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A LEARNING EFFECT MODEL OF INVESTMENT: AN ALTERNATIVE INTERPRETATION OF TOBIN'S Q

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ABSTRACT

The purpose of this paper is to develop and to estimate a learning effect model of investment and to offer an alternative interpretation of Tobin's q. Estimation results using Hayashi's [8] data are remarkably good. The model also justifies theoretically Tobin's assertion that the investment rate is equal to the natural rate of growth of the economy and that average q is equal to unity in the steady state. We also discuss the role of q as a predictor of the future growth of the economy, which the learning effect model suggests.

I. Introduction

DURING the last decade, there has been a growing interest in Tobin's q theory of investment (Tobin [15]). Tobin's idea is that investment, which is one of the most important links between the real and financial sectors of the economy, is an increasing function of q, namely, the ratio of the market value of a firm to its replacement costs and that the influence of financial markets on investment can be traced by observing the value of q.

Recently, Abel [1], Yoshikawa [19], Hayashi [8], and Summers [14] have attempted to lay the microfoundations of q by arguing that it is equivalent to the adjustment costs theory of investment developed by Lucas [11], Gould [7], Uzawa [18], and Treadway [17]. They show that, first, the optimal rate of investment derived from the adjustment costs theory is an increasing function of marginal q, namely, the ratio of the market value of an additional unit of capital to its replacement costs, and that there exists an exact correspondence between average q and marginal q under certain conditions.

Although their models are admirable for logical rigor, their estimates are far from satisfactory. Hayashi [8] and Summers [14] estimate investment functions for the U.S. economy by taking into account effects of tax policies. The ordinary least squares (OLS) estimate, equation (30) in Hayashi [8] is reproduced as follows:

$$\text{(H-I)} \qquad 100(I/K)_t\% = \underset{(0.840)}{9.80} + \underset{(0.912)}{4.23}\,\tilde{q}_t \qquad \text{D.W.} = 0.43 \quad R^2 = 0.46 \quad \text{S.E.} = 1.03$$

where the numbers in the parentheses are standard errors and $\tilde{q}$ is the tax adjusted q.

First, as we will see in section 3 (Equation H-II, Table 3), lagged capacity utlization rate (CU) has strong predictive power beyond that of contemporaneous q, which casts doubt on the validity of the model because the model indicates that all the information that is relevant to the investment decision is summarized by q.

Second, the Durbin-Watson statistic (D.W.)indicates a strong positive serial correlation in the error term, that is to say, there exists significant inertia in the investment decision.

Third, their models suggest that the investment rate compatible with q=1 is equal to zero, while, according to the estimate, investment rate when q is unity, is significantly positive and even greater than the average rate of investment over the sample period.[1]

The third point mentioned above is particularly worthy of attention in connection with the alleged equivalence between q theory and adjustment costs theory. Tobin's assertion is that the investment rate is equal to the natural rate of growth of the economy when q is unity, which is different from that of adjustment costs theory [15, 16]. Of course, there exists several ways of reconciliation between the two.[2]

Summers [14] attempts to solve it by assuming "that adjustment is costless until some normal level of investment is reached ([14, p. 88])." This assumption, however, is questionable. First, the rate of investment becomes indeterminate when q is below unity, or to put it another way, q is never below unity. Second, what is meant by "normal level" is quite ambiguous. If we interpret it as a technical condition, it coincides with the natural rate of growth only by accident. Perhaps the most appropriate interpretation is that adjustment is costless as long as the current rate of investment is equal to the average rate in the past. The idea underlying it is that the adjustment costs function shifts as the current rate of investment moves, or the longer a firm expands at a constant rate, the less costly capital accumulation at that rate becomes as though there exists a learning effect in the process of capital stock adjustment: the firm learns how to invest efficiently. If this is the case and "normal level" is determined endogenously, then we should incorporate the learning effect explicitly in the optimization of a firm.

