Economic Growth & Development: Part 2
Horizontal Innovation Models

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Horizontal Innovation Model of Growth:

- Dynamic extensions of the Dixit-Stiglitz model.

- R&D activities introduce new products that are horizontally differentiated. The range of goods available in the market expands over time in the Dixit-Stiglitz composite.

- R&D cost is paid upfront. Once innovated, the innovator will enjoy the monopoly power, which gives an incentive to innovate.

- Free entry to R&D activities, so that the present discounted value of future monopoly profits will be equal to the R&D cost. This determines the level of R&D and the space of innovation, and hence aggregate growth.

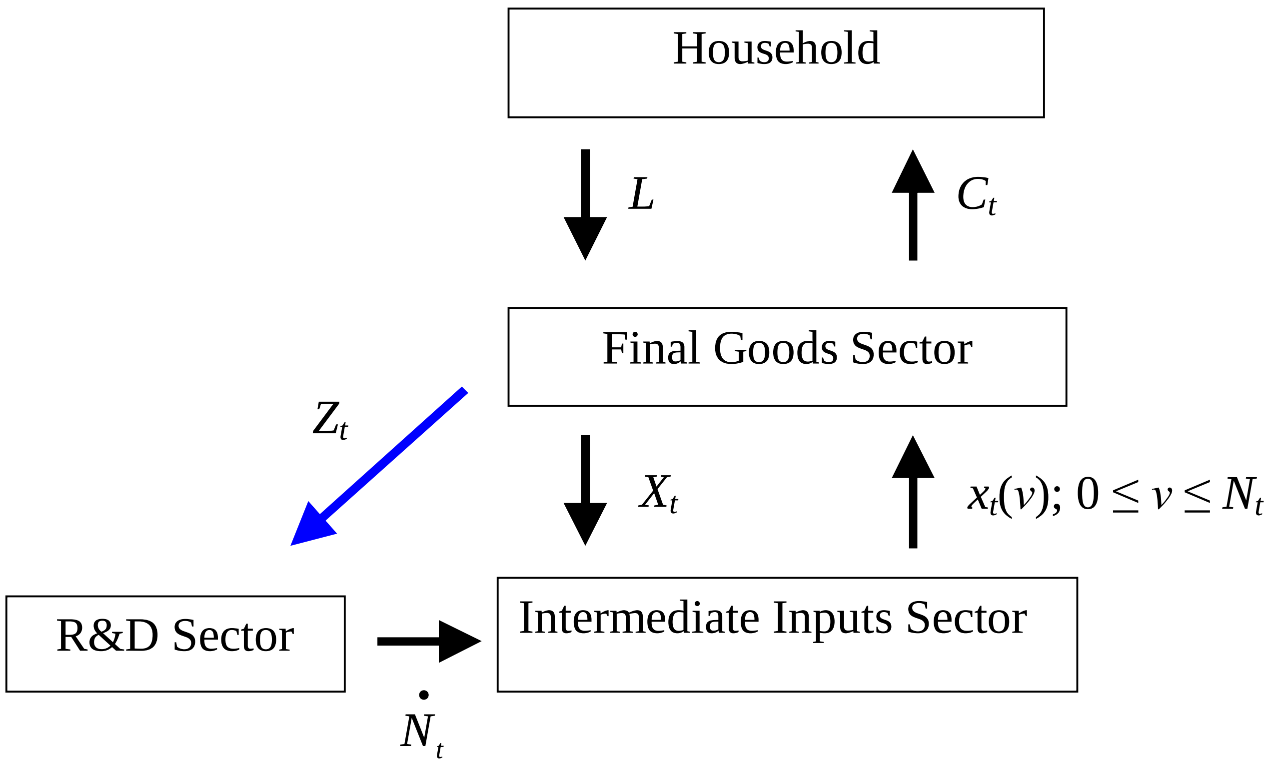
- We will look at three classes of models.

