Explaining Diversity: Symmetry-Breaking in Complementarity Games

By Kiminori Matsuyama*

Strategic complementarity games found applications in many fields, including macro, development, and labor economics.[1] They provide an useful framework within which to address questions like, “what generates the disparity across regions and countries?”; “why are there booms and recessions?”; and “what causes gender and race discrimination in the labor market?” In short, they help us think about the diversity and variations across space, time and groups.

The literature on complementarity games emphasized *coordination failures* as the key notion to understand these questions. This paper argues that such emphasis is misplaced; the key to understand the diversity is *symmetry-breaking*. The notion of coordination failures is not only irrelevant but also misleading when thinking about the diversity.

1. The Logic of Coordination Failures

Figure 1a shows a simple complementarity game. When the players engage in complementary activities, each has an incentive to increase its effort level, x, if the others are expected to increase theirs, x^e, as indicated by the upward-sloping reaction function, $x = R(x^e)$. The equilibrium is given by $x^* = R(x^*)$. This complementarity game has multiple stable equilibria, x^H and x^L, which suggests that the worst of the two, say x^L, may prevail. The beliefs that the bad equilibrium would prevail, once widely held, become self-fulfilling. When this happens, the players suffer from their coordination failures; they fail to coordinate their expectations to play a better equilibrium, x^H.

The co-existence of Pareto-ranked multiple equilibria and the possibility of coordination failures are commonly suggested as an explanation for the observed diversity across space, time and groups. In development, they argue that the regional disparity occurs because the poor are trapped in a bad equilibrium, while the rich manage to play a good one. In macroeconomics, they argue that the economy suffers from the coordination failures during recessions, but succeeds in solving the coordination problem during booms. In labor economics, it is said that discrimination occurs because the employer has self-fulfilling negative perception against some, but not all, groups of workers.

While insightful, the economics of coordination failures has some drawbacks as an explanation for the diversity across space, time, and groups.

First, strategic complementarity is not sufficient for multiple stable equilibria. Their existence requires the presence of an unstable equilibrium, $x^* = 1$, which in turn requires sufficiently strong complementarity, $R'(1) \equiv \theta > 1$. This raises the question of the empirical relevance of coordination failures.

Second, the logic of coordination failures offers no compelling reason why we should expect to observe the diversity. Nothing in Figure 1a suggests that different equilibria should prevail in different regions, periods, or markets. When we argue that, in Figure 1a, one country is at x^H and the other is at x^L, we implicitly assume that there is no interaction between the two countries. However, this assumption also allows us to argue that the same equilibrium should prevail in both countries. This approach is thus subject to a common criticism against a model of multiple equilibria; the model can explain anything, which makes it empirically irrefutable. This limitation can be seen more clearly in Figure 1b. If each country can be analyzed independently by means of Figure 1a, this two-country world economy has indeed four stable equilibria, (x^L, x^H), (x^H, x^L), (x^L, x^L), and (x^H, x^H). The only first two, the asymmetric ones, exhibit the diversity, while the last two, the symmetric ones, do not. Only the asymmetric equilibria are useful for explaining uneven development. The symmetric ones are nothing but a nuisance, which merely weakens the prediction of the theory.

The problem of the above approach is that the game is not designed to explain the observed diversity across space, time and groups; any implication of coordination failures on this matter is given merely as an after-thought. To be able to impose any restriction on the equilibrium diversity, we must explicitly model how different regions, periods, or groups interact with one another. As long as one treats them separately, there is no way of generating stable asymmetric equilibria without generating stable symmetric ones.

2. The Logic of Symmetry-Breaking[2]

Let us now consider the following game, taken from Matsuyama (1999c). The players engage in two activities and choose its effort level in each. Each activity is complementary across the players. The two activities are linked, because the players can substitute their efforts across the two. Formally, the player chooses $x = (x_1, x_2)$ to maximize the payoff: $V(x; x^e) = R(x^e_1)x_1 + R(x^e_2)x_2 - [x_1^2 + x_2^2 - (\beta/2)(x_1 - x_2)^2]/2$, where $x^e = (x^e_1, x^e_2)$ is the choice of the other players.

