Endogenous Ranking and Equilibrium Lorenz Curve Across (ex-ante) Identical Countries

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1. Introduction:

- In cross-section of countries, the rich tend to have higher TFPs and higher *K/L* ratios than the poor.

- These findings are typically interpreted as the *causality* from TFPs and/or *K/L* ratios to per capita income, under the maintained hypotheses
  - These countries offer independent observations
  - Variations across countries would disappear without any exogenous variations.

- Here, I offer a **trade economist's perspective** by showing how interaction through trade among countries might lead to:
  - Endogenous country ranking
  - *Joint* distributions and correlations in per capita income, TFPs, & *K/L* ratios

  emerging as stable equilibrium patterns.

- **Key Features** of the model:
  - A finite number (*J*) of (ex-ante) identical countries
  - A unit interval [0,1] of tradeable goods *à la* Dornbusch-Fischer-Samuelson
  - Endogenous productivity due to the variety of nontradeable differentiated intermediates, "producer services," *à la* Dixit-Stiglitz
  - Tradeable goods differ in the dependence on local services.

- **Key Mechanism: Two-way causality** between patterns of trade and productivity;
  - ➢More variety of local services gives a country CA in tradeable sectors that depend more on such services.
  - ➢Having CA in tradeable sectors that depend more on services means a larger market for such services and hence more variety.

- In any *stable* equilibrium,
  - ➢**Endogenous comparative advantage**: Different countries sort themselves into different sets of tradeable goods; The unit interval [0,1] is partitioned into J subintervals.
  - ➢**Strict ranking** of countries in income, TFP, and K/L ratios.
  - ➢**Unique equilibrium distribution:** As $J \to \infty$, analytically solvable, and all key parameters entering in log-submodular way, easy to show their changes cause a Lorenz-dominant shift of the cross-country distribution.

  Key parameters:
  - ➢Degree of differentiation among local producer services
  - ➢Fraction of the consumer goods that are tradeable
  - ➢Fraction of factors whose supplies respond to productivity changes.

- Welfare effects of trade; What fraction (if any) of countries lose from trade?

2. Basic Model: All Factors in Fixed Supply, All Consumer Goods Tradeable

J (inherently) identical countries in the World Econonmy

Representative Consumers:

- Endowed with V units of the (nontradeable) primary factor of production, which may be a composite of capital, labor, etc., as $V = F(K, L, \ldots)$.
- Cobb-Douglas preferences over **Tradeable Consumer Goods** $s \in [0,1]$

$$\log U = \int_0^1 \log(X(s))dB(s) = \int_0^1 \log(X(s))ds$$

Note: The goods are indexed by the cumulative expenditure share, WLOG.

Tradeable Consumer Goods Sectors $s \in [0,1]$: *Competitive*

Cobb-Douglas unit cost function: $C(s) = (\omega)^{1-\gamma(s)}(P_N)^{\gamma(s)}$

ω: the price of the primary factor of production (TFP in equilibrium).

P_N: the Dixit-Stiglitz price index of nontradeable producer services, defined by

$$P_N = \left\{\int_0^n [p(z)]^{-\frac{1}{\theta}} dz\right\}^{-\theta} \qquad \left(\theta = \frac{1}{\sigma - 1} > 0\right)$$

n: Equilibrium variety of producer services

θ: the degree of differentiation

$\gamma(s)$: the share of services in sector-s, increasing in $s \in [0,1]$

Nontradeable Producer Services Sector: *Monopolistically Competitive*

- Primary factor required to supply q units of each variety: $T(q) = f + mq$
- **Constant Mark-Up Pricing:** $p(z) = (1+\nu)\omega m$ $(0 < \nu \leq \theta)$
  - ➢ Unconstrained (Dixit-Stiglitz) monopoly pricing: $\nu = \theta$
  - ➢ Limit pricing: $\nu < \theta$
- **Free Entry-Zero Profit:** $\nu mq = f$

Unit Cost in Sector-s:

$$C(s) = (\omega)^{1-\gamma(s)} \left\{ \int_0^n [p(z)]^{-\frac{1}{\theta}} dz \right\}^{-\theta\gamma(s)} = \{(1+\nu)m\}^{\gamma(s)} (n)^{-\theta\gamma(s)} \omega$$

- ➢ Decreasing in n; productivity gains from variety *à la* Ethier-Romer
- ➢ High-indexed sectors gain more from greater variety
- ➢ This effect is stronger for a larger θ.

