CREDIT MARKET IMPERFECTIONS AND PATTERNS OF
INTERNATIONAL TRADE AND CAPITAL FLOWS*

Kiminori Matsuyama
Northwestern University†

Abstract
This paper offers two simple models to illustrate how corporate governance, contractual enforcement, and the balance sheet condition of the business sector etc. can affect the patterns of international trade and capital flows in the presence of credit market imperfections.

JEL Classification Numbers: D52, F15, F21, F40

*I thank Kripa Freitas, Gene Grossman and Jean Tirole for their feedback.
†Email Address: k-matsuyama@northwestern.edu.
1. Introduction

In a world where credit relationships are subject to a variety of agency problems, corporate governance, contractual enforcement, and the balance sheet condition of the business sector are among many factors that play an important role in the allocation of resources. This paper offers two simple models to illustrate how these factors can affect the patterns of international trade and capital flows in the presence of credit market imperfections.

2. Patterns of International Capital Flows

The following model has been inspired by Gertler and Rogoff (1990) and Boyd and Smith (1997). Consider a closed economy populated by the landowners and a continuum of identical entrepreneurs with unit measure. There is a single consumption good, produced with the CRS technology, \( F(K, L) \), using physical capital, \( K \), and land, \( L \). Let \( F(K/L, 1) \equiv f(k) \), where \( k = K/L \) and \( f \) satisfies \( f' > 0 > f'' \) and \( f'(0) = \infty \). The competitive factor markets reward each unit of physical capital by \( f'(k) \) and each unit of land by \( f(k) - kf'(k) > 0 \), paid in the consumption good.

The landowners are collectively endowed with \( L \) units of land, which they supply inelastically to consume \( [f(k) - kf'(k)]L \). Physical capital is produced by the investment projects run by the entrepreneurs. Each entrepreneur can run at most one project and is endowed with \( \omega < 1 \) units of the input. Each project requires one unit of the input and generates \( R \) units of physical capital. Thus, any entrepreneur who runs the project must borrow \( 1 - \omega \) units of the input from those who do not run the project. Let \( r \) denote the
interest rate that the borrower can promise to the lender. Thus, the entrepreneurs who run the project earn \( Rf(k) \) in the consumption good, out of which \( r(1-\omega) \) must be paid to the lenders. Thus, by running the project, they consume \( Rf(k) - r(1-\omega) \). By not running the project and lending the input instead, they consume \( r\omega \). Hence, an entrepreneur is willing to run the project if and only if \( Rf(k) - r(1-\omega) \geq r\omega \), and equivalently,

\[
\text{(PC) } Rf(k) \geq r,
\]

where PC stands for the *profitability constraint*.

Even when (PC) holds with strict inequality, some entrepreneurs may not be able to run the project due to the borrowing constraint. The borrowing limit exists because the borrower can pledge only up to a fraction of the project revenue for the repayment, \( \lambda Rf(k) \), where \( 0 \leq \lambda \leq 1 \). Knowing this, the lender would lend only up to \( \lambda Rf(k)/r \). Thus, the entrepreneur can finance the project if and only if

\[
\text{(BC) } \lambda Rf(k) \geq r(1-\omega),
\]

where BC stands for the *borrowing constraint*. The parameter, \( \lambda \), the pledgeable fraction of the project revenue, captures the quality of contractual enforcement, of corporate governance, and other factors that determine the efficiency of the credit market, and may be interpreted as the state of financial development.\(^1\) The parameter, \( \omega \), on the other hand, represents the net worth or the balance sheet condition of the entrepreneurs.

The assumption, \( f'(+0) = \infty \), ensures that a positive fraction of the entrepreneurs invests in equilibrium of this closed economy, which means that both (PC) and (BC) hold. Furthermore, one of (PC) and (BC) must be binding;
otherwise, no entrepreneur would become lenders. Hence, the equilibrium interest rate is given by

\[ r = \min\{1, \lambda/(1-\omega)\}Rf(k). \]

Since there is a unit measure of entrepreneurs, each endowed with \( \omega \) units of the input, the aggregate endowment of the input in this economy is equal to \( \omega \). Since each project requires one unit of the inputs, the resource constraint implies that the total number of the projects run is equal to \( \omega \). Since each project generates \( R \) units of physical capital, the aggregate supply of physical capital is given by

\[ k = R\omega. \]

