Engel's Law in the Global Economy:
Demand-Induced Patterns of Structural Change, Innovation and Trade*

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Abstract: Endogenous demand composition across sectors due to nonhomothetic demand (Engel's Law) affects i) sectoral compositions in employment and in value-added, ii) variations in innovation rates and in productivity change across sectors, iii) intersectoral patterns of trade across countries, and iv) migration of industries from rich to poor countries. This paper offers a unifying perspective on how economic growth and globalization affects the patterns of structural change, innovation and trade across countries and across sectors in the presence of Engel's Law. To this end, we develop a two-country model of directed technological change with a continuum of sectors under nonhomothetic preferences, which is rich enough to capture all these effects as well as their interactions. Among the main messages is that globalization amplifies, instead of reducing, the power of endogenous domestic demand composition differences as a driver of structural change.

Keywords: Isoelastically nonhomothetic CES, Implicit (direct and indirect) additivity, Dixit-Stiglitz-Krugman monopolistic competition model of production and trade, The Schmookler effect, Directed productivity change, Home market effect in employment, Home market effect in patterns of trade, The Linder effect, Vernon's product cycle hypothesis, Terms of trade change, Factor price convergence, Leapfrogging, Trade patterns reversal, Log-supermodularity, Monotone likelihood ratio, Monotone comparative statics
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1. Introduction

With income elasticity differences across sectors, the expenditure shares are more skewed towards higher income elastic sectors in richer countries. Such an endogenous demand composition due to the nonhomotheticity of demand across sectors, which we shall call *Engel's Law* for the brevity, is an important channel through which economic growth and globalization affect the patterns of structural change, innovation, and trade across countries and across sectors. For example, Engel's Law plays a central role in accounting for changing sectoral shares in employment and in value-added. It could also affect relative productivity growth across sectors, since the market size is a crucial factor in providing incentives for innovations, as pointed out by Schmookler (1966) and many others. It also affects the intersectoral patterns of trade between rich and poor countries. Linder (1961) argued that the difference in the demand composition across rich and poor countries causes the rich (poor) to develop comparative advantage in the higher (lower) income-elastic sectors, while relying on importing more from the poor (rich) in the lower (higher) income elastic sectors. It could also play a crucial role in determining the migration patterns of industries from rich to poor countries. Vernon (1966), in particular, proposed the product cycle hypothesis; industries that produce income-elastic goods are first established in high-income countries, where they find much of their demand, and then migrate to low income countries, as the world economy grows.

As will be discussed in the literature review later, some of these effects have been a subject of previous studies, but they have been mostly treated separately. This could be misleading, as these effects are *interconnected*. For example, many studies in the structural transformation literature ask to what extent the changing patterns of sectoral shares in employment and in value-added can be accounted for by the demand nonhomotheticity or by productivity growth differentials across sectors, under the (often implicit) assumption that productivity change in each sector is exogenous. Such an "income elasticity *versus* productivity growth differentials" approach is a *false* dichotomy in the presence of the Schmookler effect, because the relative productivity changes across sectors respond *endogenously* to changes in the relative market sizes caused by economic growth due to the Engel's Law. Furthermore, the existing studies typically use closed economy models, where the domestic supply is necessarily equal to the domestic demand sector-by-sector. Since the domestic supply composition does not need be equal to the domestic demand composition in an open economy, one might think, intuitively, that international trade would make the domestic demand composition *less* important as a driver of structural change. If so, one's intuition is faulty. First, international trade would generate productivity gains. The resulting income effect would cause a further sectoral shift in the expenditure through Engel's Law. Second, in the presence of the Linder effect, rich (poor) countries would allocate even *more* resources in higher (lower) income-elastic sectors under trade than under autarky, which means that the domestic demand composition would have *more than* proportional effects on the domestic supply composition. Furthermore, migration of industries from rich to poor countries would enable both rich and poor countries to achieve sectoral shifts towards more income-elastic, if those industries that migrate from rich to poor are less (more) income-elastic than those operating in rich (poor) countries, as Vernon argued. Then, product cycles should be regarded as an integral part of the interdependent patterns of structural change across rich and poor countries. For all these reasons, globalization could amplify, instead of reducing, the power of Engel's Law and the endogenous domestic demand composition differences across countries as a driver of structural change in the global economy.

The goal of this paper is to offer a unifying perspective on how economic growth and globalization affect the interdependent patterns of structural change, innovation, and trade across countries and across sectors in the presence of Engel's Law. With this goal in mind, it develops a two-country static model of directed technological change with a continuum of sectors under nonhomothetic preferences, which is rich enough to capture all these effects of Engel's Law as well as their interactions. At the same time, it deliberately abstracts from all other factors to isolate the effects of Engel's Law.

Here is a roadmap of the paper. Section 2 introduces this model and derives its equilibrium conditions. The model has a single nontradeable factor of production, labor, and two countries, which differ in the population size, N, and labor productivity, h, which also means that they differ in size, measured in the total effective labor supply, $L = hN$. There is a continuum of nontradeable consumption goods sectors, indexed by $s \in I$, where I is an open real interval, and preferences over these goods are *isoelastically nonhomothetic with constant elasticity of substitution* (CES). As explained in more detail in Appendix A, this class of preferences, which satisfies *implicit (both direct and indirect) additivity*, has several features that make it uniquely well-suited for our purpose.[1] First, it allows for any number of sectors with sector-specific income elastic parameters, $\varepsilon(s)$, while keeping the constant elasticity of substitution across sectors, η, as a separate parameter. This makes it possible to control for the income elasticity differences without affecting the price elasticity, which helps to isolate the role of income elasticity differences. Second, the CES parameter, η, can be either greater than one (the case of gross substitutes) or less than one (the case of gross complements).[2] Third, being a CES, it retains much of the tractability of the standard CES, in spite of the nonhomotheticity. Furthermore, with their income elasticity parameters being the only fundamental heterogeneity across sectors, the sectors can be indexed such that sector-specific income elasticity $\varepsilon(s)$ is increasing in $s \in I$. This implies that the weight attached on each good in our (nonhomothetic) CES utility function is *log-supermodular* in $s \in I$ and in the per capita real expenditure (and income). This in turn implies that, holding prices given, a higher per capita real income shifts the density of the expenditure shares towards higher-indexed goods in the sense of the monotone likelihood ratio (MLR), which also implies that its cumulative distribution function shifts to the right in the sense of the first-order stochastic dominance (FSD).[3] On the production and trade side, we deliberately use the standard monopolistic competition model of trade due to Dixit-Stiglitz (1977: Section I) and Krugman (1980), in order to isolate the role of Engel's Law. Each nontradeable consumption good is produced by a competitive sector, which assembles tradable differentiated intermediate inputs, using the CES aggregator. Each differentiated intermediate input is supplied by a monopolistically competitive firm, using labor for both production and

[1]In the original formulation of this class of preferences by Hanoch (1975), as well as in its recent applications by Comin, Lashkari, and Mestieri (2015), the set of consumption goods is assumed to be finite. Here, we consider the case where the set of consumption goods, *I*, is an open real interval (that is, it is a totally ordered set with a continuum of elements), as it facilitates the characterization of the equilibrium and comparative statics, as in Dornbusch, Fischer, and Samuelson (1980).

[2]The ability to deal with the case of gross complementarity across sectors, $\eta < 1$, is important, as the empirical estimates of Engel's curves are in the range of $\eta \approx 0.7 - 0.8$. In contrast, nonhomothetic preferences that rely on some notion of vertical or quality differentiation across goods within a sector, used by Flam and Helpman (1987), Stokey (1991), or Fajgelbaum, Grossman, and Helpman (2011), necessarily imply that goods are gross substitutes, and hence not well-suited for studying Engel's Law, i.e., nonhomotheticity of demand across sectors that produce complementary goods. At the same time, the ability to deal also with the case of gross substitutes, $\eta > 1$, might make our class of preferences potentially useful as a reduced form way of capturing the nonhomotheticity of demand across goods with different quality levels within a sector.

[3]See Athey (2002) and Vives (1999; Ch.2.7) for log-supermodularity and monotone comparative statics. Costinot (2009) and Costinot and Vogel (2012, 2015) are among the first to apply them in international trade.

entry (or innovation). These differentiated inputs are tradable, subject to the iceberg trade cost, as in Krugman (1980). One key feature of this setup is that productivity of each consumption goods sector in each country is endogenously determined, as it depends on the availability of differentiated inputs, which change through entry/exit of monopolistically competitive firms as well as through trade.[4]

Section 3 looks at the closed economy equilibrium. An increase in h (or N) improves welfare. In other words, the per capita real income goes up. This shifts the relative market sizes towards the higher-indexed sectors due to Engel's Law, causing a proportional change in the employment shares across sectors. This change in the relative market sizes also leads to some entries of input producers to the higher-indexed sectors as well as to some exits from the lower-indexed sectors. The resulting change in the relative productivity across sectors (the Schmookler effect) makes the higher-indexed goods relatively cheaper, which moderates (amplifies) the sectoral shift when the consumption goods are gross complements (gross substitutes).

Section 4 characterizes the cross-country variations in a trade equilibrium. The wage rate (per efficiency unit) is lower in the country smaller in size at any positive trade cost, because the smaller economy has disadvantage of having the smaller domestic market. In contrast, the country ranking (i.e., which country is richer measured in the per capita real income) is endogenously determined. When the trade cost is sufficiently high, the country with higher labor productivity could be poorer if it is sufficiently smaller in size, because of its disadvantage of being smaller. But it is richer when the trade cost is sufficiently low, which makes this disadvantage sufficiently small. At any given equilibrium, the domestic demand composition is more (less) skewed towards the higher-indexed sectors in the country that is richer (poorer) at that equilibrium, due to Engel's Law. With a positive trade cost, this cross-country difference in the domestic market compositions causes relatively more input producers to operate in the

[4]There is an alternative interpretation of our model, as was used in Matsuyama (2015, Section 2). Consumers have isoelastically nonhomothetic CES preferences over a continuum of consumption *categories*. And the utility of consuming each *category* is given by a Dixit-Stiglitz (CES) aggregator of a variety of tradeable differentiated consumer products. And each tradeable differentiated consumer product is supplied by a monopolistic competitive firm and subject to the iceberg trade cost. Here, by following Ethier (1982) and Romer (1990), we instead interpret the tradeable differentiated products as intermediate inputs to the production of nontradeable consumption goods for two reasons. First, it allows us to talk about endogenous sectoral productivity, which makes it easier to discuss its implications on the Schmookler effect and structural change. Second, much of global trade in manufacturing consists of intermediate inputs, not consumption goods; see, for example, Antras (2015).

higher(lower)-indexed sectors in the richer(poorer) country. As a result, the richer country has relatively higher productivity in higher income-elastic sectors (the Schmookler effect) and allocate *disproportionately* more labor in those sectors (the Home Market Effect in employment). This disproportionate effect on the cross-country difference in the domestic demand composition also shows up in the inter-sectoral patterns of intra-sectoral trade. Although there are two-way flows of differentiated inputs within each sector, there is a unique cutoff sector, $s_c \in I$, such that the richer country runs a trade surplus in the sectors above the cutoff and the poorer runs a trade surplus in the sectors below the cut-off. Thus, the richer (poorer) becomes a net exporter in the higher (lower) income-elastic sectors, because its domestic demand composition is more skewed towards higher (lower) income-elastics (the Linder effect).[5]

Section 5 conducts some comparative statics of the trade equilibrium. Section 5.1 looks at labor productivity growth uniform across countries, which does not affect the terms-of-trade (the relative wage) nor the ranking of the two countries. It shifts both the expenditure and employment shares towards higher-indexed sectors in both countries. It also shifts the cutoff-sector, $s_c \in I$. Thus, the richer country switches from a net exporter to a net importer in some middle sectors, generating something akin to Vernon's product cycles. The intuition behind this result is easy to grasp. As both countries become richer and shift their expenditure towards higher-indexed sectors, the weights of the higher indexed sectors, in which the richer country runs a surplus, become higher. In order to keep the overall trade account between the two countries in balance, the sectoral trade account of the richer country must deteriorate in each sector. This is why its sectoral trade balances switch from being positive to negative in some middle sectors. Furthermore, migrating sectors in the middle range from the richer to the poor countries causes the sectoral shares in employment to shift towards higher-indexed sectors in

[5]Note that it is not the relative country size but the relative per capita real income that determines the direction of the patterns of intersectoral trade in this model. The relative country size does matter but only *indirectly* through its effect on the relative per capita real income. For example, imagine that one country, say Switzerland, is much smaller but its consumers enjoy higher per capita real income than those living in the country that is much larger, say China. Our model predicts that Switzerland is a net-exporter in high income elastic sectors, even though the Chinese domestic markets might be larger than the Swiss domestic markets in all sectors, including high income elastic sectors. This is because the Linder effect or Home Market effect in this model is due to the difference in the domestic demand composition, as in Krugman (1980), and *not* in the absolute size of the domestic demand in each sector, as in Helpman and Krugman (1985, Ch.10.4).

both countries. How welfare gains from such a change are distributed across the two countries depends on the elasticity of substitution across sectors. By increasing the relative market sizes of high-indexed sectors and hence by reducing the relative prices of those sectors in which the richer country has comparative advantage through the Schmookler effect, a uniform labor productivity growth narrows (widens) the welfare (i.e., the per capita real income) gap between the two countries, when the consumption goods produced in different sectors are gross complements (substitutes).

The effects of globalization, captured by a reduction in the iceberg trade cost uniform across sectors, are similar to uniform labor productivity growth, except there are additional terms of trade effects when the two countries differ in size, measured in the total effective labor supply (or equivalently in GDP). When the two countries are equal in size (Section 5.2), the wage rates are always equalized across the countries and hence the terms of trade are not affected by a reduction in the trade cost. This means that the country with higher labor productivity has higher per capita real income in this case. And without causing any terms of trade change, the effects of globalization are isomorphic to those of uniform labor productivity growth. The intuition is, again, easy to grasp. A lower trade cost allows both countries to have better access to the differentiated inputs produced abroad, which generates productivity gains isomorphic to labor productivity growth. This income effect of productivity gains from trade causes both countries to shift their expenditure towards higher-indexed sectors, and the richer (the poorer) to switch from a net exporter (importer) to a net importer (exporter) in some middle sectors, generating product cycles, despite that the decline in the trade cost is uniform across sectors. Again, a globalization narrows (widens) the welfare gap between the two countries when the consumption goods produced in different sectors are gross complements (substitutes).

When the two countries are unequal in size (Section 5.3), the factor price is lower in the smaller country, reflecting its disadvantage of being smaller in this world of aggregate increasing returns due to the variety effect. Globalization reduces (but never eliminates) this disadvantage, and causes the factor prices to converge (but never completely equalize) and hence the terms of trade to change in favor of the smaller country.[6] This generates some additional effects. If labor

[6] This effect of globalization on the terms of trade is not due to the nonhomotheticity, as shown by Matsuyama (2015, section 3) in an alternative model, which differs from the present model only in that the domestic demand composition difference across the two countries is due to the exogenous difference in taste.

productivity is lower in the smaller country--which includes the case where the two countries have the equal population size--, this country has lower per capita real income regardless of the trade cost. However, if labor productivity is higher in the smaller country, globalization causes a *leapfrogging* due to a terms-of-trade change and a factor price convergence. At a high trade cost, the smaller country with higher labor productivity might have a lower per capita real income than the larger country with lower labor productivity, because of their disadvantage of having the smaller domestic markets. Globalization reduces this disadvantage of the smaller country enough so that its per capita real income becomes higher. In our setup, this leads to a reversal of the patterns of trade. The smaller country with higher labor productivity can be a net exporter in the lower income-elastic sectors at a higher trade cost, and a net exporter in the higher income-elastic sectors at a lower trade cost.

