Economic Growth & Development: Part 6
Non-Homothetic Preferences

By Kiminori Matsuyama

Updated on 2011-05-05, 1:17:52 PM

Overview:

- Stone-Geary Preferences and Sectoral Compositions
- Beyond Stone-Geary: Hierarchical Demand and Income Distribution
- Non-proportional gains from variety
- Non-Constant Relative Risk Aversion
- North-South Trade: Ricardian Models
- North-South Trade: Monopolistic Competitive Models
- Non-homothetic intertemporal preferences
- Fertility Transition

Motivation:

Most growth models assume homothetic preferences (and quite often Cobb-Douglas).

- Homotheticity implies that, in cross-sections, the rich & the poor consume goods in the same proportions.
- With Cobb-Douglas, each sector accounts for a fixed share of the total expenditure.
- Taste for variety, attitude towards risk or saving for the future, etc. do not change as the households become rich.

Empirically, they are clearly false. Conceptually, too restrictive for thinking about many important issues related to growth and development.

- Engel's Law
- US, EU, and Japan are the three biggest markets for SUV; China, India, and Indonesia are the three biggest markets for motorbikes.
- Fisher-Clark-Kuznets thesis; as economies develop, sectoral compositions change; The decline of agriculture, the rise and fall of manufacturing, and the rise of service sectors.
- Prebisch-Singer thesis; the long run trend that TOT moves in favor of the rich North and against the poor South.

etc. etc.

The following shows that, in additively separable utility functions, any deviation from CES would give us non-homothetic preferences.

Proposition: Suppose that the utility function, $U : R_+^J \to R$, is quasi-concave, increasing, and separable, $U(x) = \sum_{j=1}^{J} u_j(x_j)$. Then, it is homothetic if and only if

$$u_j(x) = \alpha_j \frac{(x_j)^{1-\theta}}{1-\theta} + \beta_j \text{ for } \theta > 0, \neq 1; \text{ or } u_j(x) = \alpha_j \log(x_j) + \beta_j, \text{ where } \alpha_j > 0 \text{ for all } j.$$

Sketch of the Proof: From the homotheticity, $\frac{u_j'(\lambda_j x)}{u_1'(x)}$ is independent of $x > 0$ for all $\lambda_j > 0$ for all $j > 1$. By log-differentiating it w.r.t. x,

$$\frac{\lambda_j u_j''(\lambda_j x)}{u_j'(\lambda_j x)} = \frac{u_1''(x)}{u_1'(x)} \text{ for all } x > 0 \text{ and for all } \lambda_j > 0 \text{ for all } j > 1. \;.$$

By setting $\lambda_j = 1/x$, $\frac{x u_1''(x)}{u_1'(x)} = \frac{u_j''(1)}{u_j'(1)} \equiv -\theta < 0 \Rightarrow u_1'(x) = \alpha_1 (x)^{-\theta}$ where $\alpha_1 > 0$.

By setting $\lambda_j = 1$, $\frac{u_j''(x)}{u_j'(x)} = \frac{u_1''(x)}{u_1'(x)} = -\theta \Rightarrow u_j'(x) = \alpha_j (x)^{-\theta}$; $\alpha_j > 0$ for all j,

from which the result follows.

Note:

The separability is crucial for this result. One could easily construct homothetic preferences that are not additively separable. To see this, let $f_k : R_+^J \to R_+$ (k = 1, 2, …, K) is linear homogeneous and $g : R_+^K \to R$ is homothetic. Then, $U : R_+^J \to R$ defined by:

$$U(x) = g\left(f_1(x), f_2(x), \ldots, f_K(x)\right),$$

is also homothetic, although it is not generally additively separable.

Example 1: $U(x) = A(x_1)^{\alpha}(x_2)^{1-\alpha} + Bx_1$

Example 2: $U(x) = \alpha \log(x_1 + x_2) + \beta \log(x_2)$,

and so on.

Stone-Geary Preferences and Sectoral Compositions:

Stone-Geary Preferences:

$$U(x)=\sum_{j=1}^{J}\alpha_j \frac{(x_j-\gamma_j)^{1-1/\sigma}}{1-1/\sigma} \text{ for } \sigma>0, \neq 1; \quad \text{or} \quad U(x)=\sum_{j=1}^{J}\alpha_j \log(x_j-\gamma_j),$$

where $\alpha_j > 0$ for all *j*.

The household demand, under the budget constraint, $\sum_{j=1}^{J} p_j x_j^h \leq I^h$, takes form of:

$$p_j x_j^h = \Gamma_j(p) + B_j(p) I^h \text{ for each } j.$$

Notes:

- With $\Gamma_j(p) \neq 0$, the *average* propensity to consume, $p_j x_j^h / I^h$, monotonically decreasing (a necessity) or, monotonically increasing (a luxury) in I^h. **(i.e., non-homothetic).**
- But, the *marginal* propensity to consume, $\partial p_j x_j^h / \partial I^h = B_j(p)$, is independent of I^h, which allow for aggregation across households. Thus, we can talk about the representative household or the Household Sector.

A Simple Model of Sectoral Shifts: Consider the *J*-consumption goods sectors.

Representative Household with ***Stone-Geary Preferences***:

$$U(x) = \sum_{j=1}^{J} \alpha_j \frac{(x_j - \gamma_j)^{1-1/\sigma}}{1-1/\sigma} \text{ for } \sigma > 0, \neq 1; \quad \text{or} \quad U(x) = \sum_{j=1}^{J} \alpha_j \log(x_j - \gamma_j),$$

where $\alpha_j > 0$ for all *j*.

Technologies: CRS with $x_j = A_j L_j(t)$ where A_j is the productivity of sector-j.

Resource Constraint: $\sum_{j=1}^{J} L_j(t) = L(t)$, where $L(t)$ is the supply of the unique factor, increasing over time, & $\lim_{t\to\infty} L(t) = \infty$.

Notes:

- Alternatively, $L(t)$ can also be interpreted as Hicks-neutral technical change.
- Relative prices are determined solely by A_j's.
- No means for intertemporal resource allocation, so that the equilibrium is a sequence of the static equilibrium at each *t*, which changes as $L(t)$ changes.

Solving for Equilibrium Allocation: From the f.o.c., $\alpha_j (x_j(t) - \gamma_j)^{-1/\sigma} = \lambda(t)/A_j$,

$$\Rightarrow L_j(t) = x_j(t)/A_j = (\lambda(t))^{-\sigma}(\alpha_j)^{\sigma}(A_j)^{\sigma-1} + \gamma_j / A_j$$

$$\Rightarrow L(t) = (\lambda(t))^{-\sigma}\sum_{j=1}^{J}(\alpha_j)^{\sigma}(A_j)^{\sigma-1} + \sum_{j=1}^{J}(\gamma_j / A_j)$$

$$\Rightarrow \frac{L_j(t)}{L(t)} = \beta_j\left(1 - \frac{\sum_{k=1}^{J}(\gamma_k / A_k)}{L(t)}\right) + \frac{\gamma_j}{A_j L(t)}, \quad \text{where } \beta_j \equiv \frac{(\alpha_j)^{\sigma}(A_j)^{\sigma-1}}{\sum_{k=1}^{J}(\alpha_k)^{\sigma}(A_k)^{\sigma-1}}.$$

Example 1: $\gamma_j = 0$ for all *j*. Then, preferences are homothetic, and

$$\frac{L_j(t)}{L(t)} = \beta_j.$$

Under homothetic preferences, and with constant relative prices, sectoral compositions remain constant as the economy grows.

Example 2: $J = 2$; $\gamma_1 > 0$ & $\gamma_2 = 0$.

- $\dfrac{L_1(t)}{L(t)} = \beta_1 + \beta_2 \dfrac{\gamma_1}{A_1 L(t)}$, decreasing over time with $\lim_{t\to\infty} \dfrac{L_1(t)}{L(t)} = \beta_1$;
- $\dfrac{L_2(t)}{L(t)} = \beta_2 \left(1 - \dfrac{\gamma_1}{A_1 L(t)}\right)$, increasing over time with $\lim_{t\to\infty} \dfrac{L_2(t)}{L(t)} = \beta_2$.

Interpretation: Sector-1 is the food sector; and Sector-2 is everything else.

More generally,

- $\dfrac{L_j(t)}{L(t)}$ is monotonically increasing (decreasing) if $(1-\beta_j)\dfrac{\gamma_j}{A_j} < (>) \beta_j \sum_{k\neq j}^{J} \left(\dfrac{\gamma_k}{A_k}\right)$;
- $\dfrac{L_j(t)}{L(t)}$ is constant, iff $(1-\beta_j)\dfrac{\gamma_j}{A_j} = \beta_j \sum_{k\neq j}^{J} \left(\dfrac{\gamma_k}{A_k}\right)$;

and

- $\lim_{t\to\infty} \dfrac{L_j(t)}{L(t)} = \beta_j$.

Example 3: $J = 3$; $\gamma_1 > 0$, $\gamma_2 = 0$, & $\gamma_3 < 0$. Then,

- $\dfrac{\dot{L}_1(t)}{L(t)} < 0$, as $(1-\beta_1)\dfrac{\gamma_1}{A_1} > 0 > \beta_1\dfrac{\gamma_3}{A_3}$;
- $\dfrac{\dot{L}_3(t)}{L(t)} > 0$, as $(1-\beta_3)\dfrac{\gamma_3}{A_3} < 0 < \beta_3\dfrac{\gamma_1}{A_1}$;
- $\dfrac{\dot{L}_2(t)}{L(t)} > 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} > 0$; $\dfrac{\dot{L}_2(t)}{L(t)} < 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} < 0$; $\dfrac{\dot{L}_2(t)}{L(t)} = 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} = 0$.

Notes:

- Kongsamut, Rebelo, Xie (2001) embedded this type of preferences into a standard growth model to reconcile the Fisher-Clark-Kuznets thesis with the Kaldor's balance growth view. However, Example 3 suggests that this was a futile attempt.
  - ➢ Think of sector-1 as agriculture, sector-2 manufacturing, & sector-3 services.
  - ➢ We can never have the rise and fall of manufacturing (the inverted U-patterns), because its share has to be rising, declining, or constant.
  - ➢ The share of every sector will eventually converge to a constant.

Notes:

- Stone-Geary is highly tractable, because the marginal propensity to consume for all goods is independent of the income, which allows for aggregation across households.
- However, this same feature of Stone-Geary makes it also highly restrictive:
  - Income distribution across households has no effects on the aggregate demand
  - The average propensity to consume each good is either monotonically increasing (a luxury), monotonically decreasing (a necessity), or constant for all income levels, making it ill-suited for capturing the rich patterns of structural change
  - Asymptotically homothetic, suggesting that non-homotheticity is merely a transitional problem. This feature makes it difficult to fit the long-run data, as pointed out by Buera-Kaboski (JEEA 2009).

