Engel’s Law in the Global Economy:
Demand-Induced Patterns of Structural Change, Innovation, and Trade

Kiminori Matsuyama
Northwestern University

University of Zurich, 2018-9-19
Bocconi University, 2018-9-20
Introduction
Motivation

• *Endogenous Demand Composition due to Nonhomothetic Demand* (Engel’s Law)
  ✓ Expenditure shares more skewed towards higher income elastics in richer countries.
  ✓ An important channel through which economic growth and globalization affect
    ▪ Sectoral compositions in employment and in value-added
    ▪ Variations in innovation/productivity growth rates across sectors (Schmookler)
    ▪ Patterns of intersectoral trade between rich and poor countries (Linder)
    ▪ Migration of industries from rich to poor countries (Vernon’s Product Cycles)

• These effects are *interconnected*, yet studied *separately*, which can be *misleading*.
  ✓ *False dichotomy* of income elasticity vs. productivity growth differences
    ▪ *Endogenous* productivity response to the relative market size
  ✓ *Alleged* claim that globalization *reduces* the power of domestic demand composition
    ▪ Gains from trade have *the income effect*; more important for smaller countries
    ▪ The Linder Effect *magnifies* the power of domestic demand composition
    ▪ Migration of industries from the rich to the poor *facilitate* structural change in both

• *A unifying* perspective on how economic growth and globalization affect patterns of
  structural change, innovation, and trade by capturing all these interactive effects of
  Engel’s Law.
Framework: 2-Country Directed Technical Change with Endog.Demand Compositions

- One Nontradeable Factor (labor)

- **2 Countries;** differ only in *population size* \(N\) and *labor productivity* \(h\), hence also in the country size, measured in *effective labor supply* \(L = hN\)

- **Continuum of Nontradeable Consumption Goods, in** \(s \in I\), with *Isoelastic Nonhomothetic CES*
  - Goods-specific income elasticity parameters, \(\varepsilon(s)\), increasing in \(s \in I\)
  - Constant elasticity of substitution parameter, \(\eta\), *not* linked to income elasticity parameters, \(\varepsilon(s)\); Goods can be complements \((\eta < 1)\) or substitutes \((\eta > 1)\).

- Production side deliberately standard, *a la* Dixit-Stiglitz-Krugman
  - Each nontradeable consumption good produced in a competitive sector by assembling tradeable differentiated inputs with *CES aggregators*.
  - **Tradeable differentiated intermediate inputs:**
    - ✓ supplied by *monopolistic competitive* firms with labor for both production and entry
    - ✓ subject to *iceberg trade cost*
  - **Endogenous Sectoral TFP**, depend on the availability of differentiated inputs in each sector, endogenous through entry and trade.
Isoelastic Nonhomothetic CES

\[ \bar{U} = U(C_s, s \in I), \text{implicitly additive as} \left[ \int_I (\beta_s)^{\frac{1}{\eta}} \left( \frac{\varepsilon(s) - \eta}{\eta} \right)^{\frac{\eta - 1}{\eta}} (C_s)^{\frac{\eta}{\eta - 1}} ds \right]^{\frac{\eta}{\eta - 1}} = 1 \]

- \( I = (0,1) \) and \( \int_0^1 \varepsilon(s) ds = 1 \), w.o.l.g.
- \( (\varepsilon(s) - \eta)/(1 - \eta) > 0 \), for global monotonicity & quasi-concavity
- If \( \varepsilon(s) = 1 \), homothetic CES. If \( \varepsilon(s) \neq 1 \), nonhomothetic CES.
- \( \varepsilon(s) \) increasing in \( s \in I \) \( \Rightarrow \omega(s, \bar{U}) \equiv (\beta_s)^{\frac{1}{\eta}} \left( \frac{\varepsilon(s) - \eta}{\eta} \right) \), log-supermodular in \( s \) & \( \bar{U} \)
- \( \omega(s, \bar{U}) \equiv (\beta_s)^{\frac{1}{\eta}} \left( \frac{\varepsilon(s) - \eta}{\eta} \right) \), isoelastic in \( \bar{U} \), \( \Rightarrow \) the stable slope of the Engel curve