Our justifications for a learning effect are minor modifications of Lucas' [11] arguments. Consider a firm composed of two divisions, a production and maintenance division and a planning division. The production and maintenance division uses labor and capital stock, while the planning activity needs some managerial resources as well. Suppose that, for any gross investment rate y, there exists an efficient level of managerial resources M(y). If M(y) is an increasing function of y, we lose no generality by assuming that M(y)=y with appropriate change of measure.

Suppose that the firm invests at a higher rate x than y. This has two different effects, an adjustment costs effect and a learning effect. On the one hand, planning requires a withdrawal of labor and capital from the production and maintenance division, reducing output or raising the physical depreciation rate. This cost is represented by $G_1(x-y)$ where $G_1=0$ if $x \leq y$ and G_1', $G''_1 > 0$ if $x>y$. On the other hand, an increase in planning activity contributes to the accumulation of managerial resources, say, $dy/dt = \theta(x-y)$, by, for example, on-the-job training, revisions in work procedures, i.e., the learning effect. This formula also implies that there exists dislearning effect in case of $x<y$. Furthermore, the change in the level of managerial resources causes the firm to incur organizational costs, $G_2(dy/dt)$, where $G_2(0) = G_2'(0) = 0$ and $G''_2 > 0$. Total effect can thus be written as $G(x-y) = G_1(x-y) + G_2(\theta(x-y))$. As long as the firm has the foundation day, t_0, such that $x(t) = y(t) = 0$ for all $t<t_0$, we can show from $dy/dt = \theta(x-y)$, that y is the exponentially weighted average of

the past investment rates, i.e., "normal level"of investment rate.

The present paper does not provide a rigorous theory of the learning process. Unfortunately, the lack of a theoretical model of learning is the state of the art in the extensive literature on the learning curve or "experience curve" in the field of business economics.[3] Empirical studies, however, provide undeniable evidences of the existence of the learning process. Although these findings center on the effect of accumulated output on the production costs, we can reasonably expect the similar effect of investment in the past on the adjustment costs, since the story of adjustment costs can be interpreted as that of multiproduct firm in which the two products are "output" and "installation."

The purpose of this paper is to develop and estimate a learning effect model of investment and to offer an alternative interpretation of Tobin's q. In section 2, we will develop a learning effect model of investment and see how the standard adjustment costs model is modified by introduction of a learning effect. It turns out that the model relates the market value of the firm to the change in investment rate rather than to the investment rate itself and that the long run elasticity of investment is much greater than the short run elasticity. In section 3, we will estimate the learning effect model by OLS on annual data taken from Hayashi [8]. The behavior of the investment rate derived from our model is a function not only of q, but also of lagged q and lagged investment rate. The results are remarkably good and satisfy the constraints of coefficients the model suggests. We will also test the predictive power of lagged capacity utilization rate (CU). The forward-looking behavior of the firm that forms the basis of the model suggests that lagged values of CU should have no additional explanatory value for the change in investment rate once q and lagged q are included as independent variables. The data support this view. Lagged CU has a slightly negative coefficient. Lagged change in CU has a positive but insignificant coefficient.

In section 4, we will show that our learning effect model indicates, unlike the standard adjustment costs model, that Tobin's q equals unity while the investment rate is equal to the natural rate of growth in the steady-state equilibrium of the economy. We will also discuss the role of q as a predictor of the future growth of the economy, which the learning effect model suggests.

2. Adjustment Costs and the Learning Effect

For the sake of simplicity, it is assumed that there is no taxation and that a firm issues neither debt nor new equity nor repurchases existing shares. Let the certainty equivalent required rate of return be r_t. Then the firm's net present worth is:

$$(1) \quad V_t = \int_t^{+\infty} \{F(K_s, N_s) - w_s N_s - p_s x_s K_s\} \exp\left(-\int_t^s r_u \, du\right) ds,$$

where K, N, x, w, and p are all functions of time and represent capital stock, labor input, gross investment rate, wage rate and price of investment goods in terms of product price respectively, and F is a linearly homogeneous production and satisfies $f'(k) > 0$ and $f''(k) < 0$ where $k = K/N$ and $F(K,N) = f(k)N$.