Section 6 discusses extensively the relation to the existing studies. Section 7 offers concluding remarks, including a few suggested directions for future research. Two Appendices follow. Appendix A explains why our class of nonhomothetic preferences is uniquely well-suited for our purpose. Appendix B offers two lemmas on log-supermodularity and monotone comparative statics used throughout the analysis.

2. The Model

Imagine the world economy that consists of two countries, indexed by j or k = 1 or 2. (Generally, j is used to indicate the location of production, and k that of consumption.) There is a single nontradeable factor of production, which shall be called labor. Country j is populated by N^j homogenous agents, each of whom supplies h^j units of effective labor inelastically at the wage rate, w^j. Thus, the per capita "nominal" [7] expenditure (and income) in k is $E^k = w^k h^k$ and the total effective labor supply in j is $L^j = h^j N^j$. The population size, N^j, and its effective labor supply per agent h^j, or labor productivity, are the only possible sources of heterogeneity across the two countries.

2.1 Nonhomothetic Preferences and Expenditure Shares:

[7] We call E^k the per capita "nominal" income to distinguish it from the per capita "real" income introduced later as a measure of the welfare level, $U^k = E^k / P^k$, where P^k is the exact price index in k. It is not "nominal" in the sense of being measured in some current unit. (This is not a monetary model, and there is no currency in the model.)

There is a continuum of sectors, indexed by $s \in I$, each producing a nontradeable consumption good, also indexed by $s \in I$, where $I \subset R$ is an open interval. The preferences of each agent, $\tilde{U}^k = U(C_s^k, s \in I)$, are given *implicitly* by

$$(1) \qquad \left[\int_I (\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}} \left(C_s^k\right)^{\frac{\eta-1}{\eta}} ds\right]^{\frac{\eta}{\eta-1}} \equiv 1; \quad \beta_s > 0,\ \eta > 0 \text{ and } \eta \neq 1,$$

with $(\varepsilon(s)-\eta)/(1-\eta) > 0$, which ensures that $\tilde{U}^k = U(C_s^k, s \in I)$ is globally monotone increasing and globally quasi-concave in C_s^k, $s \in I$. Without further loss of generality, let $I \in (0,1)$ and normalize $\varepsilon(s)$ such that $\int_I \varepsilon(s) ds = 1$.[8] The utility function (1) is *implicitly directly additive,* with *constant elasticity of substitution* (CES).[9] In addition, the weight of each sector, $\omega(s, \tilde{U}^k) \equiv (\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$, the coefficient on the term $\left(C_s^k\right)^{\frac{\eta-1}{\eta}}$, is *isoelastic in* $\tilde{U}^k$ (i.e., it is a power function of $\tilde{U}^k$). If $\varepsilon(s) = 1$ for all $s \in I$, (1) becomes the standard homothetic CES:

$$\tilde{U}^k \equiv \left[\int_I (\beta_s)^{\frac{1}{\eta}} \left(C_s^k\right)^{1-\frac{1}{\eta}} ds\right]^{\frac{\eta}{\eta-1}},$$

which is *explicitly directly additive*. By letting $\varepsilon(s)$ dependent on *s*, this class of utility functions, no longer explicitly directly additive but still implicitly directly additive, allows the income elasticity to differ across sectors, while keeping the price elasticity, η, constant and the same across sectors. In what follows, we assume that the sectors can be ordered such that $\varepsilon(s)$ is

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[8] To see why this is without loss of generality, suppose $\int_I \varepsilon(s) ds = c \neq 1$. Since $(\varepsilon(s)-\eta)/(1-\eta) > 0$ implies $(c-\eta)/(1-\eta) > 0$, $\hat{U}^k \equiv \left(\tilde{U}^k\right)^{\frac{c-\eta}{1-\eta}} > 0$ is an order-preserving monotone transformation of $\tilde{U}^k > 0$. Then, for $\hat{\varepsilon}(s) \equiv \eta + \frac{(1-\eta)}{(c-\eta)}(\varepsilon(s)-\eta)$, which satisfies $\int_I \hat{\varepsilon}(s) ds = 1$, $\left(\hat{U}^k\right)^{\frac{\hat{\varepsilon}(s)-\eta}{\eta}} = \left(\hat{U}^k\right)^{\frac{1-\eta}{c-\eta}\frac{\varepsilon(s)-\eta}{\eta}} = \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$. Hence, this preserves ordinal properties of the preference. Yet, this normalization has one convenient cardinal property. By imposing this normalization, the maximized value of $\tilde{U}^k$ under the budget constraint is equal to the per capita real income, as shown below.

[9] Appendix A explains different notions of additivity (explicit vs. implicit, direct vs. indirect), which are important for understanding why our thought experiment necessitates the use of this particular class of preferences.

strictly increasing in $s \in I$. Then, $\omega\left(s,\tilde{U}^k\right) \equiv (\beta_s)^{\frac{1}{\eta}}\left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$ is *log-supermodular* in s and $\tilde{U}^k$.

By applying Lemma 1 (see Appendix B) for $\hat{g}(s;\tilde{U}^k) = (\beta_s)^{\frac{1}{\eta}}\left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$, this implies that, as $\tilde{U}^k$ goes up, the agent cares more about the higher-indexed goods in the sense that the density function of the weights attached to different sectors satisfies the monotone likelihood ratio (MLR) property and hence that its cumulative distribution function satisfies the first-order stochastic dominance (FSD) property.

Each agent in k chooses C_s^k, $s \in I$, to maximize $\tilde{U}^k$, subject to (1) and the budget constraint:

(2) $$\int_I P_s^k C_s^k ds \leq E^k = w^k h^k,$$

taking P_s^k, $s \in I$, the nontradeable consumption goods prices in k, given.

The solution can be expressed in terms of the expenditure share on good s, m_s^k:

(3) $$m_s^k \equiv \frac{P_s^k C_s^k}{E^k} = \frac{\beta_s\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}}{\left(E^k\right)^{1-\eta}} = \frac{\beta_s\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}}{\int_I \beta_t\left(U^k\right)^{\varepsilon(t)-\eta}\left(P_t^k\right)^{1-\eta} dt}, \qquad \text{with } \int_I m_s^k ds \equiv 1$$

where U^k is the maximized value of $\tilde{U}^k$ and given by the indirect utility function of (1):

(4) $$\left[\int_I \beta_s\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k / E^k\right)^{1-\eta} ds\right]^{\frac{1}{\eta-1}} \equiv 1,$$

which is *implicitly indirectly additive*.[10] Recall the parameter restriction, $\left(\varepsilon(s)-\eta\right)/\left(1-\eta\right) > 0$, which ensures the global monotonicity of the utility function, (1). This restriction implies that LHS of (4) is strictly increasing in U^k, and hence U^k is strictly increasing in E^k, holding prices given. Furthermore, eq. (4) can be rewritten as:

$$U^k = \frac{E^k}{P^k}, \quad \text{where } P^k \equiv \left[\int_I \beta_s\left(U^k\right)^{\varepsilon(s)-1}\left(P_s^k\right)^{1-\eta} ds\right]^{\frac{1}{1-\eta}},$$

[10]Note that, with $\varepsilon(s) \neq 1$, (1) does *not* satisfy what is commonly called "indirect additivity," which should be called more precisely "explicit indirect additivity," because its *indirect* utility function (4) is not *explicitly* additive.

where P^k is the *exact price index* of the consumption goods for the agent in *k* because log-differentiating P^k yields

$$\partial P^k / P^k = \int_I \frac{\partial \log P^k}{\partial \log P_s^k}\left(\partial P_s^k / P_s^k\right)ds = \int_I \left(\frac{\beta_s \left(U^k\right)^{\varepsilon(s)-1}\left(P_s^k\right)^{1-\eta}}{\int_I \beta_t \left(U^k\right)^{\varepsilon(t)-1}\left(P_t^k\right)^{1-\eta} dt} \right)\left(\partial P_s^k / P_s^k\right)ds$$

and multiplying both the numerator and the denominator by $\left(U^k\right)^{1-\eta}$ leads to

$$\partial P^k / P^k = \int_I m_s^k \left(\partial P_s^k / P_s^k\right)ds$$

where m_s^k is given by (3). Hence, U^k is the real expenditure (and income) per capita.[11] In what follows, U^k shall be called interchangeably the welfare, the per capita real income, and the standard-of-living in country *k*.

Notice that the numerator of eq.(3), $\beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}$, is *log-supermodular* in *s* and U^k. Hence, by applying Lemma 1 for $\hat{g}(s, U^k) = \beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}$, eq.(3) shows that, holding the prices constant, the agent with a higher per capita real income, U^k allocates larger shares of their expenditure towards higher-indexed sectors in the sense that the density function of the expenditure share across sectors satisfies the MLR property and that its cumulative distribution function satisfies the FSD property. Note also that, from eq. (3), one could express the relative expenditure shares of any two sectors as:

$$\log(m_s^k / m_{s'}^k) = \log(\beta_s / \beta_{s'}) + \left(\varepsilon(s) - \varepsilon(s')\right)\log(U^k) + (1-\eta)\log(P_s^k / P_{s'}^k).$$

and the relative demands of any two sectors as:

[11]Needless to say, comparing the per capita real income across different countries or different periods poses an *empirical* challenge, because the expenditure share of each good, m_s^k, the weight used to calculate the aggregate price index, changes when the relative prices and the total expenditure change discretely. However, such an empirical challenge is not unique to our preferences. Even in the standard homothetic CES, a discrete change in the relative prices makes it impossible to calculate the exact price index because the expenditure share changes (unless it is Cobb-Douglas), and needs to be approximated by Laspeyres, Paassche, Divisia, or other indices. The only difference is that, in the standard practice, the empirical challenge associated with an endogenous change in the weight caused by a change in the total expenditure is *ignored* by *assuming* that the preferences are homothetic. I thank Erzo GJ Luttmer for sending me his note on this point, Luttmer (2017).

$$\log(C_s^k / C_{s'}^k) = \log(\beta_s / \beta_{s'}) + \big(\varepsilon(s) - \varepsilon(s')\big)\log(U^k) - \eta\log(P_s^k / P_{s'}^k).$$

This shows not only that the relative demand for a higher-indexed sector has higher income elasticity. It also shows that the slope of the Engel curve, $\partial\log(C_s^k / C_{s'}^k)/\partial\log(U^k) = \varepsilon(s) - \varepsilon(s')$, is independent of the per capita real income, U^k.[12] Furthermore, these income elasticity parameters are not linked to the price elasticity, unlike in other forms of nonhomothetic preferences.[13]

2.2 Production and Trade:

We keep the rest of the model deliberately standard, using the Dixit-Stiglitz-Krugman monopolistic competitive model of production and trade to isolate the role of Engel's Law.

2.2.1 Competitive Nontradeable Consumption Goods Sectors:

Each nontradeable consumption good, $s \in I$, is produced in a competitive sector, also indexed by $s \in I$, by assembling a continuum of tradable differentiated inputs, indexed by $\nu \in \Omega_s$, with the CES aggregators,

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[12] Comin, Lashkari, and Mestieri (2015) review the empirical evidence that income elasticity differences across sectors are stable over a wide range of per capita income levels. This is in strong contract to the Stone-Geary preferences, which implies that income elasticity differences across sectors decline with per capita income. This makes Stone-Geary unsuited for modelling North-South trade, as well as long run developing processes, as pointed out by Buera and Kaboski (2009). See Matsuyama (2016) for a more extensive discussion on the restrictive nature of Stone-Geary and other explicitly directly additive nonhomothetic preferences.

[13]For example, by using the Constant Ratio of Income Elasticity (CRIE) preferences, which are explicitly directly additive, both Fieler (2011, eq.(9)) and Caron, Fally, and Markusen (2014, eq.(20)) derive the relative expenditure curve, which can be rewritten as:

$$\log(m_s^k / m_{s'}^k) = const. - \big(\varepsilon(s) - \varepsilon(s')\big)\log(\lambda^k) + (1-\varepsilon(s))\log(P_s^k) - (1-\varepsilon(s'))\log(P_{s'}^k),$$

and hence its relative demand curve as:

$$\log(C_s^k / C_{s'}^k) = const. - \big(\varepsilon(s) - \varepsilon(s')\big)\log(\lambda^k) - \varepsilon(s)\log(P_s^k) + \varepsilon(s')\log(P_{s'}^k)$$

where λ^k, the Lagrange multiplier associated with the budget constraint, is inversely related to E^k at any given prices. Notice that $\varepsilon(s)$ and $\varepsilon(s')$ appear also in the coefficients of the log of prices. Under CRIE (in fact, also under Stone-Geary, or any other explicitly directly additive preferences), the ratio of income elasticity and price elasticity is constant across sectors (the so-called Pigou's Law), and hence it is infeasible to disentangle the effects of income elasticity differences and those of price elasticity differences. Furthermore, Deaton (1974) and many others who have estimated such log-linear consumption demand systems have rejected the Pigou's Law, but have not been able to reject a common price elasticity of substitution, implied by our preferences. Comin, Lashkari, and Mestieri (2015) reviews the empirical evidence in support of the log-linear Engel curves implied by our implicitly additive isoelastically nonhomothetic CES and against those implied by explicit direct additivity.

