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CUSTOM VERSUS FASHION: PATH-DEPENDENCE AND LIMIT CYCLES IN A RANDOM MATCHING GAME*

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Abstract

A pairwise random matching game is considered to identify the social environments that give rise to the social custom and fashion cycles. The game, played by Conformists and Nonconformists, can generate a variety of socially stable behavior patterns. In the path-dependence case, Conformists set the social custom and Nonconformists revolt against it; what action becomes the custom is determined by "history." In the limit cycle case, Nonconformists become fashion leaders and switch their actions periodically, while Conformists follow with delay. The outcome depends on the relative share of Conformists to Nonconformists as well as their matching patterns.

Keywords: Best response dynamics, Bifurcation, Conformity and nonconformity, Equilibrium refinement, Evolutionary process, Limit cycles, Path-dependence, Strategic complements and substitutes, The collective selection and trickle-down theories of fashion.

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"Fashion is custom in the guise of departure from custom."

Edward Sapir (1930, p.140)

"Fashion is evolution without destination."

Agnes Brooks Young (1937, p.5)

1. Introduction

Fashion is the process of continuous change in which certain forms of social behavior enjoy temporary popularity only to be replaced by others. This pattern of change sets fashion apart from social custom, which is time-honored, legitimated by tradition, and passed down from generation to generation. Fashion is also a recurring process, in which many "new" styles are not so much born as rediscovered: Young (1937). This cyclical nature, or its regularity, sets fashion apart from fads, which are generally considered as rather bizarre one-time aberrations.[1] Although most conspicuous in the area of dress, many other areas of human activity are also under the sway of fashion. Among them are architecture, music, painting, literature, business practice, political doctrines, as well as scientific ideas (not least in economic theory). Every stage of our lives is immersed in fashion, from names bestowed at birth to the forms of gravestones. Despite its pervasiveness and its apparent significance as a determinant of variations in demand, very few attempts have been made by economists, supposedly experts of cyclical behavior, to identify mechanisms generating fashion cycles.

On the other hand, there is no shortage of theories in the fields of psychology and sociology (Sapir (1930) and Blumer (1968, 1969): see also Sproles (1985)). Two psychological tendencies are often put forward as fundamental forces behind continuous changes and diffusions of fashion. Many observers point

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[1]Blumer (1968, p.344) wrote: "The most noticeable difference (from fashion) is that fads have no line of historical continuity; each springs up independent of a predecessor and gives rise to no successor."

out the importance of conformity in the establishment of fashion; that is, the desire of people to adopt and imitate the behavior of others, or to join the crowd. Such conformist attitudes may result from widely divergent motives. People may imitate out of admiration for one imitated or by the desire to assert equality with her. Those who follow fashions may do so with enthusiasm or may simply be coerced by public opinion exercised through ridicule and social ostracism. Other observers of fashion also emphasize the importance of nonconformity. That is, they find the essence of fashion in the search for exclusiveness or the efforts of people to acquire individuality and personal distinction; they treat fashion as an expression of the desire to escape from the tyranny of the prevailing social custom and to disassociate one's self from the common masses.

It should be immediately clear that, for the recurring process of fashion to emerge and persist, these two fundamentally irreconcilable desires of human beings -- the desire to act or look the same, and the desire to act or look different-- both must operate. We cannot explain continuous changes in the process of fashion by merely pointing out that it is the product of conformity and imitation, because, if everybody imitates each other, the process would eventually cease, and certain forms of behavior would emerge as the social custom, or convention. The desire for personal distinction or exclusiveness must work against universal adaptation of a style. Nor can we adequately explain the regularity of fashion cycles by saying that it is the product of nonconformity, because, if everybody seeks individuality, the result would be disorderly, and utterly unpredictable. The forces of imitation and uniformity need to be strong enough for any discernible patterns to emerge. As Simmel ([1904]1957, p.546) wrote, "two social tendencies are essential to the establishment of fashion, namely, the need of union on the one hand and the need of isolation on the other. Should one of these be absent, fashion will not be formed--its sway will abruptly end."

This paper attempts to demonstrate in a formal model that such a delicate balance between conformity and nonconformity is not only necessary but also sufficient for the emergence of fashion cycles, while too strong preference for conformity would lead to the emergence of the social custom, and too strong preference for nonconformity lead to disorder. To be more specific, I will consider a simple pairwise random matching game, played by two types of individuals: Conformists and Nonconformists. Each individual is matched with both types of individuals with some probability: there are both intergroup and intragroup matchings. Each individual must take one of two actions, Blue and Red, at any point in time, before knowing the type of the individual s/he will meet. When matched, individuals observe the choice made by their partners. One type, the Conformist, gains a higher payoff if he and his partner have made the same choice. The other type, the Nonconformist, gains a higher payoff if she has made the choice different from her partner's.[2]

Section 2 sets up the static version of this game and characterizes the Nash equilibria (Proposition 1). In general there exist multiple Nash equilibria, which depend on the ratio of Conformists to Nonconformists and their matching patterns. In order to investigate the evolution of behavior patterns in the society, as well as to test the stability of the Nash equilibria of the static game, I apply the best response dynamics to this game in Section 3. This dynamics, proposed by Gilboa and Matsui (1991), postulates that a constant

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[2]This game between Conformists and Nonconformists was inspired by Gaertner, Pattanaik, and Suzumura (1990).

fraction of those currently playing suboptimal strategies switches to the strategy that is the best response to the current distribution of strategies. Depending on the parameters, this dynamics leads to a variety of asymptotic properties, such as global convergence, path-dependence and a limit cycle (Proposition 2). This result is used to identify socially stable behavior patterns. In the global convergence case, one half of both Conformists and Nonconformists choose Blue and the other half chooses Red, so that no patterns emerge. In the path-dependence case, Conformists set the social custom, and Nonconformists revolt against it; what action becomes the custom is determined by the initial condition, or by "history." In the limit cycle case, Nonconformists become fashion leaders and switch their actions periodically, while Conformists follow with delay.

The richness of the asymptotic properties may be understood in terms of strategic complements and substitutes: see Bulow, Geanakoplos, and Klemperer (1985) for the definitions. The two actions are strategic complements for a Conformist, and strategic substitutes for a Nonconformist. Therefore, the game, if played by a Conformist and a Nonconformist, or by a pair of Nonconformists, would have a unique Nash equilibrium, in which one half of the population chooses Blue and the other half chooses Red. This equilibrium is globally stable according to the best response dynamics. On the other hand, if played only by Conformists, there would be two additional Nash equilibria, in which every individual chooses the same action. The best response dynamics (and, in fact, any evolutionary dynamics) show that these two equilibria are locally stable, while the equilibrium with mixed strategies is unstable. Allowing for both intergroup and intragroup matchings blends two games with different properties, thereby generating much richer dynamic paths of behavior patterns. In fact, limit cycles can be generated by two kinds of bifurcation in this model. The first case is when an increase in the share of Conformists leads to a loss of stability in the unique Nash equilibrium in the game with strategic substitutes; the regular patterns of fashion cycles emerge from the disorder. The second case is when a decrease in the share of Conformists eliminates the two locally stable Nash equilibria in the game with strategic complements; fashion cycles emerge as departure from custom. In other words, the transition from disorder to fashion cycles to social custom occurs as the share of Conformists increases.

The present paper is partly motivated by the recent growing interest within economics in evolutionary dynamics and equilibrium refinement in normal form games: Friedman (1991), Gilboa and Matsui (1991), Nachbar (1990), Samuelson and Zhang (1990), and van Damme (1987, Ch. 9.4). In this literature, it is typically assumed either that the game is played by the homogeneous population or that, when the population is heterogeneous, consisting of, say, males and females, all matchings are between a male and a female.[3] It should be noted that Gilboa and Matsui have demonstrated the possibility of a limit cycle in the best response dynamics, but their example is a two person 3x3 game with a homogeneous population. One can show that the best response dynamics do not produce any cycle in a two person 2x2 game if all matchings are either intergroup or intragroup.[4] The reason why a limit cycle occurs in the two person 2x2 game discussed below is that the population is heterogeneous and both inter- and intragroup matchings are possible.

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[3]For a notable exception, see Schuster, Sigmund, Hofbauer, and Wolff (1981) on the replicator dynamics.

[4]In this sense, the best response dynamics are similar to the fictitious play (Miyasawa (1961) and Shapley (1964)), but quite different from the replicator dynamics (Schuster and Sigmund (1981) ans Maynard Smith (1982, Appendix J)).

In Section 4, I address some issues concerning model specifications and discuss conjecturally the consequences of relaxing some of the assumptions. In Section 5, alternative models of custom and fashion are discussed. In particular, it is shown that a model based on class differentiation, a formalization of the so-called trickle-down theory of fashion, can also generate path-dependence and limit cycle phenomena (Proposition 3). I argue, however, that this model is of limited relevance as a general theory of fashion cycles.

2. The Matching Game

Time is continuous and extends from zero to infinity. There lives a continuum of anonymous individuals in this society. They are divided into two groups: Conformists and Nonconformists. Let $0 < \theta < 1$ be the share of Conformists in the society. Individuals randomly meet each other in pairs. The pairwise matchings occur according to the following Poisson process.[5] At any small time interval, dt, a Conformist runs into another Conformist with probability $(1-\beta)\theta dt$; he runs into a Nonconformist with probability $\beta(1-\theta)dt$, where $0 < \beta < 1$. On the other hand, a Nonconformist runs into a Conformist with probability $\beta\theta dt$; she runs into another Nonconformist with probability $(1-\beta)(1-\theta)dt$. Note that this matching process satisfies what Schelling (1978) called "the inescapable mathematics of musical chairs." That is, the frequency of a Conformist being matched to a Nonconformist is always $(1-\theta)/\theta$ times the frequency of a Nonconformist matched to a Conformist, no matter how often intergroup matchings take place.

Two additional features of the matching process deserves some emphasis. First, by controlling β, this process allows for the entire range between the two

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[5]This process is adopted from Matsuyama, Kiyotaki and Matsui (1991).

polar cases that are commonly studied in the matching game literature: that is, the case where an individual always interacts with his/her own group ($\beta = 0$) and the case where an individual always interacts with a member of a distinct group ($\beta = 1$). Second, the probability with which one is matched with an individual from a given group is proportional to the size of that group.