- Another criticism is that sectoral shifts may occur when different sectors experience different productivity growth, e.g., Baumol (1969) and Ngai-Pissaridis (2007). Hence, Stone-Geary or any other forms of non-homothetic preferences is not necessary for generating sectoral shifts. Acemoglu-Guerrieri (2008) also shows that, when sectors differ in capital intensity, sectoral shifts may occur as a result of capital accumulation; see Acemoglu (Ch.20.2).

Example 4: $J = 3$; $\gamma_1 = \gamma_2 = \gamma_3 = 0$. Let $A_j(t) = \exp(g_j t)$. Then,

$$\frac{L_j(t)}{L(t)} = \beta_j(t) \equiv \frac{\left(\alpha_j\right)^{\sigma}\left(A_j(t)\right)^{\sigma-1}}{\sum_{k=1}^{3}\left(\alpha_k\right)^{\sigma}\left(A_k(t)\right)^{\sigma-1}} = \left[1 + \sum_{k \neq j}\left(\alpha_k / \alpha_j\right)^{\sigma} e^{(g_k - g_j)(\sigma-1)t}\right]^{-1}.$$

For $g_1 > g_2 > g_3$ and $\sigma < 1$:

- $\frac{L_1(t)}{L(t)}$ is monotone decreasing and $\lim_{t\to\infty}\frac{L_1(t)}{L(t)} = 0$;
- $\frac{L_3(t)}{L(t)}$ is monotone increasing and $\lim_{t\to\infty}\frac{L_3(t)}{L(t)} = 1$;
- $\lim_{t\to\infty}\frac{L_2(t)}{L(t)} = 0$. $\frac{L_2(t)}{L(t)}$ is hump-shaped (inverted U), if $\left(\alpha_1\right)^{\sigma}(g_1 - g_2) > \left(\alpha_3\right)^{\sigma}(g_2 - g_3)$.

In the next model, differential productivity growth across sectors are combined with Stone-Geary preferences.

Agricultural Productivity & Industrialization: Matsuyama (1992); see also Acemoglu (Ch.20.3).

This model was motivated by the following questions;

- Why industrialization started early and progressed more rapidly in some countries?
- How does agricultural productivity affect industrialization? Did high agricultural productivity stimulate or hinder the industrial development? And how this relation depends on the country's openness to trade?

Two Sectors: (M)anufacturing & (A)griculture, competing for labor, whose endowment is normalized to $L = 1$,

$$X_M(t) = M(t)F(n(t)); \qquad F(0) = 0; F'(\bullet) > 0; F''(\bullet) < 0$$
$$X_A(t) = AG(1-n(t)); \qquad G(0) = 0; G'(\bullet) > 0; G''(\bullet) < 0.$$

$n(t)$: the employment share of the M-sector.

A: A-Sector productivity, exogenous (and time-invariant)

$M(t)$: M-Sector Productivity, which evolves as $\dot{M}(t) = \delta_M X_M(t)$ due to learning-by-doing (LBD), which is external to firms that generate them.

Notes:

- The assumption that productivity growth in the M-sector takes the form of LBD is not important. What is essential is that the industry size determines the pace of productivity growth.
- The assumption of no productivity growth in the A-sector is not important. It can grow exogenously, or through spillovers from the productivity growth in the M-sector, such as $A(t) = (M(t))^{\alpha} \exp(g_A t)$. What is essential is that it is independent of the size of the A-sector.

Preferences: Stone-Geary that captures the Engel's Law:

$$\beta \log(C_A(t) - \gamma) + \log(C_M(t)), \gamma > 0.$$

which implies:

$$C_A(t) - \gamma = \beta p(t) C_M(t),$$

where $p(t)$ is the price of the M-good (with the A-good being the numeraire).

Labor Market Equilibrium:

$$p(t)M(t)F'(n(t)) = w(t) = AG'(1-n(t)).$$

M-Productivity Growth: From $\dot{M}(t) = \delta_M X_M(t)$

$$g_M = \frac{\dot{M}(t)}{M(t)} = \delta_M F(n(t))$$

In Autarky:

$$C_M(t) = X_M(t) = M(t)F(n(t)) \quad \& \quad C_A(t) = X_A(t) = AG(1-n(t))$$

Combining with $C_A(t) - \gamma = \beta p(t) C_M(t)$ & $p(t)M(t)F'(n(t)) = AG'(1-n(t))$,

$$\phi(n(t)) = \frac{\gamma}{A}$$

where $\phi(n) \equiv G(1-n) - \frac{\beta F'(n)}{G'(1-n)} F(n)$ is a strictly decreasing function.

Hence,

- $n(t) = \phi^{-1}\left(\frac{\gamma}{A}\right) \equiv N(A)$ is constant, and strictly increasing in *A*.
- $g_M = \frac{\dot{M}(t)}{M(t)} = \delta_M F(N(A))$ is also strictly increasing in *A*.

Hence, more productive agriculture helps industrialization! This captures the idea that "Agricultural Revolution was a necessary precondition for Industrial Revolution."

Intuition:

- Engel's law implies that a certain amount of food must be produced first.
- With a high A, a smaller fraction of labor is needed to produce the minimal level of food, so that a larger fraction of labor can be allocated to the M-sector, which leads to higher productivity growth in the M-sector.

Notes:

- The other side of the coin is that a higher *A* is accompanied by a lower relative price of the A-good. With non-homothetic preferences, the demand for the A-good does not rise as fast as the productivity of the A-sector.

- Over time, the relative price of the A-good rises faster when A is high, because

$$p(t) = \frac{AG'(1 - N(A))}{M(t)F'(N(A))},$$

which is why the effects of non-homotheticity would not go away even the economy grows unbounded.

- Indeed, it is crucial that the relative prices respond endogenously. The effects of a higher agricultural productivity is very different if the relative price is exogenous (e.g., when the economy trades with the rest of the world and if it is too small to affect the world price.)

Small Open Economy Case:

- Imagine that this economy trades with the rest of the world (ROW), which may have different productivities; $A^* \neq A$ and $M^*(t) \neq M(t)$.
- This economy is (infinitesimally) small, so that ROW can be treated as the closed economy. Then, this economy faces the world relative price given by:

$$p(t) = \frac{A^* G'(1-N(A^*))}{M^*(t)F'(N(A^*))}, \qquad \text{where } \frac{\dot{M}^*(t)}{M^*(t)} = \delta_M F(N(A^*)).$$

Since $p(t) = \dfrac{AG'(1-n(t))}{M(t)F'(n(t))}$,

$$\frac{A}{M(t)} > \frac{A^*}{M^*(t)}$$

$$\Rightarrow n(t) < N(A^*) \qquad \text{(Due to comparative advantage)}$$

$$\Rightarrow \frac{\dot{M}(t)}{M(t)} < \frac{\dot{M}^*(t)}{M^*(t)} \qquad \Rightarrow \frac{A}{M(t)} >>> \frac{A^*}{M^*(t)}$$

Patterns of comparative advantage are hence reinforced!!

Notes:

- Suppose that, initially, $M(0) = M^*(0)$ but $A > A^*$. Then, this economy has comparative advantage in the A-sector, $n(0) < N(A^*)$. Hence, this economy lags behind the ROW in the M-sector, and will grow slowly.

- Thus, in a small open economy, high agricultural productivity leads to industrial stagnation. Or being unproductive in agriculture helps industrialization. This is consistent with regional patterns of industrialization. (E.g., Belgium industrialized ahead of Holland in the Low Countries; New England ahead of the American South.)

- The assertion often made by economic historians--"Without Agricultural Revolution that preceded it, Industrial Revolution would not have been possible."-- is perfectly consistent with the observation that, in cross-sections of countries, those countries that have higher agricultural productivity lagged behind in industrial development.

- More broadly, the A-sector may be interpreted as the Natural Resource Sector, in which case it offers the framework for thinking about the relation between the Natural Resource Abundance and Economic Growth.

- In an open economy case, the model suggests that even a temporary boom in the Natural Resource Sector can have permanent adverse effects in the M-sector, so-called "Dutch Disease."

Structural Change in an Interdependent World: A Global View of Manufacturing Decline: Matsuyama (JEEA 2009)

Here's another example suggesting that the time-series implications and cross-sectional implications of structural change should not be confused.

Two Countries: Home and Foreign (*) with labor endowment normalized as $L = L^* = 1$.
Home (Foreign) wage: w (w^*).

Three Sectors (Goods):

- Numeraire *(O)*; tradeable at zero cost;
  No production. Endowment of y units
- Manufacturing *(M)*; tradeable at zero cost;
  Home (Foreign) unit labor requirement in M; a_M (a_M*).
- Services (S): nontradeable;
  Home (Foreign) unit labor requirement in S: a_S (a_S*).

Prices:

- Home price of *S:* $p_S = a_S w$
- Foreign price of S: $p_S^* = a_S^* w^*$
- World Price of M: $p_M = a_M w = a_M^* w^*$

whenever both countries produce both M and S.

Home Households: *Stone-Geary* Preferences

$$U = \begin{cases} (c_O)^{\alpha}\left[\beta_M(c_M-\gamma)^{\theta}+\beta_S(c_S)^{\theta}\right]^{\frac{1-\alpha}{\theta}} & \text{for } \theta<1,\ \theta\neq 0, \\ (c_O)^{\alpha}(c_M-\gamma)^{\beta_M(1-\alpha)}(c_S)^{\beta_S(1-\alpha)} & \text{for } \theta=0. \end{cases}$$

If $\gamma > 0$, the income elasticity of demand for M is less than one.
If $\theta < 0$, the price elasticity of relative demand of M & S, $\sigma \equiv 1/(1-\theta)$, is less than one.

Home Budget Constraint: $c_O + p_M c_M + p_S c_S \leq y + w$

Home Demand Schedules for *O* and *S*:

$$c_O = \alpha(y+w-\gamma p_M), \qquad c_S = \frac{(\beta_S)^{\sigma}(p_S)^{-\sigma}(1-\alpha)(y+w-\gamma p_M)}{(\beta_M)^{\sigma}(p_M)^{1-\sigma}+(\beta_S)^{\sigma}(p_S)^{1-\sigma}}.$$

Likewise,

Foreign Demand Schedules for *O* and *S*:

$$c_O^* = \alpha(y+w^*-\gamma p_M), \qquad c_S^* = \frac{(\beta_S)^{\sigma}(p_S^*)^{-\sigma}(1-\alpha)(y+w^*-\gamma p_M)}{(\beta_M)^{\sigma}(p_M)^{1-\sigma}+(\beta_S)^{\sigma}(p_S^*)^{1-\sigma}}.$$

Market Clearing Conditions:

$$c_O + c_O^* = 2y; \qquad a_S c_S = 1 - L_M; \qquad a_s^* c_S^* = 1 - L_M^*,$$

where L_M ($L_M{}^*$) is Home (Foreign) Manufacturing Employment Share.