In stable equilibrium, ω and n will end up being different across countries.

Stable Equilibrium Patterns in the *J*-Country World:

Index the countries so $\{n_j\}_{j=1}^{J}$ is monotone increasing. Then,

- $\dfrac{C_j(s)}{C_{j+1}(s)} = \left(\dfrac{n_j}{n_{j+1}}\right)^{-\theta\gamma(s)}\left(\dfrac{\omega_j}{\omega_{j+1}}\right)$, is strictly increasing in *s;*

A country with a higher *n* has comparative advantage in higher-indexed sectors.

- The unit interval is partitioned into *J*-subintervals: the *j*-th exports $s \in (S_j, S_{j+1})$, where

  $\{S_j\}_{j=1}^{J}$ is given by $S_0 = 0$, $S_J = 1$ and $\dfrac{C_j(S_j)}{C_{j+1}(S_j)} = \left(\dfrac{n_j}{n_{j+1}}\right)^{-\theta\gamma(S_j)}\left(\dfrac{\omega_j}{\omega_{j+1}}\right) = 1.$

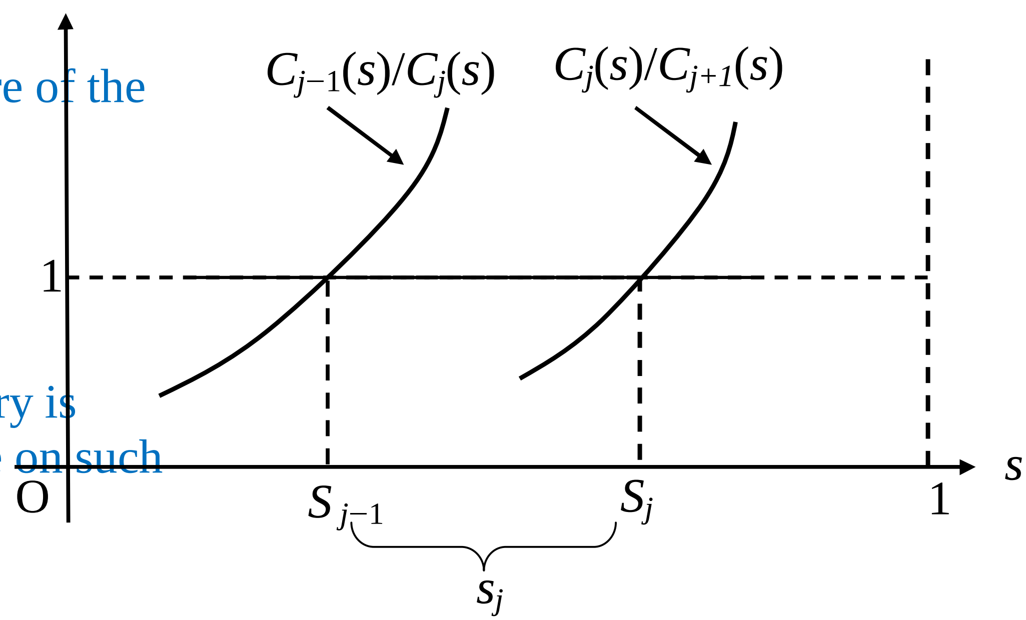
- $\{\omega_j\}_{j=1}^{J}$ is also monotone increasing. productivity gains from variety
- $Y_j = \omega_j V = (S_j - S_{j-1})Y^W$

A country's share = world's expenditure share of the consumer goods it produces.