Now we introduce adjustment costs with learning effect as indicated in section 1. It is assumed that capital stock is accumulated as follows:

$$(2) \quad \dot{K} = (x - \delta - G(x-y))K,$$

where the dot denotes the derivative with respect to time and δ is the physical depreciation rate independent of the gross investment rate and G is an adjustment costs function and satisfies $G(0) = G'(0) = 0$ and $G'' > 0$.[4] y_t is the normal level of investment rate, or the index of the past investment experience of the firm and we will specify:

(3) $y_t = \int_0^{+\infty} x_{t-s}\delta \exp(-\delta s)ds.$

δ is a positive constant and one interpretation of δ is the decay rate of the learning effect. By differentiating (3) with respect to time, we have:

(4) $\dot{y}_t = \delta(x-y).$

Thus, the firm can accumulate its experience of investment by δ% per unit of time if the current investment rate is one percent higher than the normal rate of investment.

Three points should be mentioned here. First, we assume constant returns to scale technology which is consistent with the observed variability of firm size and independence of growth rate and firm size. Second, by setting y and δ equal to zero, the model reduces to the standard adjustment costs models. Third, we implicitly assume the separability between production function and adjustment costs function by introducing the latter in a similar way with Uzawa [18]. If adjustment costs were introduced in a non-separable form as Lucas [11], the optimal investment function would depend on w/p as well as q.[5].

The firm's objective at time zero is the maximize V_0 subject to the constraints (2), (4) and K_0 and y_0. The optimal paths x_t and N_t must satisfy the following conditions:[6]

(5a) $f(k) - kf'(k) = w,$

(5b) $p = \lambda(1-G'(x-y)) + \theta\delta,$

(5c) $\dot{\theta} = \theta(r + \delta + G(x-y) - x) - (\lambda - p),$

(5d) $\dot{\lambda} = \lambda(r + \delta + G(x-y)) - (\lambda - p)x - \theta\delta(x-y) - f'(k),$

where λ and θK are the shadow prices for the constraints (2) and (4) respectively and satisfy:

(5e) $\lim_{t \to +\infty} \lambda_t K_t \exp(-\int_0^t r_u du) = 0,$

(5f) $\lim_{t \to +\infty} \theta_t K_t y_t \exp(-\int_0^t r_u du) = 0.$

Since the shadow price of capital stock is λ, λ/p is the marginal q. For the average q, i.e., V/pK, we have:

$$(6a)\quad q_t = \lambda_t/p_t - \frac{\mathcal{G}}{p_t K_t}\int_t^{+\infty} \theta_s(x_s - y_s)K_s \exp\left(-\int_t^s r_u\, du\right)ds,$$

and by differentiating (6a) and substituting it into (5),

$$(6b)\quad \dot{q} = q(r + \delta + G(x-y) - \dot{p}/p) - (q-1)x - f'(k)/p.$$

We can obtain the standard adjustment costs theory of investment by setting y and $\mathcal{G}$ equal to zero in the equations (5) and (6). From (6a), average q coincides with marginal q and from (5b), we have the investment function:

$$(7)\quad x = I(1-q^{-1}),$$

where I is the inverse function of G', therefore satisfies $I(0) = 0$ and $I' > 0$.[7] Clearly, investment is positive if and only if q is greater than unity. Standard adjustment costs theory of investment has a different characteristic from that of Tobin's q theory of investment.

On the other hand, the existence of learning effect implies:

$$(8)\quad x = y + I\,(1 - p/\lambda + \theta\mathcal{G}/\lambda).$$

Several points deserve emphasis. First, the market valuation correlates more with the change in investment rate than the investment rate itself. Secondly, the long run elasticity of investment with respect to the value of the firm is much greater than the short run elasticity. Third, we have no one-to-one correspondence between marginal q and average q in spite of the constant returns to scale technology assumption. This is because the firm's history or experience of growth as well as the existing capital stock are assets of the firm and average q contains information about market valuations of both. Therefore the functional relationship between the gross investment rate x and q is an approximate one, not an exact one.