Equilibrium Employment Shares:

$$L_M = \frac{\frac{\alpha}{2}\left(1 - \frac{a_M}{a_M^*}\right) + \gamma a_M + \left(\frac{\beta_M}{\beta_S}\right)^{\sigma}\left(\frac{a_M}{a_S}\right)^{1-\sigma}}{1 + \left(\frac{\beta_M}{\beta_S}\right)^{\sigma}\left(\frac{a_M}{a_S}\right)^{1-\sigma}};$$

$$L_M^* = \frac{\frac{\alpha}{2}\left(1 - \frac{a_M^*}{a_M}\right) + \gamma a_M^* + \left(\frac{\beta_M}{\beta_S}\right)^{\sigma}\left(\frac{a_M^*}{a_S^*}\right)^{1-\sigma}}{1 + \left(\frac{\beta_M}{\beta_S}\right)^{\sigma}\left(\frac{a_M^*}{a_S^*}\right)^{1-\sigma}}.$$

Suppose either $\gamma > 0$ & $\sigma = 1$, or $\gamma = 0$ & $\sigma < 1$. Then,

Global Productivity Growth in M: $\frac{\Delta a_M}{a_M} = \frac{\Delta a_M^*}{a_M^*} < 0 \quad \rightarrow \quad \Delta L_M < 0;\ \Delta L_M^* < 0.$

National Productivity Growth in M: $\frac{\Delta a_M}{a_M} < 0 = \frac{\Delta a_M^*}{a_M^*} \quad \rightarrow \quad \Delta L_M\ ??0;\ \Delta L_M^* < 0.$

- The model suggests **a global trend of manufacturing decline due to productivity growth in manufacturing.**
- *However*, it does *not* suggest that faster productivity growth in a country would lead to faster decline in *its* manufacturing sector.
- **In *cross-sections of countries*, manufacturing productivity might be *positively* correlated with the manufacturing employment share, due to *comparative advantage.***

e.g. Higher productivity growth in the German or Japanese manufacturing sector means that the manufacturing sectors must decline *somewhere* in the world, but *not necessarily* in Germany or Japan.

Message: Imagine

- An economist wants to test the hypothesis that productivity growth in manufacturing causes a decline in the manufacturing employment.
- He develops a *closed* economy model.
- He runs cross-country regressions under the *false* maintained hypothesis that each country is *in autarky*.

Then, he would find the evidence that reject the hypothesis *convincingly*, even though the hypothesis *is* correct.

Beyond Stone-Geary: Hierarchical Demands and Income Distribution

Recall that Stone-Geary preferences imply:

- *Average* propensity to consume varies (monotonically) with income.
- *Marginal* propensity to consume, however, is independent of income, which allows aggregation across households, hence we can assume the representative agent. household. **Very simple and convenient, but no effect of income distribution.**
- Asymptotically homothetic, suggesting that non-homotheticity is merely a transitional problem for a growing economy (unless the relative prices change as the economy grows or γ's change endogenously, like a habit formation).
- For each good, income elasticity of demand is either greater than one, or equal to one, or smaller than one at any income level. In other words, each good is either a *necessity*, or a *luxury*, or neither, for all households, regardless of their income levels. It is not possible to capture the idea that certain goods are necessities for some households, while luxuries for other households.

We now look at an alternative to Stone-Geary, Hierarchical Demand Systems.

Hierarchical Demand System in Competitive Models

Matsuyama (2002) developed a model to study:

- A mechanism behind the *Rise of Mass Consumption Societies*, where the economy grows as an increasingly large number of households enjoys an increasingly large number of goods.
- *Flying Geese Patterns*, where different industries take off one after another.
- Why some countries succeed while others fail in making such a transition.

A few quotes from Katona (1964), *The Mass Consumption Society*

> "The past few decades have seen the rise, here in America, of a new and unique phenomenon in human history, the mass consumption society."
>
> "Throughout the course of human history, poverty has been the rule, riches the exception. Societies in the past were called affluent when their ruling classes lived in abundance and luxury. Even in the rich countries of the past, the great majority of people struggled for mere subsistence. Today in this country minimum standards of nutrition, housing, and clothing are assured, not for all, but for the majority. Beyond these minimum needs, such former luxuries as homeownership, durable goods, travel, recreation, and entertainment are no longer restricted to a few. *The broad masses participate in enjoying all these things and generate most of the demand for them.*" (italics added)
>
> "We are rich compared with our grandparents and compared with most other peoples of the world. In fact, however, we are still a middle-class society, enjoying middle-class comforts. …. The drudgery of seeking subsistence has been supplanted for millions of people, not by abundance and indulgence, but rather by *a new concept of what are necessities and needs*." (italics added)

Key Features:

- Households differ in their income.
- Each product is indivisible. Each household either consumes one unit or none at all.
- *Hierarchical demands:* products are indexed according to the priority.
  - As their income go up, the households go down on their shopping list.
  - *Demand complementarities*. Lower prices of high-priority (lower-indexed) products increases demand for low-priority (high-indexed) products.
  - The poor consume a subset of what the rich consume.
- Growth driven by industry- or product-specific LBDs, without inter-industry technological spillovers. Yet, a take-off in one industry may trigger a take-off in the next because of demand complementarities. *Flying Geese*.
- Each product has gone through the same life-cycle. Initially consumed by the rich only, it gradually spreads to the poor. As the economy develops, each product changes from a luxury to an amenity, and then to a necessity.
- A two-way causality between the market size and productivity gains.
  - A larger market size → productivity gains and lower prices, which makes the products affordable to more households → a larger market size. Or:
  - A small market size → no productivity gains → small market size
- Some income inequality is needed for the economy to take off. But, with too much inequality, the process stops prematurely.

The Model:

Households (a measure *N***):** differ in the endowment of effective labor, distributed according to the cdf, $F(\bullet)$.

Goods: J+1 goods, (j = 0,1,...,J), and leisure.

- Leisure is divisible.
- Good zero is food; a homogenous, divisible good.
- J manufacturing (M) goods (j = 1,...,J). indivisible, come in discrete units.

Preferences: All the households have the same preferences:

$$U = \begin{cases} c & \text{if } c \leq 1 \\ 1 + \sum_{k=1}^{J} \left(\prod_{j=1}^{k} x_j \right) + \eta l & \text{if } c > 1 \end{cases}$$

c : the food consumption, l : leisure,

x_j an indicator, = 1 if j is consumed; = 0 if not.

- Food is a necessity, with the subsistence level normalized to be one.
- Households have a well-defined priority over the *M*-goods with lower indexed good being higher on their shopping list.
- Household demand for each *M*-good satiates after one unit.

Budget Constraint: $p_0 c + \sum_{j=1}^{J} p_j x_j + l \leq I$,

I: the household's income,
p_0: the price of food per unit, p_j: the unit price of j
leisure is a numeraire.

Individual Demand: Let $P_k \equiv \sum_{j=0}^{k} p_j$, the minimum income necessary to buy all the goods up to k. For a sufficiently small η, $\eta p_j < 1$ ($j = 1, 2,\ldots,J$),

$c = I/P_0$, $l = 0$, $x_j = 0$ ($j = 1,\ldots,J$), if $I < P_0$;
$c = 1$, $l = I - P_k$, $x_j = 1$ ($j \leq k$), $x_j = 0$ ($j \geq k + 1$) if $P_k \leq I < P_{k+1}$ ($k = 0,\ldots, J-1$);
$c = 1$, $l = I - P_J$, $x_j = 1$ ($j = 1,\ldots, J$) if $I \geq P_J$.

For $I < P_j$, good j is a luxury, which is beyond their means.
For $I > P_j$, good j is a necessity, with which they are already satiated.

What is essential that the marginal propensity to spend on a particular good is small when income is either very low or very high.

Aggregate Demand: Since only the households whose income exceed $P_j \equiv \sum_{k=0}^{j} p_k$ buy good-j, and no household buys more than one unit of good-j,

$$D_j = N[1 - F(P_j)] = N\left[1 - F\left(\sum_{k=1}^{j} p_k\right)\right].$$

- Depends on income distribution, F.
- Bounded from above by N. The market size for each M-good is limited by the number of households that can afford to consume it.
- Decreasing in p_k for $1 \le k \le j$ and independent of p_k for $k > j$. Demand complementarities running from low to high indexed goods.
- $D_1 \ge D_2 \ge \ldots \ge D_J$, since every household has the same priority across these goods.

Technology: Linear in labor:

$$L_0(t) = a_0 X_0(t)$$

$$L_j(t) = A^j(Q_j(t)) X_j(t); \quad \dot{Q}_j(t) = \delta_j\left(X_j(t) - Q_j(t)\right) \qquad (j = 1, 2, \ldots, J).$$

with industry-specific LBD in the M-goods where $A^j(Q_j(t))$ is decreasing in $Q_j(t)$, the cumulative experience in j.

Perfect competition ensures $p_0 = a_0$ & $p_j = A^j(Q_j(t))$ $\quad (j = 1, 2, \ldots, J)$.

Dynamics: $\dot{Q}_j = \delta_j\left(D_j - Q_j\right) = \delta_j\left\{N\left[1 - F\left(\sum_{k=1}^{j} A^k(Q_k)\right)\right] - Q_j\right\}$

which can be expressed in a more compact manner, as follows:

$$\dot{Q} = \Psi(Q)$$

where $Q = (Q_1, Q_2, \ldots, Q_J) \in [0, N]^J$.

Some General Properties
(P1): $[0,N]^J$ is positively invariant.
If the economy starts in $[0,N]^J$, the economy remains in $[0,N]^J$ forever.

(P2): $\Psi_{ij} \equiv \partial\Psi_i/\partial Q_j = 0$ if $i < j$; $\Psi_{ii} = \delta_i(D_{ii} - 1)$; $\Psi_{ij} = \delta_i D_{ij} \geq 0$ if $i > j$.
The system is

- *recursive*, since the dynamics of $(Q_1, \ldots, Q_j)$ is independent of $(Q_{j+1}, \ldots, Q_J)$ for all j.
- *cooperative* in the sense of Hirsch (1982), that is $\Psi_{ij} \geq 0$ for all $i \neq j$.

(P3): $M_+ \equiv \{Q \in [0,N]^J \,|\, \Psi(Q) \in \Re^J_+\}$ and $M_- \equiv \{Q \in [0,N]^J \,|\, -\Psi(Q) \in \Re^J_+\}$ are positively invariant. The cooperative system maintains the monotonicity of trajectories.

(P4): The set of steady states, $S \equiv \{Q \in [0,N]^J \,|\, \Psi(Q) = 0\} = \{Q \in [0,N]^J \,|\, Q = D(Q)\}$, is a nonempty, compact lattice, where the ordering is induced by $\Re^J_+$. Its greatest element is sup M_+ , its least element inf M_-. Tarski's (1955) fixed-point theorem applied to Q=D(Q).

(P5): For any initial condition, $Q(0) \in [0,N]^J$, $\lim_{t\to\infty} Q(t) \in S$.