➢Lab-Equipment Models (Acemoglu; Ch.13.1)
➢Knowledge-Spillover Models (Acemoglu; Ch.13.2; Ch.13.3)
➢Labor-for-intermediate Models (Acemoglu; Ch.13.4)

For a survey, Gancia-Zilibotti's chapter in Handbook of Economic Growth.

Lab-Equipment Model (Acemoglu Ch.13.1):

The final good is used as the input to R&D activities, hence called "lab-equipment model."



Final Good Sector: *numeraire***,** competitive with CRS Technology

$$Y(t) = F(\overline{X}, L) = \frac{1}{1-\beta}\overline{X}^{1-\beta}L^{\beta} \quad \text{where } \overline{X}(t) = \left[\int_0^{N(t)} [x(\nu,t)]^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}$$

$N(t)$; the range of intermediates that has been innovated by time t.
$x(\nu,t)$; supply of intermediate,

$\overline{X}(t) = \left[\int_0^{N(t)} [x(\nu,t)]^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}$ is the Dixit-Stiglitz composite.

In the special case, $\beta = 1/\sigma$,

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} [x(\nu,t)]^{1-\beta} d\nu\right] L^{\beta}$$

Thus, the marginal productivity of a particular variety of the intermediate goods is independent of the amounts of other varieties used in the production. This is clearly special, but it guarantees the existence of balanced growth.

Demand for Intermediates: $x(\nu,t) = L(p^x(\nu,t))^{-1/\beta}$;

Intermediate Inputs Sector: Monopolistically competitive;

- $N(t)$ firms, each producing its own variety as a sole producer
- ψ units of the final good are required to produce one unit of output. By normalizing $\psi = 1-\beta$,

Monopoly Pricing: $p^x(v,t)(1-\beta) = \psi \Rightarrow p^x(v,t) = 1$ for all v and t.

$\Rightarrow x(v,t) = L$ and $\pi(v,t) = \beta L$ for all v and t.

$$\Rightarrow Y(t) = \frac{L}{1-\beta}N(t); \quad X(t) = (1-\beta)LN(t); \qquad w(t) = \frac{\beta}{1-\beta}N(t).$$

Note that the economy grows with $N(t)$.

Value of Intermediate Firm: $V(t) = \int_t^\infty \pi(s)\exp\left[-\int_t^s r(u)du\right]ds$

- This assumes that, once innovated, each product is forever produced by a monopolist. (Perpetual patent protection, for example.)
- Differentiating with respect to t yields $\dot{V}(t) + \pi = r(t)V(t)$.

Representative Household: supply L to earn the wage income, $w(t)L$, and invest in the shares of the intermediate firms, $\dot{N}V$, to earn the profit income, πN, and choose consumption of the final good, C, to:

$$\text{Max } U = \int_0^\infty u[C(t)]e^{-\rho t}dt = \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta}e^{-\rho t}dt, \quad \text{subject to}$$

Flow Budget Constraint: $\dot{N}V + C = wL + \pi N = wL + (rV - \dot{V})N = wL + rNV - N\dot{V}$

➔ $(\dot{NV}) = r(NV) + wL - C$

➔ $\int_0^\infty [C(t) - w(t)L]e^{-\int_0^t r(u)du}dt = N(0)V(0) - \lim_{t\to\infty} N(t)V(t)e^{-\int_0^t r(u)du} \leq N(0)V(0)$

➔ Intertemporal Budget Constraint:

$$\int_0^\infty C(t)\exp\left[-\int_0^t r(u)du\right]dt \leq N(0)V(0) + \int_0^\infty w(t)L\exp\left[-\int_0^t r(u)du\right]dt$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{r(t)-\rho}{\theta}; \qquad \lim_{t\to\infty} N(t)V(t)\exp\left[-\int_0^t r_u du\right] = 0$$

R&D Sector (Innovation): There is free entry to the R&D sector, which produces "blue prints" for new products with the linear technology; $\dot{N} = \eta Z$. The return to innovate a new product is *V*.