The first two terms represent the benefits from participating in the two activities. The marginal benefit of x_j is $R(x^e_j)$, which is increasing in x^e_j. The third term represents the cost, where β measures the substitutability of the player's efforts across the two activities. If $\beta = 0$, $V(x; x^e) = \Sigma^2_{j=1} \{R(x^e_j)x_j - (x_j)^2/2\}$, so that the players choose the efforts in the two activities independently, and their best responses are given by $x_j = R(x^e_j)$. Thus, this game can be analyzed by means of Figure 1a. If $\beta = 1$, $V(x;x^e) = R(x^e_1)x_1 + R(x^e_2)x_2 - [(x_1 + x_2)/2]^2$, so that each player's efforts in the two activities are perfect substitutes. With $\beta \in [0, 1)$, this specification allows for the whole range of intermediate cases between these two extremes. This game is meant to capture the situations, where the players participate in two complementarity games simultaneously, but the two games compete for the player's effort and resources. This interdepedence makes the player's marginal cost of effort in one game increases with the effort in the other. For example, the players may choose the timing and the volume of durable goods purchases over time. Or they may be multinational investors, who allocate their resources across countries. In the context of the labor market, the players may be employers, who allocate their recruiting efforts across different pools of workers.

The best response of each player is given by the two first-order conditions:

(1) $V_1(x; x^e) = R(x^e_1) - (1-\beta/2)x_1 - (\beta/2)x_2 = 0.$

(2) $V_2(x; x^e) = R(x^e_2) - (\beta/2)x_1 - (1-\beta/2)x_2 = 0.$

The equilibria of this game are given by $x^* = (x_1^*, x_2^*)$ that solve (1) and (2) with $x^* = x = x^e$. We test the stablity of an equilibrium by the following dynamical system:

(D) $\dot{x}_{jt} = V_j(x_t; x_t)$ $(j = 1, 2)$.

Thus, taking the choices of the other players as given, each player moves in the direction that increases its payoff.[3] Note that $x = x^*$ is an equilibrium of this game if and only if it is a stationary point of (D). The stability of an equilibrium can be examined by linearizing (D) around the equilibrium and calculating the eigenvalues of its Jacobian matrix. The equilibrium is stable if and only if both eigenvalues have negative real parts.

Suppose $R(x) = 1 + \theta(x-1) - \mu(x-1)^3$, where $\theta, \mu > 0$, and $\mu > \theta - 1$, and $\theta \equiv R'(1)$ represents the complementarity parameter.[4] Since $R(1) = 1$, $(x_1^*, x_2^*) = (1, 1)$ is always an equilibrium. When $\theta < 1-\beta$, it is the only equilibrium and stable. When $1-\beta < \theta < 1$, it loses the stability, which generates a pair of stable asymmetric equilibria, (x^L, x^H) and (x^H, x^L), as shown in Figure 2a. Along the 45° line, however, it remains stable and there is no other symmetric equilibrium. This symmetry-breaking bifurcation thus generates the pair of asymmetric equilibria as the only stable, and hence observable, outcomes. When $1 < \theta < 1+\beta/2$, (1, 1) now becomes unstable along the 45° line to generate a pair of unstable symmetric equilibria. When $\theta > 1+\beta/2$, these two symmetric equilibria become stable, so that there are now four stable equilibria, only two of which are asymmetric, as shown in Figure 2b.

Figure 2c summarizes the results. In Case D, this model has little predictive content, because both asymmetric and symmetric stable equilibria exist. The prediction of this game, as a theory of equilibrium diversity, is most powerful in Cases B and C, where all the symmetric equilibria are unstable. In these cases, the players participate in different activities at different levels in any stable outcome. Thus, the model predicts diversity. Note that coordination failures are not responsible for the equilibrium diversity. Note also that Case B does not necessarily require strong complementarity. It can occur for an arbitrarily small θ, if the interdependence across the two activities, β, is sufficiently high.

3. Discussions

A. Globalization and Uneven Development

What explains uneven development or what Danny Quah (1993) calls "twin peaks" in the world economy? In the standard neoclassical model, any cross-country differences are explained by the variations in other variables, such as the total factor productivity, and yet the variations in these variables are left unexplained. One advantage of the economics of complementarites is that it can explain the diversity across countries without assuming inherent differences.

Many studies in the literature stress coordination failures as an explanation for uneven development. For example, in the closed economy models of Kevin Murphy, Andrei Shleifer, and Robert Vishny (1989), aggregate demand spillovers create Pareto-ranked multiple equilibria. This result is then interpreted to say that the rich play the good equilibrium, while the poor play the bad one. This explanation cannot impose any restriction on the degree of inequality observed in the world economy. It also gives the impression that the closedness of the national economy is responsible for uneven development, which seems to suggest the openness as another policy implication. Murphy, Shleifer and Vishny indeed stressed the closed economy assumption, and discussed at length the empirical importance of the domestic demand spillovers, which can be interpreted as a large θ.