In this closed economy, the aggregate investment is determined entirely by the aggregate endowment. Only \( \omega \) fraction of the entrepreneurs borrows and invests, while \( 1-\omega \) fraction of them becomes lenders. For \( \lambda + \omega \geq 1 \), they are indifferent, because the equilibrium interest rate adjusts so as to make (PC) hold with the equality; \( r = Rf(R\omega) \). For \( \lambda + \omega < 1 \), on the other hand, \( r = \lambda Rf(R\omega)/(1-\omega) < Rf(R\omega) \). That is, (BC) is binding, while (PC) holds with strict inequality. In this case, the entrepreneurs strictly prefer borrowing and running the projects themselves, instead of becoming lenders and letting their endowment invested by others. Thus, the equilibrium allocation necessarily involves credit rationing, where some random mechanism allocates the credit to the fraction, \( \omega \), of the entrepreneurs, while the rest is denied the credit. These unlucky entrepreneurs have no choice but to become the lenders; they would not be able to entice the potential lenders by promising a higher interest rate, because that would violate (BC).\(^2\)
Eq. (2) implies that a higher endowment leads to a higher aggregate investment. However, the equilibrium interest rate,

\[ r = \min\{1, \lambda/(1-\omega)\} R f'(R \omega), \]

may not be decreasing in \( \omega \). There are two competing forces. On one hand, due to diminishing returns, a higher aggregate investment comes with a lower return. On the other hand, for \( \lambda + \omega < 1 \), a higher endowment eases the borrowing constraint of the entrepreneurs, which allow them to pledge a higher return to the lender. When the second force is stronger, a higher \( \omega \) leads to both a higher investment and a higher return to the lender, in spite of diminishing return in the investment. Simple algebra can show that the condition for this is \( 1 - \lambda > \omega > \eta/(1+\eta) \), where \( \eta \equiv -\log(p')/\log(p) = - (f'/f) \) is the elasticity of the marginal productivity of the investment.

Now, suppose that the world economy consists of two countries of the kind analyzed above, North and South. They share the identical parameters, except \( \lambda \) and \( \omega \). To avoid a taxonomical analysis, let us assume \( 1 > \lambda_N \geq \lambda_S > 0 \) and \( 1 > \omega_N > \omega_S > 0 \). Furthermore, it is assumed that land is nontradeable and that only northern (southern) entrepreneurs know how to produce physical capital used in the production of the consumption good in North (South). On the other hand, the input to the investment projects and the consumption good can be traded between the two countries.

If North and South are in autarky, the investments in the two countries are simply \( k_N = R \omega_N > R \omega_S = k_S \). This is depicted by point A in Figure 1. The autarky interest rates are given by \( r_j = \min\{1, \lambda_j/(1-\omega_j)\} R f'(R \omega_j) \), where \( j = N \) or \( S \).
Now, suppose that the two countries become financially integrated and that the entrepreneurs from both countries can lend and borrow their input endowments across the borders without additional costs. This leads to an equalization of the interest rates across the two countries,

\[ \min\{1, \frac{\lambda_N}{1-\omega_N}\} f'(k_N) = \min\{1, \frac{\lambda_S}{1-\omega_S}\} f'(k_S), \]

and the resource constraint applies to the world economy as a whole:

\[ k_N + k_S = R(\omega_N + \omega_S), \]

as depicted by the downward sloping line in Figure 1. Along this resource constraint, eq. (4) determines the allocation of the inputs across the two countries.\(^3\)

If \( \frac{\lambda_S}{1-\omega_S} \geq 1 \), which also implies \( \frac{\lambda_N}{1-\omega_N} > 1 \), eq. (4) becomes simply \( f'(k_N) = f'(k_S) \), or equivalently, \( k_N = k_S \). Thus, the equilibrium allocation is given by the intersection with the 45° line, depicted by W in Figure 1. Since (BC) is not binding in either country, the movement of international capital flows is entirely dictated by the difference in marginal productivity. As a result of financial integration, the investment in South is partially financed by the lending from North, and capital flows from the rich to the poor until the difference in marginal productivity is eliminated.

