Aggregate Implications of Credit Market Imperfections (II)

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Lecture 2: Dynamic Models with Homogeneous Agents

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The Framework

A Model of Convergence

A Model of Endogenous Investment-Specific (IS) Technical Change
- Procyclical Case: Traps and Collapses
- Countercyclical Case: Takeover, Cycles, Growth Miracle

A Model with Good and Bad Projects
- Under-Investment: Inefficient Recessions and Persistence
- Over-Investment: Inefficient Booms and Volatility

A Model with Private Benefits

A Model of Asymmetric Cycles and Intermittent Volatility:
- The Good, The Bad, and The Ugly

The Framework

A Variation of the Diamond OG model

Time: Discrete ($t = 0, 1, 2,\ldots$)

Final Good: Y_t, which can be consumed or invested.

$Y_t = F(K_t,L_t)$, with physical capital, K_t and labor, L_t.

$y_t \equiv Y_t/L_t = F(K_t/L_t,1) \equiv f(k_t)$, where $k_t \equiv K_t/L_t$; $f'(k) > 0 > f''(k)$.

Competitive Factor Markets:

$\rho_t = f'(k_t)$; decreasing in k_t

$w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$, increasing in k_t.

Demography: 2-period lived OG agents.

Each generation consists of homogeneous agents with unit measure

Agents:

- Each agent has one unit of the endowment, “labor,” in the first period only, supplied inelastically.
- Each consumes only in the second. They save everything.
- Each aims to maximize the second-period consumption.

Aggregate Saving: $S_t = W(k_t)$.

Investment Technologies:

Agents can choose one (and only one) of J indivisible projects ($j = 1,2, \ldots J$).

	Period t	*Period t+1*
Type-j Project	m_j units in final good	m_jR_j units in capital & m_jB_j units in final good

m_j: the (fixed) set-up cost,

R_j: project productivity in capital

B_j: project productivity in final good

Does An Agent Want to Invest in Project-j?

By starting a project-j, $C^j_t = m_j R_j \rho_{t+1} + m_j B_j - r_{t+1}(m_j - w_t)$,

By lending, $C^0_t = r_{t+1} w_t$

Profitability Constraint: $C^j_t \geq C^0_t$ or

(PC-j) $R_j f'(k_{t+1}) + B_j \geq r_{t+1}$,

Note: *In the perfect credit market, all credit goes to the projects whose (PC-j) are among the highest.*

Can Agents Finance Project-j?

Borrowing Constraint:

(BC-j) $\lambda_j m_j R_j f'(k_{t+1}) + \mu_j m_j B_j \geq r_{t+1}(m_j - W(k_t))$,

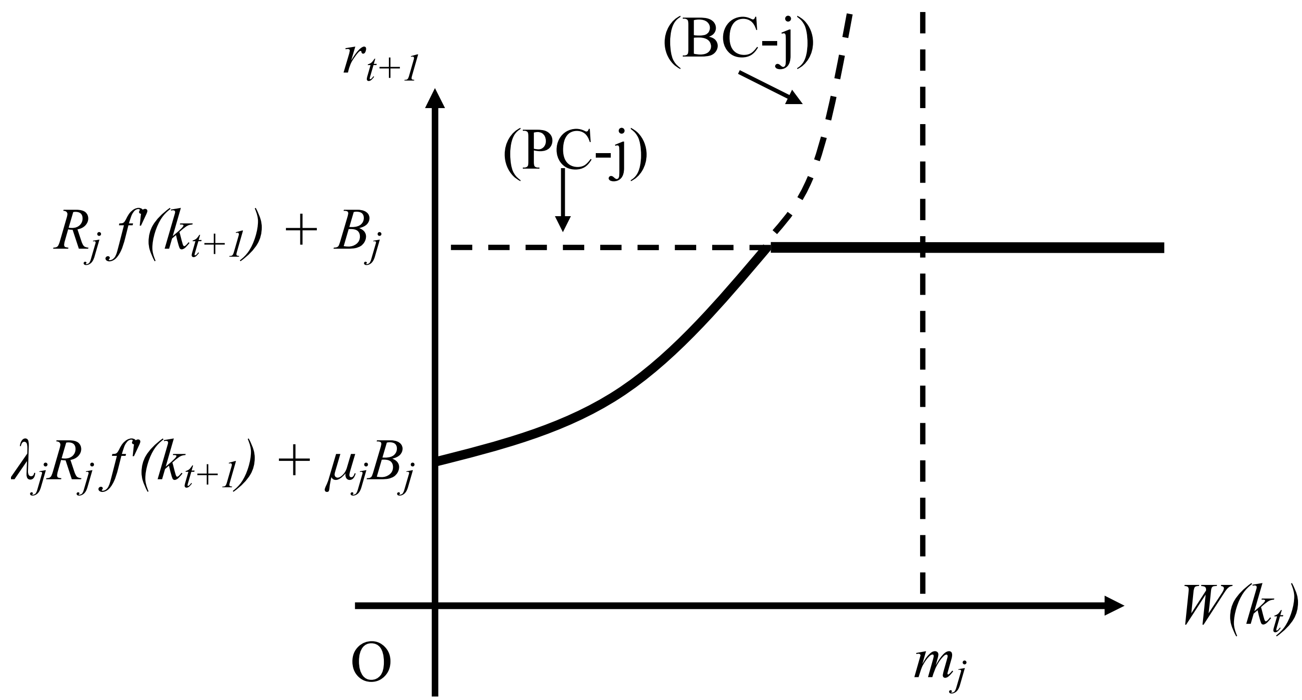
λ_j: pledgeability of capital produced by project-j

μ_j: pledgeability of the final good produced by project-j

Both (PC-j) and (BC-j) must be satisfied for the credit to flow into type-j projects.

How much can the lenders expect from a type–j project?
From (PC-j) and (BC-j),

$$\frac{1}{r_{t+1}} = Max\left\{ \frac{1 - W(k_t)/m_j}{\lambda_j R_j f'(k_{t+1}) + \mu_j B_j}, \frac{1}{R_j f'(k_{t+1}) + B_j} \right\}$$



Equilibrium Conditions;

(1) $W(k_t) = \sum_j (m_j X_{jt})$.

(2) $k_{t+1} = \sum_j (m_j R_j X_{jt})$.

(3) $$\frac{1}{r_{t+1}} \leq Max\left\{ \frac{1 - W(k_t)/m_j}{\lambda_j R_j f'(k_{t+1}) + \mu_j B_j}, \frac{1}{R_j f'(k_{t+1}) + B_j} \right\} \quad (j = 1, 2, \ldots J)$$

where X_{jt} is the measure of type-j projects initiated in period t, and $X_{jt} > 0$ (j = 1, 2,…J) implies the equality in (3).

For $k_0 > 0$, (1)-(3) determine the equilibrium trajectory.

The present framework is highly restrictive in that

- each generation consists of **homogeneous agents**
- the net worth is represented by a single scalar variable, w_t.
- the aggregate saving is inelastically supplied and equal to w_t
- there is only type of **homogeneous capital** that could enhance the net worth of the next generation, which means that the net worth can be represented as a fixed function of a single variable, k_t, i.e., $w_t = W(k_t)$.

In spite of these restrictions, the framework is rich enough to generate a wide range of dynamic behaviors, because, due to **heterogeneity of the investment projects** available to the agents, a movement of the net worth can change ***the composition of the credit***.

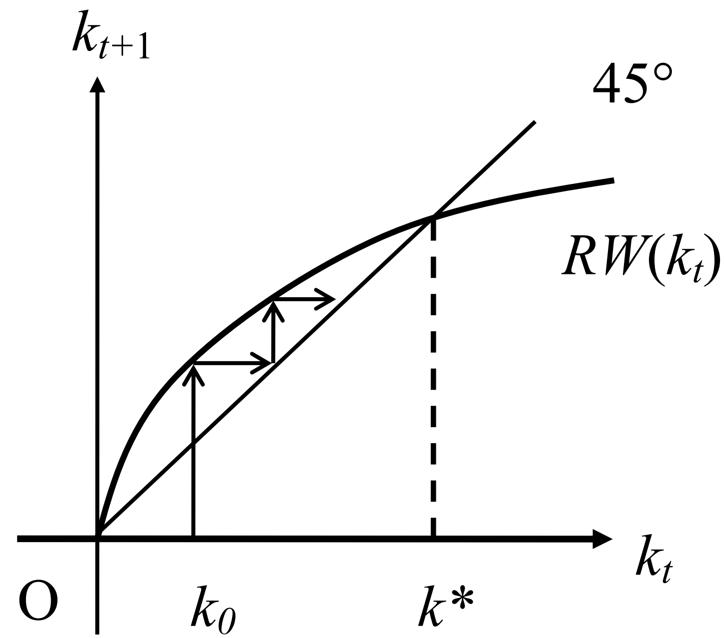
Nevertheless, let us first look at the case of homogeneous projects, which offers a benchmark against which to evaluate the composition effects.

A Model of Convergence

Let J = 1.

(1) $W(k_t) = mX_t$.

(2) $k_{t+1} = RW(k_t)$



Monotone Convergence under

- $W(k)/k$ is strictly decreasing in k,
- $\lim_{k\to+0} W(k)/k = \infty$; $\lim_{k\to+\infty} W(k)/k = 0$.

These assumptions will be maintained for the remainder of the lectures.

$$
(3)\ r_{t+1} = \begin{cases} \dfrac{\lambda Rf'(RW(k_t)) + \mu B}{1 - W(k_t)/m} & \text{if } \dfrac{W(k_t)}{m} \leq 1 - \dfrac{\lambda Rf'(RW(k_t)) + \mu B}{Rf'(RW(k_t)) + B} \\ Rf'(RW(k_t)) + B & \text{if } \dfrac{W(k_t)}{m} \geq 1 - \dfrac{\lambda Rf'(RW(k_t)) + \mu B}{Rf'(RW(k_t)) + B} \end{cases}
$$

These effects on the rate of return will not affect the dynamics, because of

- Inelastic Credit Supply
- Inelastic Labor Supply

which helps to simplify the analysis and to allow us to focus on the composition effects.

Let us now start looking at various cases of heterogeneous projects.

In the next model, the projects do not differ in terms of the composition of the goods they generate.

They differ in productivity, minimum investment requirement, and the severity of agency problems.

A Model of Endogenous Investment-Specific (IS) Technical Change:
Based on Matsuyama (AER 2007)

Let $B_j = 0$ for all j = 1,2, …J.

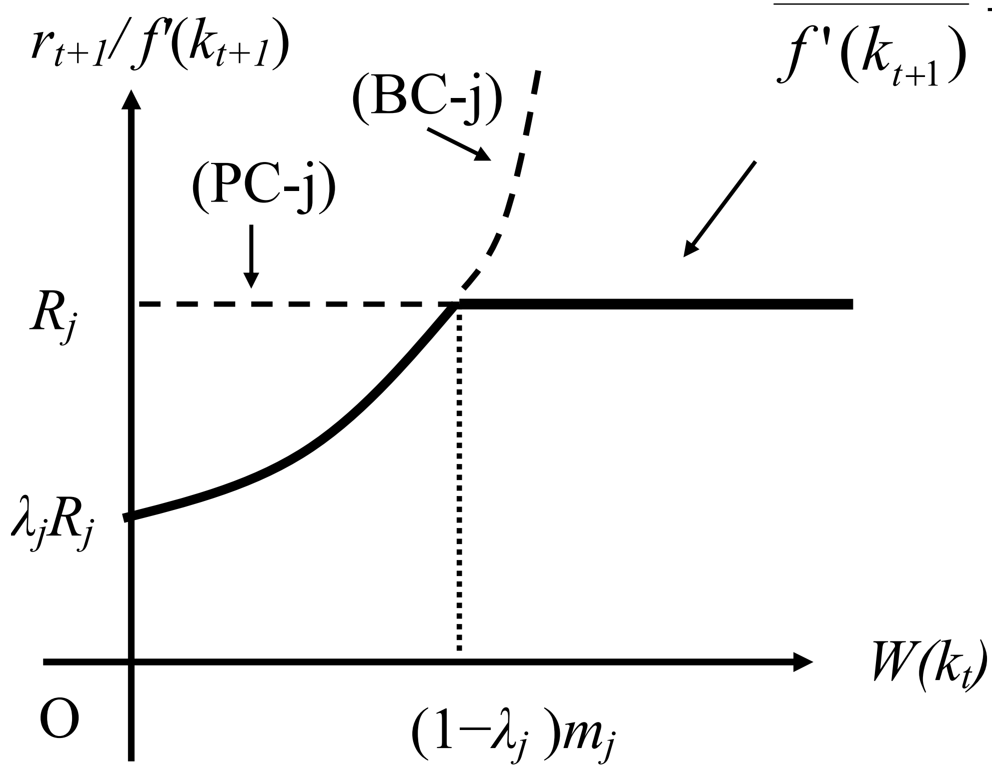
	Period t	*Period t+1*
Type-j Project	m_j units in final good	m_jR_j units in capital

m_j: the (fixed) set-up cost

R_j: project productivity

λ_j: pledgeability

The maximal rate of return the lenders can expect from a type–j project:



$$\frac{r_{t+1}}{f'(k_{t+1})} = \frac{R_j}{Max\left\{\frac{1-W(k_t)/m_j}{\lambda_j},1\right\}}$$

Equilibrium Conditions;

(1) $W(k_t) = \sum_j (m_j X_{jt})$.