(5) $$Y_s^k = \left[\int_{\Omega_s} \left(q_s^k(\nu)\right)^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}};\ s \in I;\ \sigma > \mathrm{Max}\{1, \eta\},$$

where Y_s^k is the output; $q_s^k(\nu)$ the quantity of input variety $\nu \in \Omega_s$ used in sector $s \in I$; Ω_s the set of tradeable differentiated inputs available for $s \in I$, and $\Omega_s = \Omega_s^1 \cup \Omega_s^2$, where Ω_s^j (j = 1 or 2) are the (disjoint) sets of differentiated inputs produced in country j in equilibrium. The restriction on σ, the elasticity of substitution between inputs within each sector, implies not only $\sigma > 1$ but also $\sigma > \eta$, so that differentiated inputs are closer substitutes within each sector than across sectors.[14] Given $p_s^k(\nu)$, the unit price of input variety $\nu \in \Omega_s$ in k, each competitive consumption good sector chooses the input combination to minimize its cost, which yields the unit cost (and hence the unit price) of the consumption good $s \in I$ in k:

(6) $$P_s^k = \left[\int_{\Omega_s} \left(p_s^k(\nu)\right)^{1-\sigma} d\nu\right]^{\frac{1}{1-\sigma}},$$

which is the CES price aggregator, the dual of (5), and the quantity of input variety $\nu \in \Omega_s$ used in k:

(7) $$q_s^k(\nu) = \left(\frac{p_s^k(\nu)}{P_s^k}\right)^{-\sigma} Y_s^k.$$

2.2.2 Iceberg Costs and Demand for Differentiated Inputs:

The unit price of each input variety, $p_s^k(\nu)$, $\nu \in \Omega_s$, depends on k, because of the (iceberg) trade costs; To deliver one unit of $\nu \in \Omega_s^j$ in country k, τ_{jk} units need to be shipped from j. Thus, with the unit factory price, $p_s^j(\nu)$, $\nu \in \Omega_s^j$, $p_s^k(\nu) = \tau_{jk} p_s^j(\nu) \geq p_s^j(\nu)$. Then, from (3) and (7), and using $Y_s^k = N^k C_s^k$, the demand for $\nu \in \Omega_s^j$ by country k is:

[14]For the empirically more relevant case of gross complements, $\eta < 1$, this imposes no additional restriction. For the case of gross substitutes, it is necessary to assume $\sigma > \eta > 1$. If $\eta > \sigma > 1$, two differentiated inputs used in the same sector would become Hicks-Allen complements and the entry of two monopolistic competitive firms into the same sector would become strategic complements, leading to multiple equilibria for the same reason discussed at length in Matsuyama (1995).

$$\tau_{jk}q_s^k(\nu) = \tau_{jk}\beta_s N^k\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(p_s^k(\nu)\right)^{-\sigma} = \tau_{jk}\beta_s N^k\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(\tau_{jk}p_s^j(\nu)\right)^{-\sigma}$$

$$= \rho_{jk}\, N^k\beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(p_s^j(\nu)\right)^{-\sigma},$$

where $\rho_{jk} \equiv \left(\tau_{jk}\right)^{1-\sigma} \leq 1$. Thus, the aggregate demand for $\nu \in \Omega_s^j$ can be expressed as:

(8) $\quad D_s(\nu) = A_s^j(p_s^j(\nu))^{-\sigma}$,

where

(9) $\quad A_s^j \equiv \sum_k \rho_{jk}b_s^k$;

(10) $\quad b_s^k \equiv \beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}N^k\left(P_s^k\right)^{\sigma-\eta} = \beta_s\left(U^k\right)^{\varepsilon(s)}\left(P^k\right)^{\eta}N^k\left(P_s^k\right)^{\sigma-\eta}$

where A_s^j may be interpreted as the aggregate demand shift parameter for a variety produced in sector-*s* in country *j*; b_s^k as the aggregate demand shift parameter for sector-*s* in country *k*; and ρ_{jk} is the weight attached to the aggregate spending by country *k* of varieties produced in country *j*. Eqs. (8)-(10) show that the demand curve for each variety has a constant price elasticity with its demand shift parameter, A_s^j, which depends on the trade costs in a manner familiar in the Dixit-Stiglitz-Krugman monopolistic competition models of trade. What is new is that U^k has differential impacts on the demand shift parameters across sectors due to the nonhomotheticity of preferences.

For the remainder of this paper, we follow Krugman (1980) and many others by assuming that $\tau_{11} = \tau_{22} = 1$ and $\tau_{12} = \tau_{21} = \tau > 1$, so that

(11) $\quad \rho_{11} = \rho_{22} = 1$ and $\rho_{12} = \rho_{21} = \rho \equiv (\tau)^{1-\sigma} < 1$.

Thus, $\rho \in [0,1)$ measures how much each country spends on an imported variety relative to what it would spend in the absence of the trade cost; it is inversely related to τ, with $\rho = 0$ for $\tau = \infty$ and $\rho \to 1$ for $\tau \to 1$.

2.2.3 Production and Pricing by Monopolistically Competitive Firms:

Each differentiated input variety is produced by a monopolistically competitive firm**.** Producing one unit of each differentiated input in sector-s requires ψ_s units of labor, so that the marginal cost is equal to $w^j\psi_s$ for $\nu \in \Omega_s^j$. Since Eq. (8) shows that the price elasticity of demand for each input is constant, σ, all the input varieties are priced with the same mark-up rate:

(12) $$p_s^j(\nu) = \frac{w^j \psi_s}{1-1/\sigma} \equiv p_s^j \text{ for all } \nu \in \Omega_s^j,$$

and hence they are produced by:

(13) $$y_s^j \equiv A_s^j (p_s^j)^{-\sigma}.$$

By inserting (12) into (6),

(14) $$\left(P_s^k\right)^{1-\sigma} = \int_{\Omega_s} \left(p_s^k(\nu)\right)^{1-\sigma} d\nu = \sum_j \int_{\Omega_s^j} \left(\tau_{jk} p_s^k(\nu)\right)^{1-\sigma} d\nu = \sum_j V_s^j \rho_{jk} (p_s^j)^{1-\sigma}$$

where V_s^j is the Lebesgue measure of Ω_s^j, the equilibrium measure of varieties produced (and of active firms) in sector-*s* of country *j*.

2.2.4. Free Entry Conditions and Sectoral Shares in Employment:

This equilibrium measure, V_s^j, is determined by the free entry condition. To enter sector-*s,* all monopolistically competitive firms need to pay the setup cost per variety, ϕ_s, in labor, and they have incentive to do so, as long as the profit is non-negative. Thus, in equilibrium, either a positive measure of firms (and varieties) enter, in which case they all break even ($V_s^j > 0 \Rightarrow p_s^j y_s^j = w^j\left(\psi_s y_s^j + \phi_s\right)$), or no firms (and varieties) enter, because they would earn negative profit if they were to enter ($p_s^j y_s^j < w^j\left(\psi_s y_s^j + \phi_s\right) \Rightarrow V_s^j = 0$). Using eqs. (12) and (13), this free entry condition can be written as the complementarity slackness condition:

$$V_s^j \geq 0; \qquad y_s^j = A_s^j (p_s^j)^{-\sigma} \leq (\sigma - 1)\phi_s / \psi_s.$$

This implies that each active firm in sector-s based in country *j* hires $\psi_s y_s^j + \phi_s = \sigma\phi_s$ units of labor, so that labor demand by sector-s in country *j* is $L_s^j = \sigma\phi_s V_s^j$ and its share in employment is $f_s^j \equiv L_s^j / L^j = \sigma\phi_s V_s^j / L^j$. Thus, the above complementarity slackness condition can be further rewritten as:

(15) $$f_s^j \equiv L_s^j / L^j = \sigma\phi_s V_s^j / L^j \geq 0; \quad y_s^j = A_s^j (p_s^j)^{-\sigma} \leq (\sigma - 1)\phi_s / \psi_s,$$

with

(16) $\int_I f_s^j ds = 1$,

which is nothing but the labor market clearing condition, $\int_I L_s^j ds = L^j = h^j N^j$.

In what follows, we use the following normalizations to simply the notation. First, choose the unit of each differentiated input in sector-*s* such that $\psi_s = 1 - 1/\sigma$. This simplifies (12) to

(12') $p_s^j(\nu) = p_s^j = w^j$ for all $\nu \in \Omega_s^j$ and all $s \in I$.

Second, choose the unit of variety in sector-s such that $\phi_s = 1/\sigma$. These two normalizations together simplifies (15) to:

(15') $f_s^j = V_s^j / L^j \geq 0$; $y_s^j = A_s^j (w^j)^{-\sigma} \leq 1$ for all $s \in I$ and j = 1 and 2.

In other words, *without loss of generality*, we choose the units of measurement such that each active firm produces by $y_s^j = 1$, hires labor by $\psi_s y_s^j + \phi_s = 1$ and sells its output at $p_s^j = w^j$, to break even in equilibrium, and the labor demand by sector-s of country *j* is $L_s^j = V_s^j$.

2.3 Equilibrium Conditions:

We are now ready to consolidate all the equilibrium conditions. First, using (11), (12') and (15'), eq. (14) becomes

(14') $(P_s^1)^{1-\sigma} = f_s^1 L^1 (w^1)^{1-\sigma} + \rho f_s^2 L^2 (w^2)^{1-\sigma}$; $(P_s^2)^{1-\sigma} = \rho f_s^1 L^1 (w^1)^{1-\sigma} + f_s^2 L^2 (w^2)^{1-\sigma}$, $s \in I$,

By introducing $\omega \equiv w^1 / w^2$, the relative wage or the factoral terms of trade (and also the relative prices of input varieties produced in the two countries in the same sector), (14') can be further simplified to:

(17) $(w^1 / P_s^1)^{\sigma-1} = f_s^1 L^1 + \rho f_s^2 L^2 (\omega)^{\sigma-1}$; $(w^2 / P_s^2)^{\sigma-1} = \rho f_s^1 L^1 (\omega)^{1-\sigma} + f_s^2 L^2$, $s \in I$,

where w^j / P_s^j is the TFP of sector-s in country *j*.

Second, from (9), (11), (15'), the complementary slackness condition for free entry in sector-s in each country can be written as

(18) $f_s^1 \geq 0$; $(b_s^1 + \rho b_s^2)(w^1)^{-\sigma} \leq 1$ & $f_s^2 \geq 0$; $(\rho b_s^1 + b_s^2)(w^2)^{-\sigma} \leq 1$.

This can be further rewritten as

(19) $f_s^1 \geq 0$; $d_s^1 + \rho(\omega)^{-\sigma} d_s^2 \leq 1$; & $f_s^2 \geq 0$; $\rho(\omega)^{\sigma} d_s^1 + d_s^2 \leq 1$, $s \in I$.

by introducing $d_s^j \equiv (b_s^j)(w^j)^{-\sigma}$, which is the domestic market's share in the revenue of a differentiated input producer based in *j*.[15] This variable, $d_s^j \equiv (b_s^j)(w^j)^{-\sigma}$, can be expressed in two different ways from (3) and (10). First, by eliminating P_s^k from (3) and (10),

$$(20)\qquad d_s^k \equiv b_s^k\left(w^k\right)^{-\sigma} = \left(\left(h^k\right)^{\sigma} N^k\right)\left[\beta_s\left(U^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{1-\sigma}{1-\eta}\right)}\left(m_s^k\right)^{\left(\frac{\sigma-\eta}{1-\eta}\right)}, \qquad k = 1 \text{ and } 2.$$

Alternatively, by eliminating U^k from (3) and (10) and using $E^k N^k = w^k h^k N^k = w^k L^k$,

$$d_s^k \equiv b_s^k\left(w^k\right)^{-\sigma} = m_s^k L^k (w^k / P_s^k)^{1-\sigma}.$$

Using (17), this can be further rewritten to:

$$(21)\qquad d_s^1 \equiv b_s^1\left(w^1\right)^{-\sigma} = \frac{m_s^1 L^1}{f_s^1 L^1 + \rho f_s^2 L^2 (\omega)^{\sigma-1}}; \qquad d_s^2 \equiv b_s^2\left(w^2\right)^{-\sigma} = \frac{m_s^2 L}{\rho f_s^1 L^1 (\omega)^{1-\sigma} + f_s^2 L^2}$$

Finally, the expenditure share or the market size distribution, as well as the employment share across sectors must add up to one in each country.

$$(22)\qquad \int_I m_s^k ds = 1 \text{ for } k = 1 \text{ and } 2.$$

$$(23)\qquad \int_I f_s^j ds = 1 \text{ for } j = 1 \text{ and } 2.$$

Note that eqs. (19), (20), and (21) impose the conditions on six functions of $s \in I$, and that eqs. (22) and (23) impose four additional conditions, but one of them is redundant due to the Walras' Law. These conditions, eqs. (19)-(23), altogether determine six endogenous functions of $s \in I$; d_s^k (the domestic market share in the revenue of an input producer in each sector in each country), m_s^k (the market size distribution in each country), and f_s^k (the employment share in each country), as well as three endogenous variables, U^k (the welfare, the per capita real income, or the standard-of-living in each country) and ω (the terms-of-trade).

Before proceeding to solve for the equilibrium, it is worth pointing out one notable (or perhaps unusual) feature of this set of the equilibrium conditions; it contains U^k, $k = 1$ and 2.

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[15] On the other hand, the export market's share in the revenue of an input producer in *j* is $\rho(w^j / w^k)^{-\sigma} d_s^k$, $(k \neq j)$, which is equal to the domestic market's share in the revenue of an input producer in k, d_s^k, multiplied by $(w^j / w^k)^{-\sigma}$, due to the relative price between these two producers, and multiplied by ρ due to the trade cost.

Normally, when we analyze a general equilibrium model, we first solve for the equilibrium allocations (and prices) by conducting a positive analysis. Then, we plug those equilibrium allocations into the utility functions to obtain the welfare levels by conducting a normative analysis. Here, due to the implicit nature of the utility function, the consumer demand depends on the welfare level, which in turn affect the equilibrium allocations, which in turn affect the welfare level. Therefore, it is more efficient to solve for the equilibrium allocations and prices and for the welfare levels together, without the separation of the positive and normative analyses. Indeed, when solving for the equilibrium below, U^k, k = 1 and 2 are among the first endogenous variables that will be pinned down.

3 Patterns of Structural Change in a Closed Economy Equilibrium

First, let us consider the case of $\rho = 0$, where each country must produce all differentiated inputs used in every sector. Thus, for all $s \in I$ and for k = 1 and 2, $f_s^k > 0$, and hence, from (18), $d_s^k = 1$. Inserting this to (19) and (20) yields

$$(24)\qquad f_s^k = m_s^k = \left((h^k)^\sigma N^k\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left[\beta_s \left(U_0^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Subscript "0" is added here to indicate that U_0^k is the equilibrium per capita real income achieved when $\rho = 0$. Note also that eq. (24) shows that the employment (and value-added) is distributed proportionately with the market size across sectors in a closed economy.

By integrating (24) across all the sectors and using (22) or (23), we can pin down U_0^k as

$$\int_I \left((h^k)^\sigma N^k\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left[\beta_s \left(U_0^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} ds = 1,$$

which can be written more compactly as

$$(25)\qquad U_0^k = u\left(x_0^k\right) \qquad \text{with } x_0^k \equiv \left(h^k\right)^\sigma N^k = \left(h^k\right)^{\sigma-1} L^k,$$

where $u(\bullet)$ is defined implicitly by

$$(26)\qquad (x)^{\left(\frac{1-\eta}{\sigma-\eta}\right)} \equiv \int_I \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} ds.$$

Lemma 2-i) in Appendix B shows that $u(\bullet)$, defined in eq.(26), is a strictly increasing function. Thus, the welfare, the per capita real income, or the standard-of-living, in the closed economy increases with $x_0^k \equiv (h^k)^\sigma N^k = (h^k)^{\sigma-1} L^k$. (Again, subscript "0" is used to indicate $\rho = 0$.) Eq.(25) shows that $U_0^k = u(x_0^k)$ increases not only in labor productivity, h^k, but also in N^k. This is due to the aggregate increasing returns to scale in the presence of "love for variety" and the fixed cost, a familiar feature of the Dixit-Stiglitz monopolistic competition model. This can be also seen, from eq.(17) and $\rho = 0$, $w^k / P_s^k = (f_s^k L^k)^{1/(\sigma-1)} = (L_s^k)^{1/(\sigma-1)}$, so that the sectoral TFP is increasing in the total employment in that sector. Notice also that the condition for $U_0^1 = u(x_0^1) <$ $U_0^2 = u(x_0^2)$ can be expressed as $(h^1)^{\sigma-1} L^1 < (h^2)^{\sigma-1} L^2$, which occur even if $h^1 > h^2$ when $L^1 / L^2 < (h^1/h^2)^{1-\sigma} < 1$. In other words, the country with higher labor productivity may have a lower per capita real income when it is sufficiently smaller. This is because those living in a small country has disadvantage in the presence of aggregate increasing returns.[16]

Next, plugging (25) and (26) into (24) yields the equilibrium density functions of employment and market sizes across sectors as follows:

$$(27) \qquad f_s^k = m_s^k = \frac{\left[\beta_s \left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\left(x_0^k\right)^{\left(\frac{1-\eta}{\sigma-\eta}\right)}} = \frac{\left[\beta_s \left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\int_I \left[\beta_t \left(u(x_0^k)\right)^{(\varepsilon(t)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} dt}.$$

The numerator of (27) is *log-supermodular* in *s* and x_0^k. Thus, by applying Lemma 1 for $\hat{g}(s, x_0^k) = \left[\beta_s \left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$, eq.(27) shows that, for $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$, country 2, whose per capita real income is higher than country 1, spend relatively more on higher-indexed goods in the sense that m_s^1 / m_s^2 is strictly decreasing in *s* (that is, the density functions of equilibrium market size distribution across sectors satisfies the MLR property) as well as in the

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[16] This result does not contradict what we noted earlier, i.e., eq.(4) shows that the agent's utility is increasing in E^k and hence in h^k, *holding the prices given*. When comparing the two countries in equilibrium, the prices differ across the two countries because the measure of varieties used in each sector in each country is endogenously determined by the free entry condition.

sense that the cumulative distribution function for country 2 first-order stochastically dominates (FSD) the cumulative distribution function for country 1.