(P6): If $Q(0) = (0,0,\ldots,0)$, $Q(t) \in M_+$ for all $t > 0$, and $\lim_{t\to\infty} Q(t) = \inf M_-$.
The first follows from $Q(0) = (0,0,\ldots,0) \in M_+$ and the positive invariance of M_+. The second follows from (P4).

1st part of (P6): If the economy starts with little manufacturing experiences, all the industries grow monotonically in productivity. Since $D_j(Q)$ is increasing in Q for all j and $D_1 \geq D_2 \geq \ldots \geq D_J$, the dynamics shows the Flying Geese pattern, if it starts close to =(0,…,0).
2nd part of (P6); Monotone growth of industries may stop prematurely and the economy may fail to develop to reach its full potential. If S contains more than one element, the economy will be trapped into the lowest of them. The economy may be trapped.

To sum up, the dynamics show the Flying Geese pattern, in which a series of industries take off one after another. How high they can fly depends on the structure of the economy.

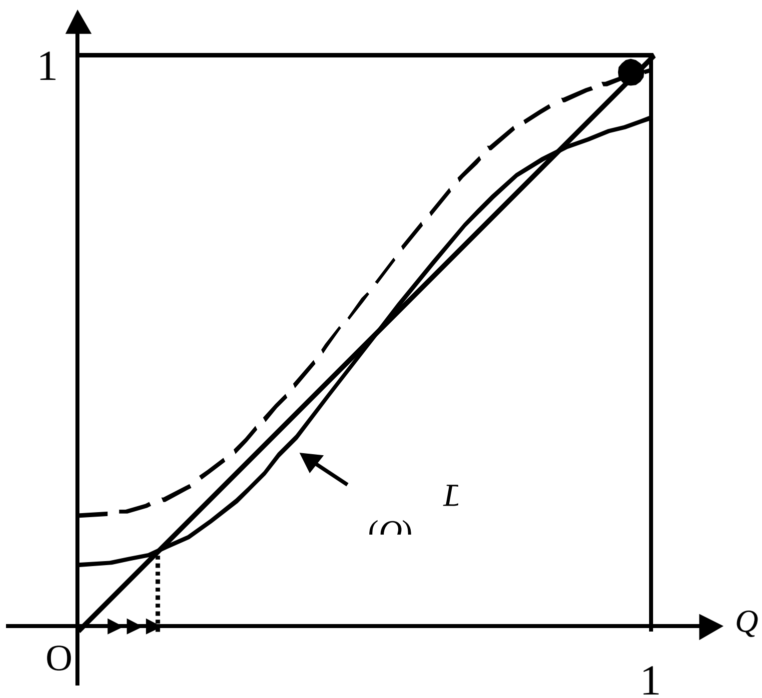
1. There may be multiple steady states, and, in that case, the economy will converge to the lowest of them.
2. Even if the steady state is unique, its level may be low. In other words, the economy may fail to transform itself to a mass consumption society.

Some important questions need to be addressed.

1. What determines the structure of S?
2. How does it depend on income distribution?
3. What kind of redistributive policies, if any, could eliminate low-level steady states?

For these questions, we need to look at special cases.

Case of $J = 1$: $\dot{Q} = \delta[D(Q) - Q]$, where $D(Q) \equiv 1 - F(a_0 + A(Q))$, $Q(0) = 0$.
($N = 1$ for simplicity.)



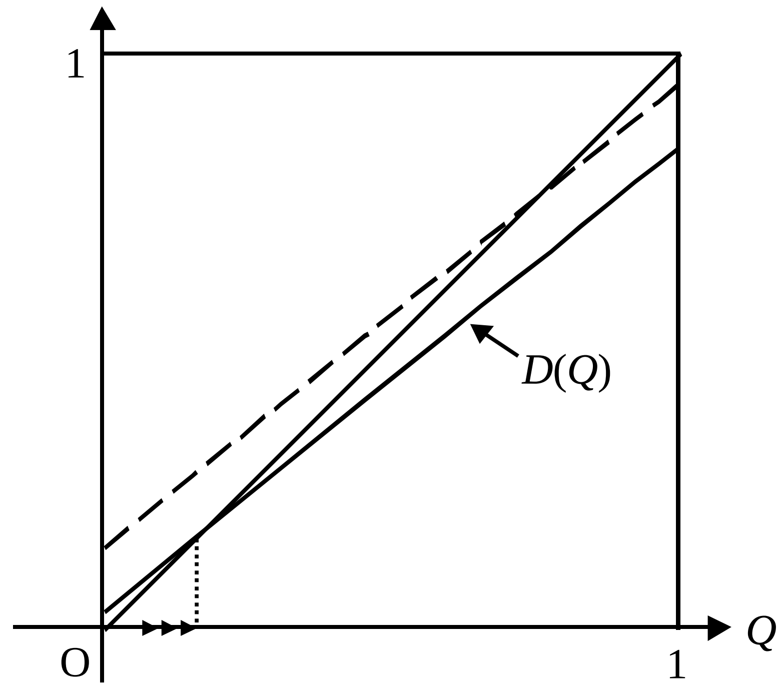
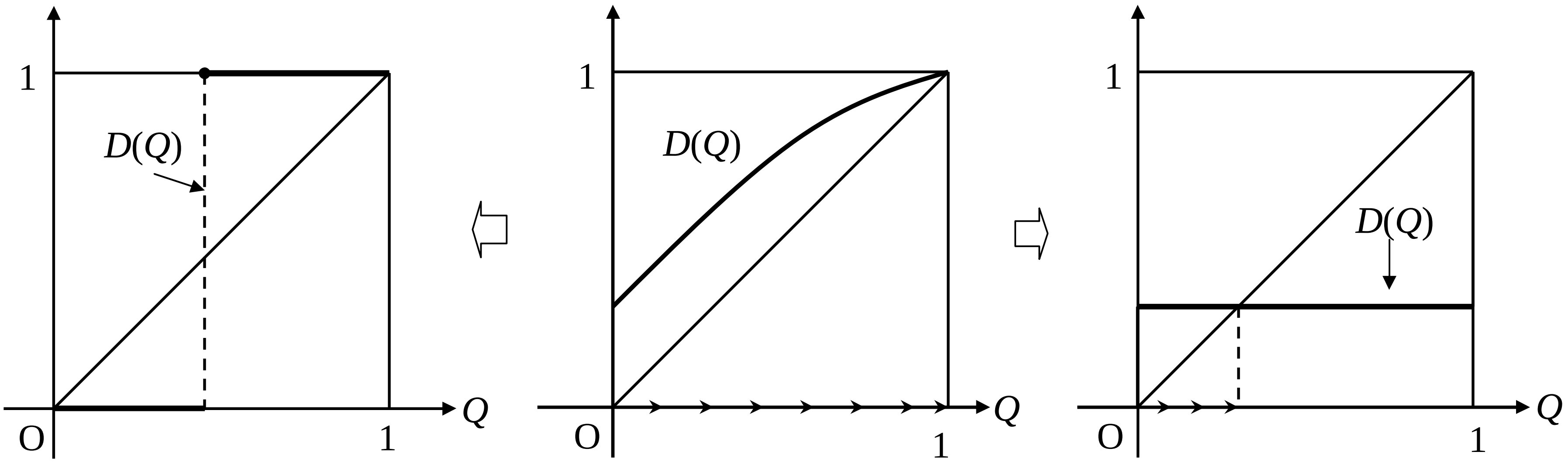


Figure shows how a small shift in the D curve could lead to a big change in the lowest steady state.

- A change in agricultural productivity, or a food aid, the Marshall Plan
- A first-order stochastic dominance

Effects of Income Inequality

- The standard measure or notion of inequality, e.g., the Gini coefficient the seconc-order stochastic dominance, are of no use.
- One can say, however, neither too much equality nor too much inequality give rise to a mass consumption society.



Starting from the situation depicted by the middle panel,

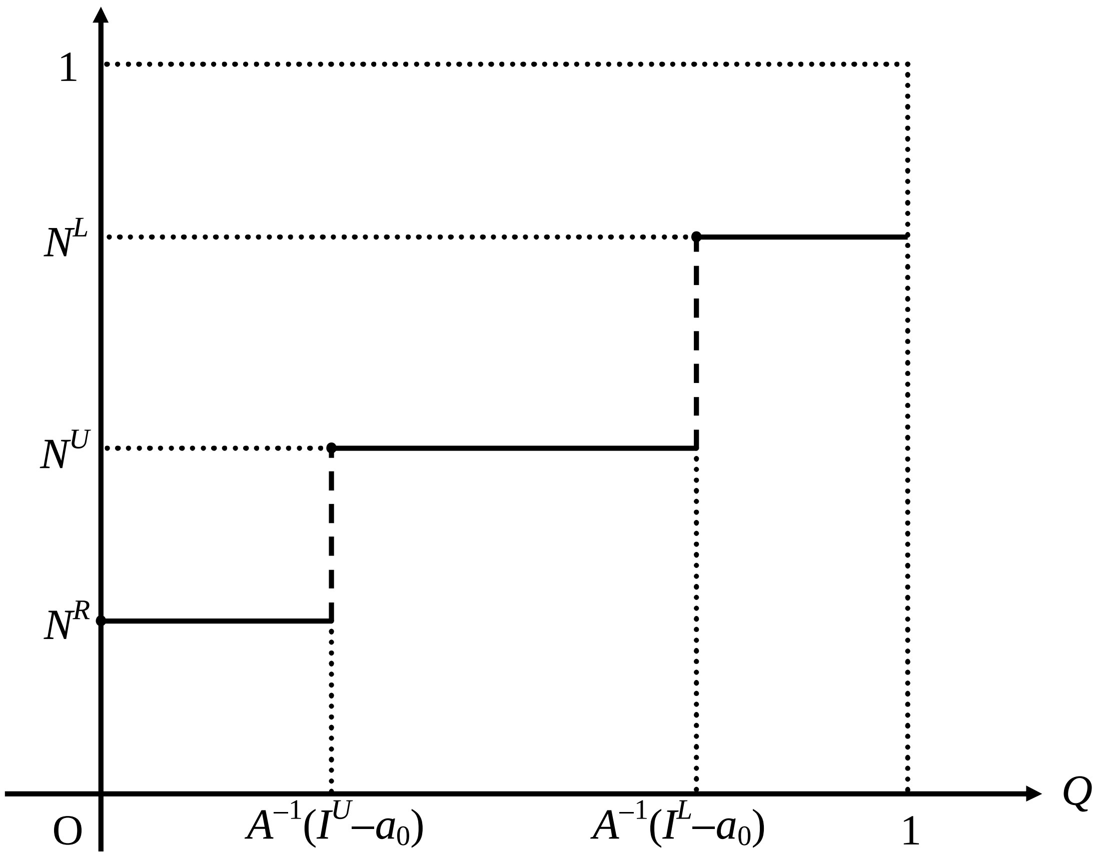
- The left panel shows the effect of Redistributing to perfect equality. A trap at Q = 0.
- The right panel shows the effect of redistributing to a polarized distribution. The process stops prematurely.