- $n_j \propto \Gamma_j \equiv \Gamma(S_{j-1}, S_j) \equiv \dfrac{1}{S_j - S_{j-1}} \displaystyle\int_{S_{j-1}}^{S_j} \gamma(s)ds,$

  hence monotone increasing, as assumed.

The size of local service sector in each country is proportional to the average expenditure share on such services among its (active) tradeable sectors.



This can be summarized as:

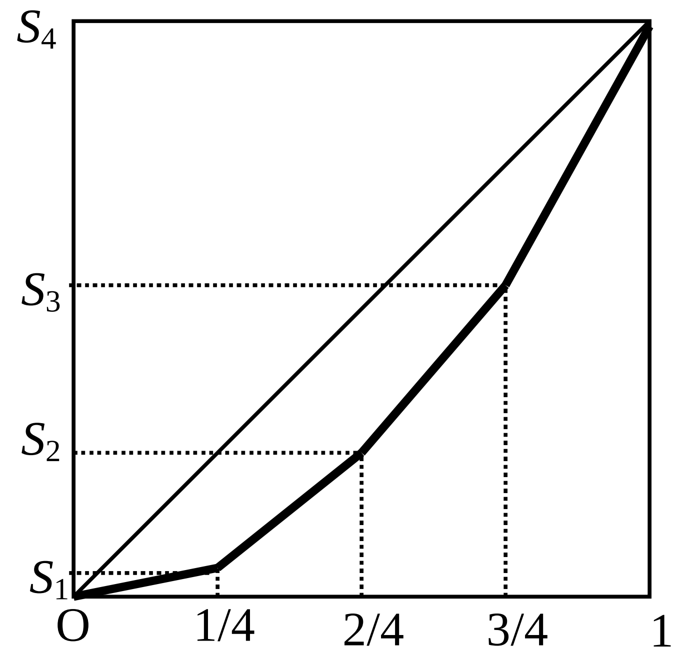
Proposition 1 (the *J*-country case):
$\{S_j\}_{j=0}^{J}$ solves:

$$\frac{S_{j+1}-S_j}{S_j - S_{j-1}} = \left(\frac{\Gamma(S_j, S_{j+1})}{\Gamma(S_{j-1}, S_j)}\right)^{\theta\gamma(S_j)} > 1 \text{ with } S_0 = 0 \ \& \ S_J = 1,$$

where $$\Gamma(S_{j-1}, S_j) \equiv \frac{1}{S_j - S_{j-1}} \int_{S_{j-1}}^{S_j} \gamma(s)ds.$$

The Lorenz curve, $\Phi^J : [0,1] \to [0,1]$, is the piece-wise linear function, $\Phi^J(j/J) = S_j$. Clearly,

- Φ^J is strictly increasing & convex;
- $\Phi^J(0) = 0$ & $\Phi^J(1) = 1$.



But, it is not analytically solvable.

- Uniqueness?
- Comparative statics?
- Welfare evaluations?

These problems disappear by $J \to \infty$.

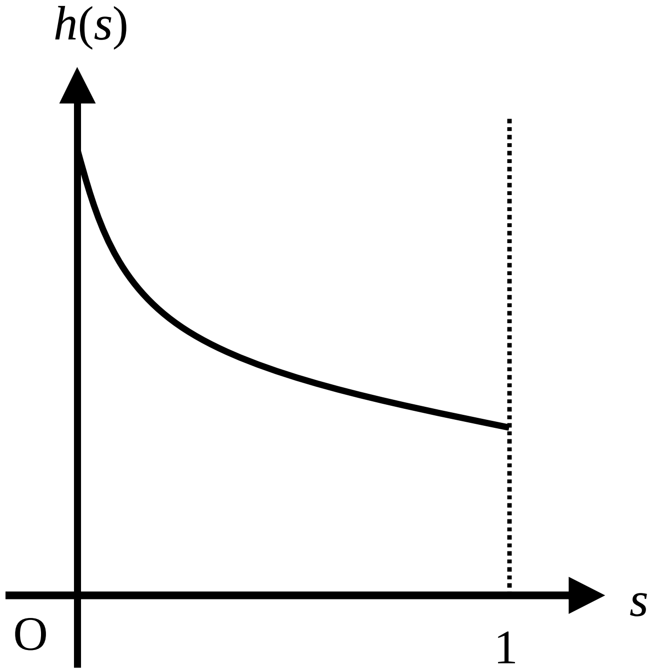
Proposition 2 (Limit Case; *J* → ∞)

