set of entering firms in any equilibrium is $K(\pi)$.*

Proof. Suppose that there exists an equilibrium E in which with positive probability the set of entering firms is different from $K(\pi)$. Denote by A the set of firms that enter with positive probability in this equilibrium, by B the set of firms that enter with probability less than 1 in this equilibrium (including those that enter with probability 0), and by C the set $K(\pi) \cap B$. Note that $N = A \cup B$. Suppose C is empty (so all firms in $K(\pi)$ enter with probability 1 in the equilibrium), and let i be the highest-ranked firm among the firms in $A \setminus K(\pi)$ (such a firm exists because A is assumed different from $K(\pi)$). By Lemma 10 and the fact that firm i enters with positive probability in E , firm i is ranked higher than all the firms in $K(\pi)$ that follow it in π . By Corollary 7 and the definition of i , firm i 's interview decisions and profits conditional on entry in the equilibrium E are exactly what they would be if only the firms in $K(\pi)$ that precede firm i in π and are ranked higher than firm i entered. We will show that the latter profits are non-positive, so that firm i can not recover its entry costs in E . Because firm i is ranked higher than all the firms in $K(\pi)$ that follow it in π , firm i is not in N^{n-1} (otherwise it would be in $K^{n-1} = K(\pi)$). Therefore, it must be the firm p^q removed from the set of firms N^q for some $q \geq 1$. By Lemma 11, the firms in $K(\pi)$ that precede firm i in π and are ranked higher than firm i are precisely those firms in K^q that precede firm i in π . But Corollary 7 and the fact that firm i was removed from N^q means that when this is the set of entering firms ranked higher than firm i , firm i cannot recover its entry costs for any positive interview costs. Thus, firm i would not enter with positive probability in the equilibrium E , a contradiction. This shows that the set C is not empty.

Consider now the highest-ranked firm j in $C \subseteq K(\pi)$. In the equilibrium E , firm j enters with probability less than 1 and all firms in $K(\pi)$ ranked higher than firm j enter

with probability 1. We will show that for low entry and interview costs firm j would be better off entering with probability 1. There are two possibilities to consider. The first is that one or more firms not in $K(\pi)$ that are ranked higher than firm j enter with positive probability in E . In this case, denote by i the the highest-ranked firm among these firms. Because firm i is not in $K(\pi) = K^{n-1}$, one of two things is true. If firm i is in $N^{n-1} \setminus K^{n-1}$, then there is a firm l in K^{n-1} ranked higher than firm i that appears after firm i in π . Because firm i is ranked higher than firm j , by definition of firm j we have that firm l enters with probability 1 in E . But then Lemma 10 and the fact that firm l appears after i in π imply that firm i would not enter in E , a contradiction. If firm i is not in N^{n-1} , then it was removed from N^q for some $q \geq 1$. But then an argument similar to the one used in proving that C is not empty shows that firm i would not enter in E . The second possibility is that the only firms not in $K(\pi)$ that enter with positive probability in E are ranked lower than firm j . In this case, by Corollary 7 and the definition of j , firm j 's interview decisions and profits conditional on entry in the equilibrium are exactly what they would be if only the firms in $K(\pi)$ preceding it in the ordering π entered. But then, by definition of $K(\pi)$ and Corollary 7, firm j can enter and make strictly positive profits for low entry and interview costs. ■

Theorem 3 is an immediate implication of Corollary 7 and Lemma 12.

B.3 Statement and Proof of Proposition 3

The following proposition describes the connection between the hiring outcome with sequential interviews and the hiring outcome with simultaneous interviews. We use it to show that the hiring outcomes we obtained for simultaneous interviews with revelation and with no revelation correspond to the outcomes of sequential interviews with the orderings $(n, n-1, \dots, 1)$ and $(1, 2, \dots, n)$.