Effects of Income Redistribution: Case of Four Classes

Rich (R), Upper-middle (U), Lower-middle (L), the Poor (P).

$N^R < N^U < N^L < N^P = 1$
$I^R > I^U > I^L > I^P$,
$I^R > a_0 + A(0)$; $I^P < a_0 + A(1)$.

D curve in the Four-Class Economy
($J = 1$)

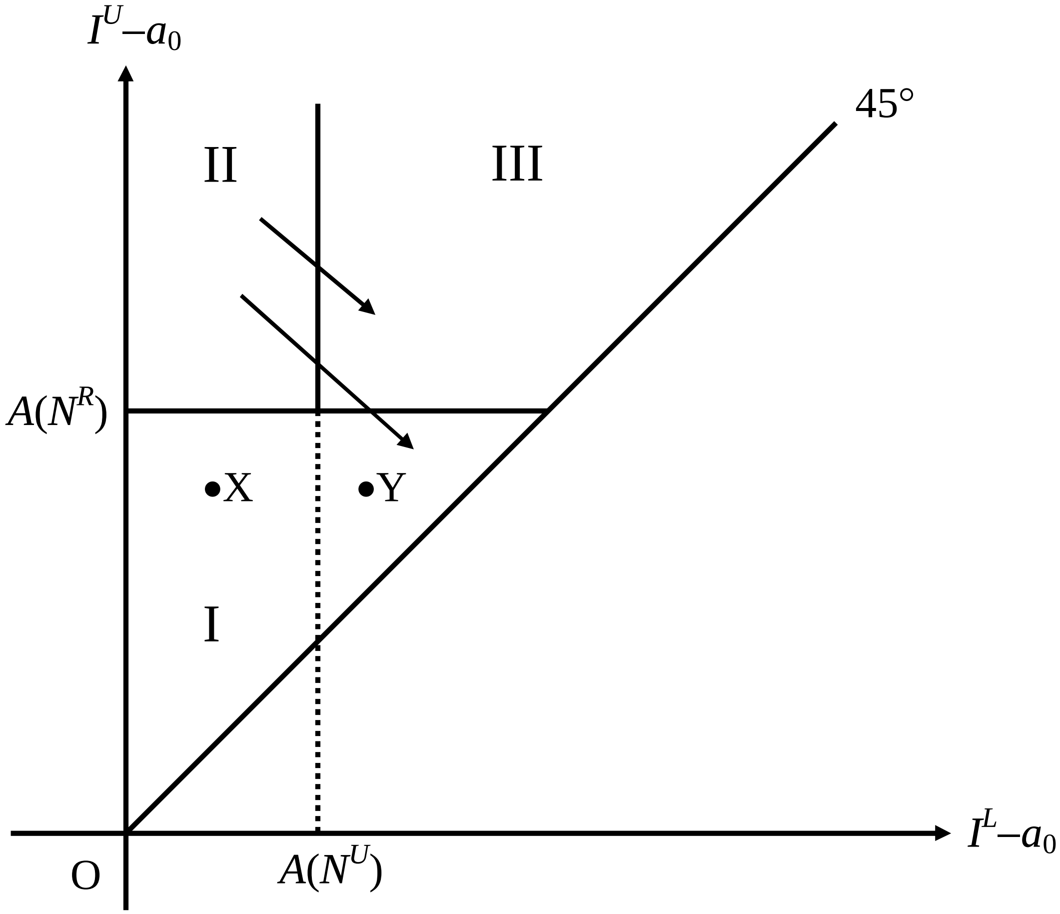


(Lowest) Steady State in the Four-Class Economy (J = 1)

I. If $I^U < a_0 + A(N^R)$, $\lim_{t\to\infty} Q(t) = N^R$,

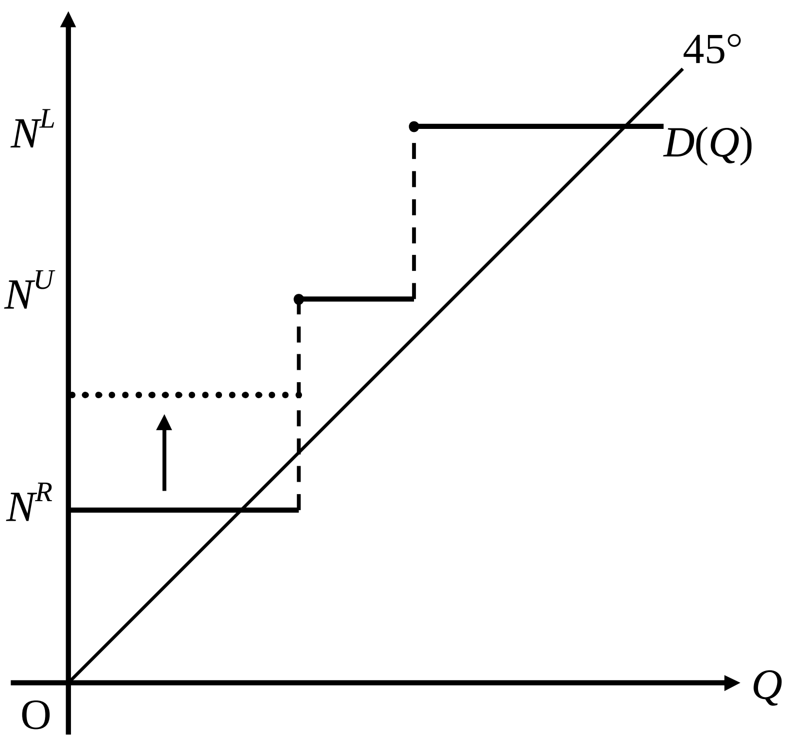
II. If $I^U \geq a_0 + A(N^R)$ and $I^L < a_0 + A(N^U)$, $\lim_{t\to\infty} Q(t) = N^U$;

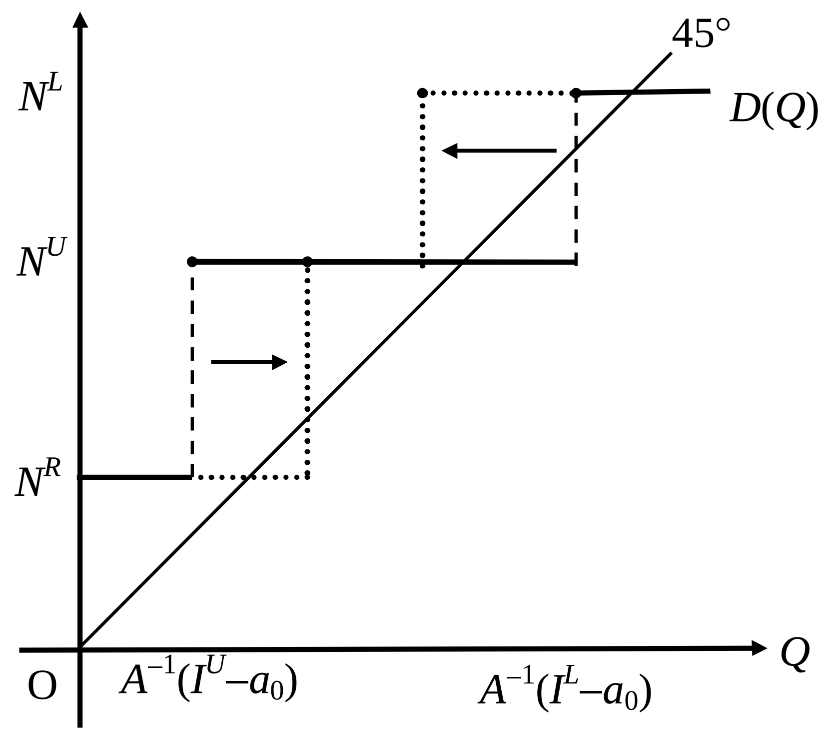
III. If $I^U \geq a_0 + A(N^R)$, $I^L \geq a_0 + A(N^U)$, $\lim_{t\to\infty} Q(t) = N^L$.



On the left panel:
Some U become R at the expense of P.
A higher N^R lowers $A(N^R)$
At X, I → II; At Y, I → III.
The rich's wealth *trickles down*.

On the right panel
An income transfer from U to L.
Narrower gaps between U & L, II → I
Wider gaps between R & U, II → III





Case of $J = 2$. **(unfinished)**

$$\dot{Q}_1 = \delta_1 [D_1(Q_1) - Q_1] \, ; \qquad \dot{Q}_2 = \delta_2 [D_2(Q_1, Q_2) - Q_2]$$

where $D_1(Q_1) \equiv 1 - F(a_0 + A_1(Q_1)) \geq D_2(Q_1, Q_2) \equiv 1 - F(a_0 + A_1(Q_1) + A_2(Q_2))$.

Figure 8

Dynamics of Q_1 can be analyzed as a one-dimensional system.

Dynamics of Q_2, depends on Q_1:

$D_2(Q_1, Q_2)$ shifts up from $D_2(0, Q_2)$ to $D_2(Q_1^*, Q_2)$.

A Trickle-up process causes Flying Geese pattern.

Figure 9 (The Phase Diagram) The Effects of a decline in a_0

The case of four classes, $I^R > a_0 + A_1(0) + A_2(0)$ and $I^P < a_0 + A_1(1)$.

$A_1(N^R) < A_1(N^L) + A_2(N^R)$; $A_1(N^U) < A_1(N^L) + A_2(N^U)$.

I. If $I^U - a_0 < A_1(N^R)$, $\lim_{t\to\infty} Q(t) = (N^R, N^R)$.

II. If $A_1(N^R) \leq I^U - a_0 < A_1(N^U) + A_2(N^R)$; $I^L - a_0 < A(N^U)$, $\lim_{t\to\infty} Q(t) = (N^U; N^R)$.

III. If $A_1(N^R) \leq I^U - a_0 < A_1(N^L) + A_2(N^R)$; $I^L - a_0 \geq A_1(N^U)$, $\lim_{t\to\infty} Q(t) = (N^L, N^R)$.

IV. If $I^U - a_0 \geq A_1(N^U) + A_2(N^R)$; $I^L - a_0 < A(N^U)$, $\lim_{t\to\infty} Q(t) = (N^U; N^U)$.

V. If $I^U - a_0 \geq A_1(N^L) + A_2(N^R)$; $A_1(N^U) \leq I^L - a_0 < A_1(N^L) + A_2(N^U)$, $\lim_{t\to\infty} Q(t) = (N^L, N^U)$.