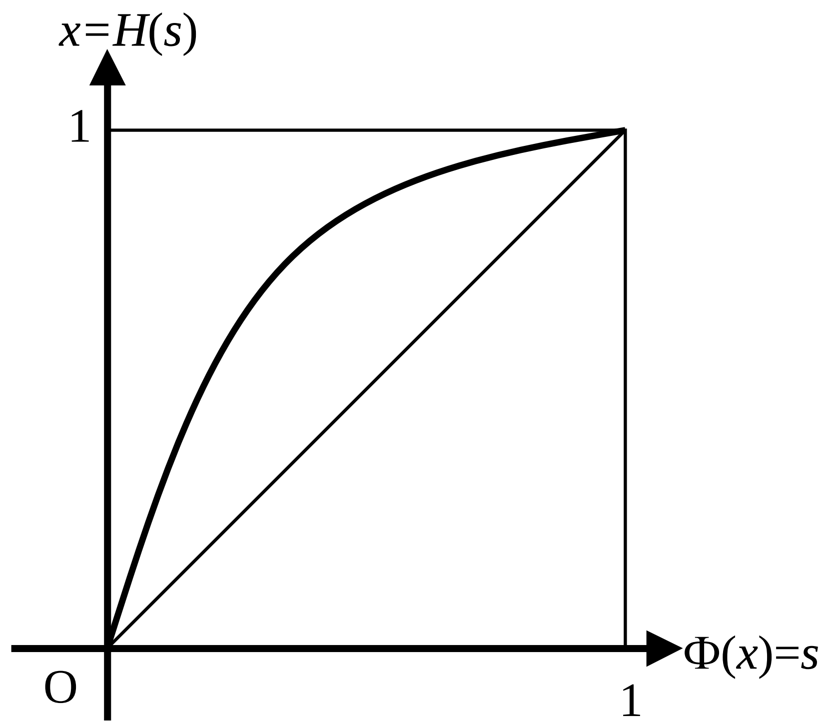
The limit equilibrium Lorenz curve, $\lim_{J\to\infty} \Phi^J = \Phi$, solves:

$$\frac{\Phi''(x)}{\Phi'(x)} = \theta\gamma'(\Phi(x))\Phi'(x) \text{ with } \Phi(0)=0 \ \& \ \Phi(1)=1$$

Its unique solution is

$$x = H\left(\Phi(x)\right) \equiv \int_0^{\Phi(x)} h(s)ds, \qquad \text{where } h(s) \equiv \frac{e^{-\theta\gamma(s)}}{\int_0^1 e^{-\theta\gamma(u)}du}.$$





Power-Law (Truncated Pareto) Examples:

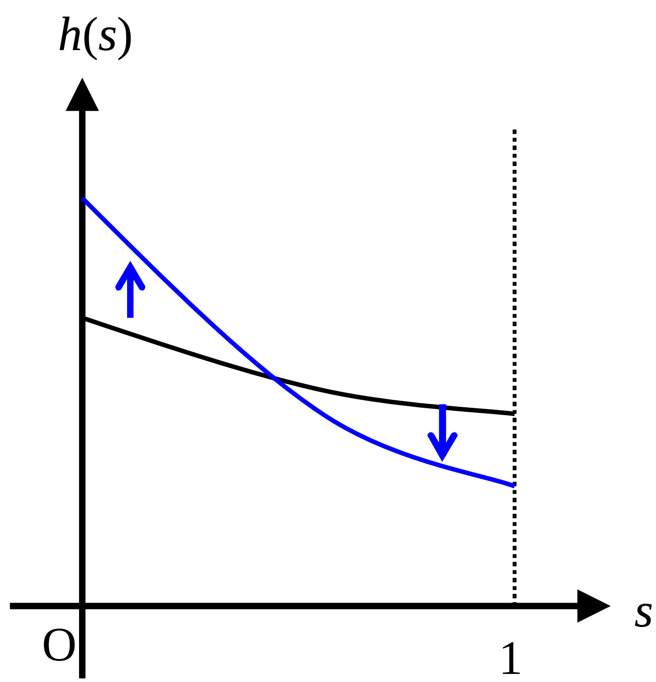
	Example 1: $\gamma(s) = s$	Example 2: $\gamma(s) = \log\left[1+(e^{\theta}-1)s\right]^{\frac{1}{\theta}}$	Example 3: $\gamma(s) = \log\left[1+(e^{\lambda}-1)s\right]^{\frac{1}{\lambda}}$ $(\lambda \neq 0; \neq \theta)$
Inverse Lorenz Curve: $x = H(s)$	$\dfrac{1-e^{-\theta s}}{1-e^{-\theta}}$	$\log\left[1+(e^{\theta}-1)s\right]^{\frac{1}{\theta}}$	$\dfrac{\left[1+(e^{\lambda}-1)s\right]^{1-\frac{\theta}{\lambda}}-1}{e^{\lambda-\theta}-1}$
Lorenz Curve: $s = \Phi(x)$	$\log\left[1-(1-e^{-\theta})x\right]$	$\dfrac{e^{\theta x}-1}{e^{\theta}-1}$	$\dfrac{\left[1+(e^{\lambda-\theta}-1)x\right]^{\frac{\lambda}{\lambda-\theta}}-1}{e^{\lambda}-1}$
Cdf: $x = \Psi(y)$ $= (\Phi')^{-1}(y)$	$\dfrac{1}{1-e^{-\theta}} - \dfrac{1}{\theta y}$	$\dfrac{1}{\theta}\log\left(\dfrac{e^{\theta}-1}{\theta}y\right)$	$\dfrac{\left(\dfrac{y}{y_{Min}}\right)^{\frac{\lambda}{\theta}-1}-1}{e^{\lambda-\theta}-1} = 1 - \dfrac{1-\left(\dfrac{y}{y_{Max}}\right)^{\frac{\lambda}{\theta}-1}}{1-e^{\theta-\lambda}}$
Pdf: $\psi(y) = \Psi'(y)$	$\dfrac{1}{\theta y^2}$	$\dfrac{1}{\theta y}$	$\left[\dfrac{(\lambda/\theta)-1}{(y_{Max})^{(\lambda/\theta)-1}-(y_{Min})^{(\lambda/\theta)-1}}\right](y)^{\frac{\lambda}{\theta}-2}$
Support: $[y_{Min}, y_{Max}]$	$\dfrac{1-e^{-\theta}}{\theta} \le y \le \dfrac{e^{\theta}-1}{\theta}$	$\dfrac{\theta}{e^{\theta}-1} \le y \le \dfrac{\theta e^{\theta}}{e^{\theta}-1}$	$\left(\dfrac{\lambda}{e^{\lambda}-1}\right)\left(\dfrac{e^{\lambda-\theta}-1}{\lambda-\theta}\right) \le y \le \left(\dfrac{\lambda}{e^{\lambda}-1}\right)\left(\dfrac{e^{\lambda-\theta}-1}{\lambda-\theta}\right)e^{\theta}$