(2) $k_{t+1} = \sum_j (m_j R_j X_{jt})$.

(3) $$\frac{r_{t+1}}{f'(k_{t+1})} \geq \frac{R_j}{Max\left\{\frac{1 - W(k_t)/m_j}{\lambda_j}, 1\right\}} \qquad (j = 1, 2, \ldots J)$$

where X_{jt} is the measure of type-j projects initiated in period t, and $X_{jt} > 0$ (j = 1, 2,…J) implies the equality in (3).

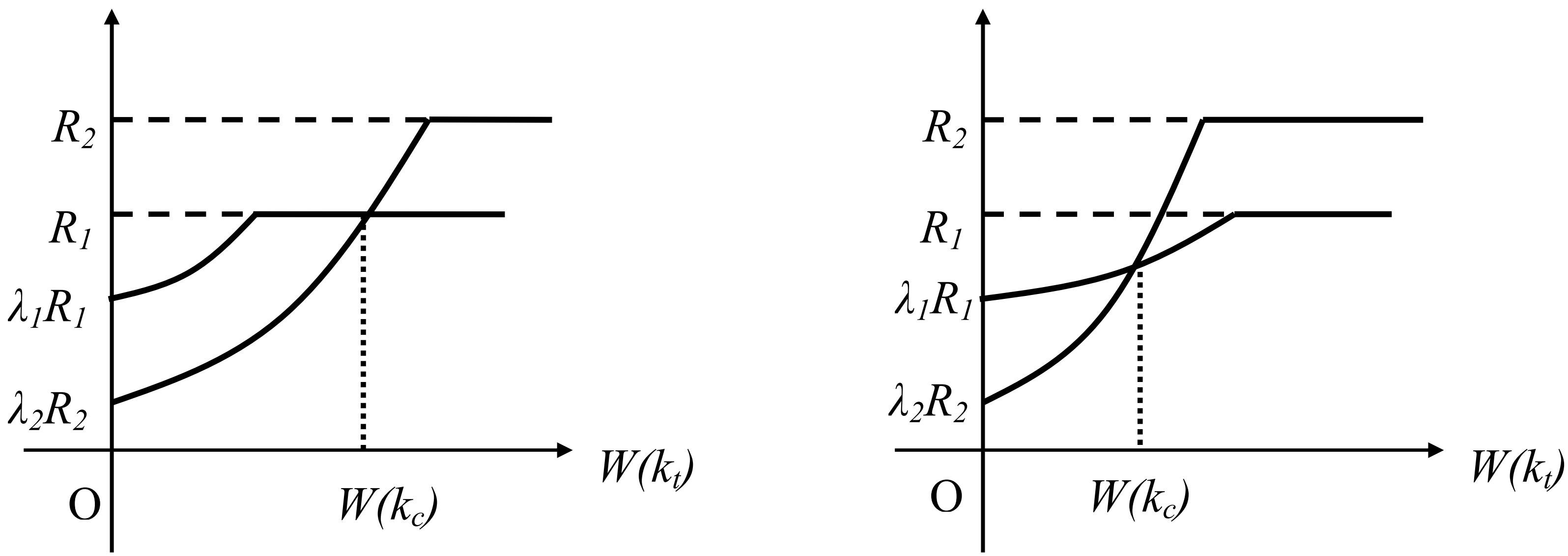
Note: *The projects are ranked according to the RHS of (3), which does not depend on the allocation of the credit.* → Generally, X_{jt} = 1 or 0.

Pro-Cyclical IS Technical Change: Traps and Collapses

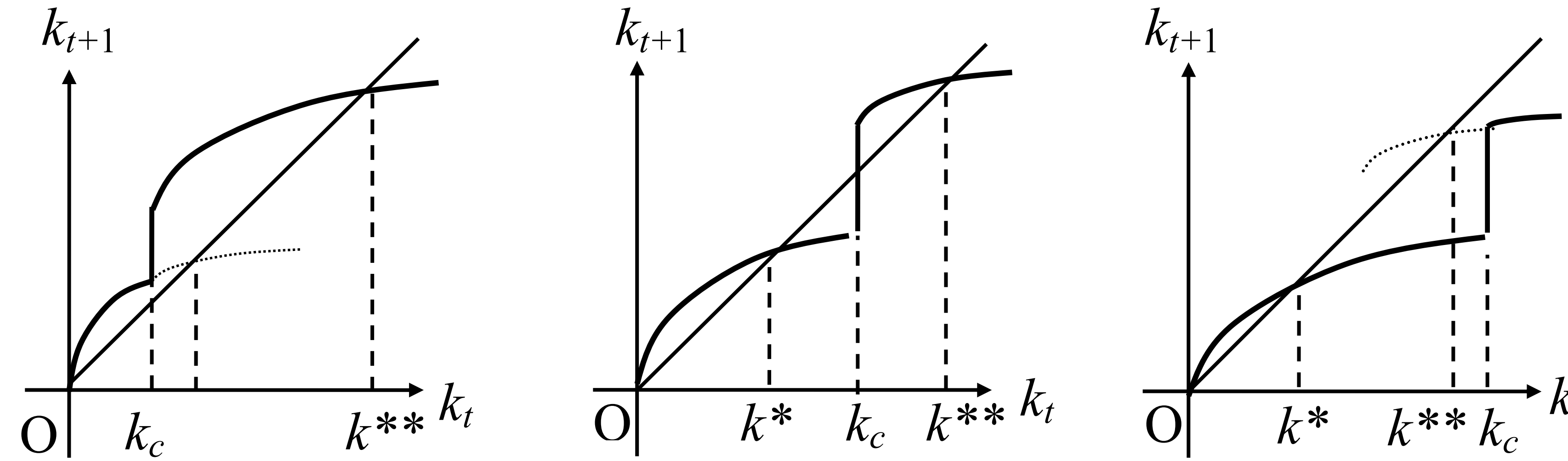
Let J =2 and $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$.

Key Trade-Off: Productivity vs. Agency Problem:

e.g. Some advanced projects that use leading edge technologies (Project-2) may be subject to bigger agency problems than some mundane projects that use well-established technologies (Project-1).



$$k_{t+1} = \begin{cases} R_1 W(k_t) & \text{if} \quad k_t < k_c, \\ R_2 W(k_t) & \text{if} \quad k_t > k_c. \end{cases}$$



Credit-Constrained Growth Credit Traps Credit Collapse

Implications on the Rate of Return Movement:

With a higher k,

- borrowers can pledge more with higher net worth (procyclical)
- credit composition may shift towards more productive projects (procyclical)
- neoclassical capital deepening effect (countercyclical)

In the Credit Trap case, the last two effects exactly offset each other when f is a Cobb-Douglas, in which case the total effect is procyclical.

The rate of return may be higher in the developed or in a booming economy than the undeveloped or in a stagnating economy.

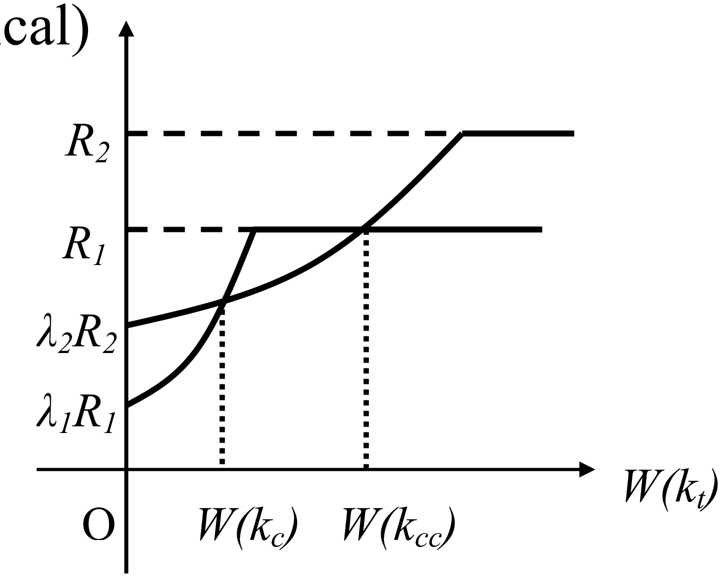
Effect of Improving λ's:
A higher λ_1 can make things worse by increasing k_c, thereby creating a credit trap or causing a credit collapse.

Countercyclical IS Technical Change: Takeover, Cycles, Growth Miracle

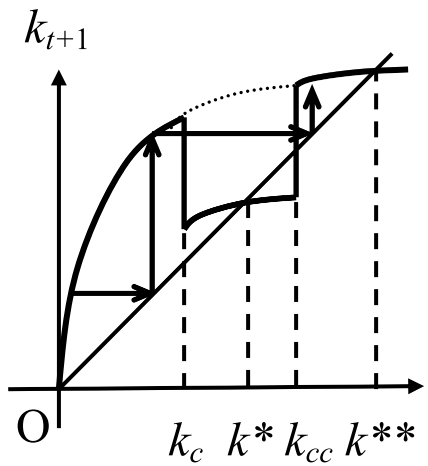
Let J = 2 and $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$, $m_2/m_1 > (1-\lambda_1)/(1-\lambda_2 R_2/R_1)$.

Key Trade-Off

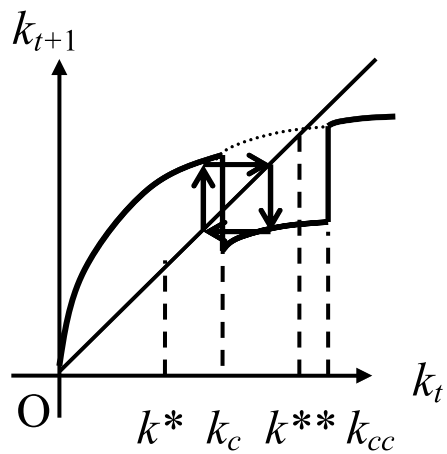
- Project 1 is less productive with more agency problem than Project 2.
- Project 1 requires the smaller set-up cost than Project 2.
  e.g. Project 1: family operated firms or other small businesses,
  Project 2: the investments in the corporate sector.
  e.g. Project 1: traditional light industries (textile and furniture)
  Project 2: modern heavy industries (steel, industrial equipments, petrochemical, and pharmaceutical)



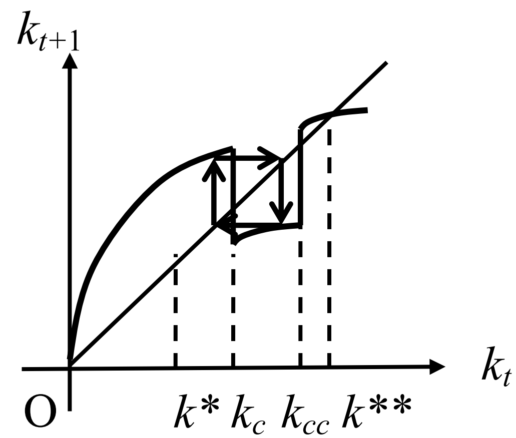
$$k_{t+1} = \begin{cases} R_2 W(k_t) & \text{if} \quad k_t < k_c \ \text{ or } k_t > k_{cc} \\ R_1 W(k_t) & \text{if} \quad k_c < k_t < k_{cc}. \end{cases}$$



Take-Over



Credit Cycles



Cycles as a Trap
Growth Miracle

One special (peculiar) feature of the model

- All the credit goes to one type of the project at any level of the net worth.
- A change in the net worth causes a bang-bang shift in the composition.

This feature disappears if we allow

- projects to differ in the composition of the goods produced (the rest of Lecture 2)
- heterogeneous agents (Lecture 3)

In the next model, the projects differ in terms of the composition of the goods they generate. More specifically,

There are two types.

The first type produces the inputs that are complementary with the inputs endowed by the next generation of the agents. This type of projects helps to improve the net worth of the next generation. We shall call them ***"Good"*** *projects.*

The other produces the good that can be directly consumed. This type of projects does not help to improve the net worth of the next generation. We shall call them ***"Bad"*** *projects.*

They are "Good" and "Bad" from the perspective of the next generation, not necessarily from the welfare perspective.

A Model with Good and Bad Projects: Based on Matsuyama (2004)

Let J =2 and $R_1 = R$ and $B_1 = 0$ and $R_2 = 0$ and $B_2 = B$.

	Period t	*Period t+1*
Type-1 *(The Good)*	m_1 units in final good	m_1R units in capital
Type-2 *(The Bad)*	m_2 units in final good	m_2B units in final good

Semantic Notes:

"Capital," here is actually meant to designate the type of capital goods that are complementary to the input owned by the next generation, hence improving the net worth of the next generation. What "Type-2" projects generate are also capital goods, except that they do not help to improve the net worth of the next generation.