It turns out that the relative frequency of intergroup matchings to intragroup matchings plays a crucial role in the following analysis. From a Conformist's point of view, the ratio of inter- versus intragroup meetings is equal to

(1a) $$m = \beta(1-\theta)/(1-\beta)\theta \ ,$$

while, for a Nonconformist, it is equal to

(1b) $$m^* = \beta\theta/(1-\beta)(1-\theta) \ .$$

Inverting Equations (1a) and (1b) yields

$$0 < \theta = \frac{\sqrt{m^*}}{\sqrt{m} + \sqrt{m^*}} < 1 \quad \text{and} \quad 0 < \beta = \frac{\sqrt{mm^*}}{1 + \sqrt{mm^*}} < 1 \ ,$$

so that any pair of positive ratios, (m, m^*), can be consistent with this matching process. There are six generic cases to be distinguished:

Case 1:	$m < 1 < mm^*$	$(1/2 < \beta < \theta)$,
Case 2:	$m < mm^* < 1$	$(1-\theta < \beta < 1/2)$,
Case 3:	$mm^* < m < 1$	$(\beta < \theta < 1-\beta)$,
Case 4:	$m > 1 > mm^*$	$(1/2 > \beta > \theta)$,
Case 5:	$m > mm^* > 1$	$(1-\theta > \beta > 1/2)$,
Case 6:	$mm^* > m > 1$	$(\beta > \theta > 1-\beta)$.

Case 6 is further divided into the two cases:

Case 6a: $m \geq m^* > 1$ $(1/2 \geq \theta > 1-\beta)$,

Case 6b: $m^* > m > 1$ $(\beta > \theta > 1/2)$.

For the sake of brevity, I will not discuss nongeneric cases, $\beta = 1/2$ ($mm^* = 1$), $\theta = \beta$ ($m = 1$), or $\theta + \beta = 1$ ($m^* = 1$).

At any moment, every individual has to choose between two actions, Blue (B) or Red (R). When matched, an individual observes the choice of his/her partner. A Conformist gains the payoff, S, if his partner has made the same choice as his, and otherwise gains D, where $S > D$. A Nonconformist gains S^* if her partner has made the same choice with hers, and otherwise gains D^*, where $S^* < D^*$. Each individual receives zero when no matching takes place. Note also that this is an anonymous game; the payoffs do not depend on the type of the matching partner.

The expected payoffs from taking each action depend on how often one is matched to those in Blue or those in Red. Let (λ_t, λ_t^*) be the <u>behavior patterns</u> in the society as of time t, where λ_t (λ_t^*) denotes the fraction of Conformists (Nonconformists) that chooses Strategy B. Then, the (instantaneous) probability with which a Conformist meets an individual who chose B and R are

$$p_{Bt} = \lambda_t(1-\beta)\theta + \lambda_t^*\beta(1-\theta) \quad \text{and} \quad p_{Rt} = (1-\lambda_t)(1-\beta)\theta + (1-\lambda_t^*)\beta(1-\theta)$$

respectively. Likewise, the probability with which a Nonconformist meets an individual who chose B and R are

$$p_{Bt}^* = \lambda_t\beta\theta + \lambda_t^*(1-\beta)(1-\theta) \quad \text{and} \quad p_{Rt}^* = (1-\lambda_t)\beta\theta + (1-\lambda_t^*)(1-\beta)(1-\theta) .$$

Thus, a Conformist's expected payoffs per unit of time if he chooses B or R are

$$\Pi_{Bt} = p_{Bt}S + p_{Rt}D \qquad \text{and} \qquad \Pi_{Rt} = p_{Bt}D + p_{Rt}S ,$$

and, for a Nonconformist,

$$\Pi^*_{Bt} = p^*_{Bt}S^* + p^*_{Rt}D^* , \qquad \text{and} \qquad \Pi^*_{Rt} = p^*_{Bt}D^* + p^*_{Rt}S^* .$$

In this section, let us further assume that each individual is free to choose between the two actions at any point in time. Then, being atomistic and anonymous, they could always play this game as if it is a static, one-shot game. Dropping the time subscript, note that $S > D$ implies $\Pi_B > \Pi_R$ if and only if $p_B > p_R$. This is to say that a Conformist chooses B if and only if he expects to see those in B more often than those in R. On the other hand, $S^* < D^*$ implies $\Pi^*_B < \Pi^*_R$ if and only if $p^*_B > p^*_R$; that is, a Nonconformist chooses R if and only if she expects to see those in B more often than those in R. More formally, the best responses by Conformists can be given by $\lambda = 1$ if $p_B > p_R$; $\epsilon\ [0,1]$ if $p_B = p_R$; $= 0$ if $p_B < p_R$; and the best responses by Nonconformists are $\lambda^* = 1$ if $p^*_B < p^*_R$; $\epsilon\ [0,1]$ if $p^*_B = p^*_R$; $= 0$ if $p^*_B > p^*_R$. After some algebra, these conditions can be written as

$$\text{(2a)} \qquad \lambda \in \begin{cases} \{1\} & \text{if } (\lambda-1/2) + m(\lambda^*-1/2) > 0 , \\ [0,1] & \text{if } (\lambda-1/2) + m(\lambda^*-1/2) = 0 , \\ \{0\} & \text{if } (\lambda-1/2) + m(\lambda^*-1/2) < 0 , \end{cases}$$

$$\text{(2b)} \qquad \lambda^* \in \begin{cases} \{0\} & \text{if } m^*(\lambda-1/2) + (\lambda^*-1/2) > 0 \ , \\ [0,1] & \text{if } m^*(\lambda-1/2) + (\lambda^*-1/2) = 0 \ , \\ \{1\} & \text{if } m^*(\lambda-1/2) + (\lambda^*-1/2) < 0 \ . \end{cases}$$

Figure 1 shows the conditions defined by (2a) and (2b) on the (λ,λ^*) space for each of the six generic cases. The two loci, $p_B = p_R$ and $p_B^* = p_R^*$, represent the behavior patterns to which Conformists and Nonconformists are respectively indifferent between B and R. They are both negatively sloped and pass through (1/2, 1/2). The slope of locus of $p_B = p_R$ is 1/m; a Conformist's best response is B above the locus and R below it. The slope of locus of $p_B^* = p_R^*$ is m^*; a Nonconformist's best response is R above the locus and B below it. The Nash equilibria, defined as the fixed points of (2a) and (2b) and depicted by the black dots in Figure 1, are summarized in the following proposition.

<u>Proposition 1</u>. Depending on the parameter values, the Nash equilibria of the static game are described by one of the following cases:

Case 1: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,0)$, $(0,1)$, $((1-m)/2,1)$ and $((1+m)/2,0)$.

Case 2: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,0)$ and $(0,1)$.

Cases 3 and 4: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,(1-m^*)/2)$ and $(0,(1+m^*)/2)$.

Cases 5 and 6: $(\lambda,\lambda^*) = (1/2,1/2)$.

Multiplicity of Nash equilibria of the static game poses some conceptual problems. To justify studying a Nash equilibrium, it is commonly assumed that all players know the entire structure of the game and also agree on which equilibrium is being played. In other words, the strategy profile is assumed to be common knowledge among the players, so that they know how to coordinate or to focus on a specific equilibrium. This assumption, which is often justified by pre-play negotiation, seems too heroic in a game with a continuum of players, such as ours. In order for players to hold confident conjectures about the actions of others, some sorts of inertia must be introduced in the behavior patterns of the society. Evolutionary processes are considered appealing as a way of explaining how a particular equilibrium is chosen and emerge in a dynamic context. Although the literature is mainly interested in the power of evolutionary dynamics in equilibrium selection, I will be also concerned with the dynamic path of behavior patterns itself.

3. Socially Stable Behavior Patterns

In an attempt to examine the dynamic stability of Nash equilibria, many evolutionary dynamics have been proposed, all of which share the following three properties. First, as in a perfectly competitive market, each player is atomistic and anonymous and thus maximizes his/her expected payoffs without getting involved in complicated strategic interactions such as retaliation or reputation. Second, it is assumed that some sorts of inertia are present in changing one's behaviors, and that only a tiny fraction of players are changing their actions at each moment. Thus, a change in behavior patterns is made in a continuous way. Third, actions that are more successful, given the current behavior patterns, replace less successful actions. These properties as well as some regularity conditions are often sufficient to test asymptotic stability of Nash equilibria.

In order to investigate the limit properties of evolution of behavior patterns, however, one needs to specify the dynamic process in detail. Here, I will consider the best response dynamics proposed by Gilboa and Matsui (1991).

According to this dynamics, the behavior patterns change only in the direction of the best response to the current behavior patterns. For example, if (1,0) is the best response profile (i.e., B is the Conformist's best response and R is the Nonconformist's best response) to (λ_t, λ_t^*), then

$$(\lambda_t, \lambda_t^*) = \alpha\{(1,0) - (\lambda_t, \lambda_t^*)\} = (\alpha(1-\lambda_t), -\alpha\lambda_t^*)$$

for a constant $\alpha > 0$, where the upper dot denotes the (right hand) derivative with respect to time. More generally, by (2a)-(2b),

$$\text{(3a)} \qquad \lambda_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & \text{if } (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) > 0 \ , \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & \text{if } (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) = 0 \ , \\ \{-\alpha\lambda_t\} & \text{if } (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) < 0 \ , \end{cases}$$

$$\text{(3b)} \qquad \lambda_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & \text{if } m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) > 0 \ , \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & \text{if } m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) = 0 \ , \\ \{\alpha(1-\lambda_t^*)\} & \text{if } m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) < 0 \ . \end{cases}$$

For any initial condition, (λ_0, λ_0^*), a dynamic path of behavior patterns is given by a solution of (3a) and (3b). Note that the set of the stationary points of dynamical system (3) is equal to the set of the Nash equilibria of the static game given in Proposition 1. Instead of the Nash equilibria, I will focus on the <u>socially stable behavior patterns</u>, i.e., the long run behavior patterns of (3) that are robust with respect to small perturbations of initial conditions.[6]

Economists often find evolutionary dynamics too mechanical and ad-hoc for a description of the human behavior. In this respect, the best response dynamics is more attractive than others, as it can be interpreted as the limiting case of the following forward looking dynamics. Suppose that an individual maximizes the expected discounted sum of payoffs, but has to make a commitment to a particular action in the short run. The opportunity to change one's action follows the Poisson process with α being the mean arrival rate, which is independent across individuals. When the opportunity arrives, an individual chooses the action which results in a higher expected discounted payoff over the next duration of commitment, knowing the future path of behavior patterns.[7] Under this formulation, the behavior patterns evolve according to

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[6]Formally, the socially stable behavior patterns can be defined by the set of the ω-limit of (3) for an open set of initial conditions, (λ_0, λ_0^*). Strictly speaking, this is different from the way Gilboa and Matsui defined the best response dynamics and the associated notions of accessibility, cyclically stable sets and social stability. In order to formalize perturbations, or mutations, they allow agents to use any distribution when randomizing, and they introduce trembling by considering the best response to the ϵ-neighborhood of the current behavior patterns. See Gilboa and Matsui (1991) for more detail.