If \( \frac{\lambda_S}{1-\omega_S} < 1 \), (BC) is binding in South, and this leads to \( f'(k_N) < f'(k_S) \) or equivalently, \( k_N > k_S \). Financial integration does not eliminate the difference in marginal productivity. Furthermore, if

\[ r_N = \min\{1, \frac{\lambda_N}{1-\omega_N}\} Rf'(R\omega_N) > r_S = \{\frac{\lambda_S}{1-\omega_S}\} Rf'(R\omega_S), \]

we have

\[ k_N > R\omega_N > R\omega_S > k_S. \]
This situation is depicted by the arrow in Figure 1. As a result of financial integration, the investment in South declines and the investment in North increases, which are partially financed by the lending from South. Thus, capital flows from the poor to the rich to magnify the difference in marginal productivity. Not surprisingly, eq. (6) holds when $\lambda_S$ is sufficiently smaller than $\lambda_N$. Poor corporate governance and any other factors contributing to financial insecurity in South lead to capital flight from South to North. What may not be so obvious is that the reverse capital flow occurs even when $\lambda_N = \lambda_S = \lambda$. For example, suppose $1-\lambda > \omega_N > \omega_S > \eta/(1+\eta)$. Then, eq. (6) is satisfied, because $f'(R\omega)/(1-\omega)$ is increasing in $\omega$ for $1-\lambda > \omega > \eta/(1+\eta)$. Even though the state of financial development does not differ between North and South in this case, southern entrepreneurs are poorer. This makes them more dependent on external finance, which in turn makes them less credit-worthy than richer northern entrepreneurs. An integration of financial markets forces southern entrepreneurs to compete with northern entrepreneurs when financing their investments, which put the former in disadvantage.

In the above analysis, the two key elements of the model, $\lambda$ and $\omega$, are treated as exogenous parameters. In the model of Matsuyama (2004a), the investment made by the current generation of the entrepreneurs in one country improves the balance sheet condition of the next generation of the entrepreneurs living in the same country. This introduces the positive feedback from $k$ to $\omega$. The result is the possibility of endogenous inequality, i.e., all stable steady states may be characterized by uneven distributions of the wealth and capital stock across the countries, even when all the countries are
inherently identical. Endogenous inequality may also arise with positive feedback from k to λ. For example, one could allow for some sorts of learning-by-doing, which make the expertise gained from the past experiences in the investment help controlling the agency problems of the future investment.  

3. **Patterns of International Trade**

The following model has been inspired by the empirical findings of Beck (2002) and Freitas (2004). \(^5\) Consider a variation of the Ricardian model with a continuum of tradeable goods, indexed by \(z \in [0,1]\), à la Dornbusch-Fischer-Samuelson (1977). The economy is populated by a continuum of identical agents, which is endowed with \(\omega < 1\) units of labor. The preferences are given by symmetric Cobb-Douglas, so that demand for good \(z\) is \(D(z) = E/p(z)\), where \(p(z)\) is the price of good \(z\) and \(E\) is the aggregate expenditure in this economy. To produce any tradeable good, the agents must run a project. Each project in sector \(z\) requires one unit of labor and generates \(R\) units of good \(z\). Each agent may run one project or may simply become a worker, by supplying the labor endowment to other agents.

Since any project requires one unit of labor, and the labor endowment of any agent is \(\omega < 1\), each agent who runs the project must employ \(1-\omega\) units of labor supplied by those who do not run the project. Let \(w\) be the wage rate, which the employers can pledge to pay to the workers after the project has been completed and the output has been sold. By running a project in sector \(z\), the entrepreneur earns \(p(z)R\), out of which they pay the wage bill, \(w(1-\omega)\), so that they consume \(p(z)R - w(1-\omega)\). By not running the project and supplying
labor, they consume $w_0$. Hence, any agent is willing to run the project in sector $z$ if and only if $p(z)R - w(1-\omega) \geq w_0$, and equivalently,

$$ (PC-z) \quad p(z)R \geq w, $$

where PC again stands for the profitability constraint. This constraint may not be binding, because the employers can pledge only a fraction of the project revenue for the wage payment. More specifically, it is assumed that the employers in sector $z$ can pledge only $\lambda(z)p(z)R$, where $\lambda(z)$ is continuous and strictly increasing with the range from zero to one. Because of the partial pledgeability, the projects in sector $z$ take place if and only if they satisfy

$$ (BC-z) \quad \lambda(z)p(z)R \geq w(1-\omega), $$

where BC again stands for the borrowing constraint. Note that the pledgeable fraction of the project revenue, $\lambda(z)$, is now sector-specific. The assumption that it is strictly increasing means that the sectors are indexed such that the agency problems underlying the borrowing constraint are bigger in lower indexed sectors.