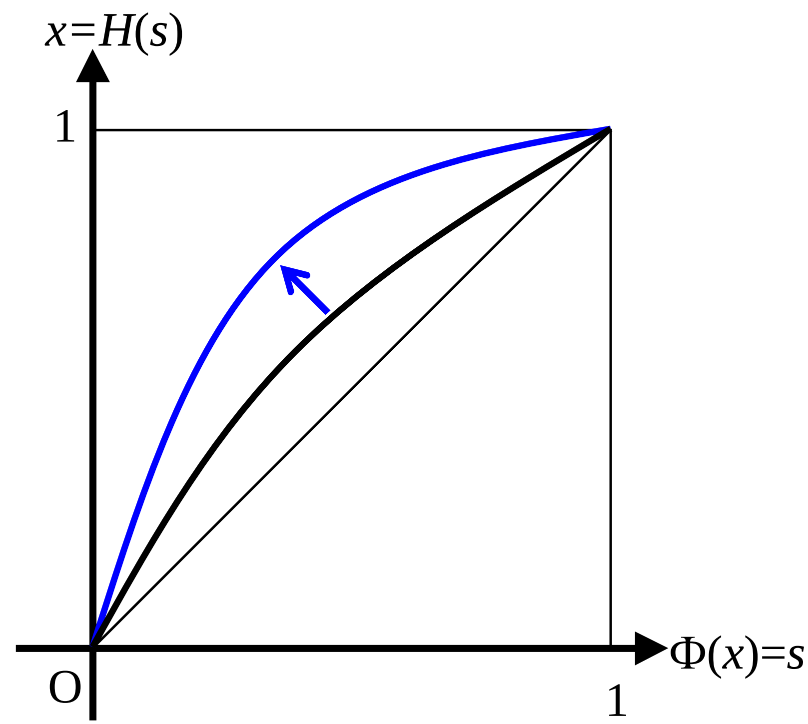
A lower λ makes the density function drop faster.

Log-submodularity and Effect of a higher θ:

Since $h(s) = \hat{h}(s) / \left[\int_0^1 \hat{h}(u)du \right]$, with $\hat{h}(s) \equiv e^{-\theta\gamma(s)}$ being *log-submodular* in θ and s, a higher θ rotates $h(s)$ "clockwise."

→ Lorenz curve "bends" more (a Lorenz-dominant shift), hence a greater inequality.





Welfare Effects of Trade

Proposition 3 (the *J*-country case): The welfare of the k-th poorest country is

$$\log\left(\frac{U_k}{U^A}\right) = \sum_{j=1}^{J} \log\left(\frac{\omega_k}{\omega_j}\right)(S_j - S_{j-1}) + \theta \sum_{j=1}^{J} \Gamma_j \log\left(\frac{\Gamma_j}{\Gamma^A}\right)(S_j - S_{j-1}).$$

- 1st term: effects on the country's relative productivity, negative for some countries.
- 2nd term; gains from trade (conditional on productivity differences), positive for all.

Proposition 4 (Limit case, *J* → ∞): The welfare of the country at 100x^*% is:

$$\frac{\log\left(U(x^*)/U^A\right)}{\theta} = \gamma(\Phi(x^*)) - \Gamma^A + \int_0^1 \gamma(s)\log\left(\frac{\gamma(s)}{\Gamma^A}\right)ds.$$

- 1st term; Productivity effect through the effect on the size of the local service.
- 2nd term; gains from trade, conditional on productivity differences, always positive.