In order to obtain further characteristics of the learning effect model, it is necessary to specify the expected paths w_t, r_t, and p_t, as we will see in section 4.

3. Some Empirical Evidence

We will estimate the investment function derived from the learning effect model in this section. To do so, we show first the modified investment function by taking into consideration the same tax system as that of Hayashi [8]. Because the derivation of the modified equation can be done in a similar way as Hayashi [8], we do not elaborate on it here.

Let u_t be the corporate tax rate, ITC_t the rate of the investment tax credit, z_t the present value of tax savings due to deprecation allowance on one unit of product value of capital installed at time t, and a_t the present value of current and future tax deductions attributable to past investments divided by $p_t K_t$.[8] Then the investment equations are modified as follows:

$$(9)\quad x = y + I(1 - \frac{p(1-ITC-z)-\theta\mathscr{G}}{\lambda}),$$

and:

$$(10)\quad \frac{\lambda t}{p_t} = q_t - a_t + \frac{\mathscr{G}}{p_t K_t}\int_t^{+\infty}\theta_s(x_s - y_s)K_s \exp(-\int_t^s r_u du)ds,$$

$$(11)\quad \theta_t = \frac{1}{K_t}\int_t^{+\infty}(\lambda_s - p_s(1-ITC_s - z_s))K_s \exp(-\int_t^s r_u du)ds.$$

Unfortunately, we are not able to observe λ_t/p_t and θ_t/λ_t separately, which inevitably requires approximation in order to estimate the equation. By denoting the third term on the right hand side of (10) as A_t,

$$(12)\quad 1 - \frac{p(1-ITC-z)-\theta\mathscr{G}}{\lambda} = 1 - \left[\frac{1-ITC-z}{q-a}\right](1+A/(q-a))^{-1} + \frac{\theta\mathscr{G}/p(q-a)}{1+A/(q-a)}.$$

It can be shown that $A/(q-a) \doteq 0$, and $\theta/(q-a) \doteq 0$ in the vicinity of the steady state. Therefore, we have approximately:

$$(13)\quad 1 - \frac{p(1-ITC-z)-\theta\mathscr{G}}{\lambda} \doteq 1-\tilde{q}^{-1},$$

where $\tilde{q} = (q-a)/(1-ITC-z)$, Hayashi's [8] tax adjusted q.

Hence, (9) can be rewritten as:

(14) $\dot{x} = y + I(1 - \tilde{q}^{-1})$.

The discrete-time analogues of (14) and (3) are, if we rewrite $I^*(q) = I(1-q^{-1})$,

(15) $x_t \doteq y_t + I^*(\tilde{q}_t)$, (t is an integer)

(16) $y_t = \sum_{i=1}^{+\infty} \delta x_{t-i} (1 - \delta)^{i-1}$, $(0 < \delta < 1)$

Using (16) to eliminate y_t in (15) we have:

(17) $x_t \doteq x_{t-1} + I^*(\tilde{q}_t) - (1 - \delta) I^*(\tilde{q}_{t-1})$.

Finally, we specify the functional form of I^* as follows:

(18) $I^*(q) = a + bJ(q) \equiv a + b(q - 1)$

which permits the model of firm to be applied to aggregate data.[9] If our model is valid, $a = 0$, and $b > 0$. Estimations are to be done for the following forms:

(19) $x_t = c_o + c_1 J(\tilde{q}_t) + c_2 J(\tilde{q}_{t-1}) + (1+c_3) x_{t-1} + \varepsilon_t$,

where $c_o = a$, $c_1 = b$, $c_2 = -(1-\delta)b$, and $c_3 = 0$.[10]

The predictions should meet the following criteria:

(i) The intercept term, c_o, should not be significantly different from zero.

(ii) The coefficients of current and lagged q_t, c_1 and c_2, should satisfy significantly, $c_1 > 0 > c_2$, $c_1 + c_2 > 0$.

(iii) The coefficient of lagged investment rate should not be significantly different from unity, or c_3 should not be significantly different from zero.