The Cobb-Douglas preferences ensure that, in autarky, the economy produces in all the sectors. Thus, both (PC-z) and (BC-z) must be satisfied for all $z$. Furthermore, for each $z$, one of them must be binding; otherwise, no agent would become workers. Therefore,

$$ (8) \quad p(z)/w = \max \{1, (1-\omega)/\lambda(z)\}/R. $$

It is decreasing in $\lambda(z) < 1-\omega$ and constant for $\lambda(z) > 1-\omega$. Note that, for $\lambda(z) < 1-\omega$, (BC-z) is binding and $p(z)R > w$. In the sectors plagued by big agency problems, each project must earn higher revenues in order to assure the workers for their wage payment. The higher prices and higher project revenues
in these sectors are due to the difficulty of obtaining the credit, which restricts the entry in these sectors. 7 Let \( n(z) \) denote the number of projects run in sector \( z \). Then, the total output in sector \( z \) is \( n(z)R \), which must be equal to \( D(z) \) in autarky. Thus, \( E = p(z)D(z) = p(z)n(z)R \). Hence, (8) becomes

\[
(9) \quad n(z) = \min\{1, \frac{\lambda(z)}{(1-\omega)}\}\frac{E}{w},
\]

which is increasing in \( \lambda(z) < 1-\omega \) and constant for \( \lambda(z) > 1-\omega \). Since each project requires one unit of labor, and the aggregate labor endowment is equal to \( \omega \), the resource constraint in this economy is given by

\[
(10) \quad \int_{0}^{1} n(z)dz = \omega.
\]

Summing up (9) for all \( z \) and using (10) yields

\[
(11) \quad n(z) = \min\{1, \frac{\lambda(z)}{(1-\omega)}\}\omega \left[ \int_{0}^{1} \min\{1, \frac{\lambda(z)}{(1-\omega)}\} ds \right],
\]

which implies \( n(z) < \omega \) for low \( z \) and \( n(z) > \omega \) for high \( z \). 8

Now, suppose that the world economy consists of two countries of the kind analyzed above, North and South. They have identical parameters except \( \lambda(z) \) and \( \omega \). To avoid a taxonomical analysis, let \( \omega_N > \omega_S \). Furthermore, it is assumed that \( \lambda_S(z) = \lambda_S \Lambda(z) \) and \( \lambda_S(z) = \lambda_S \Lambda(z) \), where \( \Lambda(z) \) is continuous and increasing in \( z \) with the range from zero to one, and \( 1 > \lambda_N \geq \lambda_S > 0 \). This means that the agency problems underlying the borrowing constraint have two components; \( \Lambda(z) \) depends on the technologies and other sector-specific factors, and \( \lambda_N \) and \( \lambda_S \) depend on corporate governance, legal enforcement and other country-specific factors that determine the overall level of financial development in these economies.
From (8), the autarky prices in North and South, $p_N(z)$ and $p_S(z)$, are now given by

\begin{equation}
\frac{p_j(z)}{w_j} = \max\{1, \frac{(1-\omega_j)/\lambda_j \Lambda(z)}{\lambda_j} \}/R \quad (j = N, S).
\end{equation}

Since $(1-\omega_N)/\lambda_N < (1-\omega_S)/\lambda_S$, eq. (12) implies that $p_N(z)/w_N \leq p_S(z)/w_S$ for all $z$ and $p_N(z)/w_N < p_S(z)/w_S$ for $z$ such that $\Lambda(z) < (1-\omega_S)/\lambda_S$, as shown in Figure 2. This means that the credit market imperfections effectively become the source of North’s absolute advantage over South.