Notice the difference between the two expressions of m_s^k, eq.(3) and eq.(27), in particular how it depends on the welfare or per capita real income. Eq.(3) implies that, *holding the prices given,* the relative market size of two sectors, $s > s'$, responds to an increase in U^k as

$$\frac{\partial \log(m_s^k / m_{s'}^k)}{\partial \log\left(U^k\right)} = \varepsilon(s) - \varepsilon(s') .$$

In contrast, eq.(27) shows that, *in equilibrium,* the relative market size of two sectors responds as

$$\frac{d \log(m_s^k / m_{s'}^k)}{d \log\left(u(x_0^k)\right)} = \left(\varepsilon(s) - \varepsilon(s')\right)\left(\frac{\sigma - 1}{\sigma - \eta}\right).$$

This is due to the Schmookler effect. A change in the relative market size causes some entries into higher-indexed sectors and some exits from lower-indexed sectors, which leads to a higher (lower) productivity in higher-(lower)-indexed sectors, which reduces the relative prices of higher-indexed goods. Formally, by setting ρ = 0 in eq.(17), $(w^k / P_s^k)^{\sigma-1} = f_s^k L^k$, so that

$$P_{s'}^k / P_s^k = \left(f_{s'}^k / f_s^k\right)^{1/(1-\sigma)} = \left(m_{s'}^k / m_s^k\right)^{1/(1-\sigma)} .$$

This change in the relative price moderates (amplifies) the shift in expenditure shares if different sectors produce gross complements (gross substitutes). Indeed, from eq.(3) and using the above expression, the total effect can be calculated as

$$\frac{d \log(m_s^k / m_{s'}^k)}{d \log\left(u(x_0^k)\right)} = \frac{\partial \log(m_s^k / m_{s'}^k)}{\partial \log\left(u(x_0^k)\right)} + (1-\eta)\frac{d \log(P_s^k / P_{s'}^k)}{d \log\left(m_s^k / m_{s'}^k\right)} \frac{d \log(m_s^k / m_{s'}^k)}{d \log\left(u(x_0^k)\right)}$$

$$= \frac{\partial \log(m_s^k / m_{s'}^k)}{\partial \log\left(u(x_0^k)\right)} + \frac{1-\eta}{1-\sigma} \frac{d \log(m_s^k / m_{s'}^k)}{d \log\left(u(x_0^k)\right)},$$

from which

$$\frac{d \log(m_s^k / m_{s'}^k)}{d \log\left(u(x_0^k)\right)} = \frac{\partial \log(m_s^k / m_{s'}^k)}{\partial \log\left(u(x_0^k)\right)} \left(\frac{\sigma - 1}{\sigma - \eta}\right),$$

which is exactly what we obtained from eq.(27). Here, the moderation effect of Schmookler under gross complements is captured by $(\sigma - 1)/(\sigma - \eta) < 1$ for $\eta < 1$, and the amplification effect of Schmookler under gross substitutes is captured by $(\sigma - 1)/(\sigma - \eta) > 1$ for $\eta > 1$.

In the literature of structural transformation, it is common to treat the income elasticity difference across sectors and the productivity growth difference across sectors as two separate exogenous causes of structural change. The above result suggests that, in the presence of the Schmookler effect, such a dichotomy can be misleading, as some productivity growth differences may be induced by the income elasticity differences.

The above amplification or moderation effect also affects the welfare impact of a change in x_0^k. From Lemma 2-ii) shown in Appendix B, $d\log u(\lambda x)/d\log\lambda = \lambda x u'(\lambda x)/u(\lambda x)$ $\equiv \zeta(\lambda x)$ is increasing (decreasing) in x if η >(<) 1. In words, welfare gains from a percentage increase in x_0^k is higher (lower) at a higher x if η >(<) 1. This implies, among other things, that a uniform labor productivity growth, $\partial h^1/h^1 = \partial h^2/h^2 > 0$, reduces (magnifies) the welfare (per capita real income) gap between the two countries, $U_0^2/U_0^1 = u(x_0^2)/u(x_0^1) > 1$, if different sectors produce gross complements (gross substitutes).

Before proceeding, it is worth pointing out that, when $\rho = 0$, the equilibrium conditions, eqs. (19), (20), (21), (22) and (23), do not depend on ω. In other words, ω is indeterminate; there is nothing to pin down the relative wage of the two countries that are isolated from each other. This is no longer the case, when $\rho > 0$. Indeed, as the first step to solve for a trade equilibrium, we need to determine the relative wage or the terms-of-trade between the two countries.

4 Trade Equilibrium: Cross-Country Variations

This section focuses on how the two countries differ in the trade equilibrium for a given set of the parameter values. The next section will deal with comparative statics.

4.1 (Factoral) Terms of Trade

In what follows, let us focus on the case where $f_s^1 > 0$ and $f_s^2 > 0$ for all $s \in I$. This simplifies eq. (19) to $d_s^1 + \rho(\omega)^{-\sigma} d_s^2 = 1$ and $\rho(\omega)^{\sigma} d_s^1 + d_s^2 = 1$, from which

(28) $$d_s^1 = \frac{1-\rho(\omega)^{-\sigma}}{1-\rho^2} \quad \text{and} \quad d_s^2 = \frac{1-\rho(\omega)^{\sigma}}{1-\rho^2} \quad \text{for all } s \in I.$$

Inserting (28) into (21) yields

(29) $f_s^1 L^1 + \rho f_s^2 L^2 (\omega)^{\sigma-1} = \dfrac{(1-\rho^2) L^1 m_s^1}{1-\rho(\omega)^{-\sigma}}$ and $\rho f_s^1 L^1 (\omega)^{1-\sigma} + f_s^2 L^2 = \dfrac{(1-\rho^2) L^2 m_s^2}{1-\rho(\omega)^{\sigma}}$ for $s \in I$.

Integrating these expressions across all sectors and using (22) and (23),

$$L^1 + \rho L^2 (\omega)^{\sigma-1} = \frac{(1-\rho^2) L^1}{1-\rho(\omega)^{-\sigma}} \text{ and } \rho L^1 (\omega)^{1-\sigma} + L^2 = \frac{(1-\rho^2) L^2}{1-\rho(\omega)^{\sigma}}.$$

These two expressions are equivalent. Indeed, either of them can be rewritten as:

(30) $\dfrac{L^1}{L^2} = \Lambda(\omega;\rho) \equiv (\omega)^{2\sigma-1} \dfrac{1-\rho(\omega)^{-\sigma}}{1-\rho(\omega)^{\sigma}}$,

where $\Lambda(\bullet;\rho)$ is strictly increasing in $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$ and satisfies $\lim_{\omega \to \rho^{1/\sigma}} \Lambda(\omega;\rho) = 0$, $\Lambda(1;\rho) = 1$, and $\lim_{\omega \to \rho^{-1/\sigma}} \Lambda(\omega;\rho) = \infty$. Figure 1 illustrates eq.(30), which determines the (factor) terms of trade $\omega \equiv w^1/w^2$ as a function of the relative labor supply, L^1/L^2, for a given $\rho \in (0,1)$. It shows that $\omega \equiv w^1/w^2$ is strictly increasing in L^1/L^2 and $\omega \equiv w^1/w^2 < 1$ if and only if $L^1/L^2 < 1$. Thus, the factor price is higher in the larger economy, which reflects the aggregate increasing returns to scale pointed out earlier.[17] It also shows the lower and upper bounds for the terms of trade, $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$. The arrows indicate the effects of an increase in ρ. As shown, it flattens the graph, thereby causing a factor price convergence. This is because globalization, captured by a reduction in τ and hence an increase in ρ, reduces the smaller country's disadvantage.

It is also worth pointing that, because ω is strictly increasing in L^1/L^2 with the range $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$ and $\omega = 1$ for $L^1/L^2 = 1$ for any $\rho \in (0,1)$, eq.(28) implies:

i) d_s^1 (d_s^2) is strictly increasing (decreasing) in L^1/L^2: that is, the domestic market accounts more for the revenue of the input producers based in the larger country;

[17] Note that eq.(30) implies $w^1 L^1 / w^2 L^2 = \omega\Lambda(\omega;\rho) = ((\omega)^{\sigma} - \rho)/((\omega)^{-\sigma} - \rho)$, which is strictly increasing in ω (hence also in L^1/L^2) and $w^1 L^1 / w^2 L^2 < 1$ if and only if $\omega < 1$ (hence also if and only if $L^1/L^2 < 1$). Thus, the larger economy is larger regardless of whether it is measured in the total labor supply or in the aggregate GDP.

ii) $d_s^1 \to 0$ and $d_s^2 \to 1$ as $L^1/L^2 \to 0$; $d_s^1 \to 1$ and $d_s^2 \to 0$ as $L^1/L^2 \to 1$: that is, the domestic (export) market account for most of the revenue of those operating in a very large (small) country;

and

iii) $d_s^1 = d_s^2 = 1/(1+\rho) > ½$: that is, when the two countries are equal in size, the domestic market accounts more than a half of their revenue in the presence of the trade cost.

4.2 Per Capita Real Income and Market Size Distributions

Next, combining (28) and (20) yields

$$(31)\qquad m_s^1 = \left(\frac{(1-\rho^2)(h^1)^\sigma N^1}{1-\rho(\omega)^{-\sigma}}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\beta_s \left(U_\rho^1\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)},$$

$$m_s^2 = \left(\frac{(1-\rho^2)(h^2)^\sigma N^2}{1-\rho(\omega)^{\sigma}}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\beta_s \left(U_\rho^2\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Here, the subscript "ρ" is added to indicate that U_ρ^k, the equilibrium per capita real income achieved in each country under trade, depends on ρ. By integrating (31) across all the sectors and using (22), we obtain

$$(32)\qquad U_\rho^1 = u\left(x_\rho^1\right), \qquad \text{with } x_\rho^1 \equiv \frac{(1-\rho^2)\left(h^1\right)^\sigma N^1}{1-\rho(\omega)^{-\sigma}} \equiv \frac{(1-\rho^2)x_0^1}{1-\rho(\omega)^{-\sigma}};$$

$$U_\rho^2 = u\left(x_\rho^2\right), \qquad \text{with } x_\rho^2 \equiv \frac{(1-\rho^2)\left(h^2\right)^\sigma N^2}{1-\rho(\omega)^{\sigma}} \equiv \frac{(1-\rho^2)x_0^2}{1-\rho(\omega)^{\sigma}},$$

where $u(\bullet)$ is the same increasing function defined implicitly by (26). Note that the welfare effects of globalization on each country are summarized by a single index, x_ρ^k. Note also that the lower and upper bound on the terms of trade established earlier, $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$, which can be seen in Figure 1, ensures gains from trade for both countries; $\omega < \rho^{-1/\sigma}$ implies $x_\rho^1 > x_0^1$, hence $U_\rho^1 = u\left(x_\rho^1\right) > U_0^1 = u\left(x_0^1\right)$ and $\omega > \rho^{1/\sigma}$ implies $x_\rho^2 > x_0^2$, hence $U_\rho^2 = u\left(x_\rho^2\right) > U_0^2 = u\left(x_0^2\right)$.

Plugging (32) back into (31) and using the definition of $u(\bullet)$, given by (26), yields the equilibrium density function of the market size distribution across sectors in each country as follows.

$$(33)\qquad m_s^k = \frac{\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\left(x_\rho^k\right)^{\left(\frac{1-\eta}{\sigma-\eta}\right)}} = \frac{\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\int_I \left(\beta_t \left(u(x_\rho^k)\right)^{(\varepsilon(t)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} dt} \quad \text{for } k = 1 \text{ and } 2.$$

Note that $\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$ is *log-supermodular* in s and x_ρ^k. Hence, by applying Lemma 1 for $\hat{g}(s, x_\rho^k) = \left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$, it follows from eq. (33) that, for $U_\rho^1 = u\left(x_\rho^1\right) < U_\rho^2 = u\left(x_\rho^2\right)$, country 2, whose per capita real income is higher than those in country 1, spend relatively more on higher-indexed in the sense that m_s^1 / m_s^2 is strictly decreasing in s (that is, the density functions of the equilibrium market size distribution across sectors satisfies the MLR property) as well as in the sense that the cumulative distribution function for country 2 first-order stochastically dominates (FSD) the cumulative distribution function for country 1. In short, the domestic demand composition is more skewed towards the higher-indexed in the country with higher per capita real income. The MLR property can also be seen by taking the ratio from (33) to obtain

$$(34)\qquad \frac{m_s^1}{m_s^2} = \left(\frac{x_\rho^1}{x_\rho^2}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\frac{u(x_\rho^1)}{u(x_\rho^2)}\right)^{(\varepsilon(s)-\eta)\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Clearly, this is strictly decreasing in s if $U_\rho^1 = u\left(x_\rho^1\right) < U_\rho^2 = u\left(x_\rho^2\right)$ and strictly increasing in s if $U_\rho^1 = u\left(x_\rho^1\right) > U_\rho^2 = u\left(x_\rho^2\right)$.

4.3 Home Market Effect in Employment and in Patterns of Trade

Unlike in the closed economy case, the employment distribution in each country is no longer proportional to the market size distribution in that country. By solving (29) for f_s^1 and f_s^2 and using (30), we obtain the equilibrium density function of employment across sectors in each country as follows:

(35) $$f_s^1 = \frac{m_s^1 - \rho(\omega)^{-\sigma} m_s^2}{1-\rho(\omega)^{-\sigma}} > 0; \qquad f_s^2 = \frac{m_s^2 - \rho(\omega)^{\sigma} m_s^1}{1-\rho(\omega)^{\sigma}} > 0,$$

which requires $\rho(\omega)^{-\sigma} < m_s^1/m_s^2 < \rho^{-1}(\omega)^{-\sigma}$. Furthermore, the ratio of the two,

(36) $$\frac{f_s^1}{f_s^2} = \left(\frac{1-\rho(\omega)^{\sigma}}{1-\rho(\omega)^{-\sigma}}\right)\left(\frac{m_s^1/m_s^2 - \rho(\omega)^{-\sigma}}{1-\rho(\omega)^{\sigma} m_s^1/m_s^2}\right)$$

is increasing in m_s^1/m_s^2 and satisfies $f_s^1/f_s^2 > m_s^1/m_s^2 > 1$, $f_s^1/f_s^2 = m_s^1/m_s^2 = 1$, or $f_s^1/f_s^2 < m_s^1/m_s^2 < 1$.