(Double-log) Demand System: \( m_s \) : expenditure share of \( s \); \( E/P \): per capita real income

\[ \log(m_s / m_{s'}) = \log(\beta_s / \beta_{s'}) + (\varepsilon(s) - \varepsilon(s')) \log(E/P) + (1 - \eta) \log(P_s / P_{s'}) \]

- Higher-indexed more income elastic
- Price elasticity is not linked to sector-specific income elasticity
  (unlike Stone-Geary or CRIE preferences)
Main Results in a Closed Economy

- Sectoral shares in employment (also valued-added) = Sectoral shares in expenditure

- A higher $h$ or $N$ leads to

  ✓ A higher per capita real income (welfare) $U$
  ✓ Sectoral shares in expenditure and employment shift toward higher-$s$ in the sense of **Monotone Likelihood Ratio** (MLR)
  ✓ Productivity growth faster in higher-$s$ through entry and exit (Schmookler effect)
  ✓ Relative prices changes would moderate (amplify) sectoral shifts when consumptions goods are complements (substitutes)

The usual dichotomy of income elasticity vs. productivity growth differences in the structural change literature would be false in the presence of Schmookler effect.

- Per capita real income would be lower in a country with higher $h$ if it is sufficiently smaller in $L = hN$
Main Results in a Trade Equilibrium: Cross-Country Variations-I

*Endogenous Terms of Trade:* The wage rate lower in the country smaller in \( L = hN \)

*Endogenous Country Ranking.* The country higher in \( h \) but smaller in \( L = hN \) may be poorer at a high trade cost but richer at a low enough trade cost.

\[
0 < \rho < 1: \text{ degree of globalization, inversely related to the iceberg trade cost.}
\]
Main Results in a Trade Equilibrium: Cross-Country Variations-II

- **Sectoral Composition in Expenditures**, $m_1^s/m_2^s$. More skewed towards higher-income elastic sectors in the richer country.
- Trade *amplifies* the effects of domestic demand composition via ***Home Market Effect***
  - **Sectoral Composition in Employment (or value-added)**, $f_1^s/f_2^s$.

$$\frac{f_1^s}{f_2^s} > \frac{m_1^s}{m_2^s} > 1 \text{ or } \frac{f_1^s}{f_2^s} < \frac{m_1^s}{m_2^s} < 1$$

- *The Linder Effect*; a cut-off sector, $s_c \in I$, above which Rich is a net-exporter.

For $U_1^s < U_2^s$
Main Results in a Trade Equilibrium: Comparative Statics-I

- **A uniform increase in** \( h \) (or \( N \))
  - ✓ No effect on ToT nor on Country Ranking
  - ✓ **Structural Change:** Expenditure & employment shares shift towards higher-s in both
  - ✓ **Product cycles:** a cut-off sector, \( s_c \in I \), goes up, turning the richer country from a net-exporter to a net-importer in the middle.
  - ✓ **Per capita real income gaps narrow** if sectors produce complements
- **A (uniform) decline in iceberg trade cost** (when the countries equal in \( L = hN \))
  - ✓ Isomorphic to a uniform increase in \( h \).

*Under Engel’s Law, globalization, through its productivity effect, causes structural change and product cycles.*

![Graph showing sectoral trade balances]

The richer country’s sectoral trade balances switch from surpluses to deficits.
Main Results in a Trade Equilibrium: Comparative Statics-II

- **A (uniform) decline in iceberg trade cost** (when the countries differ in $L = hN$).
  - ✓ *ToT change in favor of the smaller country* $\rightarrow$ *Factor Price Convergence*
  - ✓ *Leapfrogging*: The country higher in $h$ but smaller in $L$ may be poorer is a less globalized world, becomes richer with globalization
  - ✓ *Reversal of the patterns of trade*

\[
\omega \equiv \frac{w^1}{w^2} \quad \rho^{-1/\sigma} \quad 1 \quad \rho^{1/\sigma} \quad L^1/L^2
\]

\[
\frac{1-2\sigma}{\rho^{\sigma(\sigma-1)}} \quad h^1/h^2
\]
Literature Review

**Structural Change:** Matsuyama (2008); Herrendorf-Rogerson-Valentinyi (2014)
- **Nonhomotheticity:** Buera-Kaboski (2012); Kongsamut-Rebelo-Xie (2000)
- **Both:** Matsuyama (2009); Comin-Lashkari-Mestieri (2015)
- **Open Economy:** Matsuyama (2009); Uy-Yi-Zhang (2013)