For simplicity, let us assume $f'(0) = \infty$ to ensure that $X_{1t} > 0$.

Then,

Equilibrium Conditions

(1) $W(k_t) = m_1X_{1t} + m_2X_{2t}$

(2) $k_{t+1} = m_1RX_{1t}$

(3) $$\frac{Rf'(k_{t+1})}{Max\left\{\dfrac{1-W(k_t)/m_1}{\lambda_1},1\right\}} = r_{t+1} \geq \frac{B}{Max\left\{\dfrac{1-W(k_t)/m_2}{\mu_2},1\right\}};\ X_{2t} \geq 0.$$

The mapping from k_t to k_{t+1} takes a very different form, depending on whether Type-2 is active or not.

If $X_{2t} > 0$, $k_{t+1} = m_1 R X_{1t} = R(W(k_t) - m_2 X_{2t}) < RW(k_t)$ and

$$f'(k_{t+1}) = \frac{B Max\left\{\frac{1 - W(k_t)/m_1}{\lambda_1}, 1\right\}}{R Max\left\{\frac{1 - W(k_t)/m_2}{\mu_2}, 1\right\}}$$

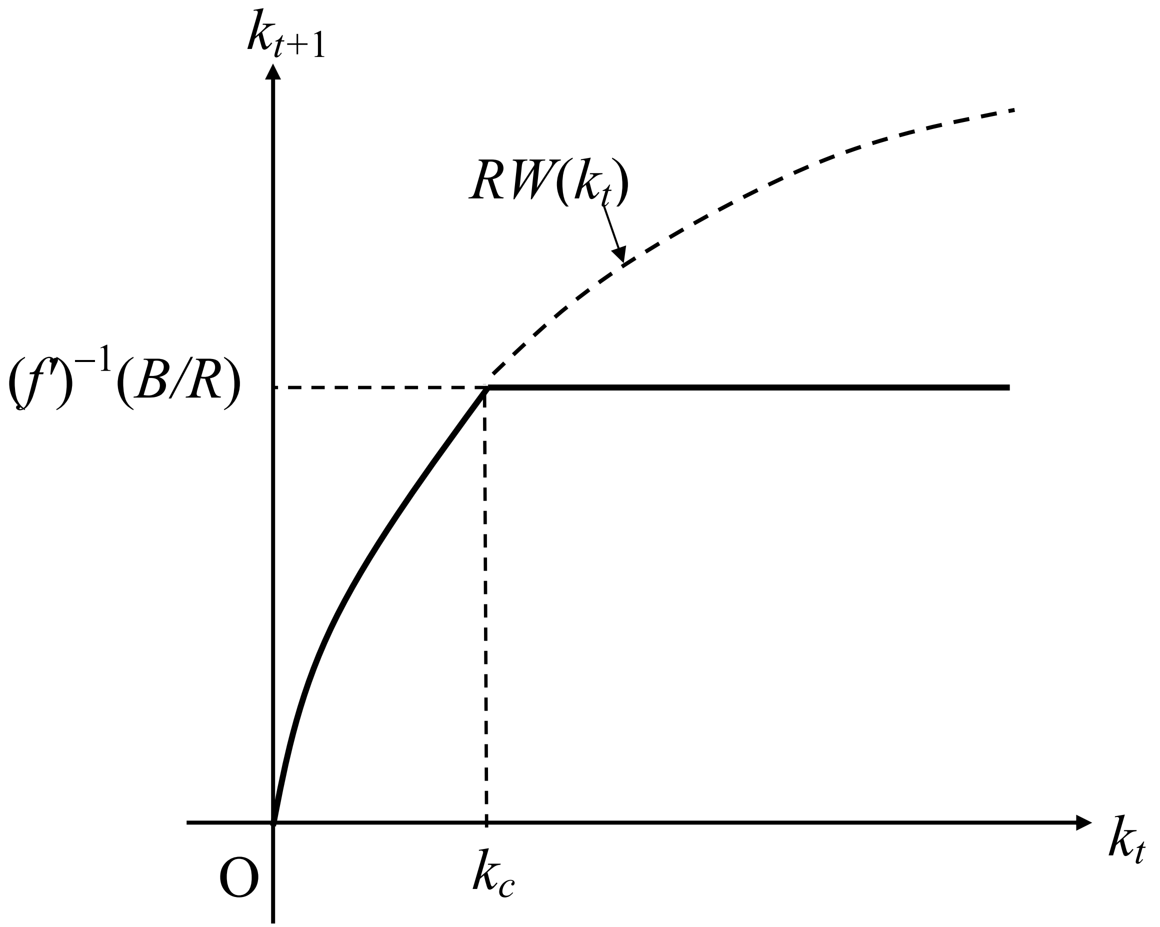
If $X_{2t} = 0$,

$$k_{t+1} = RW(k_t).$$

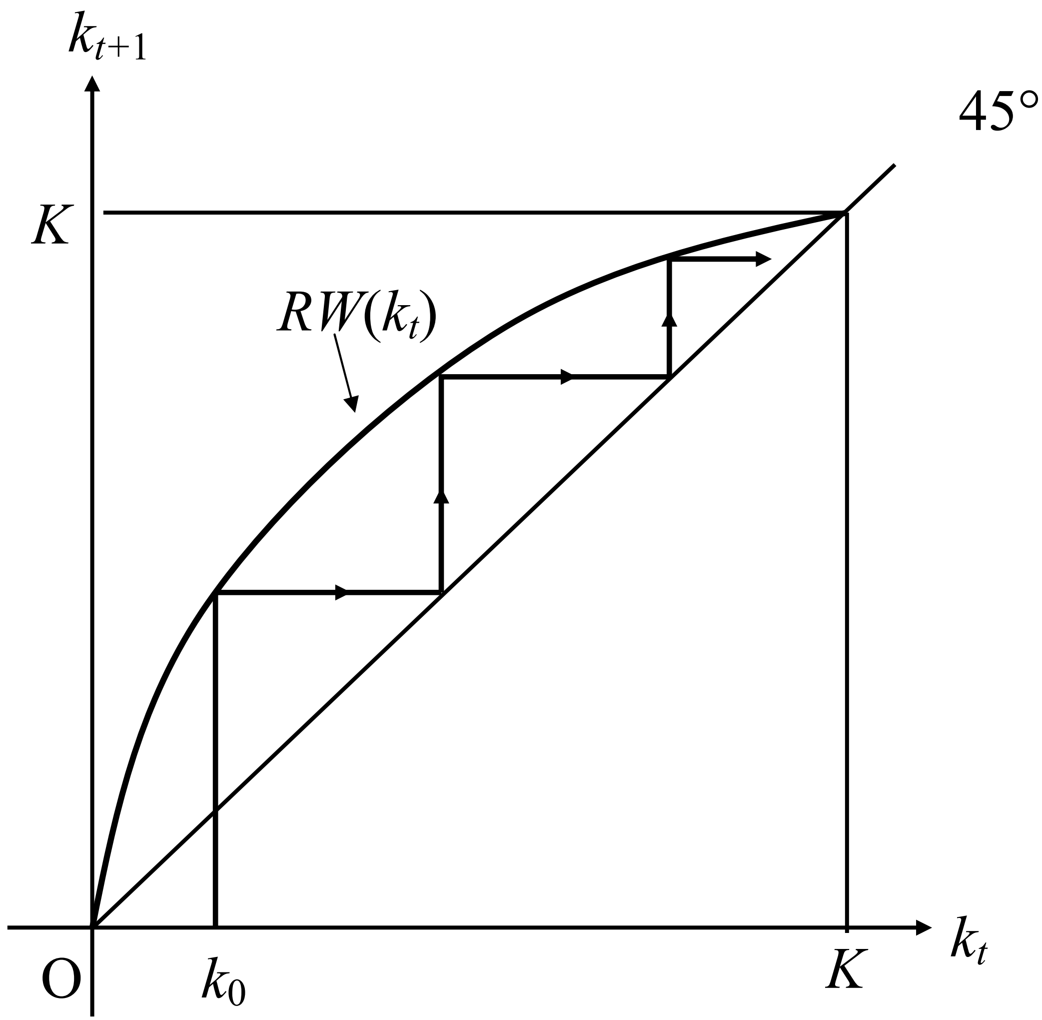
First, let us look at the perfect credit market case to give us a benchmark.

Perfect Credit Market: $\lambda_1 = 1$, $\mu_2 = 1$.

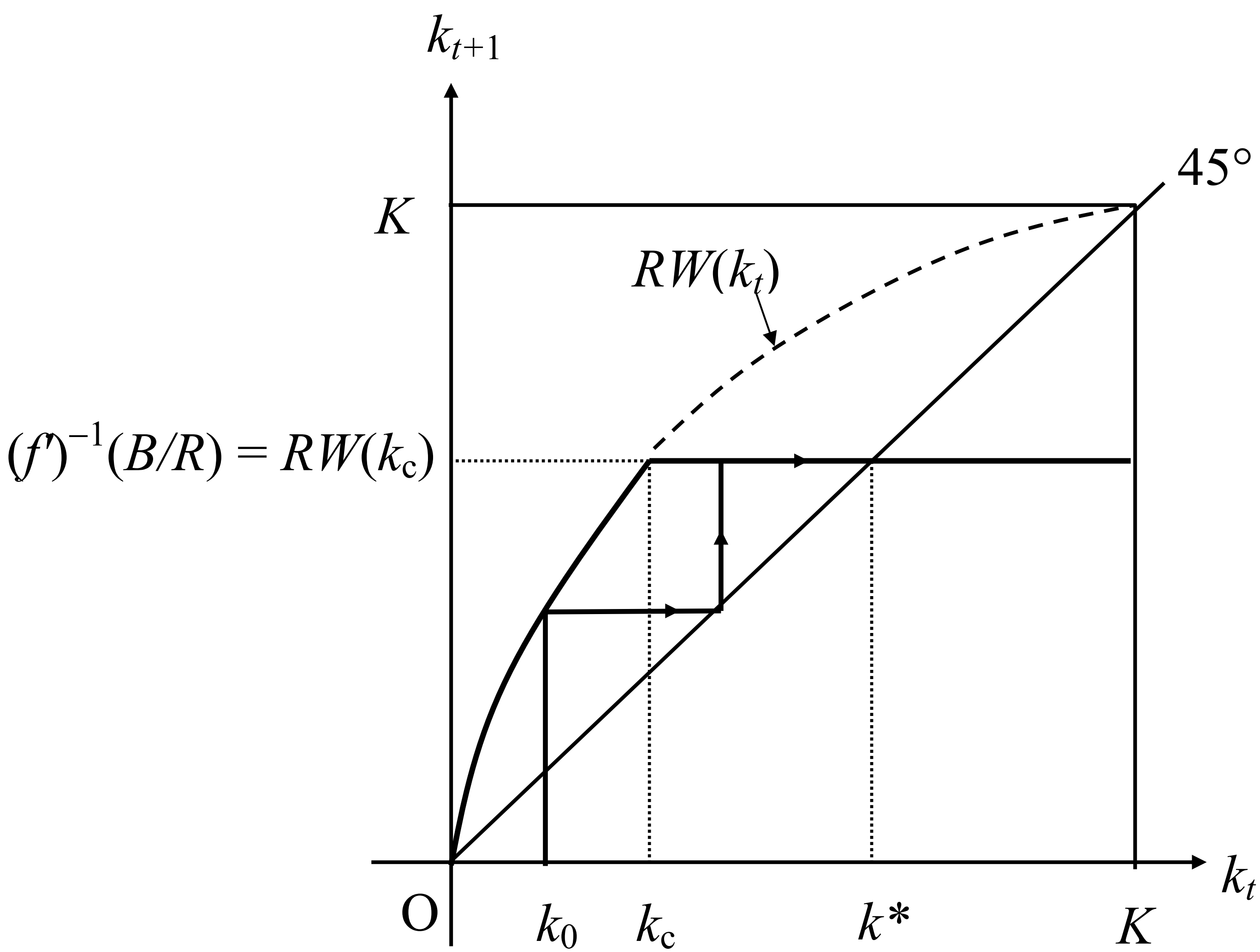
$$k_{t+1} = RW(k_t) \text{ if } k_t \le k_c \text{ and } Rf'(k_{t+1}) = B \text{ if } k_t \ge k_c$$



For a small B, **Neoclassical Convergence**



For a large B.



Next, let us see what happens if only Type-1 (Good) projects suffer from credit market imperfections.

Inefficient Recessions and Persistence: Let $\lambda_1 < 1$ and $\mu_2 = 1$.