[7]Alternatively, one can give the following interpretation, used in Matsuyama (1991, 1992). The society is populated by overlapping generations of individuals. Each individual faces an instantaneous probability of death, α, replaced by an individual of the same type. Each individual has to choose his/her strategy at the time of birth, and is restricted to stay with the strategy of his/her choice during his/her lifetime. This interpretation, while unrealistic in the context of fashion in dress, may be reasonable in other contexts, such as fashions in art, lifestyle, political ideology, and so on.

$$(4a)\qquad \lambda_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & \text{if } \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds > 0 \ , \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & \text{if } \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds = 0 \ , \\ \{-\alpha\lambda_t\} & \text{if } \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds < 0 \ , \end{cases}$$

$$(4b)\qquad \lambda_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & \text{if } \int_t^\infty \{\Pi_{Bs}^* - \Pi_{Rs}^*\} e^{(\alpha+\delta)(t-s)} ds > 0 \ , \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & \text{if } \int_t^\infty \{\Pi_{Bs}^* - \Pi_{Rs}^*\} e^{(\alpha+\delta)(t-s)} ds = 0 \ , \\ \{\alpha(1-\lambda_t^*)\} & \text{if } \int_t^\infty \{\Pi_{Bs}^* - \Pi_{Rs}^*\} e^{(\alpha+\delta)(t-s)} ds < 0 \ , \end{cases}$$

where $\delta > 0$ is the discount rate. It can be easily verified that (3) has the same phase portrait with the limit case of (4), where δ/α goes to infinity. The best response dynamics can thus be used to approximate the long run behavior patterns in the perfect foresight dynamics when the expected duration of commitment is sufficiently long. The two additional desirable properties of the best response dynamics should be noted. First, it does not rule out revival of "extinct" strategies, unlike the standard formulation of evolutionary dynamics.[8] Second, socially stable behavior patterns due to the best response dynamics are independent of α, or the degree of inertia assumed.

As shown in (3a) and (3b), the best response dynamics leads to a piecewise

[8]For example, in the replicator dynamics, developed in the evolutionary and population biology (e.g., Maynard Smith (1982) and Hofbauer and Sigmund (1988)) and widely adopted in economics (e.g., van Damme (1987, Ch. 9.4)), the growth (reproduction) rate of strategies (species) depends linearly on their relative payoffs (fitness). One implication of this specification is that any extinct strategy cannot be revived, except the possibility of mutations. It seems hard to defend this implication in our context. For example, even if all Conformists choose B at the beginning, they might get the idea of choosing R for a variety of reasons, such as watching Nonconformists choose R. And if they start switching from B to R, there is no reason to suppose that the pace of switching should be slower initially when a relatively small fraction of Conformists chooses R.

linear dynamical system and thus permits a simple geometrical analysis. Figure 2 depicts the phase portrait of (3) for each of the six generic cases. The vector always points to the vertex of the unit square that corresponds to the best response in any of the four subspaces, in which the best response is unique. When the best response is not unique, the vector belongs to the cone formed by the best response direction.

The main results can now be put forward.

Proposition 2.

The socially stable behavior patterns of (3) are given by:

Cases 1 and 2 ($m^* > 1 > m$): $(\lambda, \lambda^*) = (0,1)$ and $(1,0)$,

Cases 3 and 4 ($m^* < 1$, $m < 1/m^*$): $(\lambda, \lambda^*) = (1, (1-m^*)/2)$ and $(0, (1+m^*)/2)$,

Cases 5 and 6a ($m \geq m^* > 1/m$): $(\lambda, \lambda^*) = (1/2, 1/2)$,

Case 6b ($m^* > m > 1$):

The limit cycle, which is the parallelogram defined by the four vertices,

$$P = ((1+X_\infty/m^*)/2,\ (1-X_\infty)/2) , \qquad Q = ((1-X_\infty)/2,\ (1+X_\infty/m)/2) ,$$

$$P' = ((1-X_\infty/m^*)/2,\ (1+X_\infty)/2) , \qquad Q' = ((1+X_\infty)/2,\ (1-X_\infty/m)/2) ,$$

where $X_\infty = (m^*-m)/(mm^*-1) = (2\theta-1)\beta(1-\beta)/(2\beta-1)\theta(1-\theta)$.

As the proof, Figure 2 is sufficient for Cases 1 through 5. In Case 6, dynamic paths circle around (1/2, 1/2), as shown in Figure 2. To determine whether the paths converges to (1/2,1/2) or to the limit cycle requires some algebra, the detail of which is given in Appendix. Figure 3 also summarizes socially stable behavior patterns in terms of the parameter spaces, (m, m^*) or (θ, β).

The intuition behind Proposition 2 is easy to grasp. In Case 1 and Case 2, socially stable behavior patterns are $(\lambda, \lambda^*) = (0,1)$ and $(1,0)$. All Nash equilibria that involve mixed strategies are dynamically unstable. In these cases, the share of Conformists is sufficiently large ($\theta > \beta$, $1-\beta$), so that all individuals are matched more frequently with a Conformist, rather than with a Nonconformist ($m^* > 1 > m$). Thus, Conformists play the game primarily among themselves, while Nonconformists play the game primarily against Conformists. In these two stable equilibria, the Conformist, the majority, sets the social custom and the Nonconformist, the minority, revolt against it and acts like a social misfit. Multiplicity of stable outcomes is not surprising because Conformists form the majority and because the two actions are strategic complements from a Conformist's point of view. Which of the two stable equilibria the society would converge to depend on the initial conditions (or in other words, whether B or R becomes the social custom is determined by history.) Thus, the dynamic process exhibits path-dependence phenomena. As shown in Figure 2, if the Conformist's initial best response is B (that is, (λ_0, λ_0^*) is such that $p_{B0} > p_{R0}$), B emerges as the social custom and the society converges to (1,0). If $p_{B0} < p_{R0}$, on the other hand, R becomes the social custom and the society converges to (0,1).

In Case 3 and Case 4, socially stable behavior patterns are $(\lambda, \lambda^*) = (1,(1-m^*)/2)$ and $(0,(1+m^*)/2)$, while $(\lambda, \lambda^*) = (1/2,1/2)$ is unstable. The mixed strategies by Conformists are ruled out by the dynamic stability, but not those by Nonconformists. Note that $\beta < 1 - \theta$, or equivalently $m^* < 1$, in these cases. Because the share of Nonconformists is sufficiently large and the matching process is sufficiently biased toward intragroup matchings, a Nonconformist is matched with another Nonconformist more frequently than with a Conformist. This implies mixed strategies by Nonconformists. On the other hand, a Conformist meets another Conformist more frequently than a Nonconformist does ($\beta < 1/2$, or

$m < 1/m^*$). This implies that, for any behavior patterns to which Nonconformists are indifferent between the two actions, a Conformist follows what the majority of Conformists does. As a result, λ_t converges to either 0 or 1 along the locus of $p_B^* = p_R^*$, depending on the initial condition. In Case 3, it converges to $(1,(1-m^*)/2)$, if $p_{B0} > p_{R0}$, and to $(0,(1+m^*)/2)$, if $p_{B0} < p_{R0}$. In Case 4, it converges to $(1,(1-m^*)/2)$, if $\lambda_0 + \lambda_0^* > 1$, and to $(0,(1+m^*)/2)$, if $\lambda_0 + \lambda_0^* < 1$.

In Case 5, the dynamics is globally stable and converges to $(\lambda,\lambda^*) = (1/2,1/2)$. As in Cases 3 and 4, $m^* < 1$ so that Nonconformists play primarily among themselves, which implies mixing. Unlike Cases 3 and 4, however, the matching process is sufficiently biased toward intergroup matching ($\beta > 1/2$) so that a Conformist runs into Nonconformists more often than a Nonconformist does ($m > 1/m^*$). This implies that, for any behavior patterns to which Nonconformists are indifferent between the two actions, a Conformist follows what the majority of Nonconformists does. As a result, λ_t converges to 1/2 along the locus of $p_B^* = p_R^*$.

In Case 6, the best response dynamics generate a spiral path around $(\lambda,\lambda^*) = (1/2,1/2)$, as shown in Figure 2. For any Conformist-Nonconformist ratio, this case occurs if the matching process becomes sufficiently biased toward intergroup matchings ($\beta > \theta$, $1-\theta$ or m, $m^* > 1$). Whether the fluctuation persists forever or eventually settles down, however, depends on the ratio. If there are more Conformists than Nonconformists ($\theta > 1/2$), so that Nonconformists are more concerned with intergroup matchings than Conformists ($m^* > m > 1$), then socially stable behavior patterns become cyclical. Along the cycle, a Nonconformist, wishing to differentiate herself from the masses, changes her action, before it becomes too conventional. A Conformist, whose matchings are more often intragroup than a Nonconformist's, follows the continuing trend for a while.[9] Then he switches his action only after sufficiently many Nonconformists switch their actions. Nonconformists act as fashion leaders and Conformists act as followers.[10] On the other hand, if there are more Nonconformists than Conformists ($\theta \leq 1/2$), then Conformists are more concerned with intergroup matchings than Nonconformists ($m \geq m^* > 1$). Conformists are much quicker to follow Nonconformists in this case, so that Nonconformists cannot maintain the lead forever. The distribution of strategies eventually settles down to $(\lambda,\lambda^*) = (1/2,1/2)$. The best response dynamics converges globally to the unique Nash equilibrium.