As Case B suggests, however, neither the closed economy assumption ($\beta = 0$) nor significant complementarities (a large θ) may be essential for the inequality. With small domestic complementarities, a greater interdependence between countries makes the inequality more likely, which suggests drastically different implications; globalization magnifies the inequality, instead of reducing it. Furthermore, this approach predicts the emergence of "twin peaks," instead of merely suggesting the possibility.

Those working in international economics have naturally looked at the problem of uneven development from a global perspective. Paul Krugman (1981) and Matsuyama (1996) demonstrated that free trade causes the inequality of nations with (arbitrarily) small country-specific externalities. In models of wealth-constrained investment, Mark Gertler and Kenneth Rogoff (1990) and Matsuyama (2001) showed that financial market globalization leads to capital flows from the poor to the rich countries, thereby amplifying the inequality. In a trade model with transport costs, Krugman and Anthony Venables (1995) showed that the inequality of the symmetric regions occurs with small transport costs. In Matsuyama (1999b), geographical asymmetry across regions, however small, is magnified to create a clustering of the industries, as the transport cost goes down.

B. Intertemporal Substitution and Business Cycles

In the standard model, business cycles are driven by some exogenous shocks. While useful for understanding the propagation mechanisms, this approach cannot answer questions like, "why are there booms and recessions?" or "are there any role of automatic stabilization policies?". One advantage of the economics of complementarities is that it requires no exogenous shocks. Cooper and Andrew John (1988) is the most influential work. From a model with multiple equilibria or multiple steady states, one can construct fluctuating equilibrium paths by letting the players in the economy play different equilibria in different periods. A jump from one equilibrium to another is coordinated by "sunspots," a random variable with no intrinsic effect on the economy.   One common criticism against the sunspot theory is that it allows too many equilibria, including both fluctuating and nonfluctuating ones. Case B suggests that introducing large intertemporal substitution (a large β) could make the no-fluctuation equilibrium unstable and lead to cyclical behaviors, with small intratemporal complementarities (a small θ).

Many recent studies have taken such approach. The players choose the timing of innovation in Shleifer (1986) and Matsuyama (1999a). With some intratemporal complementarities, this leads to a synchronization of these activities. The presence of inventories, intertemporal substitution of labor, or durable goods, can also lead to a production bunching; see Cooper (1998, Ch.6). In the search externality model of Peter Diamond and Drew Fudenberg (1989), business cycles are generated partly because the players must alternate between production and search.

These models also differ in policy implications. In the sunspot models, the recession is a waste, and can be avoided. In models based on the instability of the non-fluctuation equilibrium, the recession is just one phase of inevitable business cycles, which may not be a waste in that it prepares the economy for booms in the future. An attempt to eliminate the recession may not only be counterproductive. It may simply end up shifting the recession from one period to another.

C. Race and Gender Gaps in the Labor Market

A common explanation for racial and gender gaps in the labor market is that they reflect the group differences in human capital investments. However, the differences in human capital themselves may be a result of discrimination. The recent work on statistical discrimination demonstrates that, with imperfect information, differential treatments of inherently identical groups might occur. For example, Stephen Coate and Glenn Loury (1993) constructs a model with Pareto-ranked multiple equilibria. The employer's belief that the workers lack necessary skills discourages them from acquiring the skills, and hence becomes self-fulfilling. The belief that they are highly skilled is also self-fulfilling. The group differences arise endogenously when different equilibria prevail for different groups of workers. The model treats the labor market for each group separately, hence it is consistent with both the prevalence and the absence of discrimination. Furthermore, the separation ensures that any attempt to help the disadvantaged group has no ramification for the other groups; i.e., any possibility of "reverse discrimination" is ruled out by assumption.

There are many reasons to think, however, that different labor markets are inseparable. For example, the labor markets for different ethic groups may be linked on the demand side, because the employers need to allocate their recruiting efforts across ethic groups as in George Mailath, Larry Samuelson, and Avner Shaked (2000). Or the male and female labor markets may be linked on the supply side. The married couple may decide jointly how to allocate their times between their carriers and household production, as in Patrick Francois (1998). The parents may choose how much to invest in the education of their son and daughter. Case B suggests that, if such linkages are strong enough, the equal treatment equilibrium become unstable with even small informational problems, leaving differential treatments as the only stable outcome. Unlike the models that treat different groups separately, the equilibrium in these models imposes the restriction on the race and gender gaps, which makes it possible to make predictions concerning the changing trends in discrimination.