**Market Size in Sectoral Variations in Productivity Growth:** Schmookler (1966)
- **Nonhomotheticity:** Murphy-Shleifer-Vishny (1989); Matsuyama (1992; 2002)

**Engel’s Law with Exog. Patterns of Trade & Exog. Country Ranking**
- **Ricardian:** Matsuyama(2000), Fieler(2011)
- **Factor Endowment:** Markusen(1986), Caron-Fally-Markusen(2014)

**Home Market Effect in Patterns of Trade:** Linder (1961)
- **Exg. Market Size Distribution Difference Across Sectors:** Krugman (1980)
- **Exg. Market Size Difference Across Countries:** Helpman-Krugman (1985)
Literature Review. Continue…

**Product Cycles:** Vernon (1966)
✓ **Technology-Diffusion:** Krugman (1979), Grossman-Helpman (1991); Antras (2005)
✓ **Demand-Induced:** Matsuyama (2000)

**Intra-Industry Trade with Nonhomothetic Preferences:**
✓ **Alternatives to Dixit-Stiglitz:** VES, Pricing to the Market; Parallel Trade, etc
✓ **Quality Differentiated**
  o Demand-Induced Patterns of Trade: Fajgelbaum-Grossman-Helpman (2011)

**Log-supermodularity & monotone comparative statics in Trade:**
The Framework
One Nontradeable Factor (Labor)

Two Countries ($j$ or $k = 1$ or $2$): populated by $N^j$ identical agents each endowed with $h^j$ units of effective labor, inelastically supplied at $w^j$.

- $N^j$ (the population size) & $h^j$ (labor productivity) are the only possible sources of heterogeneity across the two countries.
- $E^k = w^k h^k$; Per capita “Nominal” Income (and Expenditure) in $k$,
- $L^j = h^j N^j$; Total Effective Labor Supply in $j$: 

Continuum of Nontradeable Consumption Goods, \( s \in I \subset R \) with preferences:

\[
\tilde{U}^k = U(C^k_s, s \in I), \text{ implicitly additive as } \left[ \int_I (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\varepsilon(s) - \eta}{\eta}} (C^k_s)^{\frac{\eta - 1}{\eta}} ds \right]^{\frac{\eta}{\eta - 1}} \equiv 1
\]

- \( I = (0,1) \) and \( \int_I \varepsilon(s) ds = 1, \text{ w.o.l.g.} \)
- \( (\varepsilon(s) - \eta)/(1 - \eta) > 0 \), for global monotonicity & quasi-concavity
- If \( \varepsilon(s) = 1 \), for all \( s \in I \), standard homothetic CES.
- If \( \varepsilon(s) \neq 1 \), nonhomothetic CES. No longer explicitly additive

- Index sectors so \( \varepsilon(s) \) increasing in \( s \in I \) \( \rightarrow \omega(s, \tilde{U}^k) \equiv (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\varepsilon(s) - \eta}{\eta}} \), log-supermodular in \( s \) & \( \tilde{U}^k \) (The happier agents put more weights on the higher-indexed.)

Utility Maximization: Given \( P^k_s, s \in I \) and \( E^k \), choose \( C^k_s, s \in I \) and \( \tilde{U}^k \) to

\[
U^k = \max \tilde{U}^k \text{ s.t. } \left[ \int_I (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\varepsilon(s) - \eta}{\eta}} (C^k_s)^{\frac{\eta - 1}{\eta}} ds \right]^{\frac{\eta}{\eta - 1}} \equiv 1 \text{ and } \int_I P^k_s C^k_s ds \leq E^k
\]
Properties of the Consumption Demand System:

Expenditure Shares: \( m^k_S \equiv \frac{p^k_s c^k_s}{E^k} = \frac{\beta_s(U^k)^{\varepsilon(s)-\eta} (p^k_s)^{1-\eta}}{(E^k)^{1-\eta}} = \frac{\beta_s(U^k)^{\varepsilon(s)-\eta} (p^k_s)^{1-\eta}}{\int_1 \beta_t(U^k)^{\varepsilon(t)-\eta} (p^k_t)^{1-\eta} dt} \)

Indirect Utility Function: Implicitly additive (Not explicitly additive with \( \varepsilon(s) \neq 1 \))

\[ \left[ \int_1 \beta_s(U^k)^{\varepsilon(s)-\eta}(p^k_s/E^k)^{1-\eta} ds \right]^{1/(1-\eta)} \equiv 1 \]

(Per capita) Real Income:

\( U^k = E^k/P^k, \) where \( P^k \equiv \left[ \int_1 \beta_s(U^k)^{\varepsilon(s)-1}(p^k_s)^{1-\eta} ds \right]^{1/(1-\eta)} \) is the exact price index.