To understand the emergence of the limit cycle further, it would be useful to consider the following two thought experiments. First, starting from the case, $1-\beta < \theta < 1/2$ (or $m > m^* > 1$), where $(\lambda,\lambda^*) = (1/2,1/2)$ is the unique, globally stable Nash equilibrium, let θ increase. As the society crosses the line $\theta = 1/2$ (or $m = m^*$), $(\lambda,\lambda^*) = (1/2,1/2)$ loses its stability and bifurcates into a limit cycle. Although the Conformist is still concerned with intergroup matchings more than intragroup ones, he becomes less so than the Nonconformist is, which makes it possible for the Nonconformist to take the lead in switching actions. The regular patterns of fashion cycles thus emerge out of the disorder,

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[9]One can show that $\text{sgn}\,[\theta\lambda_t + (1-\theta)\lambda_t^*] = \text{sgn}\,\lambda_t$ along the limit cycle: that is, the fraction of the population that chooses B goes together with the fraction of Conformists that chooses B.

[10]Although the matching process needs to be biased toward intergroup for the existence of cycles, there also needs to be some intragroup matchings. When β approaches the unity, the cycle eventually shrinks to the Nash equilibrium. If there is no matching between a pair of Conformists, the game would be one of strategic substitutes and the mixed equilibrium would become globally stable.

as the forces of conformity increase. Second, starting from Case 1 ($\theta > \beta > 1/2$, or $mm^* > 1 > m$), where $(\lambda,\lambda^*) = (0,1)$ and $(\lambda,\lambda^*) = (1,0)$ are two locally stable Nash equilibria, let θ decline or β increase. As the society crosses the line $\theta = \beta$ (or $m = 1$), the two Nash equilibria first lose their stability and then disappear.[11] A Conformist becomes more concerned with a Nonconformist rather than another Conformist, and begin to imitate her. This bifurcation creates a limit cycle. Fashion cycles thus emerge as departure from custom in this case, as the forces of nonconformity increase.

As easily seen from the two Propositions, the Nash equilibrium and the socially stable behavior patterns coincide only in Case 5 and Case 6a, when the best response dynamics are globally stable. In Cases 1 through 4, only a subset of Nash equilibria are selected so that the best response dynamics serve as an equilibrium refinement. In Case 6b, a globally stable cycle emerges, which is not captured by the Nash equilibrium of the static game. This is not to be interpreted as a flaw of the best response dynamics; rather it seems to suggest that the Nash prediction of the static game is unrobust with respect to a natural perturbation of the game into a dynamic setting.

4. Discussions

I now address some issues concerning model specifications and discuss conjecturally the consequences of relaxing some of the assumptions.

i) The Matching Process

The matching technology assumed above is restrictive in two respects. First, the symmetry is imposed concerning intragroup matchings. That is, the matching between any particular pair of Conformists is equally likely as the

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[11]One can show that, in the borderline case, $\theta = \beta$ (or $m = 1$), a pair of "heteroclinic" orbits appear, which connect (0,1) and (1,0).

matching between any particular pair of Nonconformists. This restriction can be removed at the cost of complicated notations. Suppose now that a Conformist is matched with another Conformist at the rate equal to $\gamma\theta$ and with a Nonconformist at the rate $\beta(1-\theta)$, while a Nonconformist is matched with a Conformist at the rate $\beta\theta$ and with another Nonconformist at the rate $\gamma^*(1-\theta)$. Then, $m = \beta(1-\theta)/\gamma\theta$ and $m^* = \beta\theta/\gamma^*(1-\theta)$. Applying these ratios for Proposition 2, one can derive the condition for a variety of socially stable behavior patterns. For example, suppose that $\beta^2 > \gamma\gamma^*$ (that is, the process is biased toward intergroup matchings). Then, the dynamic path converges to (1/2, 1/2) if $\theta \leq \sqrt{\gamma^*}/(\sqrt{\gamma} + \sqrt{\gamma^*})$; to a unique limit cycle if $\sqrt{\gamma^*}/(\sqrt{\gamma} + \sqrt{\gamma^*}) < \theta < \beta/(\beta+\gamma)$; to (0,1) or (1,0) if $\theta > \beta/(\beta+\gamma)$. Again, the transition from disorder to fashion cycles to the social custom take place as the share of Conformists increases. One implication of this condition is that fashion cycles may occur even when Conformists do not form the majority of the society, if Nonconformists tend to avoid each other more than Conformists do ($\gamma > \gamma^*$).

Second, all matchings are assumed to be pairwise. This restriction makes the Conformist-Nonconformist ratio matter in this model. If more than two individuals are allowed to meet at the same time, as is the case with conferences, then each individual could influence others multilaterally. In such a context, it is also plausible that some individuals (e.g. speakers) have more influences than others (e.g. people in the audience). Even more generally, one could depart from a matching framework altogether and make an individual's payoff depend directly on (λ,λ^*). Such a situation may arise if an individual observes others in the crowd or through news media. Furthermore, one may assume that $\theta\lambda$ and $(1-\theta)\lambda^*$ could have differential impacts on an individual's payoffs by arguing, say, that the action taken by a Nonconformist is more conspicuous than the action by a Conformist. These extensions, however, make the model too loose to have any predictive context. It should be emphasized that one major advantage of the present approach is that a variety of social phenomena can be generated within the confinement of a pairwise matching framework, so that the outcomes can be tightly linked to the factors, such as the composition of different groups in the population and their matching patterns.

One argument for departing from the pairwise matching framework, suggested by a referee, is that in a more general setting we do not need to assume that the process be sufficiently biased toward intergroup matchings to generate fashion cycles. However, it is not difficult to imagine situations in which such a bias arises naturally. For example, suppose that men are Conformists and women Nonconformists (or the other way around). Then, it is not implausible to assume that a man and a woman tend to be matched more often than two men or two women. Or consider the society consisting of travelling salesmen and their customers. All we need is to come up with a situation in which an individual's tendency to be conformist or nonconformist is affected by or correlated with his/her occupational or other roles in the society.[12]

ii) Conformists and Nonconformists

In the formal model presented above, a Conformist (Nonconformist) simply means the type of an individual who gains positive (negative) consumption

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[12]This would also help to justify another implicit assumption, the exogeneity of the matching patterns. Of course, it would be interesting to see how an attempt to change the matching patterns by, say, forming exclusive clubs would affect the outcomes. Another extension that seems worth pursuing is to incorporate spacial factors when defining the matching patterns. Without spacial considerations, the present game cannot explain the geographical distribution of different social customs, such as the co-existence of different subculture groups, and the propagation of fashion cycles across regions.

externalities by others taking the same action. These terminologies are thus close parallels to what Leibenstein (1950) called the "bandwagon effect" and "snob effect" in his classic study on the static market demand curve. By the bandwagon (snob) effect he referred to the extent to which the demand for a commodity is increased (decreased) because others are consuming the same commodity. In the game presented above, Conformists personify the bandwagon effect and Nonconformists the snob effect.[13] Many other researchers have also studied the implications of payoff externalities of this kind: see Becker (1974, 1991), Frank (1985), Jones (1984), and Schelling (1978).

Needless to say, both conformist and nonconformist behaviors often result from widely diverse motives and, for some purposes, it would be important to identify different mechanisms that lead to such behaviors. For example, people may conform out of desire to stay in the good graces of other people and to avoid punishment: see Akerlof (1980), Kuran (1989), and Bernheim (1991).[14] Or they may be motivated by the desire to be correct and imitate those who are believed to be better informed: see Conlisk (1980), Banerjee (1989) and Bikhchandani, Hirshleifer, and Welch (1990). The preferences assumed for the Conformist in the game should be considered as a reduced form that is meant to encompass a variety

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[13]Leibenstein's analysis is static and thus he did not discuss any dynamic implication of combining these two effects, although he stated "[i]n all probability, the most interesting parts of the problem, and also those most relevant to real problems, are its dynamic aspects (p.187)."

[14]Some may prefer to call this kind of conformity "compliance" or "obedience," although, in the present model, Conformists have no intrinsic preferences over the two actions and hence they cannot be forced to conform. Allowing individuals to have diverse preferences over the two actions can be done at the cost of more complicated algebra: it makes the loci along which <u>marginal</u> players are indifferent between the two actions nonlinear.

of situations that lead to conformist behaviors.[15]

It should be added, however, that there is nothing irrational or pathological about following others in the absence of any reward, punishment, or information transmission. People may simply prefer to do things together, or they may want to go along with the crowd just for pure excitement, to share the great moment. Such a pure conformity seems particularly natural in the context of fashion in dress, in which people are motivated by desire of being in fashion, or at least by desire of not being out of fashion. It is also perfectly normal to feel embarrassed by being seen wearing the same cloth with many others. In fact, one could argue in the context of our formal model that both Conformists and Nonconformists are driven by the same desire: to be in fashion; they may only have different notions of what is fashionable.

iii) <u>The Strategy Space</u>

In the formal analysis, it is assumed that each individual chooses between the only two actions. This is clearly a strong assumption, but there are many situations in which such a binary restriction seem quite reasonable. For example, when we talk about fashion in economic thought (see Viner (1991)), we often think in terms of two alternative schools or approaches, such as Monetarist versus Keyesian, Historical versus Analytical, Rational versus Evolutionary. Even within the field of economic theory, we often debate the pros and cons of two alternative styles of writing, such as Algebraic versus Geometrical Approach. At certain times, the generality of a model tends to be valued, at other times, the simplicity tends to be regarded as a virtue. In many of these situations

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[15]Another often-mentioned reason for conformity, technological increasing returns, is not appropriate in our context, which requires that only a subgroup of the population exhibits conformist behavior. The prevalence of increasing returns should lead to the universal adaptation of a particular strategy: see Arthur (1990).

pursuing a middle ground may not be a practical option.

Nevertheless, it is important to think about the implication of the binary assumption. In an insightful comment, one reviewer questions the robustness of fashion cycles. "If there are a large number of alternative strategies (i.e., the colors of shirts), ... all the conformists would choose one particular strategy and the nonconformists would split in many small subgroups, each of which plays a different strategy.... unless the population of the conformists is very small or the matching structure is really skewed." This seems to be correct if an individual gains the identical payoff whenever matched with someone with a different strategy; which strategy the matching partner has actually chosen does not matter, as long as it is different.