In Table 1, we show four estimates, i.e., 2x2 = 4, for the cases in which the constraints $c_o = 0$ and $c_3 = 0$ are imposed or not. The data are annual data for the period 1952-76 for the U.S. corporate sector from Table 1 in Hayashi [8]. The figures in parentheses are t-values for the null hypothesis that the coefficients are zero. The figures in column 8 represent Durbin's h statistics for the estimates with no constraints on lagged investment rates terms and Durbin-Watson statistics for the estimates with $c_3 = 0$.[11]

TABLE 1

INVESTMENT EQUATIONS[a]

	c_1	c_2	$1+c_3$[b]	c_a[b]	c_1+c_2	c_3[b]	SE[c]	DH/DW[d]	DF[e]
L-I	7.18	-5.79	0.955	0.60	1.39	-0.045	0.55	-1.06*	20
	(7.69)	(-5.12)	(7.41)	(0.33)	(2.00)	(-0.35)			
L-II	7.29	-6.05	0.997		1.24	-0.03	0.54	-1.04*	21
	(8.55)	(-7.66)	(115.70)		(2.42)	(-0.31)			
L-III	7.30	-6.07		-0.04	1.23		0.54	2.43	21
	(8.55)	(-7.60)		(-0.29)	(2.35)				
L-IV	7.32	-6.03			1.29		0.53	2.40	22
	(8.83)	(-7.83)			(2.77)				

[a] Data source is Hayashi [8 Table 1]. Dependent variables are 100 $(I/K)_t$%. The figures in the parentheses are t-values.

[b] Blanks in the third, fourth and sixth columns indicate the constraints $c_g = 0$ $c_3 = 0$. Explanatory variables in the third column are also normalized to 100 $(I/K)_{t-1}$%.

[c] Standard errors of regressions.

[d] The figures with asterisks are Durbin's h statistics and those without them are Durbin-Watson statistics.

[e] The final column indicates degrees of freedom.

The results obtained are very satisfactory and strongly verify the learning effect model. We can also calculate the decay rate of the learning effect and adjustment costs from the estimates above. Calculation results are shown in Table 2. The figure in the left column represents $\hat{\delta} = (\hat{c}_1 + \hat{c}_2)/\hat{c}_1$. Thus, the annual rate of decay is about 0.18. The figure in the right column is obtained by substitution of $c_1 = b$ and $X = 0.01$ into the adjustment cost equation in footnote 9. This indicates that the physical depreciation rate rises about 0.06% when the current gross investment rate is one percent higher than the normal level.[12]

TABLE 2
DECAY RATE AND ADJUSTMENT COSTS

Investment equations	$\hat{\delta}$	G(1%)
L-IV	0.176	6.27×10^{-4}

Finally, the predictive power of lagged capacity utilization rate (CU) is tested. If lagged CU has substantial predictive power beyond that of q and lagged q, then it would support the alternative views that the firms use an ad hoc, nonoptimal criterion in investment decisions.[13] The data support our model. Equation L-V in Table 3 presents regression of the form, $x_t = x_{t-1} + c_1 J(\tilde{q}_t) + c_2 J(\tilde{q}_{t-1}) + c_4 CU_{t-1}$, testing the predictive power of CU_t, Federal Reserve index of capacity utilization rate in manufacturing. The coefficient of lagged CU have a slightly negative value. It turns out that, among a variety of regressions testing the predictive power of lagged CUs, lagged change in CU has the most substantial predictive power. Table 3 also presents regression of the form, $x_t = x_{t-1} + c_1 J(\tilde{q}_t) + c_2 J(\tilde{q}_{t-1}) + c_5 (CU_{t-1} - CU_{t-2})$. Equation L-VI shows that the null hypothesis that lagged change in CU has no predictive power is not rejected at the significant level of five per cent (one-tailed test).[14]

This is not the case with Hayashi's investment equation. Equation H-II in Table 3 shows the regression of the form $x_t = (c_0 - c_1) + c_1\tilde{q}_t + c_4 \cdot CU_{t-1}$ and the coefficient on lagged CU is highly significant. Including a CU variable in regression drastically increases the predictability of Hayashi's investment equation.