$$V\dot{N} - Z = (\eta V - 1)Z = 0 \quad \Rightarrow \eta V = 1 \text{ if } \dot{N} = \eta Z > 0 \text{ or } \dot{N} = Z = 0 \text{ if } \eta V < 1.$$

Suppose $\dot{N}(t) > 0$. Then, $V(t) = 1/\eta \quad \Rightarrow r(t) = \dfrac{\pi}{V} = \eta\beta L$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}(\eta\beta L - \rho) \equiv g^*.$$

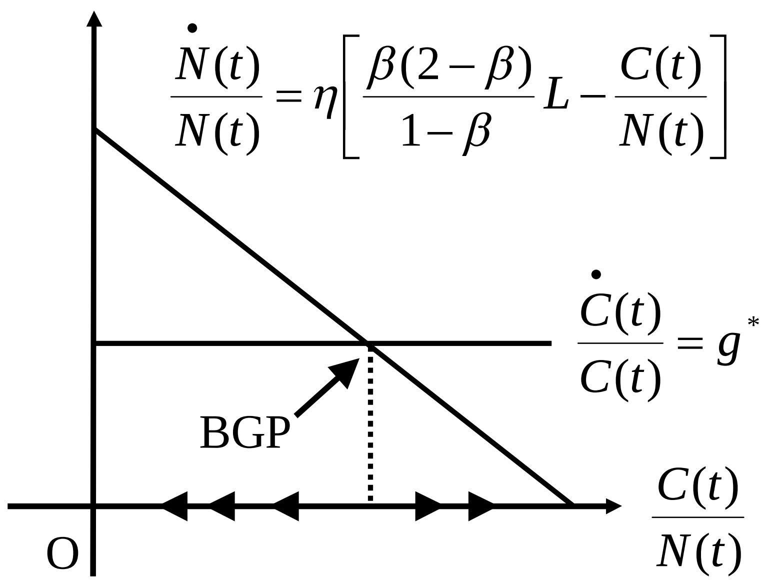
Aggregate Resource Constraint: $Y(t) = C(t) + X(t) + Z(t)$

$$\Rightarrow \frac{L}{1-\beta}N(t) = C(t) + (1-\beta)LN(t) + \frac{\dot{N}(t)}{\eta}$$

$$\Rightarrow \frac{\dot{N}(t)}{N(t)} = \eta\left[\frac{\beta(2-\beta)}{1-\beta}L - \frac{C(t)}{N(t)}\right]$$

Dynamics: A Graphic Illustration

$$\left(\frac{\dot{C}}{N}\right) = \left(\frac{C}{N}\right)\left(\frac{\dot{C}}{C} - \frac{\dot{N}}{N}\right) = \left(\frac{C}{N}\right)\left(g^* - \eta\left[\frac{\beta(2-\beta)}{1-\beta}L - \frac{C}{N}\right]\right)$$



Balanced Growth Path (BGP):

$$\frac{\dot{N}}{N} = \frac{\dot{C}}{C} = \frac{1}{\theta}(\eta\beta L - \rho) \equiv g^* > 0 \quad \text{with } \frac{C}{N} = \frac{\beta(2-\beta)}{1-\beta}L - \frac{g^*}{\eta} > 0.$$

We need $(1-\theta)\beta\eta L < \rho < \beta\eta L$.

- 1st inequality to ensure $g^* < r$ so that $\lim_{t\to\infty} N(t)V(t)\exp\left[-\int_0^t r_u du\right] = 0$;
- 2nd inequality to ensure $g^* > 0$.

Key properties of BGP: $g^* = \frac{1}{\theta}\left[\beta\eta L - \rho\right]\Uparrow$,

as $L\Uparrow$ (scale effect), $\theta\Downarrow, \rho\Downarrow$ (taste effects), and $\eta\Uparrow$ (the efficiency effect).