Figure 2 illustrates eq.(34) and eq.(36) for the case of $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$. In this case, m_s^1/m_s^2 is strictly decreasing in *s*. This causes f_s^1/f_s^2 to be also strictly decreasing in *s*. Furthermore, there is a unique cutoff sector, $s_c \in I$, such that $f_s^1/f_s^2 > m_s^1/m_s^2 > 1$ holds below the cutoff and $f_s^1/f_s^2 < m_s^1/m_s^2 < 1$ above the cutoff. And the graph of f_s^1/f_s^2 is steeper than the graph of m_s^1/m_s^2. Thus, *disproportionately larger* fractions of labor are employed in lower (higher) income elastic sectors in the country with lower (higher) per capita real income, precisely because its domestic demand composition is more skewed towards markets in lower (higher) income elastic sectors. This is in strong contrast to the closed economy case, where labor is allocated across sectors proportionately to the market size distribution across sectors, so that $f_s^1/f_s^2 = m_s^1/m_s^2$. In other words, international trade *magnifies* the power of the domestic demand composition in dictating the allocation of resources across sectors.

This result might come as a surprise to those who address the questions of structural change within a closed economy setting. In a closed economy, the domestic supply is necessarily equal to the domestic demand in each sector, and hence a change in the domestic demand composition across sectors would cause a *proportional* change in the composition of production and hence the sectoral allocation of resources. Many people seem to believe that, in an open economy, the domestic demand composition becomes *less* important, because the domestic supply *need not be equal to* the domestic demand in each sector. This logic is false. That the domestic supply is no longer equal to the domestic supply in each sector means that the impact of the domestic demand composition is no longer proportional; instead, it could be *more than proportional*. Indeed, as long as the trade cost is not zero, the difference in the domestic demand composition across countries give different incentives for entry of firms (or more generally innovation) across sectors in different countries in the presence of the Schmookler effect. Through such differential Schmookler effects across countries, the richer (poorer) country develops comparative advantage in higher (lower) income-elastic sectors, which is the Linder effect. And a lower trade cost causes the richer (poorer) country to allocate even more resources towards higher (lower) income-elastic sectors by importing even more from the poorer (richer) country in lower (higher) income-elastic sectors. Hence, globalization magnifies, instead of weakening, the power of the domestic demand composition differences in dictating the patterns of structural change.

4.4 The Linder Effect, or the Home Market Effect in Inter-Sectoral Patterns of Intra-Sectoral Trade

As the above paragraph suggests, the disproportional effect of the market size distribution on the employment distribution under trade manifests itself in the inter-sectoral patterns of intra-sectoral trade. Indeed, they are the two sides of the same coin. As indicated in Figure 2, the country with higher (lower) per capita real income becomes a net exporter (importer) above the cutoff and a net importer (exporter) above the cutoff. To see this, recall that country k spends $b_s^k(p_s^k)^{1-\sigma} = \rho b_s^k(p_s^j)^{1-\sigma} = \rho b_s^k(w^j)^{1-\sigma}$ *per variety* produced in sector-s of country $j \neq k$. With the measure of varieties produced in this sector, V_s^j, the total gross export value from j to k in sector-s is $V_s^j \rho b_s^k(w^j)^{1-\sigma} = \rho f_s^j b_s^k(w^j)^{1-\sigma} L^j$. Thus, the net export value from 1 to 2 in sector-s is given by $NX_s^1 = -NX_s^2 = \rho\left(f_s^1 b_s^2(w^1)^{1-\sigma} L^1 - f_s^2 b_s^1(w^2)^{1-\sigma} L^2\right)$. Using (28), (30) and (35), this can be further rewritten as:

$$(37)\qquad NX_s^1 = -NX_s^2 = \frac{\rho w^2 L^2}{(\omega)^{-\sigma} - \rho}\left(m_s^1 - m_s^2\right) = \frac{\rho w^1 L^1}{(\omega)^{\sigma} - \rho}\left(m_s^1 - m_s^2\right).$$

Thus, $NX_s^1 = -NX_s^2 > 0$ for $s < s_c$ and $NX_s^1 = -NX_s^2 < 0$ for $s > s_c$ when $U_\rho^1 = u\left(x_\rho^1\right) < U_\rho^2 = u\left(x_\rho^2\right)$. This is nothing but the Linder effect. It may also be viewed as a variant of the Home Market Effect of Krugman (1980). The key difference is that the cross-country difference in the market size distribution across sectors is due to nonhomothetic preferences in this model, not due to the exogenous cross-country variations in taste as assumed in Krugman (1980).

It is also worth emphasizing that country 1 becomes a net exporter in sectors where $m_s^1 > m_s^2$ holds, which are not necessarily sectors where $m_s^1 w^1 L^1 > m_s^2 w^2 L^2$ holds. What determines the direction of net sectoral trade flows in a general equilibrium model of the home market effect is *not* the cross-country difference in the market size in each sector. What matters is the *cross-country difference in the demand compositions,* i.e., in the *cross-country difference in the market size distributions across sectors.*[18]

4.5 Ranking the Countries: Trade-off between Labor Productivity and Country Size

Our remaining task is to rank the two countries in terms of the per capita real income. This is simple when the two countries are in equal size, $L^1 = L^2 = L$. In this case, $\omega = 1$ so that $x_\rho^k = (1+\rho)x_0^k = (1+\rho)\left(h^k\right)^\sigma N^k = (1+\rho)\left(h^k\right)^{\sigma-1} L$, and hence, $x_\rho^1 / x_\rho^2 = \left(h^1/h^2\right)^{\sigma-1} = \left(w^1 h^1 / w^2 h^2\right)^{\sigma-1}$. Thus, the country with higher labor productivity has higher per capita income and higher per capita real income.

Generally, the condition under which Country 1 has lower per capita real income than Country 2, $U_\rho^1 = u\left(x_\rho^1\right) < U_\rho^2 = u\left(x_\rho^2\right)$ or $x_\rho^1 < x_\rho^2$ can be written as:

$$\frac{1-\rho(\omega)^{-\sigma}}{1-\rho(\omega)^{\sigma}} > \frac{x_0^1}{x_0^2} = \left(\frac{h^1}{h^2}\right)^{\sigma} \frac{N^1}{N^2} = \left(\frac{h^1}{h^2}\right)^{\sigma-1} \frac{L^1}{L^2} \Leftrightarrow (\omega)^{2\sigma-1}\left(\frac{h^1}{h^2}\right)^{\sigma-1} < 1,$$

which can be further rewritten as:

[18] The Home Market Effect is often described simply as "relatively large domestic demand gives competitive advantages to exporting firms." To this, we have heard some IO people say something to the effect that the share of the domestic sale must be trivial for many firms based in small economies like Denmark or Switzerland. The result here should explain why such a criticism is unwarranted. Even if the Swiss domestic market might be smaller than the Chinese domestic market in every sector in absolute terms, some sectors should account for larger shares in the Swiss expenditure than in the Chinese expenditure, as long as the two countries differ in the demand composition. And that is what determines the patterns of comparative advantage in a general equilibrium model of the Home Market Effect. Matsuyama (2015, section 3) demonstrated this by extending the Krugman model with a continuum of sectors and with two countries of unequal size. In the context of the present model, even if Switzerland may be much smaller than China, and consequently the domestic market accounts for a tiny share of the revenue for the Swiss firms operating in any sectors (recall the result shown earlier that $d_s^1 \to 0$ as $L^1/L^2 \to 0$), Switzerland should become a net-exporter in high-income elastic sectors and China a net-importer in low-income elastic sectors, as long as Switzerland has higher per capita real income than China.

(38) $$\frac{L^1}{L^2} = \Lambda(\omega;\rho) < \Lambda\left(\left(\frac{h^1}{h^2}\right)^{\frac{1-\sigma}{2\sigma-1}};\rho\right) \equiv \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right).$$

To understand this condition, it would be useful to compare it with the condition under which Country 1 is poorer under autarky, $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$, which can be written as:

$$\frac{L^1}{L^2} < \left(\frac{h^1}{h^2}\right)^{1-\sigma},$$

and the condition under which Country 1 has lower per capita nominal income, $E^1 = w^1h^1 < w^2h^2 = E^2$,

$$\frac{L^1}{L^2} = \Lambda(\omega;\rho) < \Lambda\left(\left(\frac{h^1}{h^2}\right)^{-1};\rho\right) \equiv \overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right).$$

Figure 3 illustrates these conditions. The black curve depicts the graph of $L^1/L^2 = \tilde{\Lambda}(h^1/h^2;\rho)$ on which $U_\rho^1 = u(x_\rho^1) = U_\rho^2 = u(x_\rho^2)$ holds. It is downward-sloping, and $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$ holds below and to the left of this curve, and $U_\rho^1 = u(x_\rho^1) > U_\rho^2 = u(x_\rho^2)$ holds above and to the right of this curve. The red curve depicts the graph of $L^1/L^2 = (h^1/h^2)^{1-\sigma}$, on which $U_0^1 = u(x_0^1) = U_0^2 = u(x_0^2)$ holds. It is also downward-sloping and $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$ holds below and to the left of this curve, and $U_0^1 = u(x_0^1) > U_0^2 = u(x_0^2)$ holds above and to the right of this curve. The blue curve depicts the graph of $L^1/L^2 = \overline{\Lambda}(h^1/h^2;\rho)$, on which $w^1h^1 = w^2h^2$ holds. It is also downward-sloping and $w^1h^1 < w^2h^2$ holds below and to the left of this curve, and $w^1h^1 > w^2h^2$ holds above and to the right of this curve. It is also easy to verify that $\overline{\Lambda}(1;\rho) = \tilde{\Lambda}(1;\rho) = 1$ and

$$\overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right) < \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right) < \left(\frac{h^1}{h^2}\right)^{1-\sigma} < 1 \qquad \text{for } \frac{h^1}{h^2} > 1;$$

$$\overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right) > \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right) > \left(\frac{h^1}{h^2}\right)^{1-\sigma} > 1 \qquad \text{for } \frac{h^1}{h^2} < 1,$$

as shown in Figure 3.

For L^1/L^2 = 1, all three curves intersect at h^1/h^2 = 1. Hence, h^1/h^2 < 1 implies $U_0^1 < U_0^2$, $U_\rho^1 < U_\rho^2$ and $w^1h^1 < w^2h^2$, while h^1/h^2 > 1 implies $U_0^1 > U_0^2$, $U_\rho^1 > U_\rho^2$ and $w^1h^1 > w^2h^2$. Thus, when the two countries are equal in size, comparing labor productivity alone can determine which country has higher per capita real income, as already pointed out. When the two countries are unequal in size, these three conditions diverge. To see this, consider the case of h^1/h^2 > 1. For $L^1/L^2 > \left(h^1/h^2\right)^{1-\sigma}$, $U_0^1 > U_0^2$, $U_\rho^1 > U_\rho^2$ and $w^1h^1 > w^2h^2$. Thus, when the country with higher labor productivity is not too smaller in size, it has higher per capita real income both under autarky and under trade. It also has higher per capita income. For $L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma} < 1$, however, the country with higher labor productivity has lower per capita real income in autarky. When the condition (38) holds, this country has lower per capita real income and is the net-exporter in the lower income elastic sectors. Notice that (38) is more stringent than $L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma} < 1$. In other words, for $\tilde{\Lambda}\left(h^1/h^2;\rho\right) < L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma} < 1$, the per capita real income in this country is lower in autarky but higher under trade, because trade reduces this country's disadvantage of being smaller. Notice also that the condition (38) is less stringent than $L^1/L^2 < \overline{\Lambda}\left(h^1/h^2;\rho\right) < 1$, the condition under which its per capita nominal income is smaller. In other words, for $\overline{\Lambda}\left(h^1/h^2;\rho\right) < L^1/L^2 < \tilde{\Lambda}\left(h^1/h^2;\rho\right) < 1$, the per capita real income in this country is lower even when its per capita nominal income is still higher in this country. This can occur because this country benefits less from the variety effect due to its smaller size.

5 Trade Equilibrium: Comparative Statics

Having characterized the cross-country variations in a given trade equilibrium, we now turn to comparative static exercises.

5.1 Uniform Labor Productivity Growth: Interdependent Patterns of Structural Change Across Countries and Product Cycles

First, consider the effects of a uniform labor productivity growth. That is, labor productivity goes up at the same rate in all the activities in both countries. This can be captured by $\partial\log(h^1) = \partial\log(h^2) \equiv \partial\log(h) > 0$. This keeps h^1/h^2 and L^1/L^2 unchanged, with $\partial\log(L^1)$ =

$\partial\log(L^2) = \partial\log(h) > 0$. Therefore, $\omega = w^1/w^2$ is also unchanged, and so are x_0^1/x_0^2 and x_ρ^1/x_ρ^2, with $\partial\log(x_0^1) = \partial\log(x_0^2) = \partial\log(x_\rho^1) = \partial\log(x_\rho^2) = \sigma\partial\log(h) > 0$.

With $\partial\log(x_\rho^1) = \partial\log(x_\rho^2) > 0$, both $U_\rho^1 = u(x_\rho^1)$ and $U_\rho^2 = u(x_\rho^2)$ go up. With their per capita real income going up, both countries shift their expenditure shares towards higher-indexed sectors in the sense of both MLR and FSD. This can be seen from eq.(33) and applying Lemma 1 for $\hat{g}(s, x_\rho^k) = \left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$.

Even though x_ρ^1 and x_ρ^2 goes up at the same rate to keep x_ρ^1/x_ρ^2 unchanged, the per capita real income in the two countries do not go up at the same rate. To see this,

$$\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)} = \frac{\partial\log(u(x_\rho^1)) - \partial\log(u(x_\rho^2))}{\partial\log(h)} = \sigma\left(\zeta\left(x_\rho^1\right) - \zeta\left(x_\rho^2\right)\right).$$

Hence, from Lemma 2-ii),

$$\text{(39)} \qquad \operatorname{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)} = \operatorname{sgn}(\eta-1)\operatorname{sgn}(x_\rho^1 - x_\rho^2).$$

Thus, the per capita real income goes up at a faster rate in the Richer country if $\eta > 1$ and in the Poorer country if $\eta < 1$. In words, welfare gaps widen (narrow) if the goods produced in different sectors are substitutes (complements) .

To see how the patterns of trade change, log-differentiate (34) to yield,

$$\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(h)} = (\varepsilon(s)-\eta)\left(\frac{\sigma-1}{\sigma-\eta}\right)\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)},$$

and then use (39) to obtain

$$\text{(40)} \qquad \operatorname{sgn}\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(h)} = \operatorname{sgn}\left(\varepsilon(s)-\eta\right)\operatorname{sgn}(\eta-1)\operatorname{sgn}(x_\rho^1 - x_\rho^2) = \operatorname{sgn}(x_\rho^2 - x_\rho^1).$$

from Lemma 2-ii) and by recalling the parameter restriction, $\left(\varepsilon(s)-\eta\right)/(1-\eta) > 0$, that ensures the global monotonicity of the utiliy function, (1).