TABLE 3

TESTING THE PREDICTIVE POWER OF LAGGED CAPACITY UTILIZATION RATE[a]

	c_1	c_2	c_4	c_1+c_2	SE	DW	DF
L-V	7.29 (8.56)	-6.06 (-7.62)	-0.04 (-0.26)	1.24 (2.39)	0.54	2.42	21
	c_1	c_2	c_5	c_1+c_2	SE	DW	DF
L-VI	7.33 (9.19)	-6.30 (-8.30)	3.80 (1.65)	1.04 (2.19)	0.51	2.71	21
	c_0-c_1	c_1	c_4	[illegible]	SE	DW	DF
H-II	1.33 (0.46)	4.10 (5.70)	15.13 (4.41)		0.75	1.38	22

[a] The capacity utilization rate is measured as Federal Reserve index of capacity utilization rate in manufacturing. See also notes for Table 1.

4. Aggregate Investment and Tobin's q in the Steady-state and in the Business Cycle

In this section, we investigate the implications of the learning effects model on Tobin's q in a macroeconomic framework. We assume a one commodity economy in a taxless world and $\xi = 0$ for simplicity. Then we have $p_t = 1$ and $\dot{p}/p = 0$ for all t. Therefore, the learning effect model is described by the transversality conditions and the following equations:

(20a) $$f(K/N^d) - (K/N^d)f'(K/N^d) = w,$$

(20b) $$x = y+I(1-(1-\theta\omega)/\lambda),$$

(20c) $$\dot{\theta} = \theta(r+G(x-y) - x) - (\lambda-1),$$

(20d) $$\dot{\lambda} = \lambda(r+G(x-y)) - (\lambda-1)x - \theta\omega(x-y) - f'(K/N^d),$$

(20e) $$\dot{q} = q(r+G(x-y))-(q-1)x-f'(K/N^d),$$

where N^d_t represents the aggregate demand for labor at time t.

In order to investigate further properties of the model, it is required to specify the expected paths r_t, and, what is more important, w_t. We assume perfect foresight concerning the real wage with continuous labor market clearing. Suppose that labor supply is inelastic with respect to the real wage and grows at the rate n_t, namely, $N^s_t = N_o \exp(\int_o^t n_u du)$. Real wage is competitively determined by continuous market clearing $N^d_t = N^s_t$ for all t. Then the model is described by the transversality conditions and (20b) and:

(21a) $\dot{y} = \mathcal{G}(x-y)$,

(21b) $\dot{k} = (x-G(x-y)-n)k$,

(21c) $\dot{\theta} = \theta(r+G(x-y)) - (\lambda-1)$,

(21d) $\dot{\lambda} = \lambda(r+G(x-y)) - (\lambda-1)x-f'(k) - \theta\mathcal{G}(x-y)$,

(21e) $\dot{q} = q(r+G(x-y)) - (q-1)x-f'(k)$.

Let $r_t = \bar{r}$ and $n_t = \bar{n}(\bar{r} > \bar{n} > 0)$ in order to investigate the steady state properties of the model.[15] If we indicate the steady state value of variables by asterisks, we have:

(22a) $x* = y* = \bar{n}$

(22b) $\lambda* = q* = 1$,

(22c) $\theta* = 0$,

(22d) $f'(k*) = \bar{r}$,

which form striking contrast to the standard adjustment costs theory for which:

(23a) $x*-G(x*) = \bar{n}$,

(23b) $q* = (1-G'(x*))^{-1} > 1$,

(23c) $f'(k^*) = \bar{r} + (1-G'(x^*))^{-1} (G(x^*) + (\bar{r}-x^*) G'(x^*)) > \bar{r}$.

Equations (22) state that in the steady state in the learning effect model, the rate of investment equals the natural rate of growth, average q equals unity, and the marginal productivity of existing capital stock equals the rate of return on equity, which is Tobin's assertion. This is because no adjustment costs are required in the steady state by virtue of the learning effect.