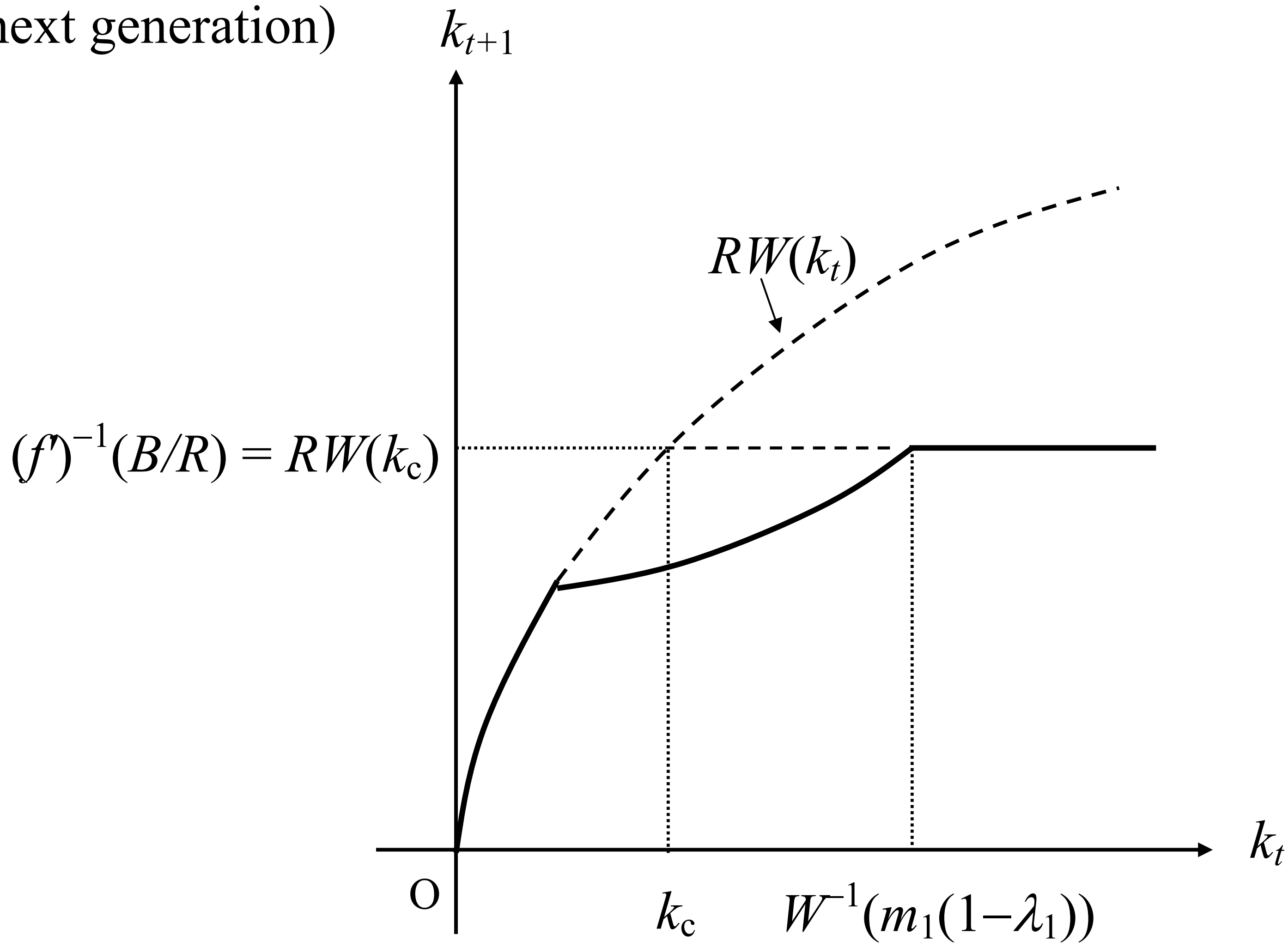
- $k_{t+1} < RW(k_t) \rightarrow Rf'(k_{t+1}) = BMax\left\{\frac{1-W(k_t)/m_1}{\lambda_1}, 1\right\} \geq B$
- $k_{t+1} = RW(k_t)$.

For a sufficiently small λ_1,

$$Rf'(k_{t+1}) > B \text{ and } k_{t+1} < RW(k_t) \text{ for } k_t \in (k_c, k_\lambda)$$

→ **Under-Investment to Type-1 (Good) Projects**

Under-Investment of Type-1 (Good) Projects (Projects that increase the net worth of the next generation)

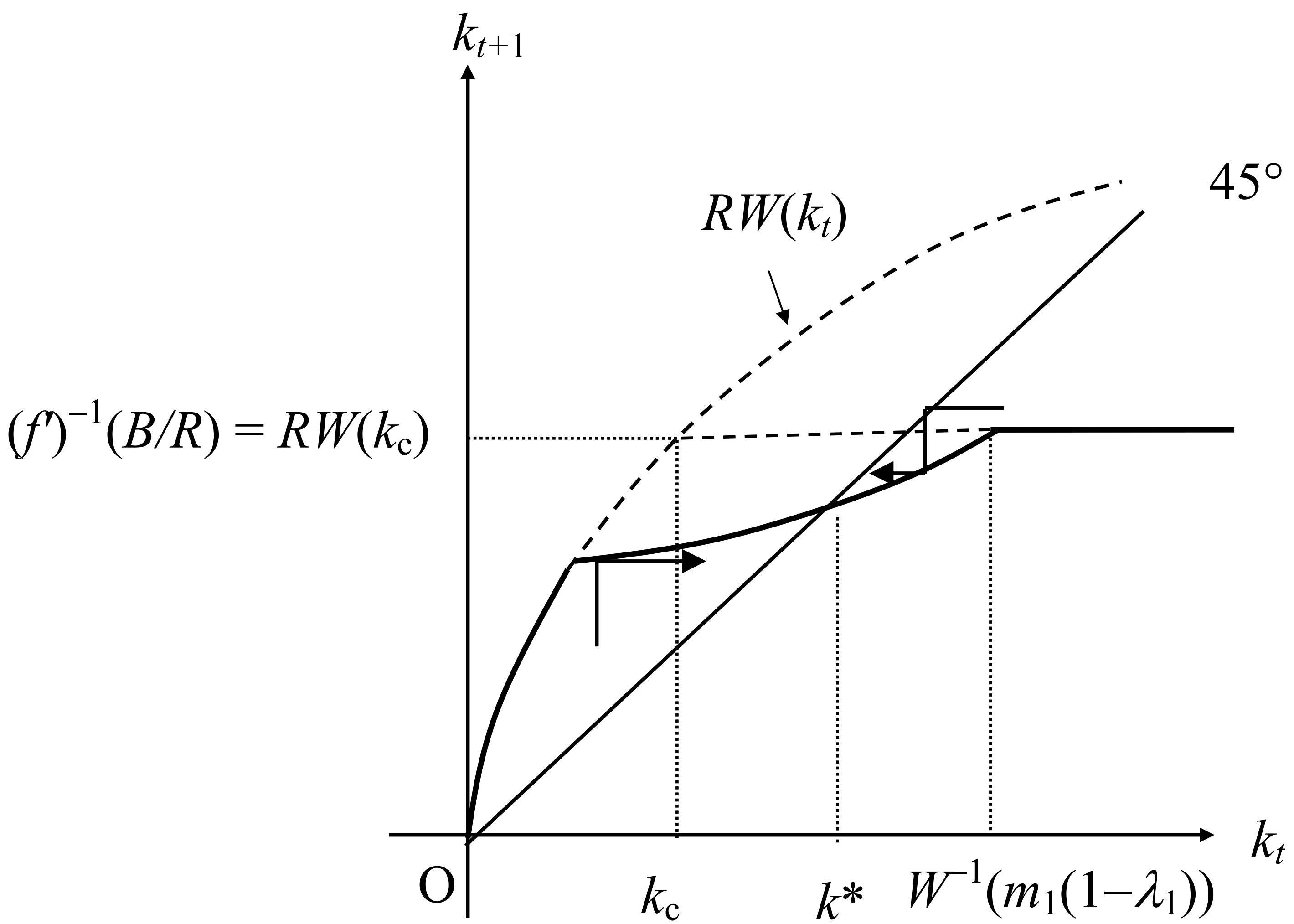


The exact dynamic implications of this under-investment depend on how this map intersects with the 45 degree line.

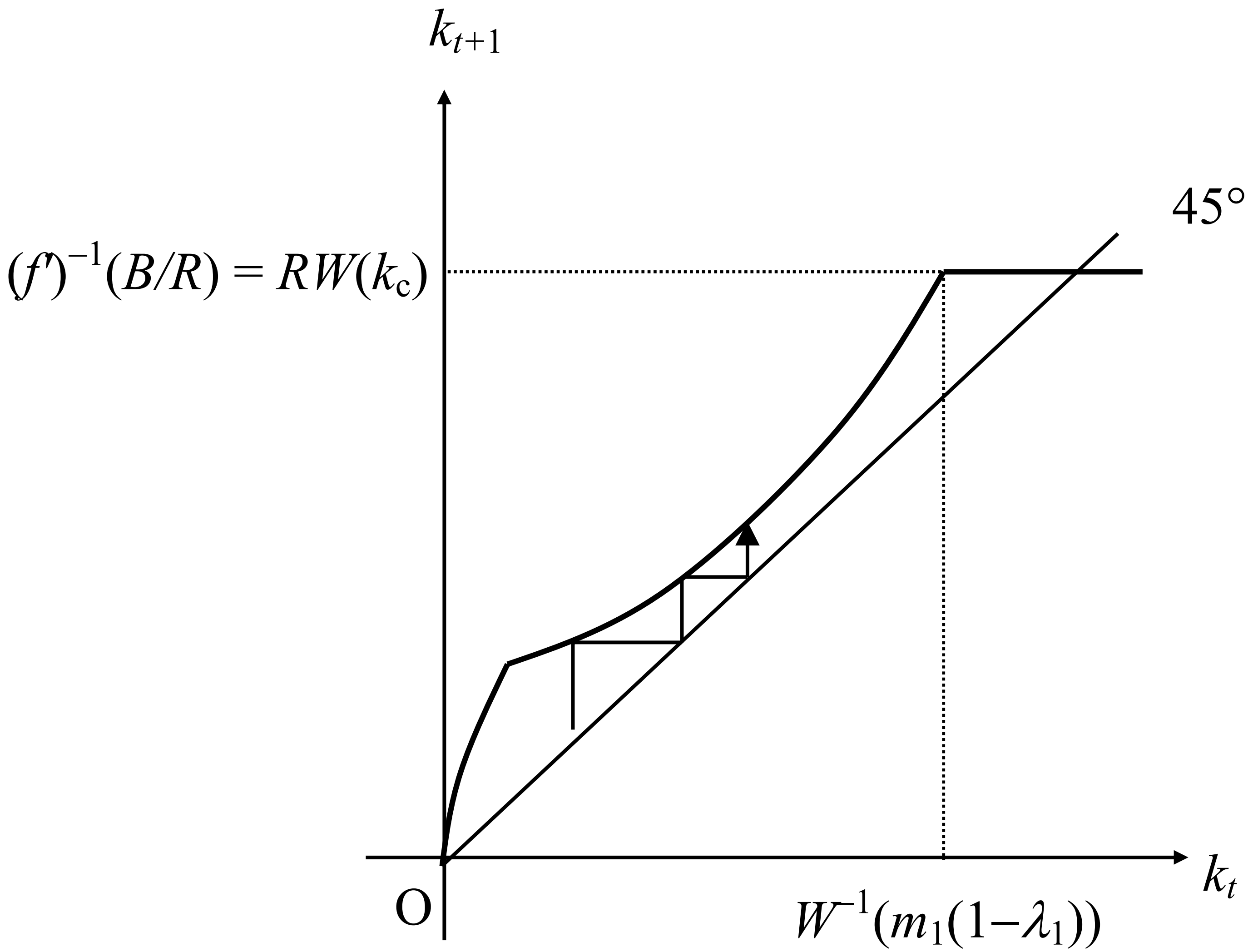
But, the following three cases all suggest ***persistence.***

Temporary Shock has an Echo Effect.