In order to ensure the robustness of fashion cycles, there seem to be several ways in which we can modify the structure of the game.[16] First, we could change the payoff structures so as to make the Nonconformist avoid not only the most common strategy but also the least common strategy in her social encounter. Then, for most initial conditions, only two strategies will survive in the long run. One may object that such a modification comes only one step short of making the two strategy assumption, but this kind of behavior seems natural and pervasive in many contexts. A brand-conscious consumer may avoid the most famous brand, but would not go for a non-brand product. In our profession, we often avoid working in the research area that we feel is too crowded. At the same time, we hesitate to work in the area that nobody is interested in, since we also need somebody to talk with about our research.

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[16]On the other hand, there seems no way of preserving the disorder, the outcome in which both Conformists and Nonconformists are equally distributed across all strategies, for a large number of strategies, and hence it should be considered as an artifact of the binary assumption.

The second possibility is to add a third group of individuals into the game, intrinsic value maximizers, those who feel loyal to a particular strategy and do not care about the behaviors of others. If there are sufficiently many intrinsic value maximizers to each strategy, then the ability for Nonconformists to separate themselves from Conformists may be restricted.

Third, we could introduce a more structure into the strategy space. For example, Green may be much closer to Blue than to Red or to Orange. It would be interesting to speculate what would happen in the case where the strategy space is a line segment, a reasonable assumption to make if we wish to ponder about the cyclical behavior of the female skirt-length (Young (1937), Richardson and Kroeber (1940), Robinson (1975), Lowe and Lowe (1985), among others.). If the payoff depends on the distance between the two matched strategies (decreasing for the Conformist and increasing for the Nonconformist), the complete separation outcome suggested by the reviewer may be less likely. It might be possible to have a different kind of fashion cycles; Nonconformists would switch between the two end points periodically, while Conformists move back and forth in the intermediate range. Instead we could assume that a Nonconformist's payoff is first increasing and then decreasing in the distance of the two strategies; she wants to look different, but not eccentric. Then, fashion cycles may be emerge, in which both Conformists and Nonconformists move together with the latter being always one step ahead.

Other strategy spaces easily come to my mind; what is the right strategy space assumption clearly depends on the context. It seems, however, that fashion cycles can easily be restored by redefining the incentive of Nonconformists appropriately (recall that the restriction of the strategy space in a game can always be replaced by an assumption on the payoffs of the players.)

5. Alternative Models of Custom and Fashion

In this section, I compare the present model with other possible models of social custom and fashion cycles. First of all, we should ask ourselves whether it is absolutely necessary to have heterogeneous individuals to model these social phenomena. Can custom and fashion emerge in the society with a homogeneous population?[17]

One possibility is to model the change of taste over time by intertemporally dependent preferences. For example, if you have been using a particular style for a long time, you would become emotionally attached to it and continue to use the same style. A model of habit formation, such as Becker and Murphy (1988), if employed at the aggregate level, can certainly explain the persistence of certain form of behaviors, similar to the social custom. Alternatively, if you have been using a particular style for a long time, you may get bored and want to switch to another style. Benhabib and Day (1981) has in fact modelled such preferences with temporary satiation, leading to alternating choices by consumers. Their model, which was meant to explain why we tend to alternate between beef and chicken for dinner or between mountains or beaches for holiday spots, could be converted to a model of fashion cycles by introducing conformity. The problem with the approach based on intertemporal dependence of preferences is, however, that it cannot identify social environments that are

[17]One obvious way to do it to let some exogenous shocks to generate fashion cycles in the society consisting of the Conformists only. There is no doubt that the fashion process reflects economic and other changes in the society. Many external forces, such as technological invention, the advent of a design genius, the changing role of women in the society, can induce shifting behaviors even by conformists. Some even pointed out the strong correlation between the female skirt-length and the business cycles: see Morris (1977: p.220-221). Nevertheless, the regularity and prevalence in many areas of human activity seem to suggest that some self-generating mechanisms are responsible for fashion cycles. Here, I am only concerned with models exhibiting endogenous fashion cycles.

conducive to emergence of a social custom or fashion cycles. Whether we get the social custom or fashion cycles outcome is predetermined at the level of preferences specification.

Another possibility is to model the intergenerational dependence of taste. Imagine that, in a discrete time model, every generation lives for two periods and commits to a particular style when young. In a such model, we could explain the social custom if all individuals are conformist and imitate the old generation when they are young. On the other hand, fashion cycles would emerge if they are all nonconformist and dislike the style chosen by the old generation. Again, the problem with this approach is that the question of custom versus fashion is ruled out in specification of preferences.

In order to explain the emergence of both custom and fashion in a unified framework and identify environments in which different social phenomena emerge, it is thus necessary and natural to consider a society with a heterogeneous population, where some conflicts between imitation and differentiation exists. In this respect, the trickle-down theory of fashion, usually associated with Simmel ([1904]1957) and more indirectly with Veblen (1899), deserves special mention. According to Simmel, fashion is driven by an imitation of the elite class by the masses. The elite class seeks to set itself apart from the masses by adopting a new style, which in turn leads to a new wave of emulation. In his theory, the social status plays a significant role; fashion is considered as a process in which a new style "trickles down" from the upper to the lower ends of the social hierarchy. Recently, Karni and Schmeidler (1990) has constructed a dynamic game based on Simmel's idea and provided a numerical example of fashion cycles. Unfortunately, their game has a different structure with the game presented above.[18] In order to facilitate a comparison, I will present a reformulation of the Karni-Schmeidler model, using the best response dynamics.

As before, there are two actions, B and R. Individuals in the society are partitioned into two social classes: the lower and upper classes. Let λ_t and λ_t^* now denote the fractions of the lower class and of the upper class that chooses B, respectively. Following Karni and Schmeidler, I assume that the evaluations of the two actions by each class is a linear function of λ_t and λ_t^*.[19] In particular, the relative merit of choosing B instead of R for the lower class is given by

$$\Pi_{Bt} - \Pi_{Rt} = (\lambda_t - 1/2) + \mu(\lambda_t^* - 1/2) ,$$

and, for the upper class,

$$\Pi_{Bt}^* - \Pi_{Rt}^* = -\mu^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) ,$$

where μ and μ^* are both positive constants and represent the extent to which each individual pays attention to the behavior of the distinct class relative to its own. These parameters could depend on a variety of factors, such as how conspicuous the behavior of the upper class is to the lower class in this

[18] Their model is a discrete time dynamic game played by two classes of individuals, α and β, who choose among three different colors every three periods. The crucial feature of their model is that the preferences of α-individuals for a given color decrease with the fraction of β-individuals that use the same color, while the preferences of β-individuals for a given color increase with the fraction of α-individuals that use the same color. Due to the complexity of the model, Karni and Schmeidler were able to generate only a numerical example of fashion cycle equilibria.

[19] I do not find a pairwise random matching framework appropriate in this context. The linearity of the payoff functions, satisfied automatically in a pairwise matching, needs to be assumed here.

society, the extent to which the lower class wants to emulate the upper class, the extent to which the upper class feel negatively about the lower class, and so on.

The best response dynamics are now given by

$$\text{(5a)} \qquad \lambda_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & \text{if } (\lambda_t-1/2) + \mu(\lambda_t^*-1/2) > 0 , \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & \text{if } (\lambda_t-1/2) + \mu(\lambda_t^*-1/2) = 0 , \\ \{-\alpha\lambda_t\} & \text{if } (\lambda_t-1/2) + \mu(\lambda_t^*-1/2) < 0 , \end{cases}$$

$$\text{(5b)} \qquad \lambda_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & \text{if } \mu^*(\lambda_t-1/2) - (\lambda_t^*-1/2) > 0 , \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & \text{if } \mu^*(\lambda_t-1/2) - (\lambda_t^*-1/2) = 0 , \\ \{\alpha(1-\lambda_t^*)\} & \text{if } \mu^*(\lambda_t-1/2) - (\lambda_t^*-1/2) < 0 . \end{cases}$$

Analytically, the only difference between (3) and (5) is that a starred individual is affected negatively by its own group in (3), and positively in (5). Figure 4 depicts the phase diagrams associated with the dynamical system (5) for four generic cases. The Nash equilibria (the black dots) are also shown in Figure 4. The locus on which the lower class is indifferent between B and R is downward sloping; its slope is equal to $1/\mu$. The best response of the lower class is B above the locus and R below it. The locus on which the upper class is indifferent, on the other hand, has the positive slope equal to $\mu*$. Its best response is B above the locus and R below it. From these diagrams, one can conclude:

Proposition 3.

The socially stable behavior patterns of (5) are given by:

Case 1 ($\mu^* > 1 > \mu$): $(\lambda, \lambda^*) = (0,1)$ and $(1,0)$,

Case 2 ($\mu^* < 1 < \mu$): $(\lambda, \lambda^*) = (1,1)$ and $(0,0)$,

Case 3 (μ, $\mu^* < 1$): $(\lambda, \lambda^*) = (0,1)$, $(1,0)$, $(1,1)$, and $(0,0)$,

Case 4 (μ, $\mu^* > 1$):

The limit cycle, which is the parallelogram defined by the four vertices,

$$P = ((1-X_\infty/\mu^*)/2, (1-X_\infty)/2) , \qquad Q = ((1-X_\infty)/2, (1+X_\infty/\mu)/2) ,$$

$$P' = ((1+X_\infty/\mu^*)/2, (1+X_\infty)/2) , \qquad Q' = ((1+X_\infty)/2, (1-X_\infty/\mu)/2) ,$$

where $X_\infty = (\mu+\mu^*)/(1+\mu\mu^*)$.

The existence of the limit cycles can be demonstrated as in Proposition 2, and hence be omitted here. The intuition behind Proposition 3 is again easy to grasp. In Case 1, which I call the case of segregation, the upper class is more affected by the behavior of the lower class than that of its own class. On the other hand, the lower class pays more attention to the behavior of its own class than that of the upper class. As a result, the force of differentiation by the upper class dominates the force of imitation by the lower class and, in any stable patterns, the two classes use different strategies. The upper class wins. In Case 2, on the other hand, the lower class wins and integration occurs; in any stable patterns, both classes use the same strategy. The lower class wins. This is because the lower class pays more attention to the behavior of the upper class and the upper class pays more attention to those in its own class; the force of imitation dominates that of differentiation in this case. In Case 3, each class pays more attention to the behavior of its own class. Whether integration or segregation occurs entirely depends on the initial conditions. Finally, in case 4, each class is affected more by the behavior of the other. Neither imitation nor differentiation dominate and, as a result, a chase-and-flight between the two classes continues forever.