Hence, when North and South trade with each other, the equilibrium relative wage must satisfy $w_N > w_S$, so that South gains comparative advantage in high indexed sectors. Figure 3 shows the patterns of comparative advantage. North, whose credit market functions better and whose entrepreneurs are richer and hence more credit-worthy, specializes and exports in the lower indexed sectors that suffer from bigger agency problems. South specializes and exports in higher indexed sectors, which are subject to smaller agency problems. The relative wage rate and the marginal sector, $\Lambda(z_c) = \Lambda_c$, are determined by the balanced trade condition. See Freitas and Matsuyama (2004) for a variety of extensions of the above model.
Figure 1: Patterns of International Capital Flows

Figure 2: North’s Absolute Advantage over South

Figure 3: Patterns of Comparative Advantage
References


Footnotes

1I have previously used similar specifications of the credit market imperfections in Matsuyama (2000a,b, 2001, 2004a,b). It is possible to give any number of agency stories to justify the assumption that borrowers can pledge only up to a fraction of the project revenue. The simplest story would be that they strategically default, whenever the repayment obligation exceeds the default cost, which is proportional to the project revenue. Alternatively, each project is specific to the borrower, and requires his services to produce $R$ units of physical capital. Without his services, it produces only $\lambda R$ units. Then, the borrower, by threatening to withdraw his services, can renegotiate the repayment obligation down to $\lambda R f'(k)$. See Hart and Moore (1994). It is also possible to use the costly-state-verification approach used by Townsend (1979) and Boyd and Smith (1997), or the moral hazard approaches used by Gertler and Rogoff (1990) and Holmström and Tirole (1997).

2Although the equilibrium credit rationing is important for understanding the working of this model, one should not make too much out of it, because it is possible to extend the model to eliminate the equilibrium credit rationing without changing the essential feature of the model. For example, suppose that
the input endowments of the entrepreneurs are drawn from a cumulative
distribution, $G(z)$, with the mean equal to $\omega$ and with no mass point. Then, the
entrepreneurs, whose endowments are greater than or equal to $z_c$, become
entrepreneurs and those whose endowments are less than $z_c$ become the
lenders, where $z_c$ is given by $G(z_c) = \frac{1}{\omega}$. What is essential for the present
analysis is whether the borrowing constraint for the marginal agent is binding
or not. The equilibrium credit rationing is a mere artifact of the homogeneity
assumption, made solely to minimize the notation and to simplify the analysis.

3The assumption, $f'(0) = \infty$, ensures that the investments are positive in both
countries. In particular, it rules out the corner solution, $k_N = R(\omega_N + \omega_S)$, $k_S = 0,$
with (4) satisfied with the inequality.

4In their moral hazard model, Gertler and Rogoff (1990) demonstrated that the
capital movement from the rich to the poor is muted in the imperfect
information case, compared to the perfect information case. It is not clear
whether the reverse capital flows occur in their model, unless the net worth of
the poor is negative. In the present model, the reverse capital flows occurs
even if the poor is just slightly poorer than the rich.

5Such an extension would bring the above model closer to the human capital
externality story of Lucas (1990). Alternatively, one might be able to model
positive feedback from $k$ to $\lambda$ by allowing for the credit market inefficiency to
be mitigated by diversification opportunities, along the line pursued by
Acemoglu and Zilibotti (1997) and Martin and Rey (forthcoming). Another
possible way of linking $\lambda$ to $k$ is to let each government to tax $k$ in an effort to
finance an improvement in the country’s financial infrastructure to improve $\lambda$, along the line pursued by Ando and Yanagawa (2002).

Kletzer and Bardhan (1987) is the seminal work exploring a theoretical link between the credit market and the patterns of international trade.

This means that the entrepreneurs are not indifferent between the sectors. They prefer running the project in lower-indexed sectors. Therefore, as in the previous model, the equilibrium allocation necessarily involves credit rationing (unless some heterogeneity among entrepreneurs is introduced; see footnote 3).

Note that the binding borrowing constraints in low-indexed sectors give rise to positive profits. The total profit in sector $z$ is equal to $E - wn(z)$, which is positive for $\lambda(z) < 1 - \omega$ and zero for $\lambda(z) > 1 - \omega$. Summing it up across all the sectors and using (10) verifies that the aggregate profit $\Pi$ is given by $\Pi = E - \omega$. Hence, the aggregate income $Y$ satisfies $Y = \omega + \Pi = E$. 