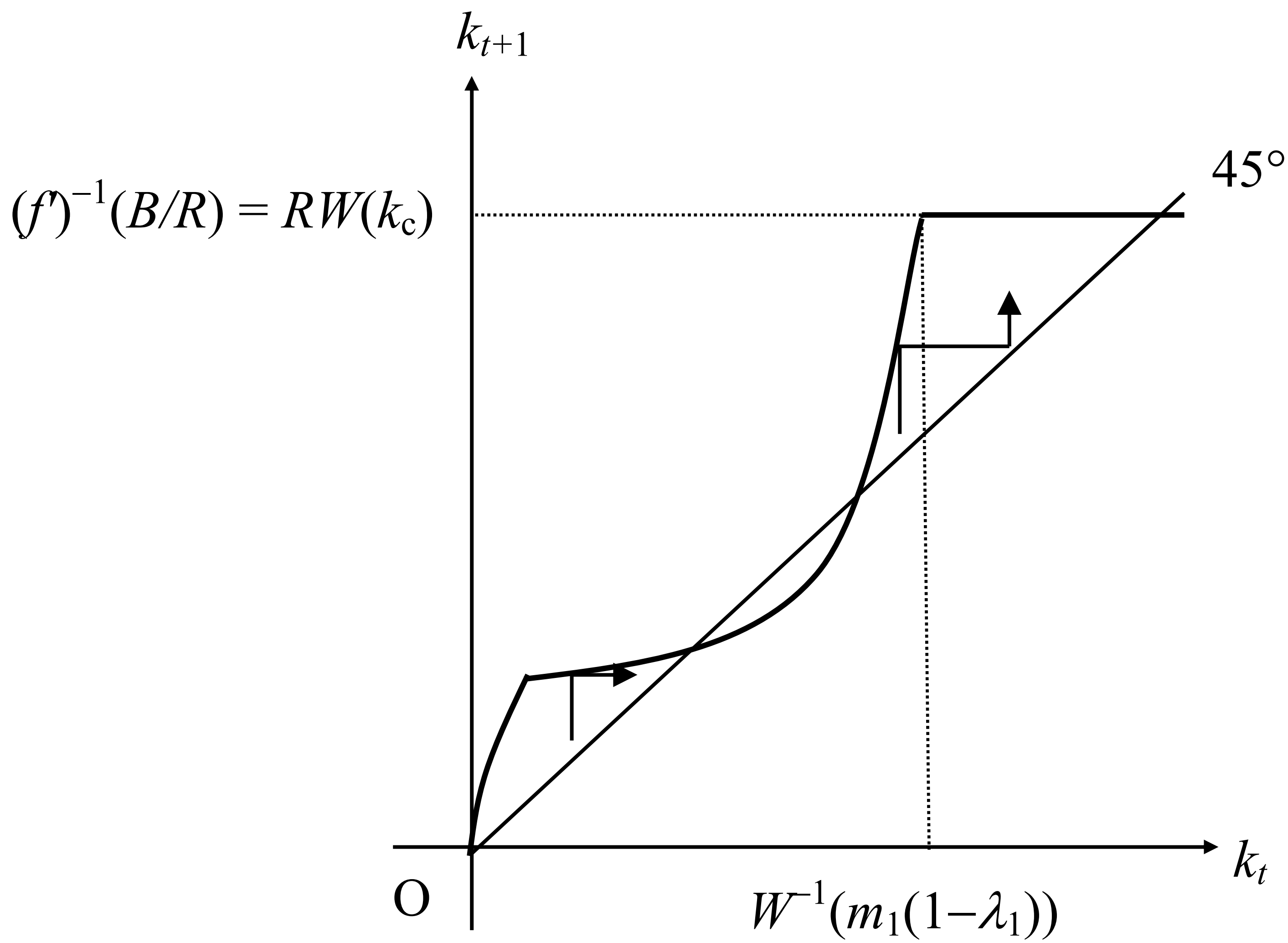
This case replicates the one discussed by Bernanke-Gertler (AER 1989)



The Case for A Slow Recovery from the Recession



The Case of Multiple Steady States



Next, let us see what happens if only Type-2 (Bad) projects suffer from credit market imperfections.

Inefficient Booms and Volatility: $\lambda_1 = 1$ and $\mu_2 < 1$; Based on Matsuyama (2004)

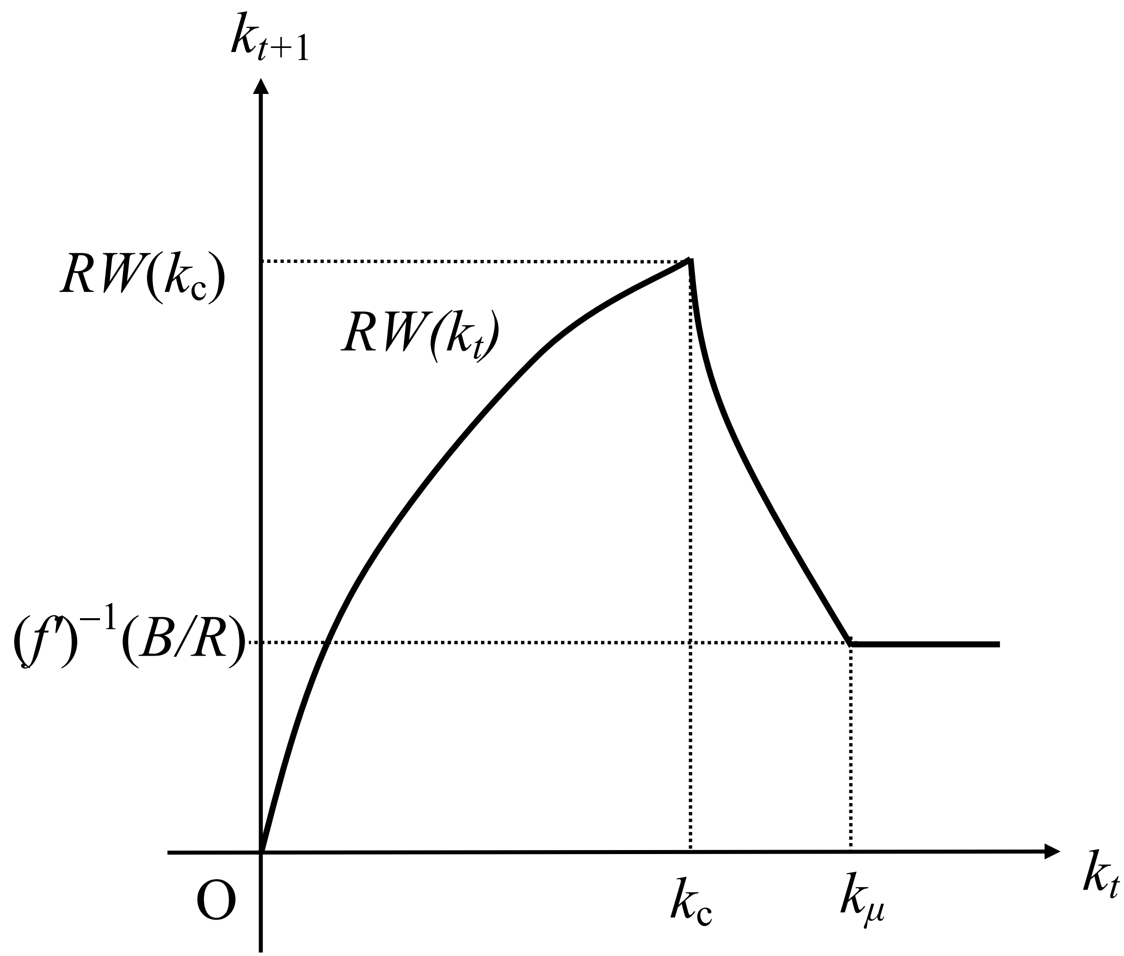
- $k_{t+1} < RW(k_t)$ → $B = Rf'(k_{t+1})Max\left\{\frac{1-W(k_t)/m_2}{\mu_2}, 1\right\} \geq Rf'(k_{t+1})$
- $k_{t+1} = RW(k_t)$, otherwise.

For a sufficiently small μ_2,

$$Rf'(k_{t+1}) < B \text{ and } k_{t+1} < RW(k_t) \text{ for } k_t \in (k_c, k_\mu)$$

→ **Over-Investment of Type-1 Projects**

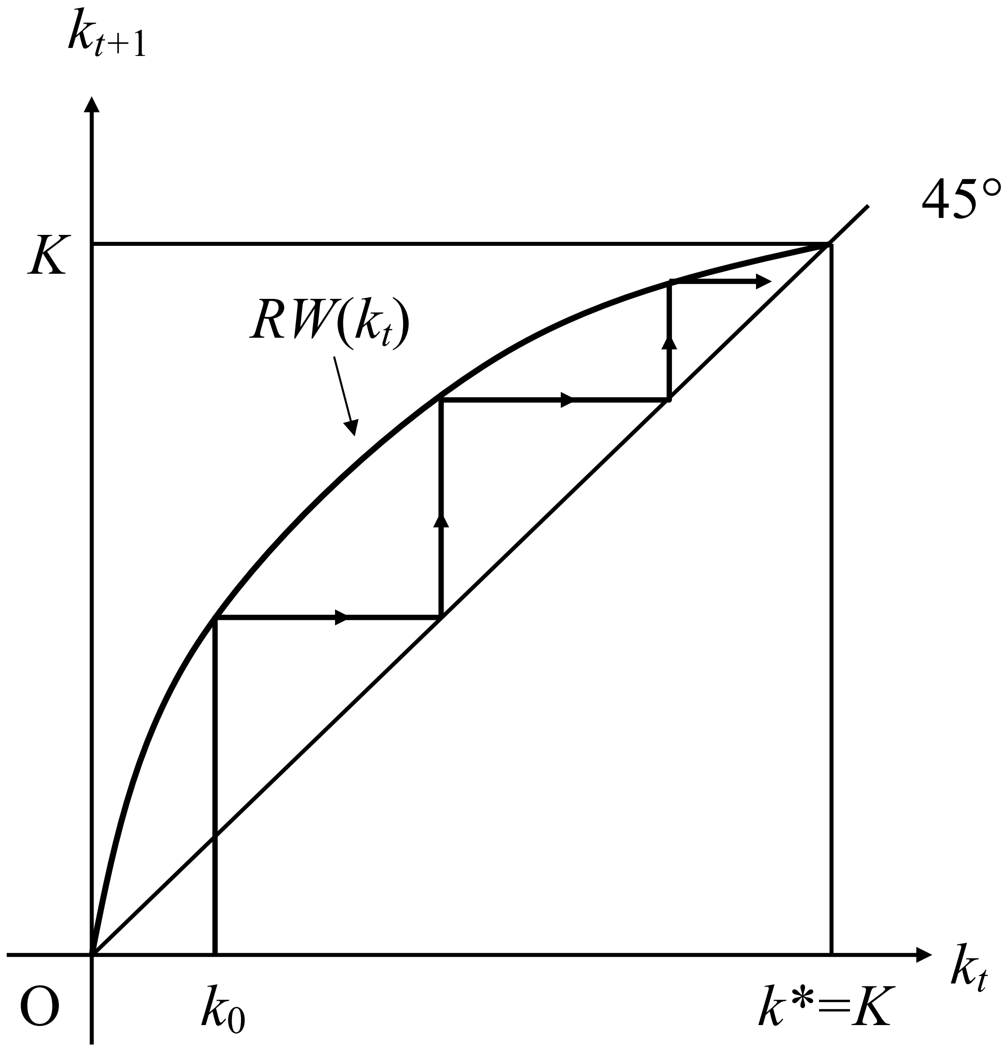
Over-Investment of Type-1 (Good) Projects (projects that increase the net worth of the next generation)



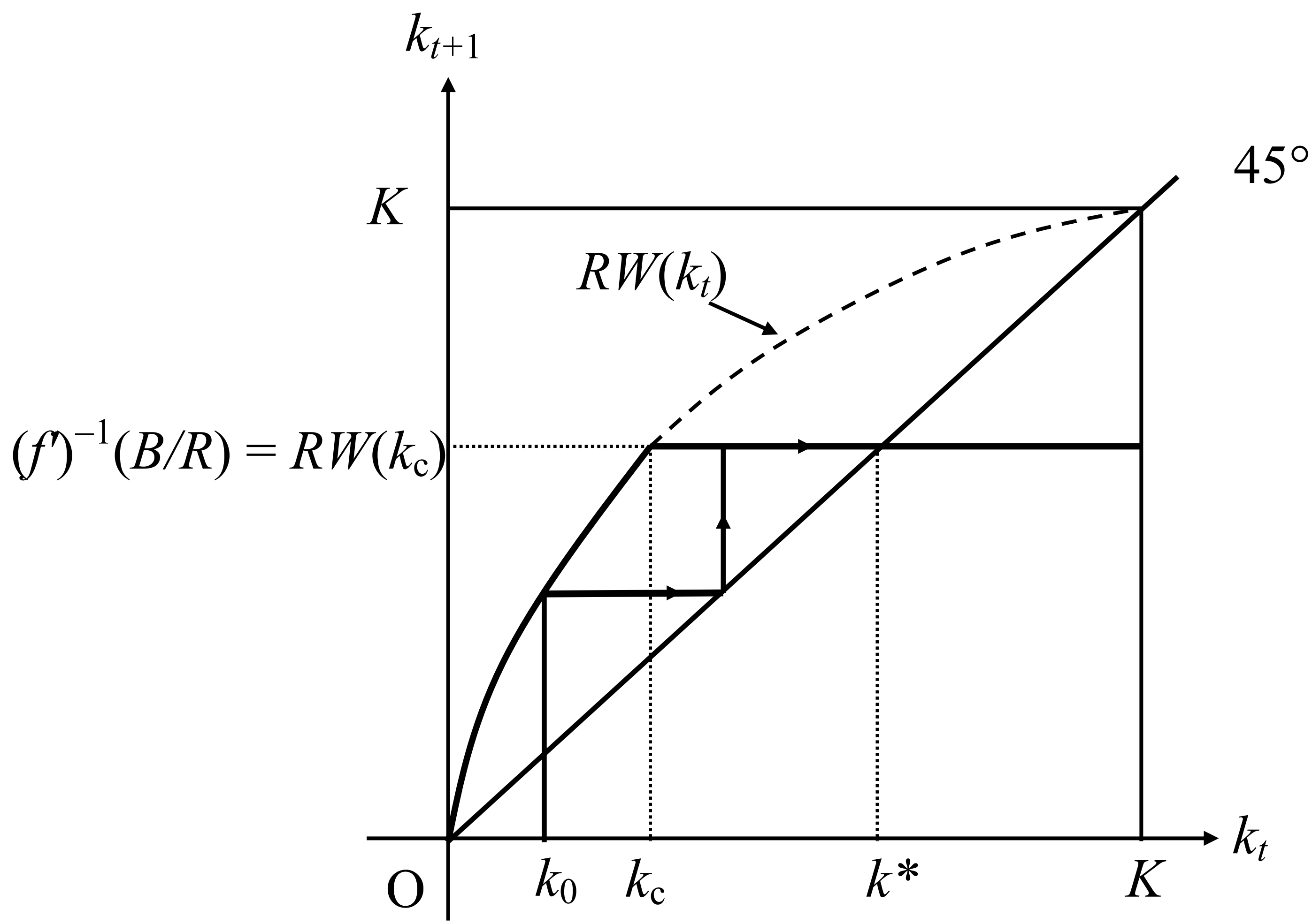
Again,

The exact dynamic implications of this over-investment depend on how this map intersects with the 45 degree line.