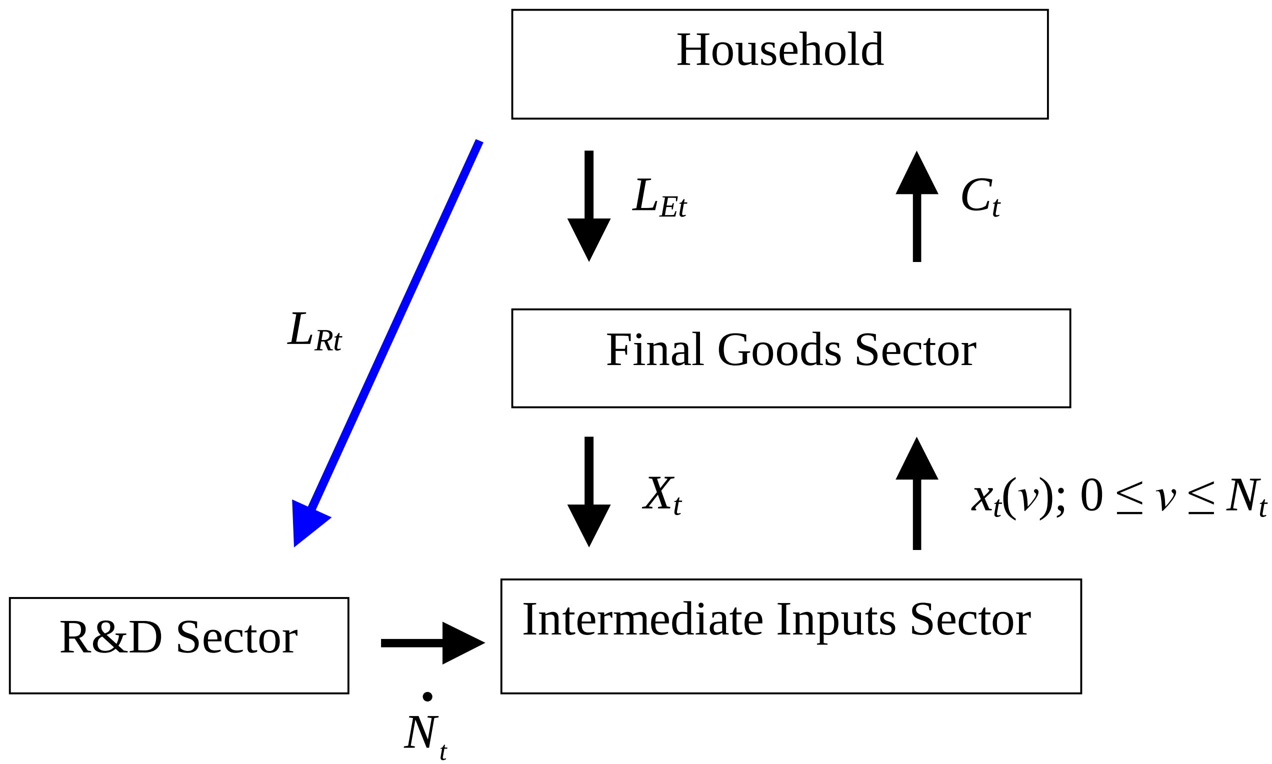
Notes: One could also show that

- There is no transition path. Starting any $N(0) > 0$, $N(t) = N(0)\exp(g^* t)$.
- The optimal path is also a balanced growth, but the rate is higher.

See Acemoglu (Ch.13.1.5) for detail.

Knowledge-Spillover Model (with Labor Input in R&D): Acemoglu Ch.13.2

Labor is used as the input to R&D activities. As the economy grows, the wage rate would go up, and so is the cost of R&D. To keep the innovation going, its productivity must go up through "Knowledge-Spillover" and "Standing on the shoulder of giants" effect.