VI. If $I^U - a_0 \geq A_1(N^L) + A_2(N^R)$; $I^L - a_0 \geq A_1(N^L) + A_2(N^U)$, $\lim_{t\to\infty} Q(t) = (N^L, N^L)$,

Figure 10

1. Some *U* become *R* at the expense of P
   (I to II, from II to IV, from I to III, from III to V or from III to IV.)
2. Income Transfer from *U* to *L*

i) Narrower gaps between *U* and *L* help the trickle-down process to reach *L* from II to III or from V to VI.

ii) Wider gap between *R* and *U* prevent the trickle-down to reach *U* from II to I or from V to III.

iii) The new possibility: *U* could gain from the income transfer, due to the trickle-down from *L*.(from II to V or to VI)

Buera-Kaboski: “The Rise of Service Economy”

Buera-Kaboski: “Scale and Origins of Structural Change”

Monopolistic Competition without Dixit-Stiglitz (unfinished)

Föllmi-Zweimüller (RES 2006) incorporated non-homothetic preferences with hierarchical needs in a horizontal innovation model.

Key Features:

As in Matsuyama (JPE 2002),

- Households differ in their income.
- Each product is indivisible. Each household either consumes one unit or none at all.
- Hierarchical preferences.
- The poor consume only a subset of what the rich consume.
- Each product has gone through the same life-cycle. Initially, the product is consumed by the rich only, and as the economy grows, it will spread to the poor.

Unlike Matsuyama (2002)

- Dynamic monopolistic competitive model of product innovation with knowledge-spillovers, *a la* Romer.
- Characterize the BGP, along which new products are innovated in the order of indices.
- In this setup, they showed that greater inequality leads to a higher equilibrium balanced growth rate.

Preferences: $U(0) = \int_0^\infty \frac{u(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho(t-\tau)} dt$ where $u(t) = \int_0^{N(t)} j^{-\gamma} c(j,t) dj$

$N(t)$; the range of differentiated consumer products innovated by time t.

$c(j,t)$; an indicator function, = 1 if product *j* is consumed, = 0 otherwise.

Budget Constraint of a Household *h*:

$$\int_0^\infty \left[\int_0^{N(t)} p(j,t)c(j,t)dj\right] e^{-R(t)} dt \le V_h(0) + \int_0^\infty w(t) l_h e^{-R(t)} dt \equiv W_h(0)$$

$p(j,t)$; the price of product *j;* $R(t) \equiv \int_0^t r(s)ds$; cumulative discount rate.

Wealth Distribution: Households differ only in $W_h(0)$.

Two classes of households, the rich (*R*) and the poor (*P*).

$\beta < 1$*:* the fraction of the poor;

$\theta < 1$*:* the household wealth of the poor relative to the average.

$\Rightarrow \frac{1-\beta\theta}{1-\beta} > 1$: the household wealth of the rich (relative to the average).

$$\Rightarrow \frac{W_R(0)}{W_P(0)} = \frac{1-\beta\theta}{(1-\beta)\theta} > 1$$

FOC: $c_h(j,t)=1$ iff $p(j,t) \le z_h(j,t) \equiv (j)^{-\gamma} e^{R(t)-\rho t}(u_h(t))^{-\sigma} / \mu_h$ (h= P or R)

where μ_h is a constant (the Lagrange multiplier).

- $z_h(j,t)$ is the willingness to pay for product j by household h, decreasing in j.
- Unless $p(j,t)$ declines faster than $z_h(j,t)$ in j, each household buys lower-indexed products. Thus, household h buys the first $N_h(t)$ products; $c_h(j,t)=1$ for $j \le N_h(t)$, $c_h(j,t)=0$ for $j > N_h(t)$. We assume that this is the case, and verify later.

$$\Rightarrow u_h(t) = \int_0^{N_h(t)} j^{-\gamma} dj = \frac{(N_h(t))^{1-\gamma}}{1-\gamma}$$

$$\Rightarrow \frac{\dot{z}_h(j,t)}{z_h(j,t)} = r(t) - \rho - \sigma(1-\gamma)\frac{\dot{N}_h(t)}{N_h(t)} \quad \& \quad \frac{\dot{z}_h(N_h(t),t)}{z_h(N_h(t),t)} = r(t) - \rho - [\sigma(1-\gamma)+\gamma]\frac{\dot{N}_h(t)}{N_h(t)}$$

Clearly, $\frac{z_R(j,t)}{z_P(j,t)} = \frac{\mu_P(u_R(t))^{-\sigma}}{\mu_R(u_P(t))^{-\sigma}} = \left(\frac{\mu_P}{\mu_R}\right)\left(\frac{N_R(t)}{N_P(t)}\right)^{-\sigma(1-\gamma)} > 1$ & $\frac{N_R(t)}{N_P(t)} > 1$ → $\mu_P > \mu_R$.

Technology:

- Labor is the only factor of production. Its total supply is normalized to one.
- Labor productivity improves due to knowledge-spillovers from innovation *a la* Romer.
  - Inventing a new product requires $\tilde{F}(t) = F / N(t)$ units of labor;
  - Producing each product requires $\tilde{b}(t) = b / N(t)$ per unit.
  - Normalize the marginal cost to one, so that $\tilde{b}(t)w(t) = 1$ → $w(t) = N(t)/b$.

Monopoly Pricing: Since $z_P(j,t) < z_R(j,t)$, the firms sell either:

- to everyone by setting $p(j,t) = z_P(j,t)$ to earn $\pi(j,t) = z_P(j,t) - 1$; or
- to only the rich by setting $p(j,t) = z_R(j,t)$ to earn $\pi(j,t) = (1-\beta)(z_R(j,t) - 1)$; or
- to no one if $z_R(j,t) < 1$.

➔

$$p(j,t) = z_P(j,t) \quad \text{with } \pi(j,t) = z_P(j,t) - 1 \qquad \text{for } j \le N_P(t),$$

$$p(j,t) = z_R(j,t) \quad \text{with } \pi(j,t) = (1-\beta)(z_R(j,t) - 1) \qquad \text{for } N_P(t) < j \le N_R(t)$$

and

$$(1-\beta)(z_R(N_P(t),t) - 1) = z_P(N_R(t),t) - 1;$$

$$N_R(t) = N(t) \text{ if } z_R(N(t),t) > 1; \text{ otherwise, } z_R(N_R(t),t) = 1 \text{ if } N_R(t) < N(t).$$

Note: This indeed means that $p(j,t)$ does not decline faster than $z_h(j,t)$ in *j*, as assumed earlier, so that the consumption patterns follow the hierarchy.

Labor Resource Constraint:

$$1 = \tilde{F}(t)\dot{N}(t) + \tilde{b}(t)[N_P(t) + (1-\beta)(N_R(t) - N_P(t))] = Fg(t) + b[\beta n_P(t) + (1-\beta)n_R(t)]$$

where $g(t) \equiv \dfrac{\dot{N}(t)}{N(t)}$ and $n_h(t) \equiv \dfrac{N_h(t)}{N(t)}$.

Balanced Growth Path (BGP): We now look for the BGP, such that

- $g(t) = g > 0$;
- $n_P(t) = n_P < n_R(t) = n_R \leq 1$;
- $r(t) = \rho + [\sigma(1-\gamma) + \gamma]g$, which implies $z_h(nN(t), t)$ is constant for any $n \leq 1$.

$$\Rightarrow z_P(N_P(t), t) \equiv z_P < z_R(N_R(t), t) \equiv z_R; \qquad \frac{\dot{z}_h(j,t)}{z_h(j,t)} = \gamma g$$

Life-cycle of product j along the BGP: Let $J(t) \equiv j / N(t)$, which declines at the rate, *g*.

- At $J(t) = 1$, product *j* is invented.
- At $J(t) = n_R$, the rich starts buying product *j* at the price, z_R.
- Then, the price increases at the rate, γg, until
- It reaches to $z_R(n_R / n_P)^{\gamma g}$ at $J(t) = n_P$, when the price is lowered to z_P and the poor also starts buying.
- From then on, the price increases at the rate, γg.

Free Entry to Innovation: At time *t*, *j* = *N*(*t*) is innovated, so that

$$\int_t^{\infty} \pi(N(t),\tau) e^{r(t-\tau)} d\tau = w(t)\tilde{F}(t) = F / b \quad \text{if} \quad \dot{N}(t) > 0 .$$

LHS is independent of *t*

(Continue…)

Föllmi-Würgler-Zweimüller; "Macroeconomics of Model T"

Saint-Paul (EJ)

Non-Constant Relative Risk Aversion:

Non-Homothetic Preferences in Ricardian Models of North-South Trade

Matsuyama (2000)

Key Features:

- Goods are indexed according to priority. As their incomes go up, the households go down on their shopping list. The rich consume more variety of goods than the poor.
- *Asymmetric demand complementarities* across goods. Lower prices of high-priority (lower-indexed) goods increases demand for low-priority (high-indexed) goods, while lower prices of low-priority goods would not increase demand for high-priority goods.
- South (North) has comparative advantage in high-priority (low-priority) goods, hence specializing in goods with lower (higher) income elasticities of demand.

Main Results:

- The ToT move against South and ***product cycles*** occur due to a faster population growth in South, a uniform productivity gains in South, and a global productivity gains.
- The welfare gains of productivity growth are unevenly distributed. North can capture all the benefits of its own uniform productivity growth, while South may lose from its own uniform productivity growth. ***(Immiserizing Growth)***
- South's *domestic* policy, which distributes income from the rich to the poor shifts the demand composition towards its own goods and hence, improve its terms of trade. This effect could be so large that *every* household in South may be better off at the cost of North. ***(Transfer Paradox)***

Basic Model

Two Countries: Home (South) and Foreign (North)*. Foreign Labor as the *numeraire.*

***Technology*:** A continuum of goods, $z \in [0,\infty)$.

(A1) $A(z) \equiv a^*(z)/a(z)$ is continuous and strictly decreasing in $z \in [0,\infty)$.

$$p(z) = wa(z),\ z \in [0,m];\ \ p(z) = a^*(z),\ z \in [m,\infty), \text{ where}$$

(PT) $w = A(m)$.

As in Dornbusch-Fischer-Samuelson (DFS), except that *the goods space is open-ended.*

Households: N household at Home; N^* households at Foreign

- A Home household with h units of labor earns wh; h is distributed as $F(h)$.
- A Foreign household with h^* units of labor earns h^*; h^* is distributed as $F^*(h^*)$.

Preferences: A Household with income I maximizes

$$V = \int_0^\infty b(z)x(z)dz \text{, subject to } \int_0^\infty p(z)x(z)dz \le I \text{,}$$

$b(z)$: the utility weight attached to good z
$x(z)$: an indicator function, equal to 1 if good z is consumed and zero otherwise.

Note: Goods come in discrete units and each household's desire of a particular good satiates after one unit.

(A2) $b(z)/a(z)$ and $b(z)/a^*(z)$ are both strictly decreasing in z,

This ensures that $b(z)/p(z)$ is strictly decreasing so that **every household consumes goods in the same order given by (A1).**

- South has comparative advantage in high priority goods, which even the poor consume.
- North has comparative advantage in low priority goods, consumed only by the rich.

Consumption and Utility Measures:

Let $E(z) \equiv \int_0^z p(s)ds = \int_0^z \min\{wa(s), a^*(s)\}ds$ and $B(z) \equiv \int_0^z b(s)ds$.