Figure 4 illustrates this for $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$. In this case, the downward-sloping curve, m_s^1/m_s^2, shifts up, which causes the cutoff sector, s_c, to move up. As a result, the

Rich's trade balances switch from net surpluses to net deficits in some middle sectors.[19] The intuition behind this result is easy to grasp. As the per capita real income goes up in both countries, both shift their expenditure shares towards the higher-indexed sectors. In response, both countries reallocate their resources towards higher-indexed sectors. In other words, the relative weights of higher-indexed sectors, in which the richer runs surpluses, go up and the relative weights of lower-indexed sectors, in which the poorer runs surpluses, go down. This means that, in order to keep the overall trade account between the two countries in balance, the richer's sectoral trade account must deteriorate in each sector. This is why the richer switches from being a net exporter to being a net importer in some middle sectors.

5.2 Globalization Without Terms of Trade Change: Interdependent Patterns of Structural Change Across Countries and Product Cycles

Next, consider the effects of globalization, captured by a trade cost reduction, or a higher $\rho = (\tau)^{1-\sigma}$. First, let us look at the case where the two countries are in equal size: $L^1 = L^2 = L$. In this case, the factor price is always equalized, $w^1 = w^2 = w$, or $\omega = 1$, independent of ρ, so that $x_\rho^k = (1+\rho)x_0^k$ = $(1+\rho)\left(h^k\right)^\sigma N^k$ = $(1+\rho)\left(h^k\right)^{\sigma-1} L$, and hence , $x_\rho^1 / x_\rho^2 =$ x_0^1 / x_0^2 = $\left(h^1 / h^2\right)^{\sigma-1}$, as noted earlier. That is, the country with higher per capita real income is the one with higher labor productivity and with higher per capita nominal income.[20] Hence, the country with higher labor productivity is always a net exporter in higher-indexed sectors and a net importer in lower-indexed sectors, precisely because they have relatively larger expenditure shares in higher-indexed sectors, which causes disproportionately larger shares of workers are employed in higher-indexed sectors due to the home market effect.

Furthermore, in this case, the effects of globalization, a higher ρ, can be seen only by looking at x_ρ^k = $(1+\rho)x_0^k$ = $(1+\rho)\left(h^k\right)^{\sigma-1} L$. Indeed, without causing any terms-of-trade change, the effects of a higher ρ is isomorphic to a uniform labor productivity growth, with $\partial \log(1+\rho)$

[19] For $U_\rho^1 = u(x_\rho^1) > U_\rho^2 = u(x_\rho^2)$, the *upward*-sloping curve, m_s^1 / m_s^2, shifts *down*, which also leads to the cutoff sector, s_c, to move up. Either way, the Rich's trade balances must switch from net surpluses to net deficits in some middle sectors.

[20] In this case, the two countries have the same aggregate GDP, but differ in GDP per capita.

> 0 equivalent to $(\sigma-1)\partial\log(h^1) = (\sigma-1)\partial\log(h^2) \equiv (\sigma-1)\partial\log(h) > 0$. Hence, by going through the analysis as done in the previous subsection, one can show that the per capita real income goes up (a higher U_ρ^k) in both countries and they shift their expenditure shares towards higher-indexed sectors both in the sense of MLR and FSD. Furthermore, one can show:

$$\mathrm{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(1+\rho)} = \mathrm{sgn}(\eta-1)\ \mathrm{sgn}(x_\rho^1 - x_\rho^2).$$

so that globalization causes the welfare gap between the Rich and the Poor to widen (narrow) if the goods produced in different sectors are substitutes (complements). One can also show:

$$\mathrm{sgn}\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(1+\rho)} = \mathrm{sgn}(x_\rho^2 - x_\rho^1),$$

so that the cutoff sector moves up (see Figure 4). Thus, the richer country, the country with higher labor productivity, switches from a net exporter to a net importer in some middle sectors, generating something akin to product cycles without any technology diffusion across countries.

In summary, when the two countries are equal in size, globalization causes no terms-of-trade change. And without any terms-of-trade change, globalization is isomorphic to the effects of uniform labor productivity growth, because it allows the consumption goods sectors in each country to better access to the varieties of inputs produced abroad. Such productivity gains from trade cause structural change through an endogenous change in the demand composition.

5.3 Globalization with Terms-of-Trade: Leapfrogging and Patterns of Trade Reversal

When the two countries are unequal in size, the factor price is lower in the smaller country, due to the disadvantage of being smaller in the presence of aggregate increasing returns. The larger the trade cost, the greater this disadvantage. Globalization reduces this disadvantage for the smaller country, thereby causing the terms of trade change in favor of the smaller country, and a factor price convergence, as shown in Figure 1.

When the smaller country has lower labor productivity, this country always has lower per capita real income, regardless of the trade cost. However, when the smaller country has higher labor productivity, it is possible that this country has lower per capita real income at a high trade cost but higher per capita real income at a low trade cost. This possibility is illustrated in Figure 5, which reproduces some parts of Figure 3. Below and to the left of the red curve, Country 1 has lower per capita real income than Country 2 in autarky. Below and to the left of the black curve, Country 1 has lower per capita real income than Country 2 under trade. Globalization, a higher ρ, rotates the black curve clockwise, as indicated by the arrows. As ρ approaches zero, the black curve converges to the red curve, which is invariant to the trade cost. As ρ approaches one, the black curve converges to the vertical line, $h^1/h^2 = 1$. Now, consider the case where country 1 has higher labor productivity, i.e., $h^1/h^2 > 1$ but it is sufficiently smaller so that $L^1/L^2 < (h^1/h^2)^{1-\sigma} < 1$. Thus, we consider the point, $(h^1/h^2, L^1/L^2)$, located to the right of the vertical line, $h^1/h^2 = 1$ and below the red curve. Then, with a sufficiently small ρ, the black curve passes above and to the right of this point, which means that Country 1 has lower per capita real income. With a sufficiently large ρ, the black curve passes below and to the left of this point, which means the Country 1 has higher per capita real income. Thus, closer to autarky, Country 1 is poorer due to its disadvantage of being smaller in the presence of aggregate increasing returns, hence running surpluses in lower-indexed sectors. Globalization reduces the disadvantage of being smaller, causing a factor price convergence, which makes it richer, hence running surpluses in higher-indexed sectors. This result thus suggests the possibility that some relatively small countries with relatively highly educated labor forces, which might initially have lower per capita real income due to their remote locations and might be net-exporters in the low income elastic sectors, benefit more from globalization and overtake other larger countries and emerge as net-exporters in the high income elastic sectors.

6. Relations to the Existing Studies

As this paper aims to offer a *unifying* perspective on the role of Engel's Law in the interdependent patterns of structural change, innovation, and trade across countries and across sectors, there are many related papers in the three distinct literatures of structural change, innovation, and trade.

For models of structural change, see Matsuyama (2008) and Herrendorf, Rogerson, and Valentinyi (2014). The latter also offers an extensive review of the empirical regularities on the changing patterns of sectoral shares in employment and in value-added. Engel's Law plays the key role in most studies in this literature, see, e.g, Kongsamut, Rebelo, and Xie (2000) and Buera and Kaboski (2012), just to name a few. A relatively few studies, Baumol (1967) and Ngai and

Pissaridis (2007), focus on an exogenous difference in productivity growth rates across sectors as an alternative driver of structural change. Both the income-elasticity and the exogenous productivity growth differences across sectors are incorporated in some recent studies, such as Matsuyama (2009) and Comin, Lashkari, and Mestieri (2015). In particular, the latter is noteworthy as it derives a clear decomposition of the income effect and the price effect as the two competing drivers of structural change with an arbitrary number of sectors, thanks to its use of isoelastically nonhomothetic CES. Most studies in this literature consider only closed economy models. Although Matsuyama (2009) and Uy, Yi, and Zhang (2013) studied open economy implications of structural change, they are primarily interested in the questions of how structural change in one country spillovers to another country. Furthermore, the use of Stone-Geary preferences makes these studies unable to isolate the role of Engel's Law. In all these models, the Schmookler effect is absent; the sectoral difference in productivity growth rates is assumed to be exogenous and unresponsive to changes in the relative market sizes across the sectors.

The Schmookler effect is central to the directed technical change literature; see, e.g., Acemoglu (1998, 2002), Acemoglu and Zilibotti (2001), Gancia and Zilibotti (2005, 2009), and Acemoglu (2009, Ch.15). In these models, the relative market sizes are given exogenously. The idea of linking Engel's Law to the Schmookler effect was pursued by Murphy, Shleifer and Vishny (1989) and Matsuyama (1992, 2002), among others, but these studies have a quite different goal. They are primarily interested in the role of nonhomotheticity on the country's aggregate growth performance. By considering models where the sectors differ not only in the income elasticity but also in the productivity growth potential, these studies showed how an endogenous shift in the demand composition towards sectors with more (less) productivity growth potential would accelerate (slow down) the aggregate growth of the economy. Furthermore, these studies use forms of nonhomothetic preferences, where the effects of income elasticity difference cannot be disentangled from those of price elasticity differences. Both Murphy, Shleifer, and Vishny (1989) and Matsuyama (2002) considered only the closed economy case. An open economy version of Matsuyama (1992) assumes no trade cost, so that producers everywhere face the same tradeable goods prices, which means the cross-country difference in the domestic demand composition cannot play any role in the allocation of resources. Taken together, these studies might have left some readers with the false impression that the role of Engel's Law would have to be less important in open economies.

In the international trade literature, many models of trade with nonhomothetic preferences have been proposed to study the role of Engel's Law in explaining the patterns of trade between rich and poor countries; Markusen (2013) for a survey. Because merely replacing homothetic preferences by nonhomothetic preferences in the standard neoclassical trade models would, *ceteris paribus*, make rich countries consume more and import more in high income elastic sectors, virtually all existing models of trade with nonhomothetic preferences *postulate* that the rich (poor) countries have comparative advantages in higher (lower) income elastic sectors. For example, in their Ricardian models of trade, Matsuyama (2000) and Fieler (2011), the technological superiority of rich countries are assumed to be greater in high income elastic sectors. In their factor proportion models of trade, Markusen (1986) and Caron, Fally, and Markusen (2014), rich countries are assumed to be relatively more abundant in the factors used relatively more intensively in high income elastic sectors. Such correlations between the differences on the supply side and the demand side are *not causally linked* in these models. Instead, they hold by assumption. In other words, all these models suggest that rich countries are exporters in high income elastic sectors, *despite* their domestic demand composition is more skewed towards such sectors. This is contrary to the Linder argument that rich countries are exporters in high income elastic sectors *because* their domestic demand composition is more skewed towards such sectors, which is central to our analysis. All these studies use nonhomothetic forms, in which the effects of income elasticity differences cannot be disentangled from those of price elasticity differences. Furthermore, the ranking of countries, i.e., which country is richer, is exogenously determined. In our analysis, leapfrogging can occur, because globalization allows the smaller country with higher labor productivity to catch up and take over the other country.

The idea that, in the presence of small but positive trade costs, the structure of an economy responds and adjusts more to the domestic markets than to the export markets, and hence the cross-country difference in the demand composition could become a source of comparative advantage was first formalized by Krugman (1980), who called it "the Home Market Effect." In Krugman (1980), the cross-country difference in the demand composition is due to the exogenous variations in taste across countries.[21] In our analysis, on the other hand, it arises endogenously due to Engel's Law.

There have been some attempts to model product cycles, or migration of industries from rich to poor countries. In Krugman (1979), they occur exogenously, as new products are innovated in the rich country as an exogenous rate, and products produced in the rich country are migrated to the poor country at an exogenous rate. Grossman and Helpman (1991) endogenized this process by assuming that the rich has comparative advantage in innovation, while the poor has comparative advantage in imitation. In both models, product cycles are driven by technology diffusions. Furthermore, all products enter symmetrically and product cycles affect the relative number of products produced in the two countries, which remains constant along the balanced growth path. Hence, there is no structural change, and the income elasticity difference across sectors, or Engel's Law, is not a factor in these models, contrary to Vernon's idea. In the Ricardian model of Matsuyama (2000), different sectors produce complementary consumption goods, which are ranked according to the priority, and hence the richer country has higher expenditure shares in sectors that produce low-priority goods. It is also assumed that the richer countries has comparative advantages in those sectors. In this setup, it is shown that uniform labor productivity growth causes structural change, i.e., labor allocations shifts towards sectors producing lower-priority goods in both countries, and this is achieved partly by product cycles, i.e., migration of sectors producing middle-priority goods from the richer to the poor countries. However, the effects of a trade cost reduction, or globalization, are not explored in Matsuyama (2000), because there is no trade cost in that model. To the best of our knowledge, the possibility that gains from a trade cost reduction and the resulting income effect alone can cause structural change as well as product cycles in the presence of Engel's Law has never been

[21] Krugman (1980) demonstrated the Home Market Effect in a two-country, two-sector model, in which the world demand for the two sectors are equal in size, but distributed unevenly across the two countries of equal size, what he called "the mirror-image" assumption. This unfortunately left the factor behind the Home Market Effect ambiguous. This is because, under this assumption, the sector in which one country develops comparative advantage is not only the sector in which this country expenditure share is larger than the other country's, but it is also the sector in which this country's expenditure is larger than the other country's, and it is also the sector in which this country's expenditure is larger than in the other sector. To resolve this ambiguity, Matsuyama (2015, section 3) extends Krugman's model to the case where there are many sectors of unequal size and two countries of unequal size, and showed that the country develops comparative advantage in sectors, neither because its expenditures in these sectors are larger than the other country's nor because they are larger than those in the other sectors, but because its expenditure *shares* in these sectors are larger than the other country's expenditure shares in the same sectors.

demonstrated before. And the Linder effect is absent in any of these existing product cycles models.

Finally, some remarks should be made of two lines of research, both of which are, in spite of the prominent role played by demand nonhomotheticity, orthogonal to our analysis.

The first explores various alternatives to the Dixit and Stiglitz (1977, Section I) model of monopolistic competition with CES, by using non-CES aggregators of differentiated products within a sector; e.g., Behrens and Murata (2007), Bertoletti and Etro (2015), Foellmi and Zweimueller (2006), and Zhelobodko, Kokovin, Parenti, and Thisse (2012). See Parenti, Thisse, and Ushchev (2017) and Thisse and Ushchev (2016) for unified treatments. Some studies in this literature explore the implications on intra-industry trade; see Behrens and Murata (2012a, 2012b), Bertoletti, Etro, and Simonovska (2016), Foellmi, Hanslin, and Kohler (2014), Foellmi, Hepenstrick, and Zweimueller (forthcoming), and Simonovska (2015). By departing from the CES aggregator, these models generate some income effects on the nature of monopolistic competition and intra-industry trade that are absent in the Dixit-Stiglitz-Krugman model of trade. The nonhomotheticity in these models is all about the consumers' "love for variety," or their willingness to pay for variety, varying with their income. With its focus on the intra-sectoral allocations and on the issues like variable mark up and "pricing to the market," this literature abstracts from inter-sectoral issues by using models with a single sector. In contrast, we abstract from their issues by keeping the Dixit-Stiglitz-Krugman structure of monopolistic competition and trade within each sector, and instead focus on the implications of Engel's Law, by using the implicitly additive nonhomothetic CES preferences across sectors.