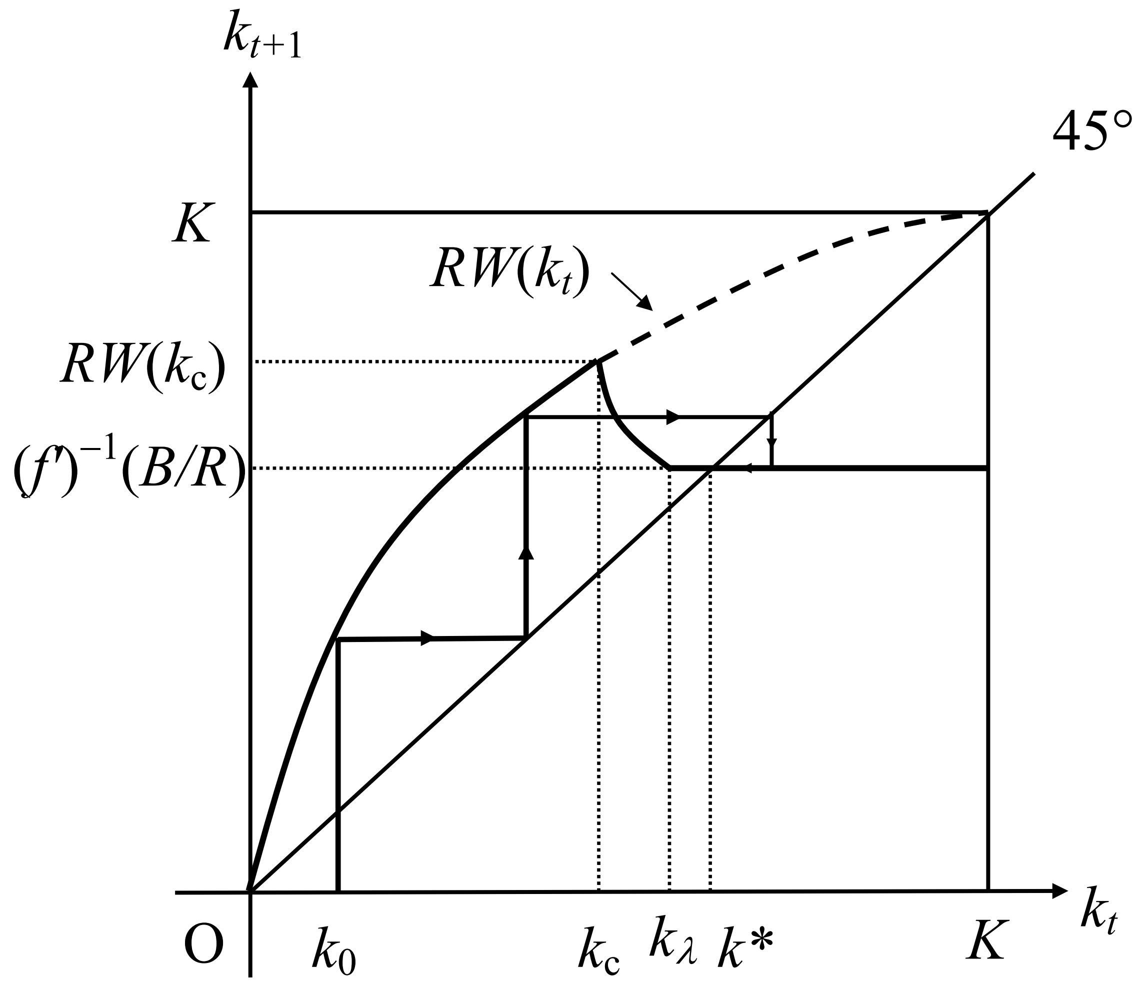
A: For a small B or a small μ, Type-2 projects are never financed.



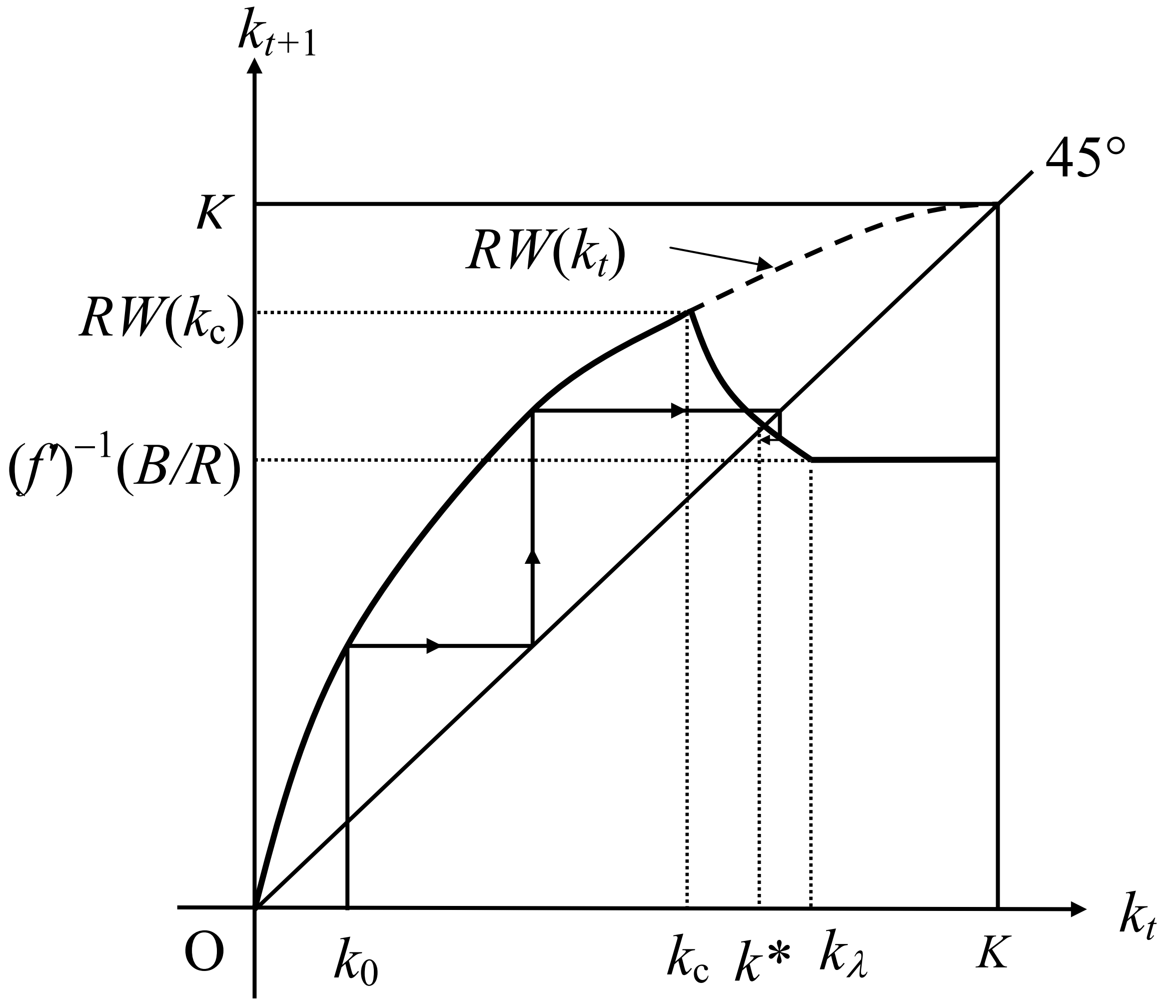
B: For a high B and a high μ, Type-2 are financed as soon as they become more productive than Type-1.



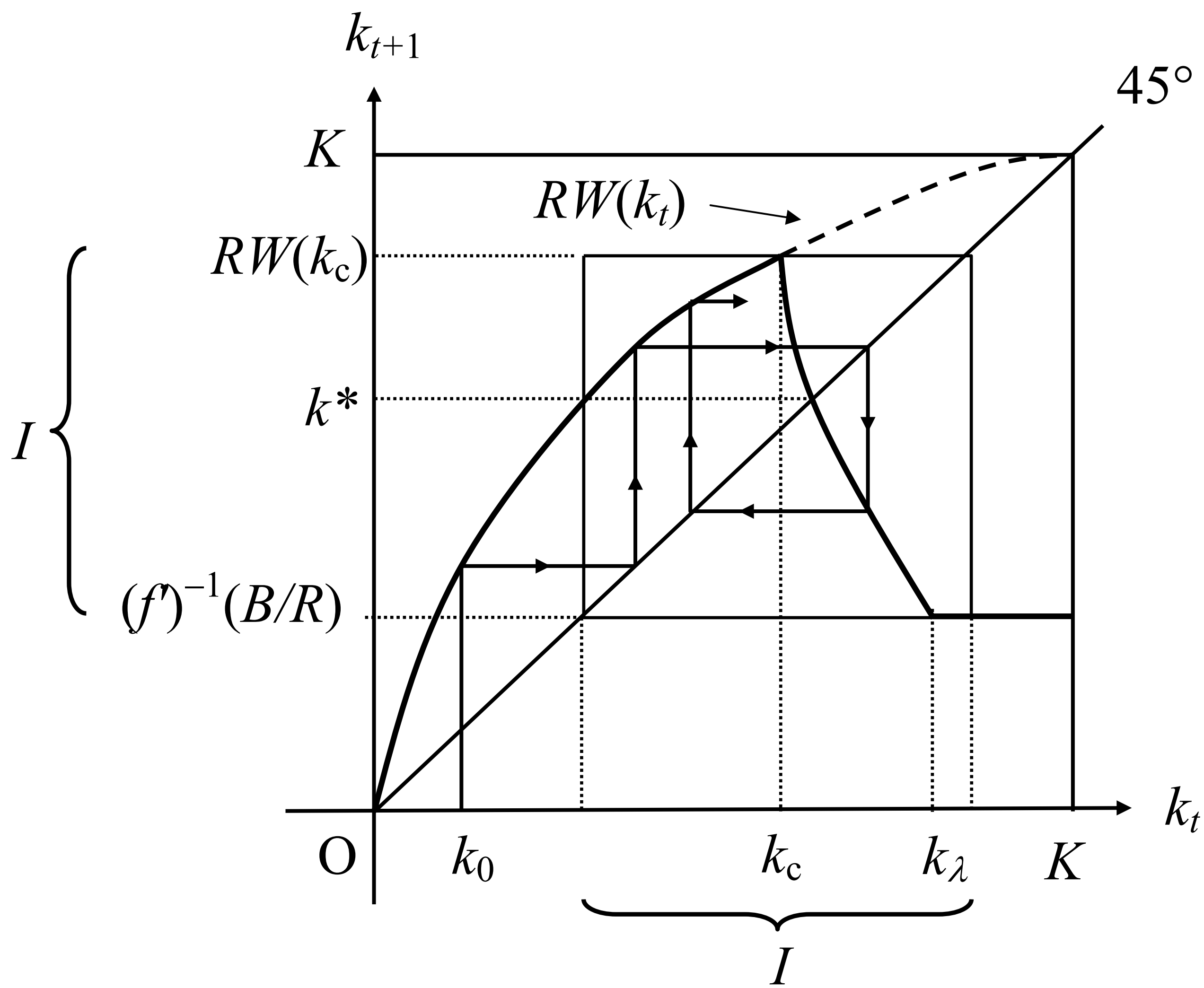
C: Overshooting (as μ becomes smaller from Case **B**)