Proposition 3 *Choose an ordering π . When entry and interview costs are low, the hiring outcome corresponding to the equilibrium E of Theorem 3 (sequential interviews with ordering π) when the set of firms is N is the same as the hiring outcome corresponding to the equilibrium \bar{E} of Theorem 1 (simultaneous interviews with no revelation) when the set of firms is $K(\pi)$ instead of N . In both equilibria precisely the firms in $K(\pi)$ enter and make positive profits. The expenditures on interviews are lower and firms' profits are higher with sequential interviews.*

Proof. Suppose $K(\pi) = \{i_1, \dots, i_k\}$, where $i_j < i_l$ if $j < l$. We prove by induction on j that for low entry and interview costs, in the equilibrium \bar{E} of Theorem 1 when the set of firms is $K(\pi)$ (i) firm i_j enters and (ii) firm i_j hires the worker for the same vectors of signals for which it hires the worker in the equilibrium E of Theorem 3 when the set of firms is N and interviews are conducted sequentially according to the ordering π . For $j = 1$, recall that firm 1 is in K^r for all $r \geq 1$, so firm 1 is in $K(\pi)$ and $i_1 = 1$. By assumption A1, for low entry costs firm 1 enters in \bar{E} . As discussed in Section 4, for low interview costs and with no revelation firm 1 interviews (and makes an offer) when the

expected value of the worker conditional on the signal is positive ((1) holds strictly with $c_1 = 0$). Therefore, (ii) holds for firm 1, and firm 1's interview expenditures and profits are identical in E and in \bar{E} . Suppose the claim holds for $j \leq l$, and consider firm i_{l+1} . Suppose that firm i_{l+1} enters in \bar{E} . As in the proof of Theorem 1, for low interview costs firm i_{l+1} interviews and make an offer to the worker at signal $s'_{i_{l+1}}$ in \bar{E} if and only if

$$\Pr\left(B|s_{i_{l+1}} = s'_{i_{l+1}}\right) E\left[v|B, s_{i_{l+1}} = s'_{i_{l+1}}\right] > 0, \quad (20)$$

where B is the set of signal vectors at which none of the firms i_1, \dots, i_l interviews the worker in \bar{E} . Firm i_{l+1} succeeds in hiring the worker if, in addition, the realized vector of signals is in B . B is also the set of signal vectors at which none of the firms i_1, \dots, i_l hires the worker in \bar{E} (because with no revelation every firm that interviews makes an offer). By the induction hypothesis, B is the set of signal vectors at which none of the firms i_1, \dots, i_l interviews the worker in E (because in E a firm that interviews makes an offer). Therefore, by the description of E in Theorem 3, firm i_{l+1} interviews and hires the worker at signal $s_{i_{l+1}}$ in E if and only the signals of firms i_1, \dots, i_l are in B and (20) holds. Finally, firm i_{l+1} enters in \bar{E} if and only if the set of signals $s'_{i_{l+1}}$ for which (20) holds is non-empty, and this set is non-empty if and only if firm i_{l+1} enters in E (by the description of E). This shows that (i) and (ii) hold for firm i_{l+1} , and also shows that firm i_{l+1} 's interview expenditures are lower and its profits are higher in E than in \bar{E} . Because a firm enters in \bar{E} if and only if it expects positive profits and every firm in $K(\pi)$ makes positive profits in E , the proof is complete. ■

B.4 Proof of Corollary 1

Let $\pi = (n, n-1, \dots, 1)$. The hiring outcome of Theorem 2 is as if only firm 1 were present in the market (in which case revelation is irrelevant). Therefore, the statement of the corollary follows from Proposition 3 if $K(\pi) = \{1\}$. But $K^1 = K^{n-1} = \{1\}$, because firm 1 is the only firm in N that is not followed by a higher-ranked firm in the ordering π , and by assumption A1 firm 1 interviews and makes offers for some signals, so $K^q = K^1$ for all $q > 1$.

B.5 Proof of Corollary 2

Let $\pi = (1, 2, \dots, n)$. By Proposition 3, to prove the statement about the hiring outcome it suffices to show that the equilibrium \bar{E} of Theorem 1 when the set of firms is N is the same as the equilibrium of Theorem 1 when the set of firms is $K(\pi)$. For this it suffices to show that the firms in $N \setminus K(\pi)$ do not enter in \bar{E} . First note that $K^q = N^q$ for all $q \geq 1$, because any firm in any subset of N is not followed by a higher-ranked firm in the ordering π . Therefore, $N \setminus K(\pi) = \cup_{q \geq 1} p^q$. Considering the definition of p^q and the proof of Theorem 1, we see that firm p^q does not enter in \bar{E} . The reason for the ranking of expenditures is given in the text following the statement of the corollary.

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