We have q=1 in the steady state partially due to the expectation that there will be no change in the rate of investment, which is seldom the case. Therefore, it should be kept in mind that there is nothing surprising about the long run value of q, namely, the sample mean of average q, being significantly different from unity.

Our model suggests the values of q in the business cycle are likely to be higher (lower) than unity when the economic growth is expected to accelerate (decelerate) and likely to be more volatile. When the anticipations of the future economic growth is changed, q will respond fully and instantaneously, while investment rate will not because it costs firms to deviate from the normal level of investment rate and this implies a series q displays a phase lead over the investment rate series.

To illustrate this, suppose that the economy is initially in the steady state where $x_t = y_t = \bar{n}$, and anticipations concerning the future economic growth of the economy are revised such that optimal investment paths shifts downward to $x_t = g < \bar{n}$ at time zero for some reason, for instance, oil price shocks. Since $g < y_t$ for all $t > 0$, the marginal valuation of y, θ will plummet from zero and gradually rise as y_t falls. From (10) and (11), this implies that $q_t < \lambda_t < 1$. Suppose now that this newly anticipated course of the economy turns out to be correct. Then y_t will eventually equal g and q will return to unity. This means that plunging q will surge despite continuing pessimism, a phenomenon unique to the learning effect model. Similar argument holds for an upward revision of anticipations as well.[16]

We believe that the relation between q and expected growth discussed above is plausible because a hike in stock prices is often regarded as a harbinger of recovery.[17]

5. Concluding Remarks

As we have seen above, our learning effect model of investment has great advantage over the standard adjustment costs theory of investment as an explanation of actual movements in investment and firms' value and as the microfoundation of Tobin's q theory. Moreover, we can claim that the learning effect model offers a solution of a "problem for the student of investment behavior, to construct a tractable abstraction which is consistent with steady-state or average q being the same in both a high investment and a low investment economy and with q varying considerably in the short run (Prescott [13, pp. 96-97])," even though what he has in mind is not the learning effect, but the time to build technology.[18]

The analysis of the steady state equilibrium in section 4, however, still remains incomplete as a model of market economy, in the sense that it assumes the path of required rate of return be exogenously given, and does not incorporate the product market equilibrium.

To make it complete, the analysis of the consumption-savings decision by households must be carried out just as Uzawa [18] and Abel-Blanchard [2] do with the standard adjustment costs theory. Obviously, one of the major differences from their results is that the steady state in case of learning effect model is characterized by the golden rule where the net marginal product of capital equals the natural rate of growth plus the rate of time preference.

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FOOTNOTES

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1. In other words, Hayashi's [8] tax adjusted q is less than unity in many years, which is a striking contrast with Summers' [14] tax adjusted q.

Utilizing average q as a proxy for marginal q, as well as assumptions of homogeneous capital, perfect capital market and aggregation, is sometimes cited as a potential cause of the empirical difficulties of the theory. We do not regard it as an attractive approach, however, to investigate the relation between investment and marginal q. This is because, first, Abel-Blanchard [3], which construct direct estimates of marginal q, did not find significant differences between marginal q and average q, and secondly, if average q is a poor proxy for marginal q, then q theory loses its raison d'être, for Tobin's basic idea is to explain investment by the observable market valuation, instead of unobservable marginal efficiency of investment and the cost of capital.

2. Once we recognize that the adjustment costs theory deals with investment by an individual firm while q theory is concerned with aggregate investment, we hit upon the following two ideas.

One is a stochastic gimmick. If q for firms are widely different across the economy and the investment functions are convex, then aggregate investment can be positive even when the average of q is unity. But this does not assure the natural rate of growth of the capital stock.

The other is the case in which the increase in the aggregate capital stock is brought about by an increase in the number of rims, which necessitates a new theory explicitly incorporating the costs of entry, as Lucas [11, p. 330] suggested. This is because the investment rate would be indeterminate and q could not differ from unity in the case of free entry.