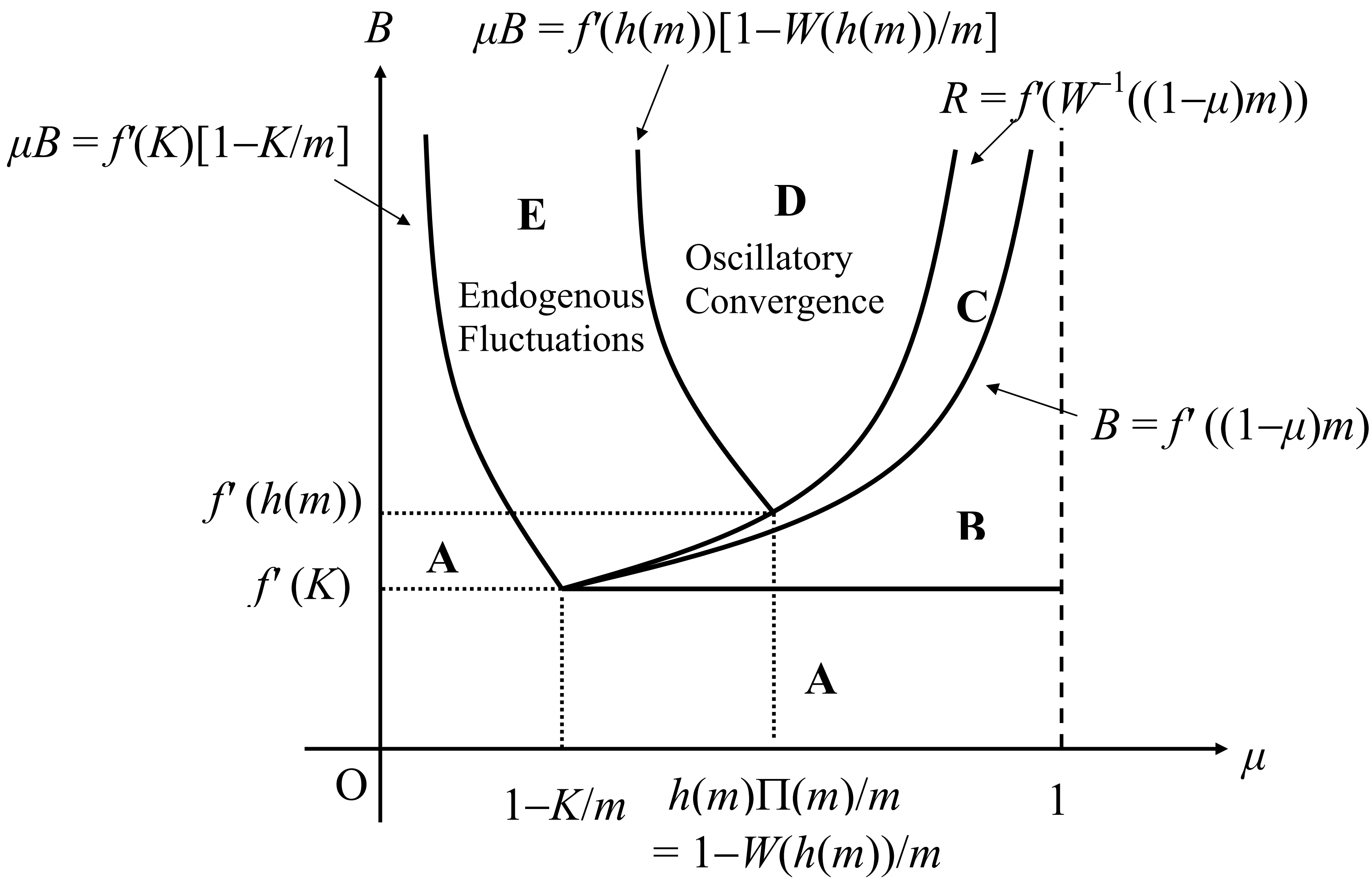
D: Oscillatory Convergence (as μ becomes more smaller)



E: Endogenous Fluctuations (as μ becomes even more smaller)



Parameter Configuration:



For a sufficiently large B, endogenous fluctuations (and oscillatory convergence and over-shooting) occur for an *intermediate* value of μ. Why?

- If Type-2 (Bad) projects suffer from major agency problems (a small μ), they are never financed. All the credit always goes to Type-1 (Good).
- If Type-2 (Bad) projects suffer from minor agency problems (a large μ), they are financed as soon as they become more profitable than Type-1 (Good) projects.
- Fluctuations occur when agency problems with Type-2 (Bad) projects are too big to be financed when the net worth is low, but small enough to be financed when the net worth is high.

Again, Non-Monotonicity!!

In this model, credit market imperfections cause an inefficient boom (excessive investment into Type-1), which correct itself as the net worth becomes sufficiently high.

Empirical Implications

It is often argued that

Larger volatility in developing countries would provide supporting evidence for the hypothesis that the credit market imperfections amplify business cycles fluctuations, because developing countries have poorer financial market institutions.

That is not quite true. The following two statements are not equivalent.

A. Imperfections amplify volatility, in contrast to the perfect case.
B. Greater (less) imperfections imply more (less) volatility.

Improving credit markets might allow greater exposure to the more risky investments, which might lead to more volatility.

What if Type-2 projects are subject to random shocks?

For example, μ is in an intermediate range and remain constant, but B occasionally becomes higher than f'(K) and pushes the economy from Region **A** to Region **E** (in the Parameter Configuration).

- As long as $B < f'(K)$, the economy has a unique stable steady state, $k = K$.

- When $B > f'(K)$ happens, some credit flows into Type-2 and the economy fluctuates below K.

- Then, as $B < f'(K)$, the economy starts recovering toward $k = K$.

So far, it has been assumed that each project produces only one-type of the good.

Let us now look at an example, where some projects jointly produce both the consumption and capital goods.

Imagine that capital is fully pledgeable, but the consumption good is not.

Then, projects whose generate more in the form of the consumption good are less pledgeable.

For example, some projects might generate more "satisfaction" or "private benefits" to those running them than others.

A Model with Private Benefits

Let $\lambda_1 = \lambda_2 = 1$, $\mu_1 = \mu_2 = 0$, $R_1 > R_2 > 0$, $B_1 = 0 < B_2$

- $R_2W(k_t) < k_{t+1} < R_1W(k_t) \leftrightarrow X_{1t} > 0 \;\&\; X_{2t} > 0$

$$\rightarrow Max\left\{\frac{1 - W(k_t)/m_2}{R_2 f'(k_{t+1})}, \frac{1}{R_2 f'(k_{t+1}) + B_2}\right\} = \frac{1}{R_1 f'(k_{t+1})}$$

$$\leftrightarrow Max\left\{1 - \frac{W(k_t)}{m_2}, \frac{1}{1 + B_2 / R_2 f'(k_{t+1})}\right\} = \frac{R_2}{R_1}.$$

- $k_{t+1} = R_1W(k_t) \rightarrow Max\left\{1 - \frac{W(k_t)}{m_2}, \frac{1}{1 + B_2 / R_2 f'(R_1 W(k_t))}\right\} \geq \frac{R_2}{R_1}.$

- $k_{t+1} = R_2W(k_t) \rightarrow Max\left\{1 - \frac{W(k_t)}{m_2}, \frac{1}{1 + B_2 / R_2 f'(R_2 W(k_t))}\right\} \leq \frac{R_2}{R_1}.$

If $W(k_t) < (1-R_2/R_1)m_2$, then $k_{t+1} = R_1W(k_t)$.

If $W(k_t) > (1-R_2/R_1)m_2$. Then,

- $R_2W(k_t) < k_{t+1} < R_1W(k_t)$ $\rightarrow$ $(R_1 - R_2)f'(k_{t+1}) = B_2$
- $k_{t+1} = R_1W(k_t)$ $\rightarrow$ $(R_1 - R_2)f'(R_1W(k_t)) \geq B_2$
- $k_{t+1} = R_2W(k_t)$ $\rightarrow$ $(R_1 - R_2)f'(R_2W(k_t)) \leq B_2$

For example,

if B_2 is sufficiently high, $k_{t+1} = R_2W(k_t)$ whenever $W(k_t) > (1-R_2/R_1)m_2$.

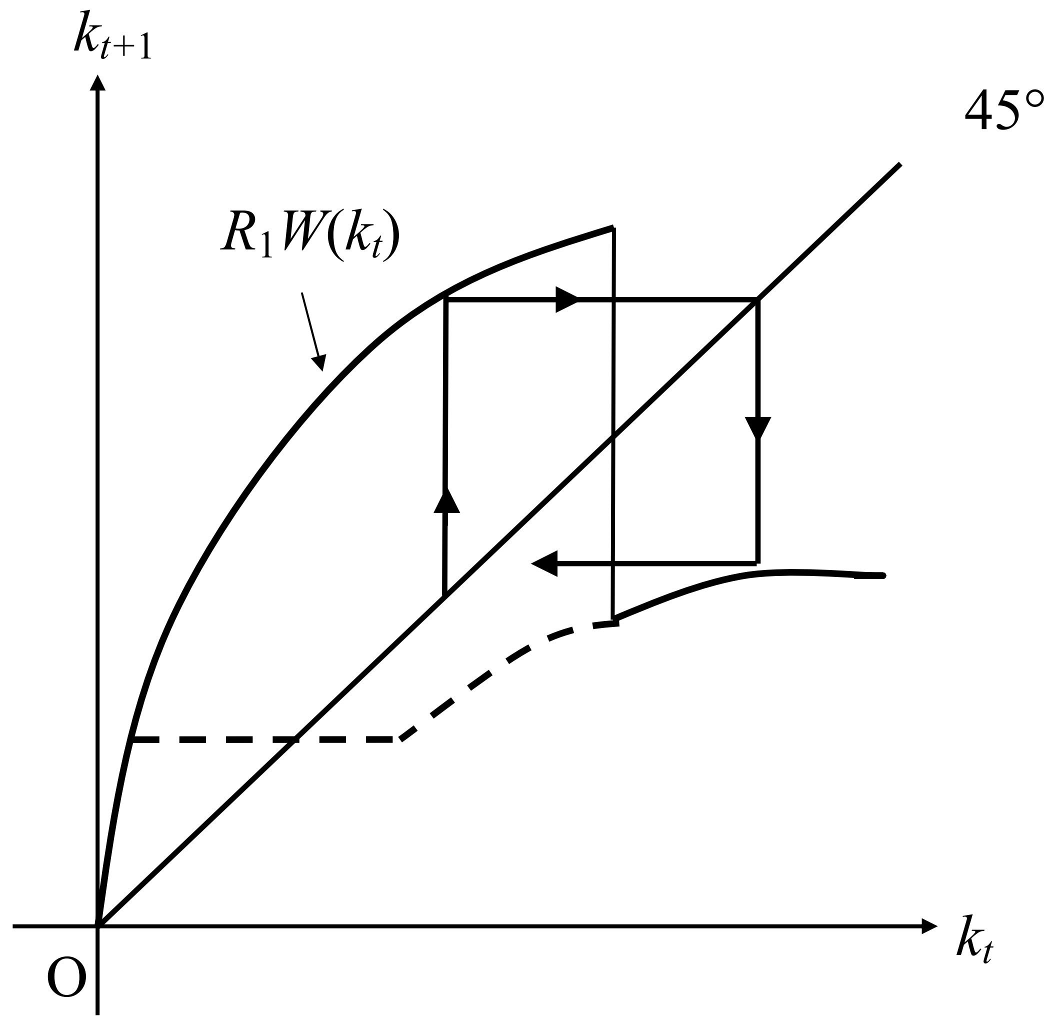
Furthermore,

if R_2 is sufficiently low, $k_t > k_{t+1} = R_2W(k_t)$.

In this model, the economy alternates between booms and recessions.

During booms, a high net worth allows the agents to pursue projects that generate personal satisfaction but less capital, which slow down the economy.

During recessions, the agents cannot pursue such projects, hence the credit goes to projecs that generate more capital.



So far, we have looked at various cases separately.

But, these cases are not mutually exclusive.

Sometimes, a combination of these cases can generate some interesting results.

A Model of Asymmetric Cycles and Intermittent Volatility: The Good, The Bad, and The Ugly; Based on Matsuyama (2004)

A Hybrid of the "Inefficient Recessions and Persistence" model and the "Inefficient Booms and Volatility" model.

Let J =3

	Period t	*Period t*+1
Type-1 *(The Good)*	m_1 units in final good	M_1R_1 units in capital
Type-2 *(The Bad)*	m_2 units in final good	m_2B_2 units in final good
Type-3 *(The Ugly)*	m_3 units in final good	m_3B_3 units in final good

Suppose $\mathbf{m_2(1-\mu_2) > m_1(1-\lambda_1) > m_3(1-\mu_3) = 0}$.