Final Good Sector: $Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} [x(\nu,t)]^{1-\beta} d\nu\right](L_E)^{\beta}$

Intermediate Input Sector:

Monopoly Pricing: $p^x(\nu,t)(1-\beta) = \psi \Rightarrow p^x(\nu,t) = 1$ for all v and t.

$\Rightarrow x(\nu,t) = L_E(t)$ and $\pi(\nu,t) = \beta L_E(t)$ for all v and t.

$\Rightarrow Y(t) = \frac{1}{1-\beta} N(t) L_E(t)$; $X(t) = (1-\beta)N(t)L_E(t)$; $w(t) = \frac{\beta}{1-\beta} N(t)$.

R&D Sector (Innovation): "Blue prints" are now produced with the linear technology;

$$\dot{N}(t) = \overline{\eta}(t) L_R(t) = \eta N(t) L_R(t).$$

In order to keep innovation going, the R&D productivity, $\overline{\eta}(t)$, must grow at the same rate with the cost of R&D, the wage rate. Hence, the inclusion of the "Knowledge spillover" term. Dependence of $\overline{\eta}(t)$ on $N(t)$ is external to each innovator.

Labor Market Equilibrium: $L_E(t) + L_R(t) = L \quad \Rightarrow \dfrac{\dot{N}}{N} = \eta L_R = \eta(L - L_E(t))$.

Suppose $\dot{N}(t) > 0$. Then, from $\dot{N}V - wL_R = (\eta NV - w)L_R = 0$,

$$V(t) = \frac{w(t)}{\eta N(t)} = \frac{\beta}{\eta(1-\beta)} \quad \Rightarrow r(t) = \frac{\pi(t)}{V} = \eta(1-\beta)L_E(t).$$

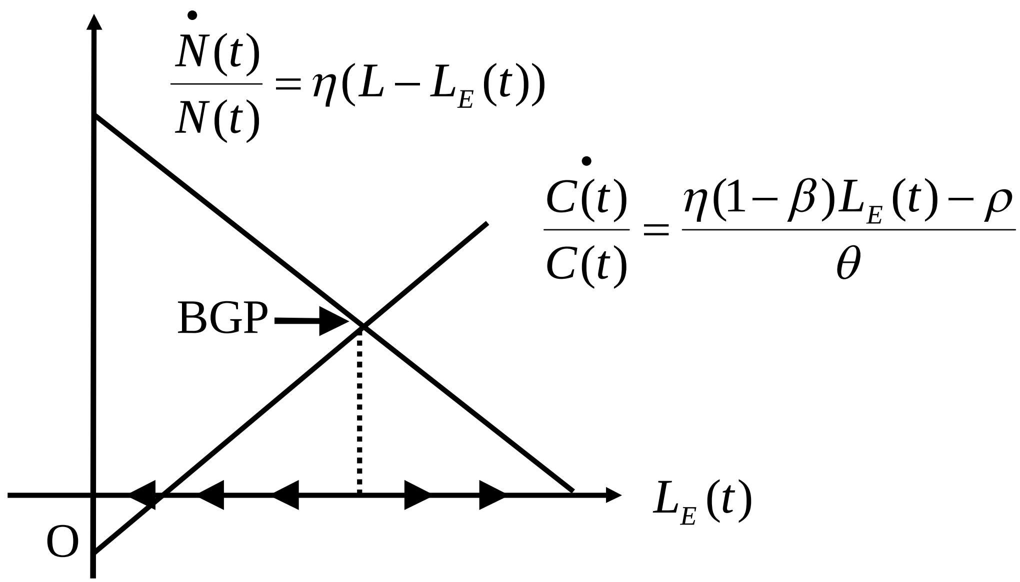
$$\Rightarrow \frac{\dot{C}}{C} = \frac{r(t) - \rho}{\theta} = \frac{\eta(1-\beta)L_E(t) - \rho}{\theta}$$

Aggregate Resource Constraint: $Y = C + X$

$$\Rightarrow \frac{1}{1-\beta} NL_E = C + (1-\beta)NL_E \quad \Rightarrow \frac{C(t)}{N(t)} = \frac{\beta(2-\beta)}{1-\beta} L_E(t)$$