The empirical literature shows that the trickle-down theory of fashion cycles was well suited to explain fashion in dress from the seventeenth to nineteenth century Europe with its particular class structure. By the latter half of the twentieth century, however, the pattern of fashion diffusion in dress has changed. Some empirical evidences (e.g., Lowe and Lowe (1985)) indicate that the variations in the female skirt length and other variables have not declined over time, despite earlier writers predicted the end of fashion as the social structure changed. Some studies also show that media exposure and mass production allowed adoption of new styles to proceed simultaneously at all levels of society, and in some cases, it actually started in the lower classes and only later spread to the upper strata. Similar patterns of "trickle-up" diffusion can be found in the other area of human activity, as new movements are often initiated by youths, blue collar workers, and blacks and other ethnic minorities. The elite class still strives for a distinction, particularly in dress, but such an effort is mainly manifested in the quality of fabrics used, rather than in the earlier adoption of a new style.

As an alternative theory, many recent writers in sociology emphasize the mass movement aspect of fashion cycles. For example, Lang and Lang (1961), Blumer (1969), and others argued that fashion must be seen as a process in which "collective tastes" are continuously defined and redefined. What is beautiful and what is appropriate are always matters of judgment, varying from time to time, depending on the Zeitgeist, the spirit of the times. People respond and adopt to changing tastes of the society, and in doing so, they inadvertently contribute to the formation of collective tastes. The recent empirical studies in marketing are now in large part devoted to the identification of "fashion conscious group," the group of consumers who play pivotal roles in shifting public tastes: King and Ring (1980) and Sproles (1981).

The random matching game between Conformists and Nonconformists presented earlier is much closer to the collective selection theory in its spirit, as each individual in that game responds to the average behavior; unlike the trickle-down theory, the identity of the carriers does not affect the relative merits of the two strategies. A Conformist simply follows the majority (in his social encounter) by desire of not being out of fashion, while a Nonconformist follows the minority (in her social encounter) by desire of being at the forefront of fashion movement. Both respond to changing collective tastes, represented by the behavior patterns in the society, and throughout fashion cycles, Nonconformists play a role of "fashion conscious group," whose choice will be imitated by Conformists.

As a general theory of fashion cycles in a modern democratic society, the model based on the desire of people to be in fashion and not to be out of fashion, where what is fashionable is defined by collective behavior, is more persuasive than the model based on the prestige of the carriers of a style defining its fashionableness. A prestigious person, despite his eminence in the society, could be easily felt to be "out of date." To carry some weight, his choice has to be approved in the eye of the public. Nor certain styles become out-of-date because they are discarded by the prestigious group. One may argue that the fashion-conscious people, those in the vanguard of the fashion process, may be prestigious in the eyes of many. But it should be remembered that they are prestigious precisely because they always look "fabulous" and "stunning." It is not their prestige per se that makes their outfits or behavior fashionable.

This is not to deny that, when many competing styles potentially meet the mood of the public, a member of the royal family or a TV personality could help the public to focus on a particular style, thereby affecting the evolution of fashion cycles. But, when their behavior or dress are against the taste of the public, they are merely greeted with a ridicule, no matter how prestigious they are. The history of fashion in dress also provides ample evidences that suggest the limited importance of the prestige as a determinant of fashionableness. Attempts by the fashion industry to plant new styles often meet with disaster. Back in the sixties, the industry introduced "midi" to check and reverse the trend toward shorter skirts. The midi-skirt project was a complete failure, in spite of the well organized and financed public campaign with the cooperation of prestigious fashion houses, fashion magazines and commentators. The same was true in the twenties. More recently, the industry experienced another failure in its attempt to promote "mini" to professional women: Blumer (1969), Reynolds and Darden (1972), and Time Magazine (1988).

In sum, the trickle-down theory has some merits. First, it stresses the importance of social structure. Second, it captures fashion as the process of change. Third, the conflict between imitation and differentiation is central to the analysis. By attributing the sources of imitation and differentiation to the prestige factors in the society, however, its applicability is substantially narrowed down. It seems to me that the trickle-down theory (and a model of cycles presented in this section) is more convincing as an explanation of chase-flight patterns between the rich and the poor in their school and neighborhood choices, rather than as a general theory of fashion cycles.

Appendix:

This appendix shows the asymptotic behaviors of the dynamical system (3) for Case 6. For any given path, define sequences $\{X_n\}_{n=0}^{\infty}$ and $\{Y_n\}_{n=0}^{\infty}$, as follows:

(6a) $P_k = ((1+X_{2k}/m^*)/2,\ (1-X_{2k})/2)$,

(6b) $Q_k = ((1-Y_{2k})/2,\ (1+Y_{2k}/m)/2)$,

(6c) $P_k' = ((1-X_{2k+1}/m^*)/2,\ (1+X_{2k+1})/2)$,

(6d) $Q_k' = ((1+Y_{2k+1})/2,\ (1-Y_{2k+1}/m)/2)$,

where P_k (P_k') is the point at which the path crosses the locus of $p_B^* = p_R^*$ to the southeast (northwest) of (1/2,1/2) for the k-th time and Q_k (Q_k') is the point at which the path crosses the locus of $p_B = p_R$ to the northwest (southeast) of (1/2,1/2) for the k-th time, as depicted in Figure 5. The sequences $\{X_n\}_{n=0}^{\infty}$ and $\{Y_n\}_{n=0}^{\infty}$ thus keep the record of the distance from the center (1/2,1/2), every time the path intersects with $p_B^* = p_R^*$ and $p_B = p_R$, respectively. Since P_kQ_k points to (0,1) and $P_k'Q_k'$ points to (1,0), these sequences satisfy

$$Y_n = \frac{m^*(1+m)X_n}{m(1+m^*) + (mm^*-1)X_n} ,$$

and, since Q_kP_k' points to (1,1) and $Q_k'P_{k+1}$ points to (0,0),

$$X_{n+1} = \frac{m(m^*-1)Y_n}{m^*(m-1) + (mm^*-1)Y_n} .$$

Note that these two recursive formula jointly generate bounded sequences $\{X_n\}_{n=0}^{\infty}$ and $\{Y_n\}_{n=0}^{\infty}$ on [0,1], when m , $m^* > 1$. Combining the two formula yields

(7) $$X_{n+1} = F\left(X_n\right) = \frac{(m^*-1)(1+m)X_n}{(m-1)(1+m^*) + 2(mm^*-1)X_n}\ , \quad Y_{n+1} = F\left(Y_n\right)\ .$$

As shown in Figure 6, any sequence generated by (7) converges to 0 if $m \geq m^* > 1$. Therefore, all paths converge to (1/2,1/2) in Case 6a. On the other hand, if $m^* > m > 1$, $\{X_n\}_{n=0}^{\infty}$ and $\{Y_n\}_{n=0}^{\infty}$ both converge to

(8) $$X_{\infty} = (m^*-m)/(mm^*-1) = (2\theta-1)\beta(1-\beta)/(2\beta-1)\theta(1-\theta)\ ,$$

unless they start from the origin. Therefore, in Case 6b, all paths except the stationary path (1/2,1/2) converge to a unique limit cycle given in Proposition 2. Q.E.D.

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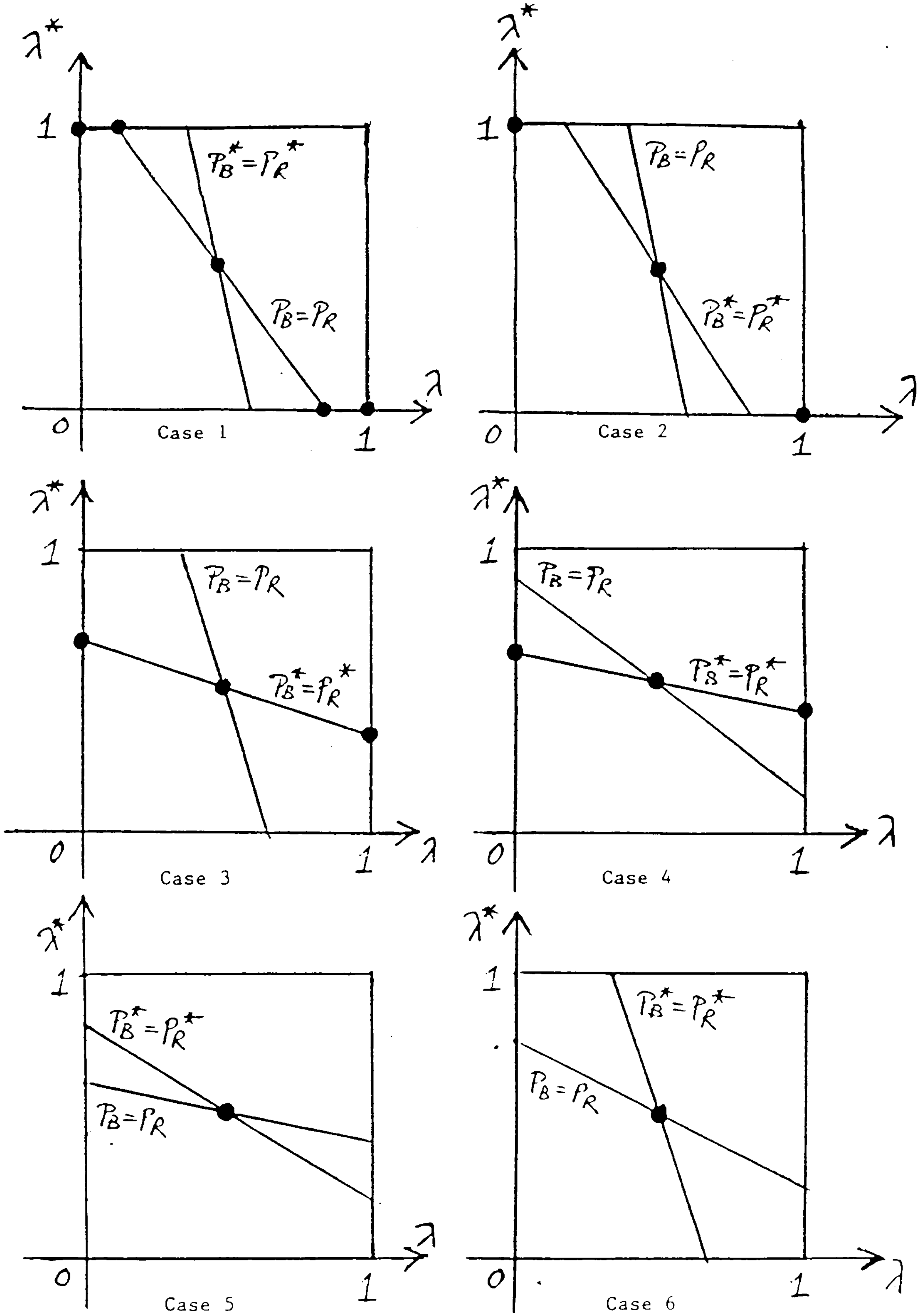