A Home household with income, wh, consumes all the goods in $[0, u(h)]$ and enjoys the utility $V(h) = B(u(h))$, where $u(h)$ is given by

Home Household's Budget Constraint: $E(u(h)) = wh$.

A Foreign household with income, h^*, consumes all the goods in $[0, u^*(h^*)]$ and enjoys the utility $V^*(h^*) = B(u^*(h^*))$, where $u^*(h^*)$ is given by

Foreign Household's Budget Constraint: $E(u^*(h^*)) = h^*$.

Note: $B(z)$ is a one-to-one mapping, so $u(h)$ & $u^*(h^*)$ may be viewed as utility measures.

Because each household whose income satisfies $I \geq E(z)$ consumes one unit of good z,

Aggregate Demand for good z: $Q(z) = N[1 - F(E(z)/w)] + N^*[1 - F^*(E(z))]$.

***Labor Market Equilibriums and the Balanced Trade*:**

Home Labor Market: $L = N\int_0^\infty h dF(h) = \int_0^m a(z)Q(z)dz$

$$\rightarrow \quad wL = wN\int_0^\infty h dF(h) = N\int_0^\infty \min\{wh, E(m)\}dF(h) + N^*\int_0^\infty \min\{h^*, E(m)\}dF^*(h^*).$$

Note: a Home household with h spends $\min\{wh, E(m)\}$ and a Foreign household with h^* spends $\min\{h^*, E(m)\}$ on the Home goods.

Foreign Labor Market: $L^* = N^*\int_0^\infty h^* dF^*(h^*) = \int_m^\infty a^*(z)Q(z)dz$

$$= N\int_0^\infty \max\{wh - E(m), 0\}dF(h) + N^*\int_0^\infty \max\{h^* - E(m), 0\}dF^*(h^*)$$

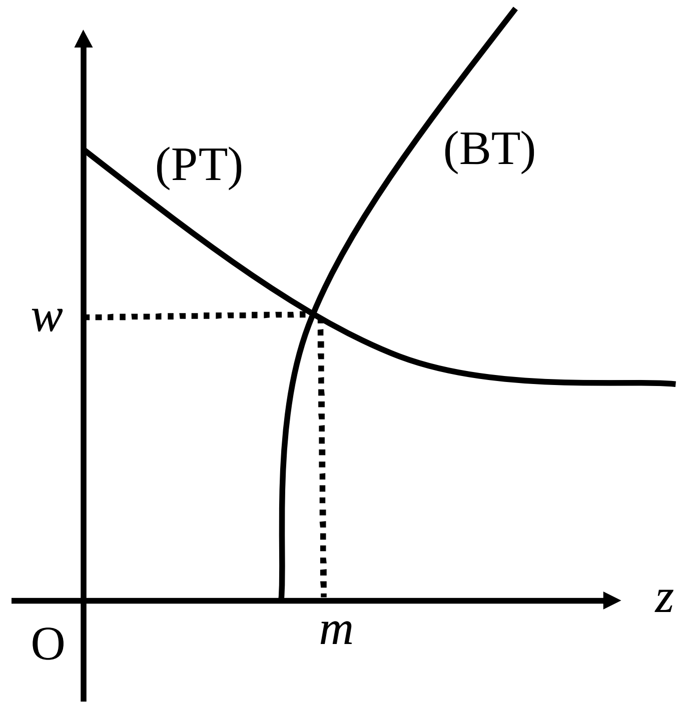
Note: Due to Walras's Law, two Labor Market Equilibriums are equivalent, which can be further rewritten as the Balanced Trade Condition (BT).

(BT) $$N\int_0^{\infty} \max\{h - \int_0^m a(s)ds, 0\}dF(h) = N^*\int_0^{\infty} \min\{\frac{h^*}{w}, \int_0^m a(s)ds\}dF^*(h^*).$$

(PT) and (BT) jointly determine *m* and *w*.

(BT) is upward-sloping, as long as some Foreign households are poor enough to consume only the Home goods.

If *w* is sufficiently small that all the Foreign households are rich enough to consume some Foreign goods, a small change in *w* does not affect the demand for Home labor. Hence, (BT) is vertical at *m*, satisfying $(N+N^*)\int_0^m a(s)ds = L$.



***A Comparison with the Dornbusch-Fischer-Samuelson (DFS) model*:**

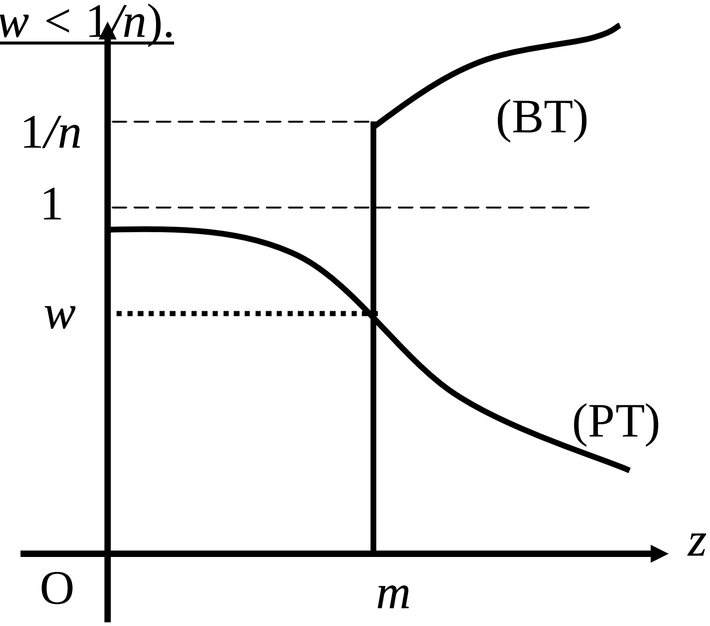
- (BT) depends on $F(h)$ and $F(h^*)$ here, but not in DFS.
- Here, the effects of L or L^* depend on whether they come from changing N or N^*, holding $F(h)$ and $F(h^*)$ constant, or changing $F(h)$ and $F(h^*)$, holding N or N^* constant.
- In DFS, (BT) goes to the origin. As the Home labor and Home goods become cheaper, demand for Home labor increases through *substitution effects*. To keep the Home labor market in equilibrium, Home's production shifts toward the bottom end of the goods spectrum. Here, as $w \to 0$ along (BT), m approaches a positive number, given by $(N+N^*)\int_0^m a(s)ds = L$. Demand for Home labor does not increase, when the Home goods prices go down. The total demand for each good is bounded by $N+ N^*$. Home must continue producing a certain range of goods to keep all the Home labor employed.
- In DFS, $a(z)$ and $a^*(z)$ do not appear in (BT), due to Cobb-Douglas. Here, they appear asymmetrically. Reducing $a(z)$ and hence the Home goods prices shifts the spending *away from* Home goods toward Foreign goods, leading to higher relative demand for Foreign labor. To restore the balance, Home must expand its range of production. On the other hand, $a^*(z)$ does not appear in (BT), because a reduction in $a^*(z)$, and Foreign goods prices only induce the household to buy other Foreign goods with higher indices, and hence does not cause a spending shift between Home and Foreign goods.

North-South Trade: Homogeneous Populations

- $h = h^* = 1$ for all households and hence $N = L$ and $N^* = L^*$. Denote $n \equiv N/(N+N^*)$.
- $A(z) < 1$ for all z, so that Foreign (North) is richer than Home (South), $w < 1$.

(BT) $\int_0^m a(s)ds = n$ (for $w < 1/n$).

(PT) $w = A(m)$.



Utilities (and the ranges of goods consumed), u and u^*, satisfy $m < u < u^*$ and are given by

$$E(u) = w\int_0^m a(s)ds + \int_m^u a^*(s)ds = w;$$

$$E^*(u^*) = w\int_0^m a(s)ds + \int_m^{u^*} a^*(s)ds = 1,$$

- $u < u^*$ because North is richer than South.
- $m < u$ because North imports $z \in [0,m]$ from South; hence, South must also imports something from North to keep the trade balanced.

Some Comparative Statics

Relative Population Sizes:

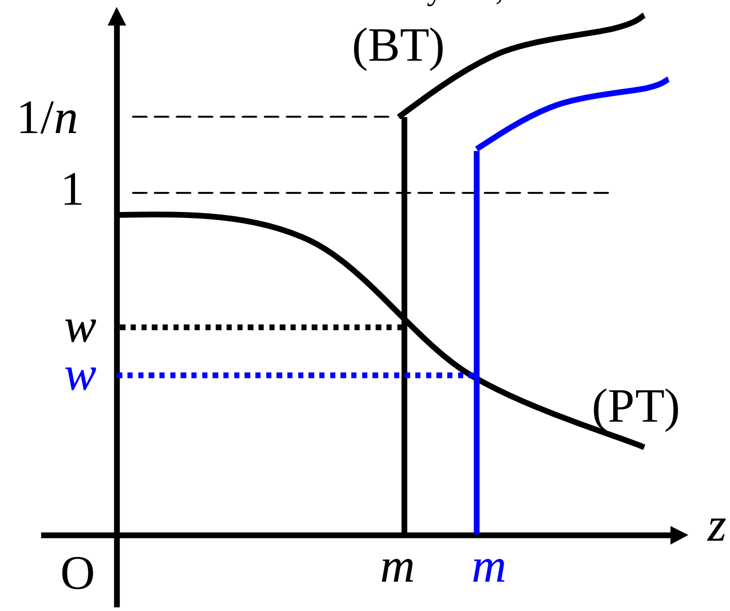
$n\uparrow \rightarrow m\uparrow$ and $w\downarrow$

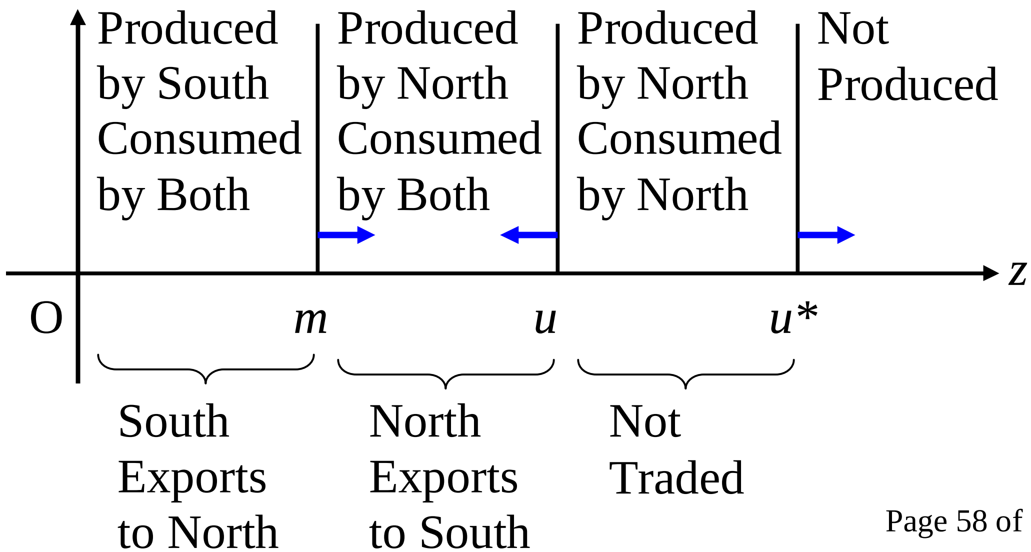
$a(m)dm = dn > 0$

$dw = A'(m)dm < 0,$

$a^*(u)du = (1-n)dw < 0,$

$a^*(u^*)du^* = -ndw > 0.$





Notes:

- The welfare effect is purely distributional, as $a^*(u)Ndu + a^*(u^*)N^*du^* = 0$.