Double-Log Demand System:

\[ \log(m^k_s/m^k_{s'}) = \log(\beta_s/\beta_{s'}) + (\varepsilon(s) - \varepsilon(s')) \log(E^k/P^k) + (1-\eta) \log(P^k_s/P^k_{s'}) \]

✓ Higher-indexed more income elastic
✓ Price elasticity is not linked to sector-specific income elasticity
✓ Empirically, common (unitary) income elasticity as well as the positive correlation between income and price elasticities implied by Pigou’s Law, are rejected, but not common price elasticity.
Rest of the model: Standard Dixit-Stiglitz-Krugman-(Ethier-Romer)

Competitive Nontradeable Consumption Goods Sectors, $s \in I$, produce $Y_s^k = N_k C_s^k$ by combining tradable intermediate inputs, $\nu \in \Omega_s$, with CES aggregators,

$$Y_s^k = \left[ \int_{\Omega_s} \left( q_s^k(\nu) \right)^{\frac{\sigma-1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}} ; s \in I; \sigma > \text{Max}\{1, \eta\},$$

Cost Minimization: given $p_s^k(\nu)$, the unit price of input variety, $\nu \in \Omega_s$ in $k$,

$$q_s^k(\nu) = \left( \frac{p_s^k(\nu)}{P_s^k} \right)^{\sigma} Y_s^k, \text{ where } P_s^k = \left[ \int_{\Omega_s} (p_s^k(\nu))^{1-\sigma} d\nu \right]^{\frac{1}{1-\sigma}},$$

Monopolistically Competitive Tradeable Differentiated Inputs Producers $s \in I$

$$\Omega_s = \Omega_s^1 + \Omega_s^2; \Omega_s^j \text{ disjoint sets of inputs produced in } j = 1, 2$$

$\phi_s$ (in labor) to set up each variety;

$\psi_s$ (in labor) to produce a unit of each variety
**Iceberg Trade Costs:** Only $\tau^{-1} < 1$ of exports survives shipping, raises export price by $\tau > 1$, reduces export demand by $(\tau)^{-\sigma} < 1$ & export revenue by $\rho \equiv (\tau)^{1-\sigma} < 1$

**World Demand for Each Input Variety:** $D_s(\nu) = A^j_s \left(p^j_s(\nu)\right)^{-\sigma}$, $\nu \in \Omega^j_s$, where

$$A^1_s \equiv b^1_s + \rho b^2_s, \ A^2_s \equiv \rho b^1_s + b^2_s,$$

where $b^k_s \equiv \beta_s N^k(E^k)\eta(U^k)\epsilon(s)\eta(p^k_s)\sigma - \eta$ Standard CES demand curve, but $U^1$ and $U^2$ affects $A^j_s$ differently across $j$ and $s$.

**Pricing:** $p^j_s(\nu) = \frac{w^j_s \psi_s}{1-1/\sigma} \equiv p^j_s < \tau p^j_s = p^k_s(\nu)$ for $\nu \in \Omega^j_s$ and $j \neq k$

**Zero Profit:** $y^j_s = A^j_s \left(p^j_s\right)^{-\sigma} = (\sigma - 1) \phi_s/\psi_s$ for $s \in I$

**Labor Demand:** $L^j_s = (\psi_s y^j_s + \phi_s)V^j_s = \sigma \phi_s V^j_s$ for $s \in I$

Labor employment $L^j_s$ is proportional to $V^j_s$, the measure of $\Omega^j_s$

**Labor Market Equilibrium:** $\int_I L^j_s ds = L^j \rightarrow \int_I f^j_s ds = 1$

$f^j_s \equiv L^j_s/L^j$; sector-$s$’s share in employment (& in value-added) in $j$. 
Patterns of Structural Change in a Closed Economy
Define \( u(\cdot) \) implicitly by
\[
\int_I x^{\frac{\eta-1}{\sigma-\eta}} \left[ \beta_s(u(x))^{\varepsilon(s)-\eta} \right]^{\frac{\sigma-1}{\sigma-\eta}} ds \equiv 1
\]