With a high B_2/R_1 and a small B_3/R_1, one could make

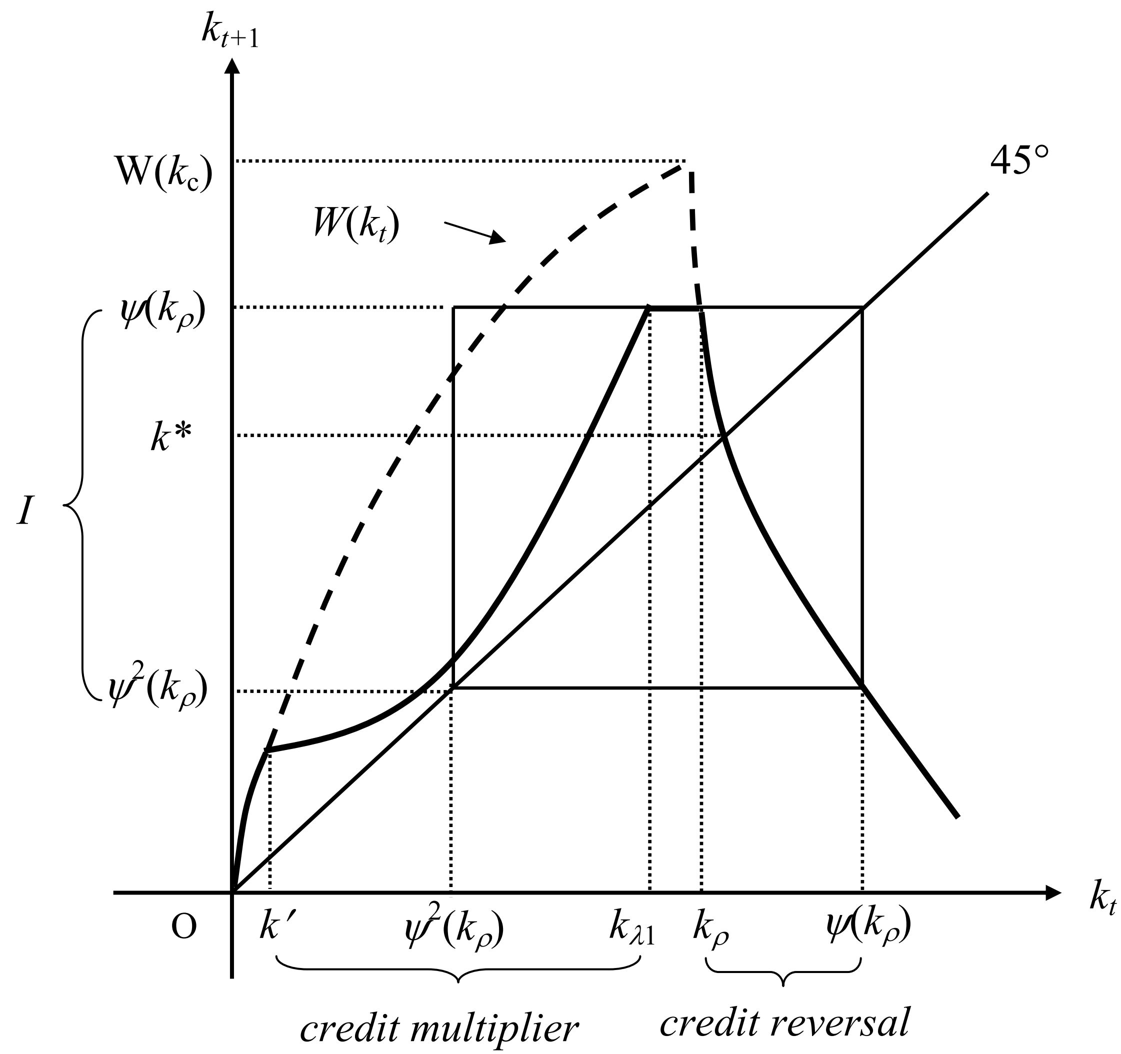
For a sufficiently low k_t,

- Type-2 projects become irrelevant.
- Type-1 compete with Type-3, and hence
- The dynamics are like in the "Persistence of Inefficient Recessions" model.

For a sufficiently high k_t,

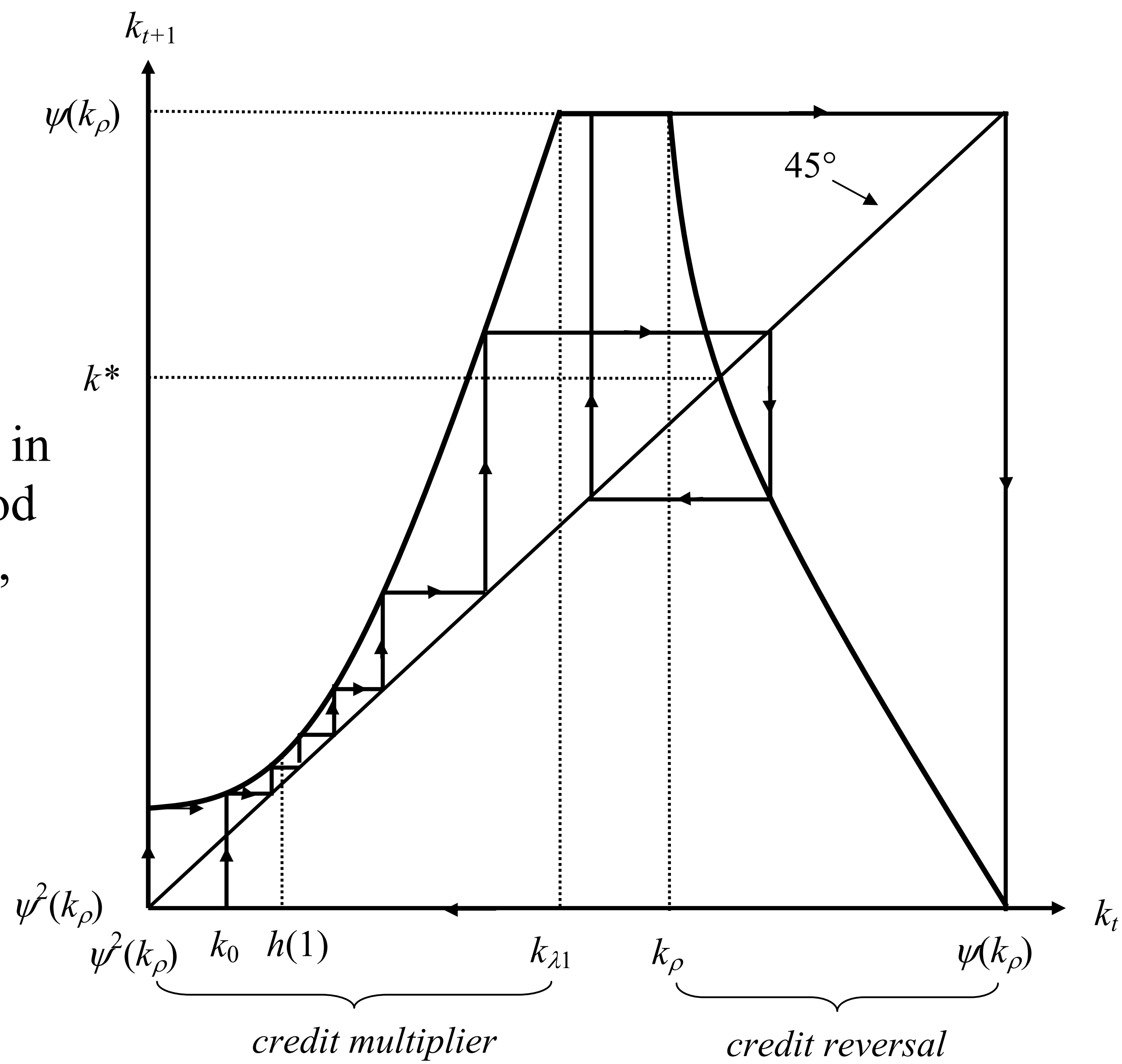
- Type-3 projects become irrelevant, and hence.
- Type-1 compete with Type-2, and hence
- The dynamics are like in the "Inefficient Booms and Volatility" model.

The map now looks like



If you magnify the box in this diagram, you will get…

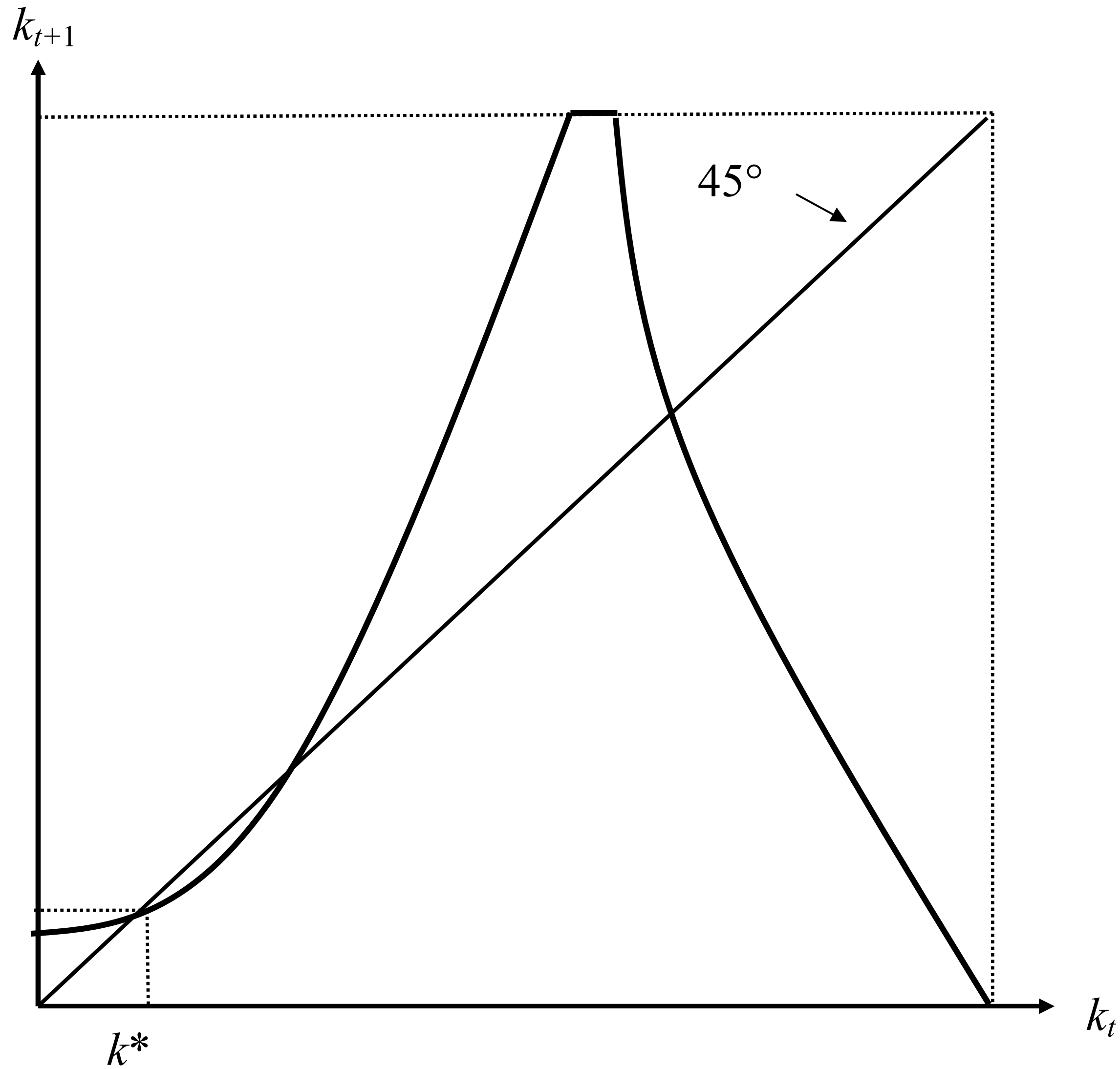
This model generates asymmetric cycles, along which the economy go through a slow recovery from recessions, and, once in booms, experiences a period of high volatility, and then, plunges into recessions.



What if random shocks are added to the production function (random TFP)?

Suppose that the parameters are such that the diagram on the right usually portrays the dynamics, but positive TFP shocks occasionally push up the map as in the diagram of the previous page.

Most of the times, the economy fluctuates around k*. Every once in a while, however, the economy experiences bubble-like booms and crashes.



Throughout Lecture 2, it has been assumed that

- *each generation consists of* **homogeneous agents***, so that the net worth is represented by a single scalar variable,* w_t.
- *there is only type of* **homogeneous capital** *that could enhance the net worth of the next generation, which means that the net worth can be represented as a fixed function of a single variable,* k_t, *i.e.,* $w_t = W(k_t)$.

In the next lecture (Lecture 3), we will deal with heterogeneous agents, where the state variables are the distribution of the net worth.

We will also look at an example with heterogeneous capital.