Dynamics: A Graphic Illustration

$$\frac{\dot{L}_E}{L_E} = \left(\frac{\dot{C}}{N}\right) \Big/ \left(\frac{C}{N}\right) = \frac{\dot{C}}{C} - \frac{\dot{N}}{N} = \frac{\eta(1-\beta)L_E - \rho}{\theta} - \eta(L - L_E)$$



Balanced Growth Path (BGP):

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} = \eta L_R = g^* = \frac{\eta(1-\beta)L - \rho}{\theta + 1 - \beta}.$$

Again, $g^* \Uparrow$, as $L \Uparrow$ (scale effect), $\theta \Downarrow, \rho \Downarrow$ (taste effects), and $\eta \Uparrow$ (the efficiency effect).

Knowledge-Spillover Model without "Scale Effects"; Acemoglu (Ch13.3)

In the above model,

- In order to sustain growth, it is not enough for the R&D productivity to increase in *N*. It is essential that it increases at the *same* rate with *N*. Innovation and hence growth will stop eventually, if $\dot{N} = \eta N^{\lambda} L_R$ ($\lambda < 1$).
- In the data, however, the total amount of resources devoted to R&D appears to increase steadiliy, and yet there is no associated increase in the growth rate, which suggests $\lambda < 1$.
- The existence of balanced growth requires, among others, *L* is constant. If *L* grows over time, the economy will experience accelerating growth.
- A large population implies higher interest rate and a higher growth. Some argue, incl. Acemoglu, that this is rejected by data, in cross-sections of countries.

These points motivate some to examine a knowledge spillover model, modified as follows:

- R&D technology is by $\dot{N} = \eta N^{\lambda} L_R$ ($\lambda < 1$)
- Labor supply grows at a constant rate; $L_t = L_0 e^{nt}$.

Along the BGP, a constant share of the labor is allocated to R&D. Hence,

$$g = \frac{\dot{N}(t)}{N(t)} = \frac{\eta L_R(t)}{(N(t))^{1-\lambda}} = \frac{\eta L_R(0)\exp(nt)}{(N(0))^{1-\lambda}\exp(g(1-\lambda)t)} \propto \exp[(n-g(1-\lambda))t],$$

which is constant if and only if

$g = \frac{n}{1-\lambda}$. In per capita term, the growth rate is $g - n = \frac{\lambda n}{1-\lambda}$

Indeed, one could show that the equilibrium is indeed characterized by BGP.

Notes:

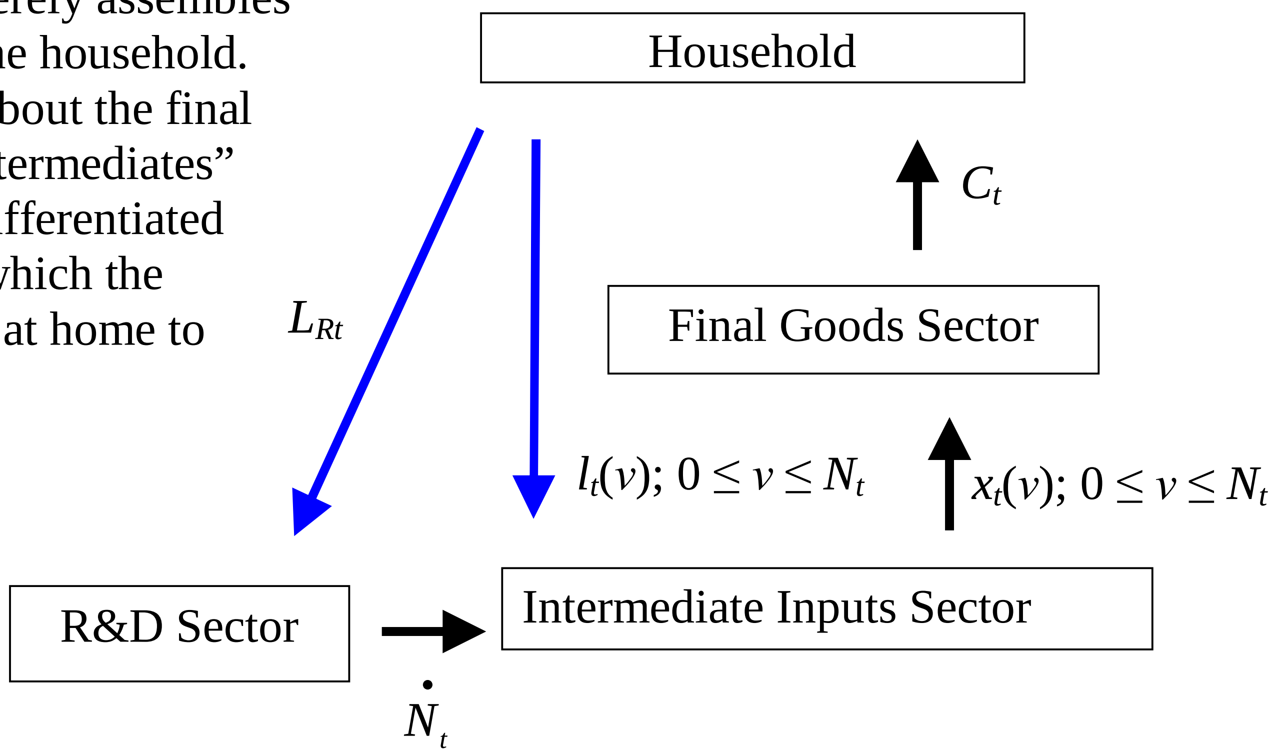
- The growth rate is no longer affected by θ, ρ (taste parameters), and η (the efficiency). In this sense, it is no longer "endogenous."
- The growth rate is also independent of L. In this sense, no scale effect on growth.
- However, there are two senses in which there are still scale effects.

➢A faster growth rate of L translates into a higher growth rate.

➢A large L leads to a higher output per capita.

Labor-for-intermediates Model: (Acemoglu; Ch.13.4)

- This was the first class of dynamic monopolistic competition model developed by Judd, Romer, Grossman-Helpman, and others.
- This specification is closest to the static Dixit-Stiglitz model, as both the fixed cost and the marginal cost of supplying intermediates are paid in labor.
- Final goods sector merely assembles "intermediates" for the household. So, we could forget about the final goods sector, and "intermediates" could be viewed as differentiated consumer products, which the household assembles at home to produce the utility.



Final (Consumption) Good Sector: numeraire;

$$C(t) = Y(t) = \overline{X}(t) = \left[\int_0^{N(t)} [x(\nu,t)]^{1-\frac{1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}}$$

Representative Household:

$$U = \int_0^{\infty} u[C(t)]e^{-\rho t}dt = \int_0^{\infty} \log C(t) e^{-\rho t} dt \quad \Rightarrow \frac{\dot{C}}{C} = r(t) - \rho$$

Note: "log" makes the marginal utility of each variety independent of other varieties.

Intermediate Input Sector:

Monopoly Pricing: $p^x(\nu,t)(1-1/\sigma) = \psi w(t) \Rightarrow p^x(\nu,t) = w(t)$ for all *v*.

$$\Rightarrow x(\nu,t) = x(t) \text{ \& } \pi(\nu,t) = x(t)w(t)/\sigma \text{ for all } v.$$

$$\Rightarrow C(t) = \overline{X}(t) = N(t)^{\frac{\sigma}{\sigma-1}} x(t) = N(t)^{\frac{1}{\sigma-1}} N(t)x(t).$$

From $P(t) \equiv \left[\int_0^{N(t)} [p(\nu,t)]^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} = N(t)^{\frac{1}{1-\sigma}} w(t) = 1,\ w(t) = N(t)^{\frac{1}{\sigma-1}}$.