**Per capita real income:** \( U_0^k = u(x_0^k) \) with \( x_0^k \equiv (h^k)^\sigma N^k = (h^k)^{\sigma-1} L^k \)
- \( U_0^k = u(x_0^k) \) increasing in \( x_0^k \) (hence both in \( h^k \) & \( N^k \)). Aggregate increasing returns
- A higher \( h^k \) has a larger effect than a higher \( N^k \)
- \( h^1 > h^2 \) and \( U_0^1 < U_0^2 \) holds if \( L^1/L^2 < (h^1/h^2)^{1-\sigma} < 1 \).
The smaller country could be poorer in spite of higher labor productivity

**Market Size Distributions:**
\[
m_s^k = \frac{\int_I \left[ \beta_t(u(x_0^k))^{\varepsilon(t)-\eta} \right]^{\frac{\sigma-1}{\sigma-\eta}} dt}{\int_I \left[ \beta_s(u(x_0^k))^{\varepsilon(s)-\eta} \right]^{\frac{\sigma-1}{\sigma-\eta}} ds} = f_s^k
\]
- Labor is allocated across sectors proportionately with their market sizes; \( f_s^k = m_s^k \)
- Sectoral TFP, \( w^k/P_s^k = (f_s^k L^k)^{\frac{1}{\sigma-1}} \), due to the Dixit-Stiglitz variety effect in each sector
Comparative Statics: An increase in \( x_{0}^{k} \equiv (h^{k})^{\sigma}N^{k} = (h^{k})^{\sigma-1}L^{k} \uparrow \)

- Higher per capita real income, \( U_{0}^{k} = u(x_{0}^{k}) \uparrow \)
- They spend relatively more on higher-indexed goods (in the sense of MLR).
- This causes entries (exits) in the higher(lower)-indexed sectors.
- The employment shares shift toward the higher-indexed (in the sense of MLR).
- Productivity grows faster in higher-indexed sectors, which moderates (amplifies) the sectoral shift if the consumption goods are complements (substitutes).

Formally,

\[
\frac{d \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}{d \log \left( u(x_{0}^{k}) \right)} = \frac{\partial \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}{\partial \log \left( u(x_{0}^{k}) \right)} + \frac{\partial \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right) d \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right)}{\partial \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right) d \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}
\]

Total Effect = Direct Income Effect + Substitution Effect*Induced Productivity Change

Since \( \frac{\partial \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}{\partial \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right)} = 1 - \eta \) and \( \frac{d \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right)}{d \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)} = \frac{d \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right)}{d \log \left( \frac{p_{s}^{k}}{p_{s'}^{k}} \right)} = \frac{1}{1-\sigma} \)

\[
\left| \frac{d \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}{d \log \left( u(x_{0}^{k}) \right)} \right| = \left( \frac{\sigma - 1}{\sigma - \eta} \right) \left| \frac{\partial \log \left( \frac{m_{s}^{k}}{m_{s'}^{k}} \right)}{\partial \log \left( u(x_{0}^{k}) \right)} \right| \begin{cases} < \eta \end{cases}
\]

for \( \eta <(>) 1. \)
Trade Equilibrium: Cross-Country Variations
Figure 1: Equilibrium (Factor) Terms of Trade:

\[
\frac{L^1}{L^2} = \Lambda(\omega; \rho) \equiv (\omega)^{2\sigma-1} \frac{1 - \rho(\omega)^{-\sigma}}{1 - \rho(\omega)^{\sigma}}
\]

The factor price lower in the smaller economy (Aggregate increasing returns) Globalization (\(\tau \downarrow\) or \(\rho \uparrow\)) reduces the smaller country’s disadvantage and hence the factor price differences.
Per capita real income: $U^k_\rho = u(x^k_\rho)$ with $x^1_\rho \equiv \frac{(1-\rho^2)x^1_0}{1-\rho(\omega)-\sigma} > x^1_0$ and $x^2_\rho \equiv \frac{(1-\rho^2)x^2_0}{1-\rho(\omega)^\sigma} > x^2_0$

