Explaining Diversity: Symmetry-Breaking in Complementarity Games

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The question: How to explain the diversity across space, time and groups;
- space; uneven development; "What generates the regional disparity?"
- time: business cycles; "Why are there booms and recessions?"
- groups: discrimination; “What causes race and gender gaps?”

The **Standard Approach**: account for the diversity by introducing the differences in other variables such as TFP, human capital investments, etc.
- The variations in these variables themselves need to be explained.

The **Economics of Coordination Failures**: strategic complementarities strong enough to generate Pareto-ranked multiple equilibria.
- It explains the diversity across space, time and groups without assuming inherent differences
- sufficiently strong complementarity is needed
- It imposes no equilibrium restriction on the degree of diversity observed. It can explain anything, which makes it empirically irrefutable

The **Economics of Symmetry Breaking**: strategic complementarities strong enough for the symmetric equilibrium to lose its stability to generate asymmetric equilibria, but too weak to generate Pareto-ranked multiple equilibria,
- It explains the diversity across space, time and groups without assuming inherence differences
- complementarities needed can be arbitrarily small
- It impose the equilibrium restriction on the degree of diversity observed
Figure 1b, the two stable asymmetric equilibria, $(x^L, x^H)$, $(x^H, x^L)$, which explain the diversity the two stable symmetric equilibria, $(x^L, x^L)$, and $(x^H, x^H)$, which weakens the prediction.
**The Logic of Symmetry-Breaking**

A game, describing

a) the players play two complementarity games simultaneously

b) the two complementarity games compete for each player’s effort.

\[
\text{Max } V(x; x^e) = R(x^e_1)x_1 + R(x^e_2)x_2 - \frac{1}{2} \left[ (x_1^2 + x_2^2) - \frac{\beta}{2} (x_1 - x_2)^2 \right]
\]

\(\beta \in [0, 1)\): the substitutability of the player’s efforts across the two.

\(\beta = 0, V(x; x^e) = \sum_{j=1}^{2} \{R(x^e_j)x_j - (x_j)^2/2\}\): the independent case, \(x_j = R(x^e_j)\).

\(\beta = 1, V(x; x^e) = R(x^e_1)x_1 + R(x^e_2)x_2 - [(x_1 + x_2)/2]^2\): the perfect substitute case.

**Interpretations**

- **time:** the agents choose the timing of durable goods purchases.
- **space:** multinational investors allocate their resources across countries.
- **groups:** employers allocate recruiting efforts across different pools of workers.
- **parents:** allocate their budget for education between sons and daughters.
- **married couples:** who jointly allocate their times between their carriers and household production.

The equilibrium:

\[
V_1(x^*; x^*) = R(x^*_1) - \left(1 - \frac{\beta}{2}\right)x_1^* - \frac{\beta}{2}x_2^* = 0, \quad V_2(x^*; x^*) = R(x^*_2) - \frac{\beta}{2}x_1^* - \left(1 - \frac{\beta}{2}\right)x_2^* = 0.
\]

The stability test

\[
(D) \quad x_i = \text{grad}V_i(x_i; x^e) \bigg|_{x_i^e = x_i} = \begin{bmatrix} V_i(x_i; x_i) \\ \frac{R(x_i)}{V_i(x_i; x_i)} \end{bmatrix} = \begin{bmatrix} R(x_{i1}) - (1 - \beta/2)x_{it} - (\beta/2)x_{2t} \\ R(x_{i2}) - (\beta/2)x_{it} - (1 - \beta/2)x_{2t} \end{bmatrix}
\]
Figure 2: The bifurcation analysis; \( R(x) = 1 + \theta(x-1) - \mu(x-1)^3 \), where \( \theta, \mu > 0 \), and \( \mu > \theta - 1 \). \( \theta \equiv R'(1) \) measure the complementarity; (1, 1) is always an equilibrium.
- Case D, this model has little predictive content; both asymmetric and symmetric equilibria exist.
- Case B and C; most powerful as a theory of endogenous diversity, the only asymmetric equilibria are stable. Coordination failures are not responsible for the equilibrium diversity. Strong complementarity is not needed.
4. Discussions

A. Globalization and Uneven Development (Twin Peaks)

The Neoclassical Model;

- requires the variations in exogenous variables
- the concept of conditional convergence.

The Coordination Failure Models (Murphy, Shleifer, Vishny 1989)

- they typically feature a) the closed economy, b) domestic externalities, which give the false impression that a) globalization facilitates a convergence, helping the poor to catch up; b) significant complementarities are needed to explain uneven development.
- The models merely suggest the possibility of “twin peaks.”

The Symmetry-Breaking

- globalization leads to uneven development, even with small domestic complementarities
- predicts the emergence of “twin peaks.”

Examples
- Globalization in capital flows: Gertler and Rogoff (1990) and Matsuyama (2001)
B. Intertemporal Substitution and Business Cycles

The Standard Approach (Real Business Cycles) needs exogenous productivity shocks.

The Coordination Failures (The Sunspots)

- generates fluctuations without exogenous productivity shocks; switching is coordinated by the sunspots; the beliefs driven by sunspots are self-fulfilling
- non-fluctuating equilibrium path also exists; the beliefs that sunspots play no role is also self-fulfilling.
- recessions are undesirable and can be and should be eliminated.

Symmetry-Breaking

- a large intertemporal substitution make the non-fluctuation equilibrium unstable and lead to cyclical behaviors, with small intratemporal complementarities.
- Recessions may not be bad; it may not be easy to eliminate recessions
  Examples:
- endogenous timing of innovation in Deneckere and Judd (1992) and Matsuyama (1999a) or of implementation in Gale (1996) and Shleifer (1986) lead to a synchronization of these activities.
- Inventories (Cooper and Haltiwanger 1992), intertemporal substitution of labor (Hall 1991), durable goods (Murphy, Shleifer and Vishny 1990), lead to a production bunching
- Search externality, together with the assumption that the players must alternate between production and search, generate business cycles (Diamond and Fudenberg 1989)
C. Race and Gender Gaps in the Labor Market

The Standard Explanation; differences in human capital investments.

The Coordination Failure Models (Statistical Discrimination)

- both multiplicity of self-fulfilling beliefs: both positive and negative stereotypes
- consistent with both the prevalence and the absence of discrimination.
- the separation rules out any possibility of “reverse discrimination”

Symmetry-Breaking

- the instability of the equal treatment equilibrium; the differential treatments arise as the only stable outcome.
- imposes the restriction on the degree of inequality across gender and race, which makes it possible to make predictions concerning the changing trends in discrimination.

Examples:
- the employers allocate their recruiting efforts across ethnic groups (Mailath, Samuelson, Shaked 2000).
- the married couple decide jointly how to allocate their times between their carriers and household production (Francois 1998).
- the parents choose how much to invest in the education of their son and daughter.