3. See Lieberman [10] for a survey of the literature.

4. The reason why we call δ the physical depreciation rate independent of gross investment rate is as follows. Uzawa's [18] original formulation of adjustment costs--i.e., the Penrose effect--is $\dot{K} = H(I,K) - \delta K$, where I is gross investment. By the assumption of linear homogeneity, we have $\dot{K} = \{h(x) - \delta\}K$ where $h(x)$ satisfies $h(0) = 0$, $h'(0) = 1$, $h'(x) > 0$ and $h''(x) < 0$. If we define $G(x) = x-h(x)$, then G satisfies the conditions and $\dot{K} = \{x-\delta-G(x)\}K$. Although K in the Penrose function is the index of production function and I is investment expenditure in Uzawa [18], we can reinterpret it as saying that the actual depreciation rate is $\delta + G(x)$, i.e., dependent on the gross investment rate. Obviously δ is the minimum depreciation rate. In fact, it seems that the rate of replacement investment is positively correlated with the rate of gross investment. See Feldstein-Foot [5, p. 54, fn 25].

5. Another slight different which the two formulations make is that, under the existence of a corporate tax, adjustment costs in Lucas' version are deductible from taxable income while those in Uzawa's version are not.

6. We assume the existence of the optimal path and price taking behavior.

7. Mention should be made of the meaning of a q theoretic investment function. Equation (7) as well as (8) below represents only the relationship between the optimal investment and the value of a firm and tell us nothing about the investment itself. To do so, we must analyze the optimal paths of $x(t)$ and $N(t)$ implicitly stated by differential equations with some assumptions on expected paths $w(t)$, $r(t)$, and $p(t)$. This will be done in section 4.

8. For details, see Hayashi [8].

9. This choice of J corresponds to the following specification of the adjustment costs function: $G(X) = X - b\log\{1+(X/b)\}$.

10. Estimations in which the dependent variable is the change in the gross investment rate are done by Malkiel, von Furstenberg and Watson [12]. Delivery lags are their justifications. But the linkage of the change in the gross investment rate to q is not derived in an optimization framework.

11. See Durbin [4] for Durbin's h statistic. This is only a large sample test but can be used for small samples as well.
For the estimates where the coefficient of lagged dependent variables are not constrained to unity, t-test is not reliable. To the best of our knowledge, however, appropriate tests have not been developed. We show these results only for reference.

12. See footnote 4 for a justification of this interpretation.

13. Of course, it does not contradict our model even if contemporaneous CU have high explanatory value. Surprisingly, however, it turns out that it has no additional predictive power with t value -0.16.

15 ~~14~~. This assumption means that perfect foresight paths are constant, not static expectations. The inequality $\bar{r} > \bar{n}$ guarantees transversality conditions.

16 ~~15~~. The following gives us a rough idea of the size of the lead of q series over the investment rate series. Let $x(t) = y(t) + bJ\{q(t)\}$ and suppose $J\{q(t)\}$ moves as $\sin \omega t$. Then it can be shown that $x(t) = g + b\sqrt{1+(\delta/\omega)^2}\,\sin\{\omega(t-h)\}$, where h is given as $\omega^{-1}\tan^{-1}(\delta/\omega)$. This allows us to calculate the lag of $x(t)$ over $J\{q(t)\}$ for any periodicity $T = 2\pi/\omega$. For example, $h = 2.6$ (months) for $T = 7$(years) and $\delta = 0.18$ per year.

17 ~~16~~. Fischer-Merton [6] provides some empirical evidences of the significance of the stock market as a predictor of the business cycle. They also discuss the exogeneity of stock prices.

18 ~~17~~. See Kydland-Prescott [9].

14 In the preliminary version of the paper, we also ran regressions with $J(q) = \log q$ and $J(q) = 1 - q^{-1}$ and found that the lagged change in CU has predictive power in these regressions. But we do not think it strong evidence against the learning model, because non-linear functional forms of $J(q)$ do not permit the model of firm to be applied for aggregated data.

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