$u(\cdot)$ defined as before.  **Gains from trade**

**Market Size Distributions: Ranked by MLR**

$$m^k_s = (x^k_\rho)^{\left(\frac{n-1}{\sigma-\eta}\right)} \left[ \beta_s(u(x^k_\rho))^{\epsilon(s)-\eta} \right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} = \frac{\left[ \beta_s(u(x^k_\rho))^{\epsilon(s)-\eta} \right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\int \left[ \beta_t(u(x^k_\rho))^{\epsilon(t)-\eta} \right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} dt}$$

$$\Rightarrow \frac{m^1_s}{m^2_s} = \left(\frac{x^1_\rho}{x^2_\rho}\right)^{\left(\frac{n-1}{\sigma-\eta}\right)} \left[ \frac{u(x^1_\rho)}{u(x^2_\rho)} \right]^{\epsilon(s)-\eta} \left(\frac{\sigma-1}{\sigma-\eta}\right)$$

is strictly decreasing in $s$ iff $U^1_\rho = u(x^1_\rho) < U^2_\rho = u(x^2_\rho)$

The Rich (Poor)’s expenditure is more skewed towards higher(lower)-indexed sectors.
Home Market Effect in Employment \( f_s^1 / f_s^2 \) is increasing in \( m_s^1 / m_s^2 \)

\[
\frac{f_s^1}{f_s^2} > \frac{m_s^1}{m_s^2} > 1 ; \quad \frac{f_s^1}{f_s^2} = \frac{m_s^1}{m_s^2} = 1 ; \quad \text{or} \quad \frac{f_s^1}{f_s^2} < \frac{m_s^1}{m_s^2} < 1.
\]

*Disproportionately* large shares of labor are employed in the sectors, in which the country spend larger shares of its expenditure relatively to the ROW.

If \( U_\rho^1 < U_\rho^2 \rightarrow m_s^1 / m_s^2 \) is strictly decreasing in \( s \rightarrow a \text{ unique cutoff sector}, s_c \in I, \) with

\[
\frac{f_s^1}{f_s^2} > \frac{m_s^1}{m_s^2} > 1 \text{ for } s < s_c ; \quad \frac{f_s^1}{f_s^2} < \frac{m_s^1}{m_s^2} < 1 \text{ for } s > s_c
\]

Home Market Effect in the Intersectoral Patterns of Intrasectoral Trade:

\[
NX_s^1 = -NX_s^2 > 0 \text{ for } s < s_c ; \quad NX_s^1 = -NX_s^2 < 0 \text{ for } s > s_c
\]

*Note:* Patterns of trade due to the cross-country difference in *the domestic market size distribution across sectors*, *not in the domestic market size in each sector*
Figure 2: HME in Employment and Patterns of Trade:

For $U_1^1 < U_2^2$

The Richer runs surpluses in higher income elastic sectors, even if it is a lot smaller so that its market size in those sectors may be smaller compared to the Poorer but bigger.
Ranking the Countries: Trade-off between Labor Productivity & Country Size:

Smaller country with higher $h$ can be poorer at a low $\rho$ but is richer at high $\rho$

Figure 3:

**Dashed Curve:** $U_0^1 < U_0^2$ below, $U_0^1 > U_0^2$ above

**Solid Curve:** $U_\rho^1 < U_\rho^2$ below, $U_\rho^1 > U_\rho^2$ above

At $\rho = 0$, Solid Curve = Dashed Curve.
A higher $\rho$ rotates Solid Curve clockwise,
At $\rho = 1$, it becomes vertical at $h^1/h^2 = 1$
Trade Equilibrium: Comparative Statics
Uniform Labor Productivity Growth: \( \partial \log(h^1) = \partial \log(h^2) \equiv \partial \log(h) > 0 \)

\( h^1/h^2, L^1/L^2, \omega = w^1/w^2, x^1_0/x^2_0, x^1_\rho/x^2_\rho \) all unchanged, with \( \partial \log(x^1_\rho) = \partial \log(x^2_\rho) = \sigma \partial \log(h) > 0 \) → Both \( U^1_\rho = u(x^1_\rho) \) and \( U^2_\rho = u(x^2_\rho) \) go up.