The second literature studies the patterns of intra-industry trade between the rich and poor countries with quality differentiated products. See, e.g., Flam and Helpman (1987), Stokey (1991), and Fajgelbaum, Grossman, and Helpman (2011). Motivated by the observations that the rich (poor) countries tend to export higher (lower) quality products within a sector, these studies developed nonhomothetic demand systems that rely on the idea that, as their incomes go up, more consumers switch from lower-quality goods to higher-quality goods. Hence, by construction, products of different quality levels are gross substitutes, which makes their demand systems unsuitable for studying Engel's Law.[22] Indeed, with their focus on the intra-industry trade, they abstract from the patterns of intersectoral trade. Nevertheless, Fajgelbaum, Grossman and Helpman (2011) deserves special mention. Unlike the other models of intra-industry trade with quality differentiation, which all assume that the rich country has comparative advantage in higher quality goods, they used a monopolistic competition model with costly trade to generate demand-induced patterns of intra-industry trade, which they also attribute to Linder (1961). Due to the presence of an outside competitive sector that produces numeraire good, their model predicts that the country becomes a net-exporter of the quality levels for which it has larger domestic market than the other country. One important implication of this prediction is that, *when the two countries are sufficiently similar in size*, the rich (poor) country becomes a net-exporter of high (low) quality products, due to the nonhomotheticity of the aggregate demand.[23] Indeed, their analysis and ours are nearly perfect complements and they cannot be more different from each other. Their analysis is all about intra-sectoral trade, designed to address IO-trade issues. They focus on within-sector quality specialization and its implications on within-country inequality. To this end, they abstract from the patterns of trade across sectors and from any

[22]For example, Fajgelbaum, Grossman and Helpman (2011) considers a single monopolistic competitive industry, which produces indivisible products, say the automobile industry. These indivisible products come in two quality levels, H & L, and different products are horizontally differentiated within each quality segment. In addition, there is an outside competitive sector that produces the divisible numeraire good tradeable at zero cost, which is big enough to kill any general equilibrium effect or the terms-of-trade effect. Each agent consumes one unit of a particular product from *either* H or L. Building on the discrete choice model of consumer behaviors, they derive a nested logit demand system, with the property that the rich consumers are more likely than the poor to choose an H-product under the assumption that marginal utility of the numeraire good is higher when combined with an H- product, which generates nonhomotheticity of demand. As is well-known, any demand system based on a discrete choice model of consumer behaviors necessarily imply that different products have to be gross substitutes. In contract, Engel's Law is about the nonhomotheticity of demand across goods that are gross complements. Food and footwear are low income elastic, and pharmaceutical products and automobiles are high income elastic, neither because food is as not good as drugs nor because shoes are not as good as cars. As consumers become richer, they may switch from low-quality food and shoes to high-quality food and shoes, and they may also spend more on drugs and cars. However, they would never stop eating food in favor of drugs nor stop wearing shoes in favor of cars, because food and drugs are not substitutes and shoes and cars are not substitutes.

[23] Notice that the Home Market Effect works very differently in their model. In our model, similar to Krugman (1980), the Home Market Effect is due to the cross-country difference in the domestic market composition, while, in FGH, similar to Helpman and Krugman (1985, Ch. 10.4), it is due to the cross-country difference in the absolute domestic market size at each quality level. This is because, in FGH, different quality segments of the monopolistic competitive sector are not competing against each other in the factor market due to the presence of a large outside good sector, which is big enough to kill the general equilibrium effect. Thus, between a very small but rich country (say Switzerland) and a very large but poor country (say, China), their model predicts that China could become a net-exporter at every quality level, including in high quality products, while Switzerland an exporter in the outside good.

effects on cross-country inequality by fixing the terms of trade and the country ranking. In contrast, our analysis is all about inter-sectoral trade, designed to address macroeconomic growth and structural change issues. We focus on the patterns of trade across sectors producing complementary goods and its implications on cross-country inequality with endogenous terms-of-trade and endogenous country ranking. To this end, we abstract from within-sector quality specialization and within-country inequality.

7. Concluding Remarks

The endogeneity of the demand composition across sectors due to nonhomothetic demand, or Engel's Law, is an important channel through which economic growth and globalization affect sectoral patterns in employment, value-added, and productivity change, as well as intersectoral patterns of trade and migration of industries across countries. Some of these effects have been studied in the past, but only separately, which could be misleading, as these effects are interconnected.

This paper offered a unifying perspective on the role of Engel's Law in the global economy, by developing a two-country model of directed technological change with a continuum of sectors under nonhomothetic preferences, which is rich enough to capture all these effects and their interactions and, at the same time, abstracts from all other factors in order to isolate the role of Engel's Law. The key ingredients of the model are i) two countries that differ in population size and labor productivity (and hence its size, measured in the total effective labor supply); ii) isoelastically nonhomothetic CES preferences over a continuum of nontradeable consumption goods, which are implicitly (both directly and indirectly) additive; iii) endogenous productivity differences across sectors and across countries, due to endogenous variety of differentiated inputs supplied monopolistically competitively with the iceberg trade cost, as in Dixit-Stiglitz (1977) and Krugman (1980). In the closed economy equilibrium, an increase in labor productivity or a population size leads to a higher per capita real income, causing a demand composition shift from lower income elastic sectors towards higher-income elastic ones. This relative market size change induces input producers to exit from the former and enter to the latter. The resulting changes in the relative productivity across sectors (the Schmookler effect) and the relative prices moderate (amplify) the sectoral composition changes if the goods produced in different sectors are gross complements (substitutes). For the trade equilibrium, in terms of cross-country variations, it was shown, among others, that the country with higher per capita real income (or standard-of-living), whose domestic demand composition is more skewed towards higher income elastic sectors, allocates disproportionately larger shares of labor in higher income elastic sectors (the Home Market Effect in employment) and becomes a net-exporter in those sectors (the Linder effect). In terms of comparative statics, it is shown, among others, that labor productivity growth (and globalization in the case of the equal country size) cause structural change towards higher income elastics in both countries; product cycles, in which the richer country switches from a net-exporter to a net-importer of the sectors in the middle range of income elasticities; and the welfare (per capita real income) gap to narrow (widen) when sectors are gross complements (substitutes) through the market size (Schmookler) effect on the relative productivity and price changes. In addition, when the countries differ in size, globalization could help the smaller country with higher labor productivity overtake the other (Leapfrogging), which leads to a reversal of the patterns of trade. For all these reasons, globalization amplifies, instead of reducing, the power of Engel's Law and the endogenous domestic demand composition differences as a driver of structural change.[24]

It would have been impossible to isolate all these effects of Engel's Law and their interactions, if we had used other classes of nonhomothetic preferences, because they would imply the strong restriction between the income and price elasticties of the goods. For example, Stone-Geary, CRIE or any other explicit direct additive form of nonhomothetic preferences, would imply Pigou's Law. This restriction not only has been rejected empirically, but also makes it impossible to disentangle the effects of income elasticity difference from those of price elasticity differences across sectors, hence to isolate the role of Engel's Law. Only an implicitly additive form of nonhomothetic preferences is free of any functional relation between the income elasticities and price elasticities, and hence allows for an arbitrary number of consumption goods

[24]Both the endogeneity of the demand composition and that of the terms of trade are crucial for most of these results. To clarify the former, Matsuyama (2015, section 3) considers an alternative model, where the domestic demand composition differences are due to the exogenous differences in taste. (This model may be viewed as an extension of the Krugman (1980)'s Home Market Effect model to the case of an arbitrary number of sectors with an arbitrary exogenous variations in taste across the two countries which are not necessarily equal in size.) To clarify the latter, Matsuyama (2015, section 4) adds a competitive outside sector, which produces a homogenous good that can be traded at zero cost, and is large enough to kill any general equilibrium terms-of-trade effect.

with good-specific income elasticity parameters, which can be controlled for independently of the price elasticity parameters.

In this paper, the model was kept deliberately as simple as possible in order to isolate the role of Engel's Law in the interdependent patterns of structural change, innovation, and trade. However, some extensions would be useful, even necessary, for other applications. Here are some suggestions for promising lines of extensions with some conjectures.

First, one could allow for multiple factors of production with some correlations between the factor intensity and the income elasticity across sectors. For example, Caron, Fally and Markusen (2014) and Buera, Kaboski, and Rogerson (2015) provided some evidence that skill intensities of sectors are positively correlated with the income elasticities of sectoral demands. Obviously, if the two countries differ in their skilled-to-unskilled ratios, this introduces the familiar Heckscher-Ohlin mechanism. But, even if the two countries have the identical factor proportion, the richer country (due to higher TFP) would become the net-exporter in higher-income elastic sectors in the presence of trade costs due to the Linder effect, which are also skill-intensive, implying higher skill premium in the richer country, and hence stronger incentive to accumulate skills in the richer country.

Second, one could allow for sector-specific trade costs, with positive correlation between the trade cost and the income elasticity. For example, higher income elastic consumption goods might have higher service components that are less tradeable. In this case, the effects of a uniform reduction in the trade cost across sectors might be partially mitigated by an endogenous shift in the demand composition towards higher-income elastic sectors, which have higher trade costs.

Third, allowing for more than two countries/regions would be necessary to capture a variety of geographical features along the line of Matsuyama (2017). For example, imagine that three countries are located along the line, but they are otherwise identical, as in Matsuyama (2017, Ex.3). Then, the country in the middle, which is centrally located, has higher per capita real income due to its geographical advantage, or the "hub" effect. This implies that it becomes a net-exporter in the higher income elastic sectors, while the two countries in the peripheries become net-importers in the lower income elastic sectors. Then, uniform labor productivity growth or globalization and the resulting shift in the demand composition towards the higher-indexed, could generate product cycles where the net trade balances in the middle-indexed sectors switch from surpluses to deficits for the country in the center. Or imagine four countries located along the circle, one of which has a bigger population size, but they are otherwise identical, as in Matsuyama (2017, Ex.2). Then, due to the economies of scale, this country has the highest per capita real income, and becomes the net-exporter in the high income elastic sectors. The two countries that are next to this country might become the net-exporters in the low income elastic sectors, due to the "shadow" effect, while the country on the opposite side of the circle might become the net-exporter in the middle range of the sectors, due to its geographical advantage of not having a big neighbor.

Finally, this paper focused on the nonhomotheticity of demand across sectors, by assuming the demand system within each sector is homothetic CES, following Dixit and Stiglitz (1977, Section I). But, it would be interesting to add nonhomotheticity within sectors to see how these two types of nonhomotheticity interact with each other. This could be achieved in a variety of ways. For example, one could use a horizontally differentiated monopolistic competition model with non-CES, similar to Behrens and Murata (2007), Bertoletti and Etro (2015), or Zhelobodko et. al. (2012). Alternatively, one could use vertically differentiated model of intra-industry trade, such as Flam and Helpman (1987), Stokey (1991), or Fajgelbaum, Grossman, and Helpman (2011). Or one could nest two (or more) isoelastically nonhomothetic CES demand structures used in this paper, with the constant elasticity of substitution being higher in the lower tier than in the upper tier.

And of course, some of these extensions can be combined to see whether they might generate some interactive effects. It is hoped that this paper stimulate further research along these lines.

<u>Appendix A</u>: Explicit vs. Implicit (Direct and Indirect) Additivity and Isoelastically Nonhomothetic CES

This appendix explains in detail why we use the particular class of preferences, isoelastically nonhomothetic CES, eq.(1), and why this must satisfy implicit (direct and indirect) additivity. To this end, we need to recall different notions of additivity.

Explicit (Direct or Indirect) Additivity:

Preference is *explicitly directly additive* if its *direct* utility function can be written *explicitly* as:

$$u = M\left[\int_I f_s(c_s)ds\right],$$

where c_s is consumption of $s \in I$, and $M[\bullet]$ is a monotone transformation. Most commonly used nonhomothetic preferences, including Stone-Geary and Constant Ratio of Income Elasticity (CRIE), are explicitly directly additive. Preference is *explicitly indirectly additive* if its *indirect* utility function can be written *explicitly* as:

$$u = M\left[\int_I g_s(p_s / E)ds\right],$$

where p_s is the price of $s \in I$ and E is the total expenditure. As shown in Samuelson (1965), the standard homothetic CES, whose direct utility function can be written as:

$$u = \left[\int_I \omega_s (c_s)^{1-1/\eta} ds\right]^{\frac{\eta}{\eta-1}},$$

and whose indirect function can be written as:

$$u = \left[\int_I (\omega_s)^{\eta} (p_s / E)^{1-\eta} ds\right]^{\frac{1}{\eta-1}} = E \Big/ \left[\int_I (\omega_s)^{\eta} (p_s)^{1-\eta} ds\right]^{\frac{1}{1-\eta}}$$

is the *only* preference that is both explicitly directly additive and explicitly indirectly additive.

As Houthakker (1960) and Goldman and Uzawa (1964) and others have pointed out, the explicit direct additivity imposes the strong restriction between the income elasticity and the price elasticity of the goods called Pigou's Law. Formally, let $\varepsilon(s)$ denote the income elasticity of $s \in I$ and $\eta(s,s')$ the Allen-Uzawa elasticity of substitution between $s,s' \in I$. Under the explicit direct additivity, $\varepsilon(s_1)/\eta(s_1,s_3)=\varepsilon(s_2)/\eta(s_2,s_3)$, for any $s_1 \neq s_2 \neq s_3 \in I$; see eq.(2.11) in Hanoch (1975). That is, the ratio of income elasticity of a good and the cross-price elasticity of that good with respect to any other good is constant across all goods. In short, Pigou's Law states that the income elasticity of a good must be proportional to the price elasticity of that good.[25] Pigou's Law is not only rejected empirically, as shown by Deaton (1974) and others. It also makes explicitly directly additive preferences conceptually unsuited for our purpose, because the effects of the income elasticity differences across sectors cannot be disentangled from those of the price elasticity differences across sectors. In particular, nonhomothetic preferences that are explicitly directly additive cannot be CES.

Likewise, the explicit indirect additivity imposes the strong restriction between the income elasticity and the price elasticity of the form, $\eta(s_1,s_3)-\eta(s_2,s_3)=\varepsilon(s_1)-\varepsilon(s_2)$, for any $s_1 \neq s_2 \neq s_3 \in I$; see eq.(3.11) in Hanoch (1975). Again, this makes it impossible to isolate the effects of the income elasticity differences across sectors from those of the price elasticity differences across sectors. In particular, nonhomothetic preferences that are explicitly indirectly additive cannot be CES.

Implicit (Direct or Indirect) Additivity:

In contrast, Hanoch (1975) showed that the income elasticity difference and the price elasticity difference can be controlled for separately under *implicit additivity*.[26] Preference is *implicitly directly additive* if its *direct* utility function can be written *implicitly* as:

$$M\left[\int_I f_s(u,c_s)ds\right]=1.$$

Preference is *implicitly indirectly additive* if its *indirect* utility function can be written *implicitly* as:

$$M\left[\int_I g_s(u,p_s/E)ds\right]=1.$$

[25] The well-known Bergson's Law, the homotheticity is equivalent to CES under the explicit direct additivity, is a special case of the Pigou's Law.