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Figure 1a

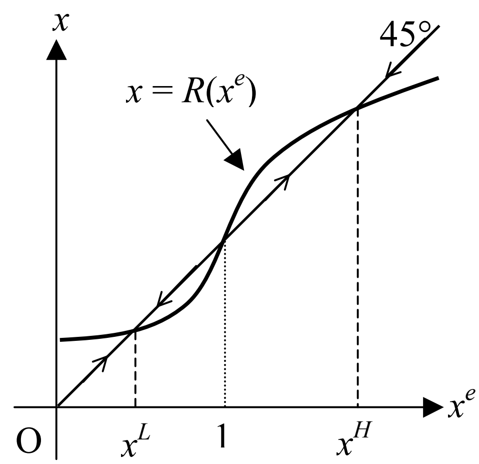


Figure 1b

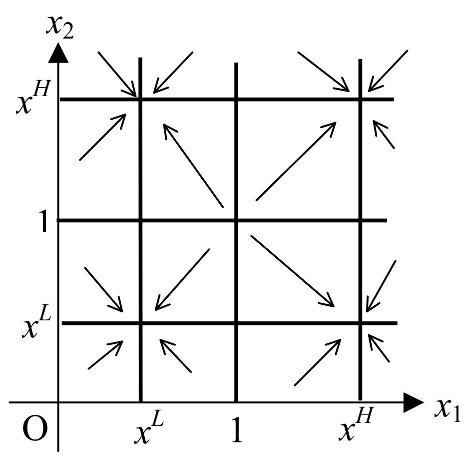


Figure 2a

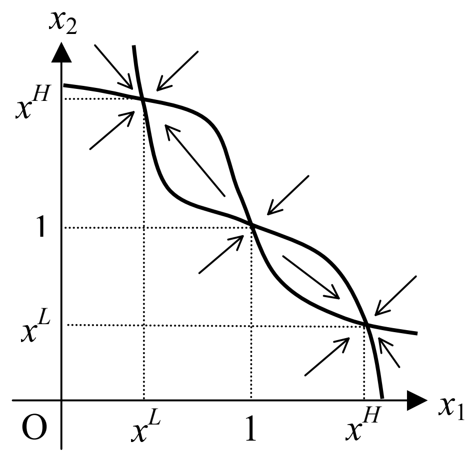


Figure 2b

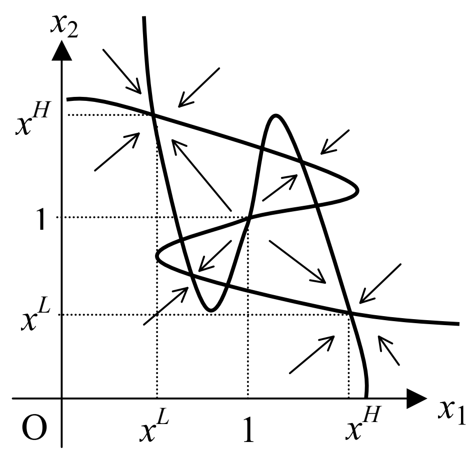
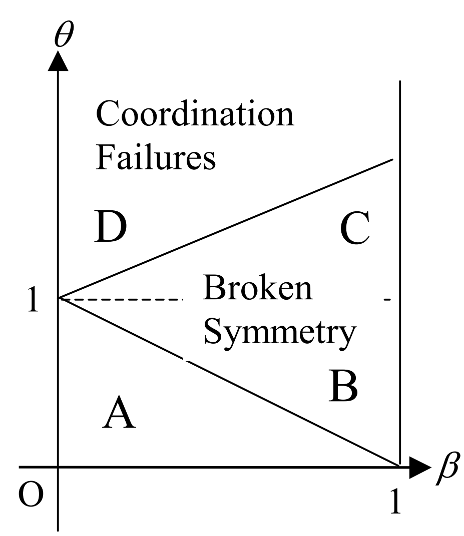


Figure 2c



FOOTNOTES:

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[1]Russell Cooper (1999) and Debraj Ray (1998) survey applications in macroeconomics and development, respectively.

[2]Matsuyama (1995) discussed the notion of symmetry-breaking from a different angle.

[3]This test is chosen for concreteness only. There are many other ways to differentiate the stability property of the equilibrium, which yield the same result.

[4]See Matsuyama (1999c) for a more general case.