Figure 1: The Nash Equilibria

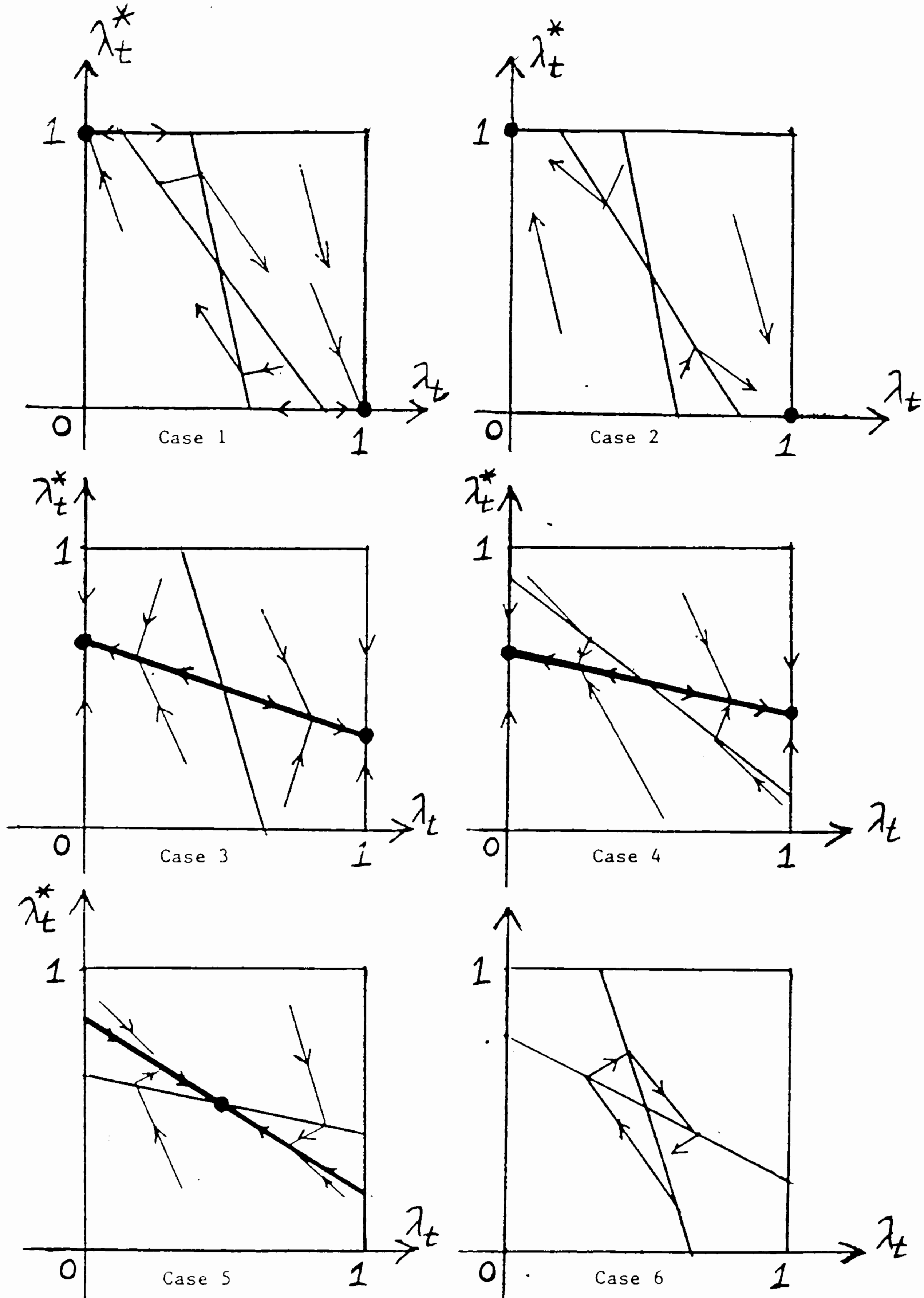


Figure 2: The Best Response Dynamics, Equation (3)