[26] This might remind the reader of the problem in macro-finance that intertemporally additive preferences impose the link between the intertemporal elasticity of substitution and the risk tolerance, and that the delinking these parameters requires the use of recursive preferences, as pointed out by Epstein and Zin (1989). I thank J. Markusen and I. Werning for this analogy.

Clearly, the explicit direct additivity implies the implicit direct additivity, and the explicit indirect additivity implies the implicit indirect additivity. Implicit additivity imposes less restriction because a change in *u* can affect the relative weights attached on different consumption goods under implicit additivity, but not under explicit additivity. In particular, implicit additivity is not subject to Bergson's law, which means that it is possible to have homothetic non-CES, as explored in Matsuyama and Ushchev (2017), as well as nonhomothetic CES, which is our focus here.

Isoelastically Nonhomothetic CES:

For the goal of this paper, it is important to isolate the role of income elasticity differences, which requires the preference to be CES. And, one can also show that implicitly CES, whose direct utility function is given by:

$$\left[\int_I \omega_s(u)(c_s)^{1-1/\eta} ds\right]^{\frac{\eta}{\eta-1}} = 1,$$

and whose indirect utility function is given implicitly by:

$$\left[\int_I (\omega_s(u))^{\eta} (p_s/E)^{1-\eta} ds\right]^{\frac{1}{\eta-1}} = E \Big/ \left[\int_I (\omega_s(u))^{\eta} (p_s)^{1-\eta} ds\right]^{\frac{1}{1-\eta}} = 1$$

is the only preference that is both implicitly directly additive and implicitly indirectly additive.[27] In spite of being a CES, this preference is *nonhomothetic* if $\partial \log \omega_s(u)/\partial u$ depends on $s \in I$. Furthermore, if sectors can be indexed such that $\partial \log \omega_s(u)/\partial u$ is monotone increasing $s \in I$, $\omega_s(u)$ becomes *log-supermodular* in *s* and *u*, which facilitates monotone comparative static exercises. In addition, empirically, the slope of the Engel's curve is stable. That is, the income elasticity differences across sectors are independent of the per capita real income, *u*. This requires that the weights of each good be *isoelastic* in *u* (i.e., a power function of *u*), hence $\partial \log \omega_s(u)/\partial \log u$ is independent of *u*. This allows us to define the sector-specific income elasticity, $\varepsilon(s)$, as a fixed parameter for each $s \in I$, which is monotone increasing, $s \in I$.

[27]We are not aware of any existing proof of this. However, it can be adopted from the proof of Proposition 4(iii) in Matsuyama and Ushchev (2017). Even though this Proposition states that homothetic implicit direct additivity and homothetic implicit indirect additivity imply homothetic CES, homotheticity does not play any role in the proof.

Appendix B: Two Lemmas

This appendix offers two lemmas, which are used repeatedly in the analysis.

Lemma 1: For a positive value function, $\hat{g}(\bullet;x): I \to R_+$, with a parameter *x*, define a density function on I by $g(s;x) \equiv \frac{\hat{g}(s;x)}{\int_I \hat{g}(t;x)dt}$, and denote its distribution function by $G(s;x)$. If $\hat{g}(s;x)$ is *log-supermodular* in *s* and *x*, *i.e.* $\frac{\partial^2 \log \hat{g}(s;x)}{\partial s \partial x} > 0$,

i) **Monotone Likelihood Ratio (MLR)**: $\frac{g(s;x_1)}{g(s;x_2)}$ is decreasing in *s* for $x_1 < x_2$;

ii) **First-order Stochastic Dominance (FSD)**: $G(s;x)$ is decreasing in *x*.

For the proof, see Matsuyama (2015, Appendix).[28]

Lemma 2: For $\eta \neq 1$, define $u(x)$ implicitly by $x^{\frac{1-\eta}{\sigma-\eta}} \equiv \int_I \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}} ds$. If $(\varepsilon(s)-\eta)/(1-\eta) > 0$,

i) $u(x)$ is strictly increasing in *x;*

ii) $\zeta(x) \equiv \frac{xu'(x)}{u(x)}$ is decreasing in *x* if $\eta < 1$ and increasing in *x* if $\eta > 1$.

Proof: Differentiating the definition yields

$$\left(\frac{1-\eta}{\sigma-\eta}\right) x^{\frac{1-\eta}{\sigma-\eta}-1} = \left(\frac{\sigma-1}{\sigma-\eta}\right) \int_I \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}-1} \beta_s (\varepsilon(s)-\eta)(u(x))^{(\varepsilon(s)-\eta-1)} u'(x) ds$$

$$x^{\frac{1-\eta}{\sigma-\eta}-1} = \left(\frac{\sigma-1}{1-\eta}\right)\left(\frac{u'(x)}{u(x)}\right) \int_I \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}} (\varepsilon(s)-\eta) ds$$

$$\frac{1}{\zeta(x)} = \left(\frac{\sigma-1}{x^{\frac{1-\eta}{\sigma-\eta}}}\right) \int_I \left(\frac{\varepsilon(s)-\eta}{1-\eta}\right) \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}} ds = (\sigma-1) \int_I \left(\frac{\varepsilon(s)-\eta}{1-\eta}\right) f(s;x) ds.$$

[28]The results in this lemma are not new. For example, they were used in Matsuyama (2013, 2014) without proof. Furthermore, ii) follows from i). Indeed, they are special cases of more general properties of log-supermodularity known in the literature: see, e.g., Athey (2002) and Vives (1999; Ch.2.7).

Hence,

(A1) $$\frac{1}{\zeta(x)} = (\sigma-1)\int_I \left(\frac{\varepsilon(s)-\eta}{1-\eta}\right) dF(s;x) > 0,$$

where $f(s;x) \equiv \dfrac{\left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}}{\int_I \left[\beta_t (u(x))^{(\varepsilon(t)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}} dt}$ is a density function, and $F(s;x)$ is its cumulative distribution function.

First, (A1) shows $\zeta(x) \equiv \dfrac{xu'(x)}{u(x)} > 0$, hence $u(x)$ is strictly increasing. Second, because $u(x)$ is strictly increasing, $\left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}$ is log-supermodular in *s* and *x*. Hence, from ii) of Lemma 1, $F(s;x)$ satisfies FSD. For $\eta < 1$, $\dfrac{\varepsilon(s)-\eta}{1-\eta}$ is increasing in *s*, so that RHS of (A1) is increasing in *x*, hence $\zeta(x)$ is decreasing in *x*. For $\eta > 1$, $\dfrac{\varepsilon(s)-\eta}{1-\eta}$ is decreasing in *s*, so that RHS of (A1) is decreasing in *x*, hence $\zeta(x)$ is increasing in x. **Q.E.D.**

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Figure 1: (Factoral) Terms of Trade Determination: $L^1/L^2 = \Lambda(\omega;\rho)$

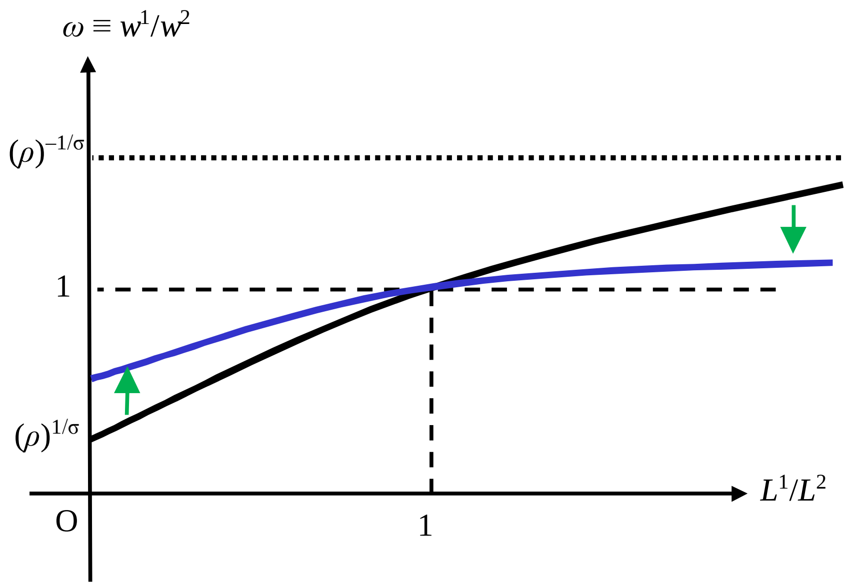

Figure 2: Endogenous Market Size Distribution and the Home Market Effect in Employment and Inter-sectoral Patterns of Intra-sectoral Trade: for $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$

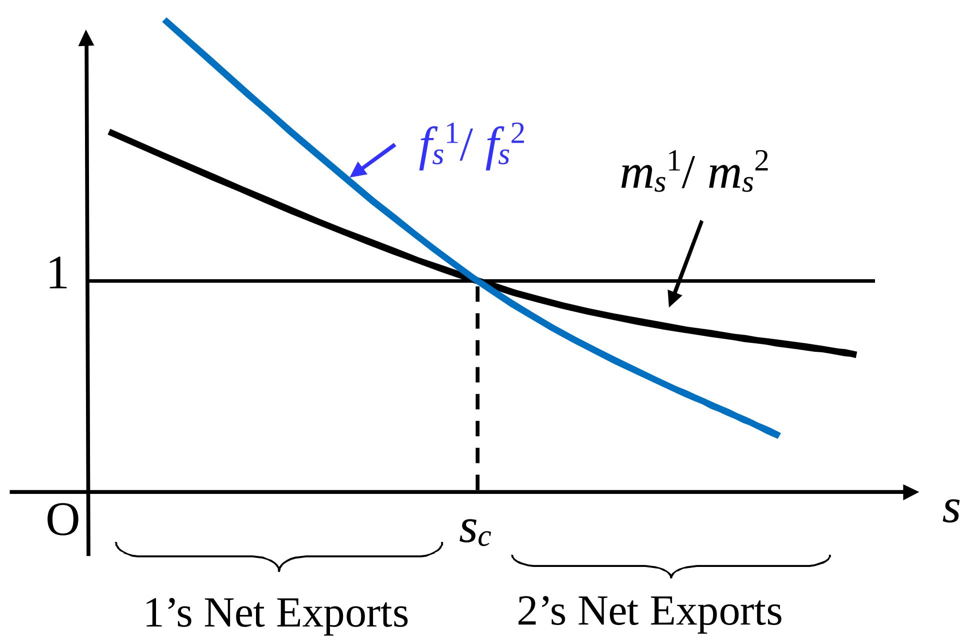

Figure 3; Ranking the Countries: Trade-off between Labor Productivity and Country Size

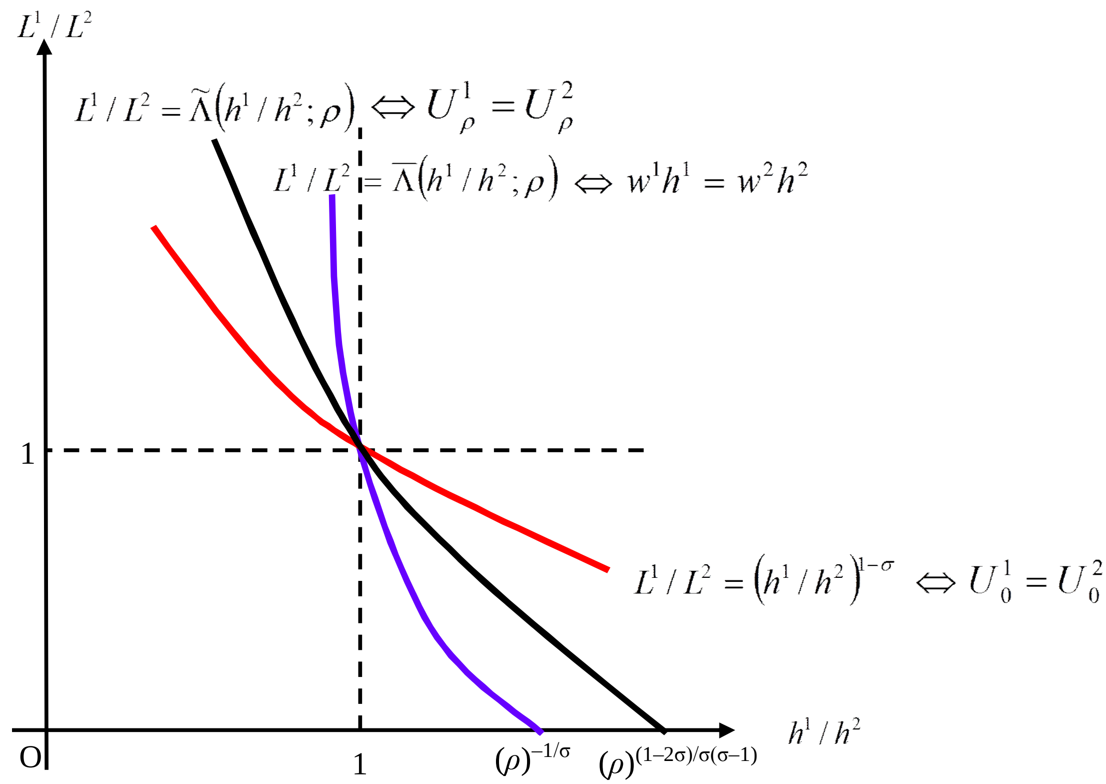

Figure 4: Interdependent Patterns of Structural Change and Product Cycles: The Effects of An Uniform Labor Productivity Growth and Globalization (when the two countries are equal in size)

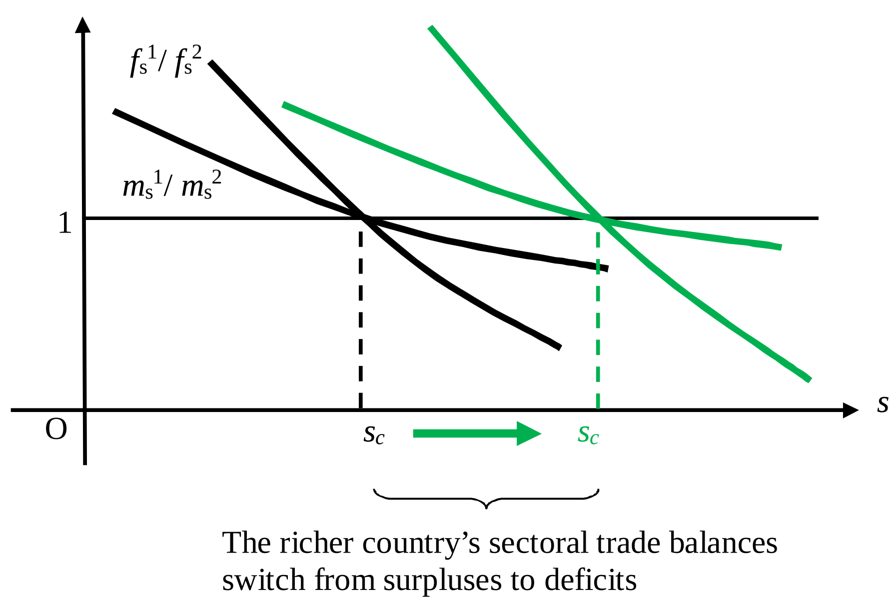

Figure 5: Leapfrogging and Reversal of Patterns of Trade: The Effects of Globalization (when the two countries are unequal in size)