Labor Market Constraint: $\int_0^{N(t)} l(\nu,t)d\nu + L_R(t) \leq L$

$$\Rightarrow L = (1-1/\sigma)N(t)x(t) + L_R(t) = L_E(t) + L_R(t),$$

where $L_E(t) \equiv (1-1/\sigma)N(t)x(t) = (1-1/\sigma)C(t)N(t)^{\frac{1}{1-\sigma}}$ is labor used in the intermediate production.

R&D Sector (Innovation): $\dot{N}(t) = \overline{\eta}(t)L_R(t) = \eta N(t)L_R(t)$.

If $\dfrac{\dot{N}(t)}{N(t)} = \eta L_R(t) > 0$, $\dot{N}V - wL_R = (\eta NV - w)L_R = 0 \Rightarrow V(t) = \dfrac{w(t)}{\eta N(t)} = \dfrac{1}{\eta}N(t)^{\frac{1}{\sigma-1}-1}$

Value of an Intermediate Firm: $\dot{V}(t) + \pi(t) = r(t)V(t)$.

$$\Rightarrow \frac{\dot{C}(t)}{C(t)} = r(t) - \rho = \frac{\dot{V}(t)}{V(t)} + \frac{\pi(t)}{V(t)} - \rho = \left(\frac{1}{\sigma-1} - 1\right)\frac{\dot{N}(t)}{N(t)} + \frac{\eta}{\sigma}N(t)x(t) - \rho$$

$$= \left(\frac{1}{\sigma-1} - 1\right)\frac{\dot{N}(t)}{N(t)} + \frac{\eta L_E(t)}{\sigma-1} - \rho$$

$$\Rightarrow \frac{\dot{L}_E(t)}{L_E(t)} = \frac{\dot{C}(t)}{C(t)} + \left(\frac{1}{1-\sigma}\right)\frac{\dot{N}(t)}{N(t)} = \frac{\eta L_E(t)}{\sigma-1} - \rho - \frac{\dot{N}(t)}{N(t)}$$

$$= \frac{\eta L_E(t)}{\sigma-1} - \eta(L - L_E(t)) - \rho = \eta\left[\frac{\sigma}{\sigma-1}L_E(t) - L\right] - \rho .$$

Balanced Growth Path (BGP): $\dot{L}_E = 0 \Rightarrow L_R = L - L_E = \frac{L}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\frac{\rho}{\eta}$

$$\Rightarrow g_N = \frac{\eta L}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\rho \qquad \& \qquad g_c = \frac{g_N}{\sigma-1} = \frac{1}{\sigma}\left(\frac{\eta L}{\sigma-1} - \rho\right).$$

Again,

- $(\sigma-1)g_C = g_N \Uparrow$, as $L \Uparrow$ (scale effect), $\rho \Downarrow$ (taste effect), $\eta \Uparrow$ (the efficiency effect).
- No transition path.

Note: Gancia-Zilibotti (Handbook chapter) discusses an alternative specification of "labor-for-intermediates" model, where the final (consumption) good is produced by the technology;

$$C(t) = Y(t) = \frac{1}{1-\beta}\left(\bar{X}(t)\right)^{1-\beta} F^{\beta} = \frac{1}{1-\beta}\left[\int_0^{N(t)} [x(\nu,t)]^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{1-\beta}{1-1/\sigma}} F^{\beta},$$

where F is the second factor (say, "land"), which is used only in the final goods sector and supplied inelastically by the representative household whose preferences are now given by**:**

$$U = \int_0^{\infty} u[C(t)] e^{-\rho t} dt = \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt.$$

Again, $\beta = 1/\sigma$ implies:

$$C(t) = Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} [x(\nu,t)]^{1-\beta} d\nu\right] F^{\beta},$$

ensuring that the marginal productivity of a particular variety independent of the amounts of other varieties used.