- Demand compositions shift toward higher-indexed in both countries (MLR)
- \( s_c \) goes up.
- Welfare gaps widen (narrow) if sectors produce substitutes (complements).

Figure 4: Uniform Labor Productivity Growth: Patterns of Structural Change and Product Cycles

The richer country’s sectoral trade balances switch from surpluses to deficits.
Globalization, a higher $\rho \equiv (\tau)^{1-\sigma}$, when two countries are equal in size: $L^1 = L^2 = L$

$$\omega = 1 \Rightarrow x^k_\rho = (1 + \rho)x^k_0 = (1 + \rho)(h^k)^\sigma N^k = (1 + \rho)(h^k)^{\sigma-1}L$$

The relative factor price fixed at $\omega = 1$ and independent of $\rho$. No ToT change
- The country with higher labor productivity is richer.
- A higher $\rho$ is isomorphic to a uniform increase in $h$.

Figure 4: Globalization: Patterns of Structural Change and Product Cycles

The richer country’s sectoral trade balances switch from surpluses to deficits
Globalization, a higher $\rho \equiv (\tau)^{1-\sigma}$, when two countries are unequal in size:

Globalization causes the ToT to change in favor of the smaller country
Leapfrogging and Reversal of the Patterns of Trade

For $h^1/h^2 > 1$ and below the Dashed Curve

$U^1_\rho < U^2_\rho$ at a low $\rho$
Closer to autarky, Country 1 is poorer due to its disadvantage of being smaller, running surpluses in lower-indexed.

$U^1_\rho > U^2_\rho$ at a high $\rho$
Globalization leads to a factor price convergence, which makes the smaller but smarter 1 richer, running surpluses in higher-indexed.
Concluding Remarks
• **Engel’s Law** with implications on
  i) Sectoral compositions in employment, innovation, and productivity growth.
  ii) Patterns of trade and migration of industries across rich and poor countries
• A unifying framework, capturing all these effects and their interactions
  ✓ **Two Countries** differ in population size and labor productivity
  ✓ **Nonhomothetic CES** over a continuum of nontradeable consumption goods
  ✓ **Endogenous Sectoral Productivity** due to the DS variety effect
  ✓ **Costly Trade**

• **Home Market Effect** in employment and patterns of trade
  ✓ Disproportionately large share of workers in higher income elastics in Rich
  ✓ **Linder Effect**: Rich (Poor) a net-exporter in higher (lower) income elastics
• **Comparative Statics**: *Labor productivity growth and globalization cause*
  ✓ **Sectoral Change** to the higher income elastics
  ✓ **Schmookler Effect**: Relative Price of high income elastics go down
  ✓ **Vernon’s Product cycles**: Rich switches from an exporter to an importer in the middle sectors
  ✓ **Leapfrogging and patterns of trade reversal** Globalization can help the smaller country with better labor force to overtake the other.

• **Implicit Additive Nonhomothetic CES** *(Explicit additivity, e.g., Stone-Geary, CRIE, are too inflexible, too restrictive, cannot isolate the effects of income effects.)*
Directions for Future Research

• Multiple factors of production (e.g., skilled & unskilled labor) with correlation btw the factor intensity (e.g., skilled intensity) and the income elasticity across sectors

• Sector-specific trade costs with correlation btw trade costs and income elasticity (e.g., high income elastic sectors have higher service components that are less tradeable)

• Multiple countries/regions with some geographical features
  ✓ Ceteris paribus, the most centrally located country/region would have the highest per capita real income, and becomes the net-exporter in the most income elastic sectors
  ✓ The countries/regions immediately next to the most centrally located country/region might become net-exporters in the least income elastics, due to the “shadow effect”
  ✓ The countries/regions in the peripheries might become net-exporters in the middle.

• Nonhomotheticity across as well as within sectors
  ✓ Nested isoelastically nonhomothetic CES, with $\eta$ higher within sectors than across
  ✓ Combining isoelastically nonhomothetic CES across sectors with horizontally differentiation with non-CES within sectors
  ✓ Combining isoelastically nonhomothetic CES across sectors with vertical (quality) differentiation within sectors.