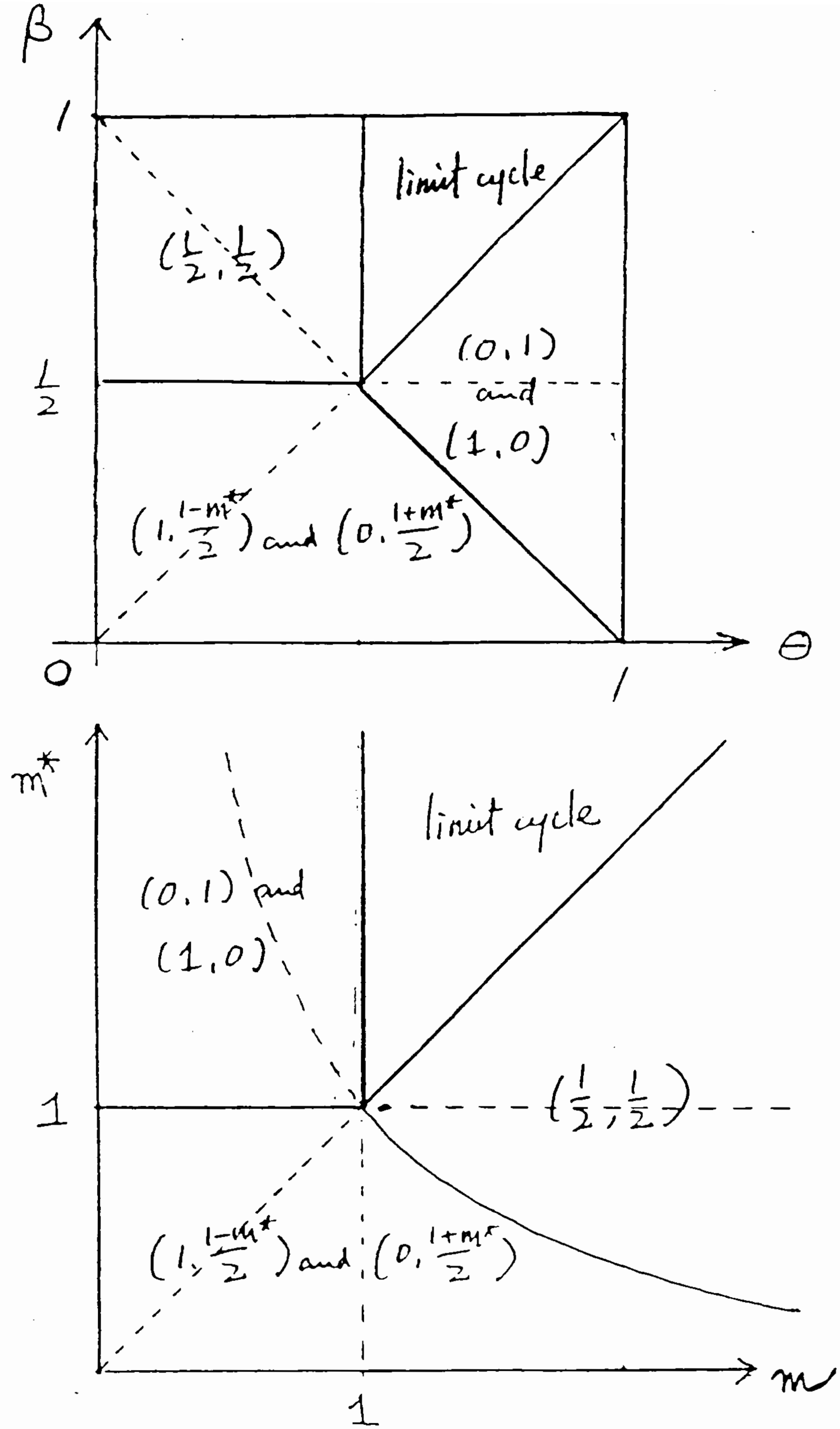


Figure 3: The Socially Stable Behavior Patterns

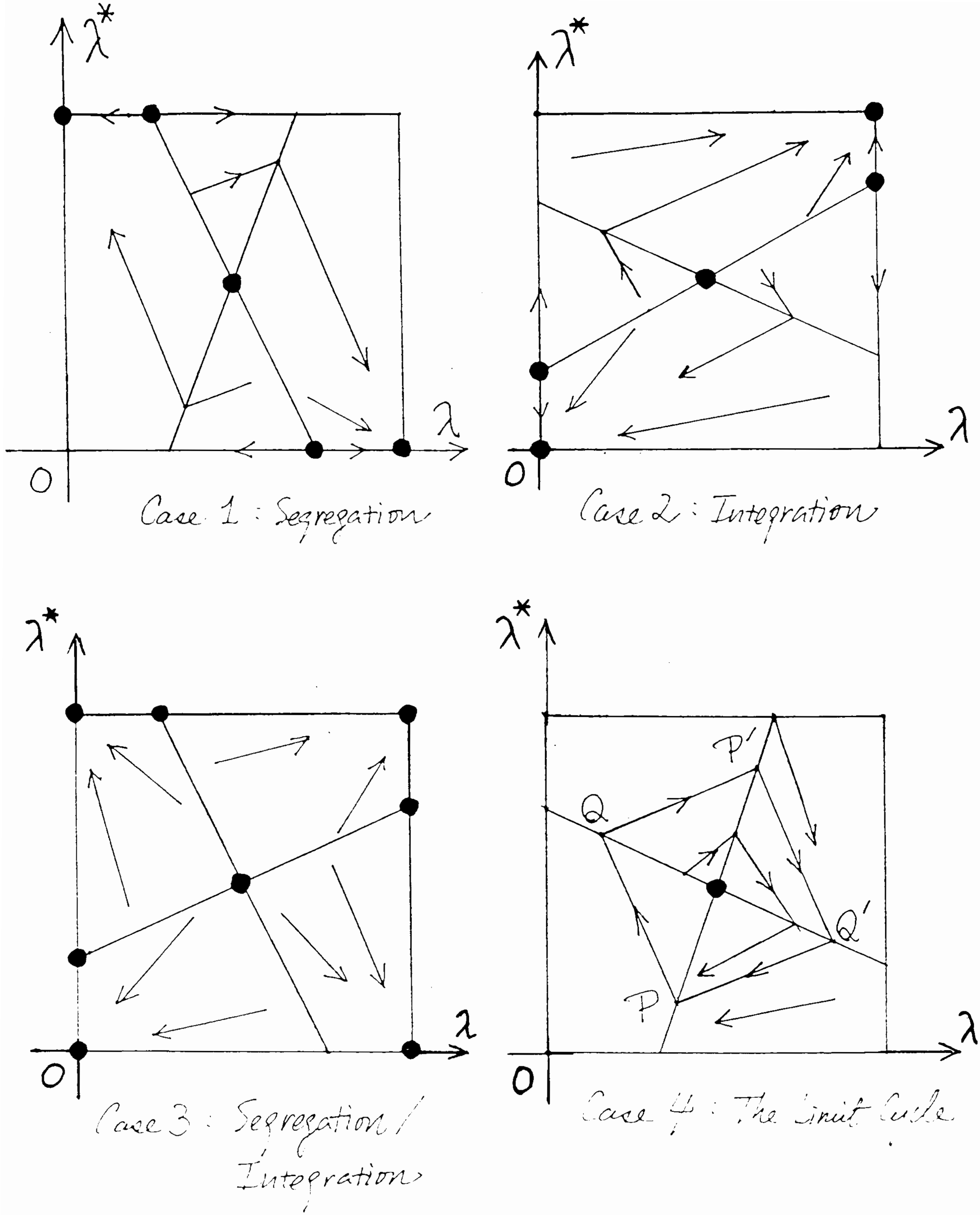
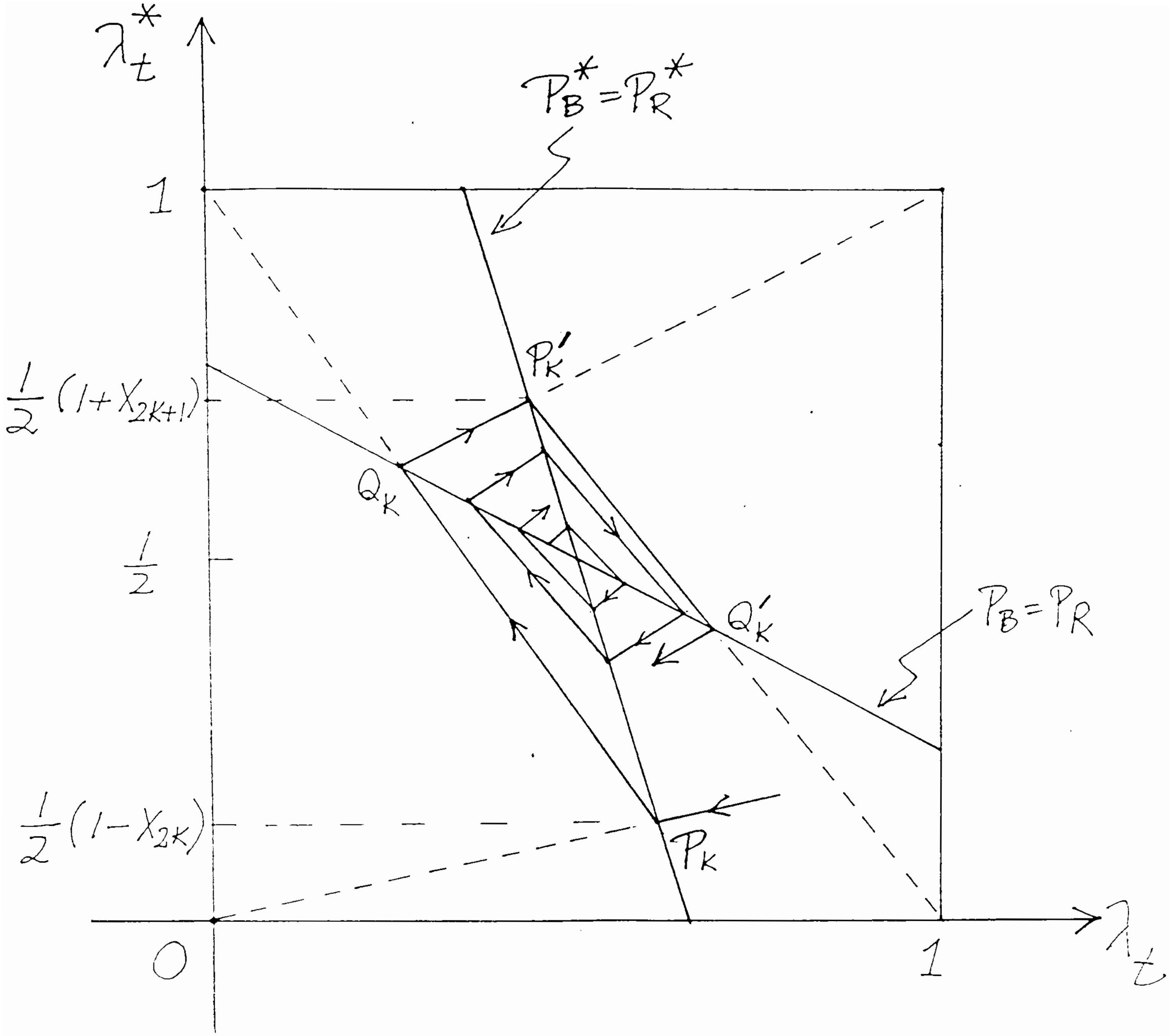


Figure 4 : The Best Response Dynamics, Equations (5)



λ_t^*
$P_B^* = P_R^*$
1
$\frac{1}{2}(1+X_{2K+1})$
P_K'
Q_K
$\frac{1}{2}$
Q_K'
$P_B = P_R$
$\frac{1}{2}(1-X_{2K})$
P_K
0
1
λ_t

Figure 5

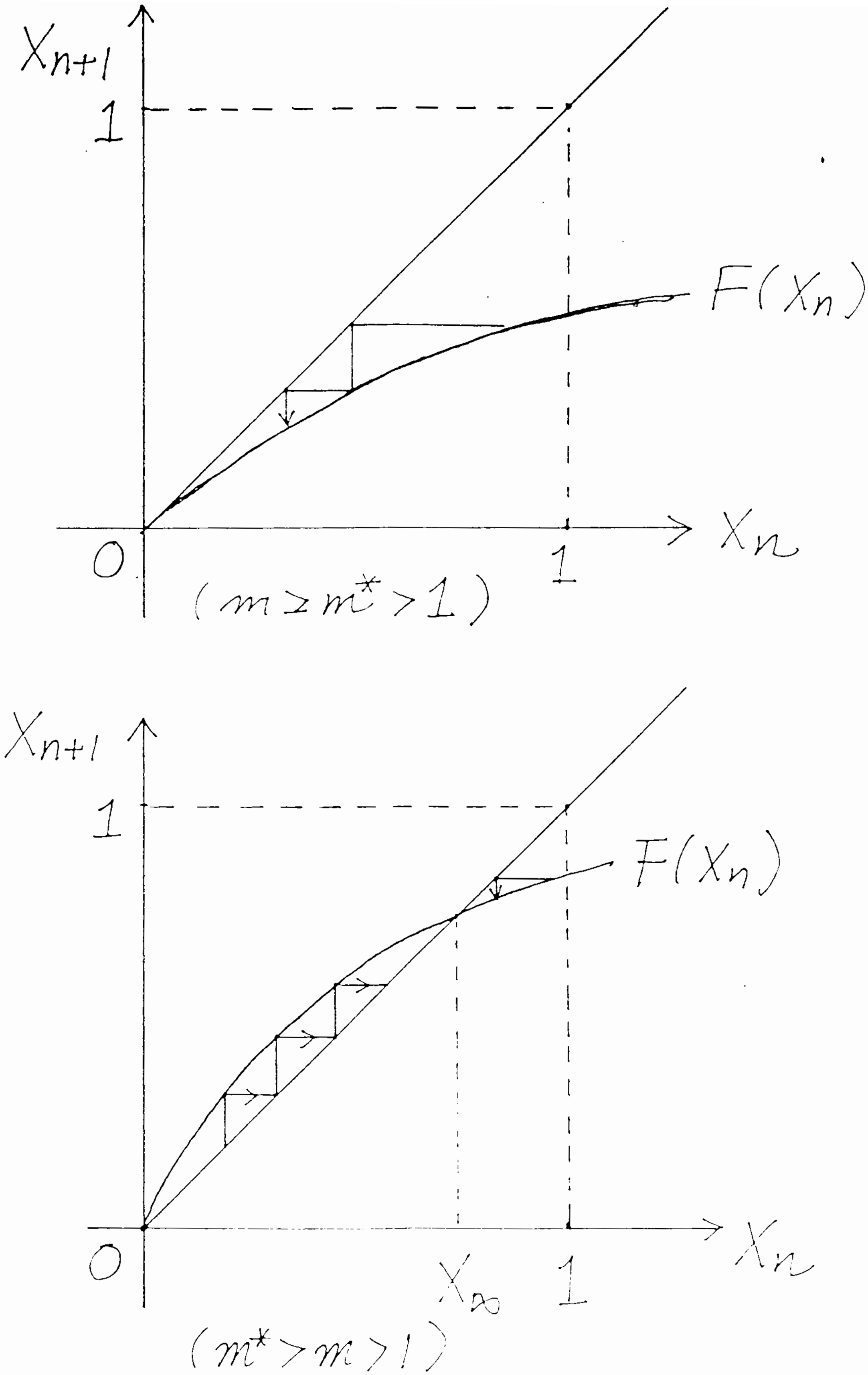


Figure 6