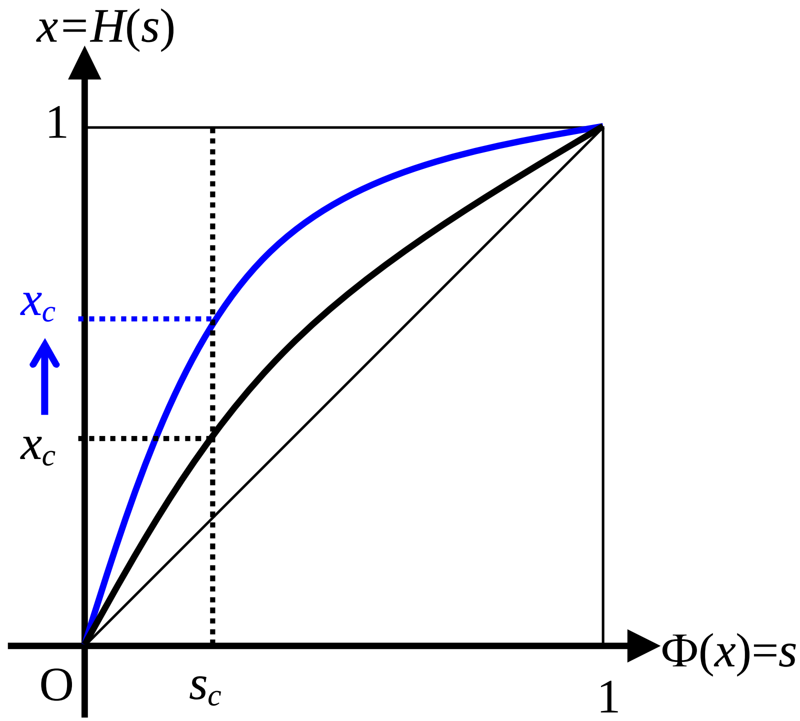
Corollary 1: All countries gain from trade iff $\frac{\gamma(0)}{\Gamma^A} \geq 1 - \int_0^1 \left(\frac{\gamma(s)}{\Gamma^A}\right)\log\left(\frac{\gamma(s)}{\Gamma^A}\right)ds.$

Corollary 2: Suppose $\frac{\gamma(0)}{\Gamma^A} < 1 - \int_0^1 \left(\frac{\gamma(s)}{\Gamma^A}\right)\log\left(\frac{\gamma(s)}{\Gamma^A}\right)ds$. Define $s_c > 0$ by $\gamma(s_c) \equiv \Gamma^A\left[1 - \int_0^1 \left(\frac{\gamma(s)}{\Gamma^A}\right)\log\left(\frac{\gamma(s)}{\Gamma^A}\right)ds\right] < \Gamma^A$.

a): All countries producing $s \in [0, s_c)$ lose from trade.

b): The fraction of the countries that lose, $x_c = H(s_c;\theta)$, is increasing in θ with $\lim_{\theta\to 0} x_c = s_c$ and $\lim_{\theta\to\infty} x_c = 1$.

Corollary 2: A Graphic Illustration



3. Two Extensions:

3.1 Nontradeable Consumption Goods:

$$\log U = \tau \int_0^1 \log(X_T(s))ds + (1-\tau)\int_0^1 \log(X_N(s))ds$$

τ; the fraction of the consumption goods that are tradeable.

A higher τ causes a Lorenz dominant shift.
Globalization through Goods Trade magnifies inequality!

3.2 *Variable Factor Supply through Factor Mobility or Factor Accumulation:*

$V_j = F(K_j, L)$ with $\omega_j F_K(K_j, L) = \rho$
Correlations between K/L and TFPs and per capita income

For $V = F(K, L) = AK^{\alpha}L^{1-\alpha}$ with $0 < \alpha < 1/(1+\theta)$,
A higher α causes a Lorenz dominant shift.
Globalization through Factor Mobility or Skill-Biased Technological Change magnifies inequality!

In both extensions, the same techniques ($J \rightarrow \infty$ to solve the Lorenz curve analytically & log-submodularity to prove the Lorenz-dominant shifts) work.

Concluding Messages:

- Countries are not close economies; they trade with one another. The world economy is the only closed economy.

- Even in the absence of ex-ante heterogeneity, interaction through trade could generate dispersion and correlations across countries in per capita income, TFPs, and *K/L* ratios as stable equilibrium patterns. *No* one-way causality from TFPs or K/L ratios to income.

- Some countries become richer (poorer) than others because they trade with poorer (richer) countries. They are *not* independent observations.

- This type of analysis does not suggest that ex-ante heterogeneity is unimportant. Instead, it suggests that even small ex-ante heterogeneity could be magnified to create huge ex-post heterogeneity.

- This paper demonstrates that this type of analysis does not have to be intractable nor lacking in prediction. *Unique, analytically solvable,* equilibrium distribution varying with parameters in intuitively ways.

- A model with many countries can be more tractable than a model with a few countries.