- In DFS, the *vertical* distance of the (BT) curve depends on the relative country size. Hence, a higher n shifts (BT) *down*, causing a less-then-proportional decline in the Home relative wage. The Home share in the world income thus goes up. Here, the *horizontal* distance of the (BT) curve depends on the relative size. Hence, a higher n shifts (BT) *to the right*, which could cause a big ToT deterioration. Hence, the Home share in the world income may go down.

- If the population continues to grow faster in South than in North, South experiences a secular decline in its terms of trade, similar to Prebisch and Singer. The lower end of industries in North continuously migrate to South, and new industries are born continuously in the North, generating *Product Cycle* phenomena, similar to those discussed by Linder (1961) and Vernon (1966).

(Infinitesimal) Productivity Changes: $g(z) \equiv -\frac{\partial a(z)}{a(z)},\ g^*(z) \equiv -\frac{\partial a^*(z)}{a^*(z)}$

$$a(m)dm = \int_0^m g(s)a(s)ds,$$

$$dw = A'(m)dm + w\{g(m) - g^*(m)\}$$

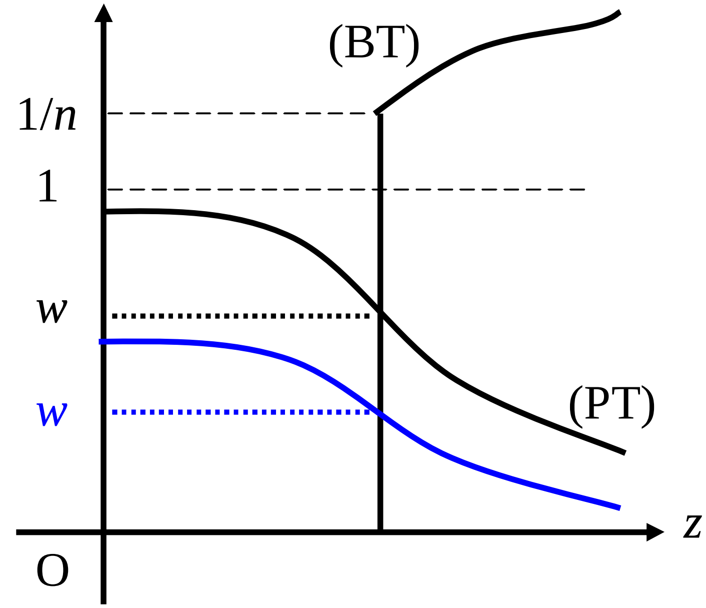
Welfare Implications:

$$a^*(u)du = w\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds + (1-n)dw$$

$$a^*(u^*)du^* = w\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds - ndw$$

The last terms represent the terms of trade effect, which determines how the overall welfare gains, $a^*(u)ndu + a^*(u^*)(1-n)du^* > 0$, are distributed between North and South.

Northern productivity growth: $g(z) = 0$; $g^*(z) > 0$.



$dm = 0$; $dw/w = -g^*(m) < 0$;

$$a^*(u)du = -(1-n)wg^*(m) + \int_m^u g^*(s)a^*(s)ds$$

$$= \int_m^u \{g^*(s) - g^*(m)\}a^*(s)ds;$$

$du^* > 0$;

Uniform Case: $g^*(z) = g^*$ for all $z \in [m, u]$, $du = 0$.

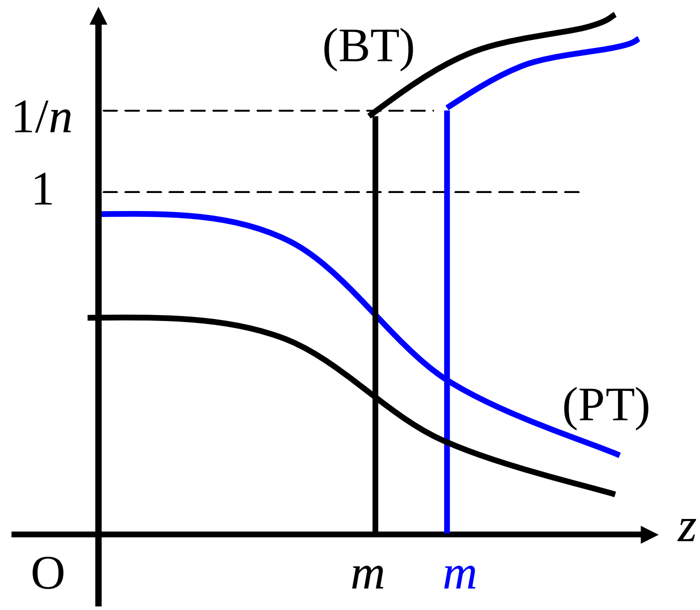
No spillover to South. A higher income of Northern households leads to more demand for the North goods. This is different from population growth in North, which leads to more demand for the South goods, hence benefits South.

Exercise: Examine the effects of increasing $h^* > 1$, while keeping $h = 1$. How is this different from the uniform productivity gains in North, discussed above?

Non-Uniform Case: $du > (<) 0$ if $g^*(z)$ is increasing (decreasing) over $[m, u]$.

South benefits when the change in North amplifies the existing patterns of comparative advantage, and loses otherwise.
Exercise: Discuss how this is different from Foreign non-uniform productivity gains in the DFS model.

***Southern productivity growth*:** $g(z) > 0$; $g^*(z) = 0$.



$$a(m)dm = \int_0^m g(s)a(s)ds > 0$$

$$dw = A'(m)dm + wg(m)$$

$$a^*(u)du = (1-n)dw + w\int_0^m g(s)a(s)ds\ .$$

$$a^*(u^*)du^* = -\frac{nA'(m)}{a(m)}\int_0^m g(s)a(s)ds + w\int_0^m \{g(s)-g(m)\}a(s)ds$$

Uniform Case: g(z) = g for all $z \in [0,m]$,

$$a(m)dm = ng > 0; \qquad \frac{dw}{w} = \frac{A'(m)}{w}dm + g = \left[1 + \frac{nA'(m)}{wa(m)}\right]g < g$$

$$a^*(u)du = (1-n)dw + nwg = \left[w + \frac{n(1-n)A'(m)}{a(m)}\right]g; \quad a^*(u^*)du^* = -\frac{n^2 A'(m)}{a(m)}g > 0 .$$

- The terms of trade move in favor of North (since $dw/w < g$).
- The cheaper South goods allow the households in North to expand their consumption.
- ***Product Cycles*** emerge (the birth of new industries in North, $du^* > 0$, and the migration of some industries from North to South, $dm > 0$)
- The effects on w and u are ambiguous.

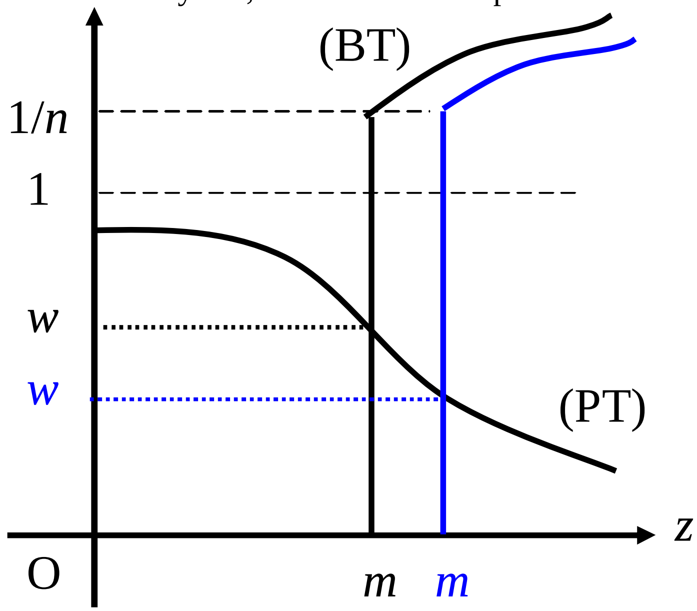
If $-A'(m) > a(m)w/n = a^*(m)/n$, the South's factor terms of trade deteriorates.
If $-A'(m) > a^*(m)/n(1-n)$, the deterioration is so large that $du < 0$; ***Immiserizing Growth.***

Exercise: Examine the effects of increasing $h > 1$, while keeping $h^* = 1$, which is small enough that $wh < 1$ continues to hold. How is this different from the uniform productivity gains in South discussed above?

Non-Uniform Case: $du^* < 0$, if $g(z)$ is sufficiently small over $[0,m]$, relative to $g(m)$.

- South captures more than 100% of all the world's productivity gains.
- North loses its industries at both ends of its spectrum.
- This situation may arise from Technology Transfers, as South has more to learn from North for higher-indexed goods.

Global productivity improvement: $g(z) = g^*(z) > 0$.



$$a(m)dm = \int_0^m g(s)a(s)ds > 0$$

$$dw = A'(m)dm < 0$$

$a^*(u)du$

$$= \left[w + (1-n)\frac{A'(m)}{a(m)} \right] \int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds$$

$$a^*(u^*)du^* = \left[w - n\frac{A'(m)}{a(m)} \right] \int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds > 0.$$

The effect on u is ambiguous, while $du^* > 0$ unambiguously.

In spite of the world-wide productivity gains, the asymmetry of demand response causes ToT to move against South and leads to Product Cycles (dm, $du^* > 0$).

Exercise: Examine the effects of ***Transfer Payments*** made from North to South, financed by lump-sum taxes in North, and distributed by lump-sum transfers in South.

Exericse:

In the above model, keep the first assumption:

- $h = h^* = 1$ for all households and hence $N = L$ and $N^* = L^*$.

but, change the second assumption to

- $A(z) > 1$ for all z, so that Foreign is poorer than Home to ensure $w > 1$.

Redo all the exercises discussed above under this alternative assumption.

Note: This may capture the situation where Home is the *Rich North*, which has comparative advantage in industrial goods consumed by the mass, while Foreign is the *Poor South*, which has comparative advantage in offering exotic Holiday Resorts, which only the rich people can afford.

North-South Trade: The Case of Heterogeneous Populations

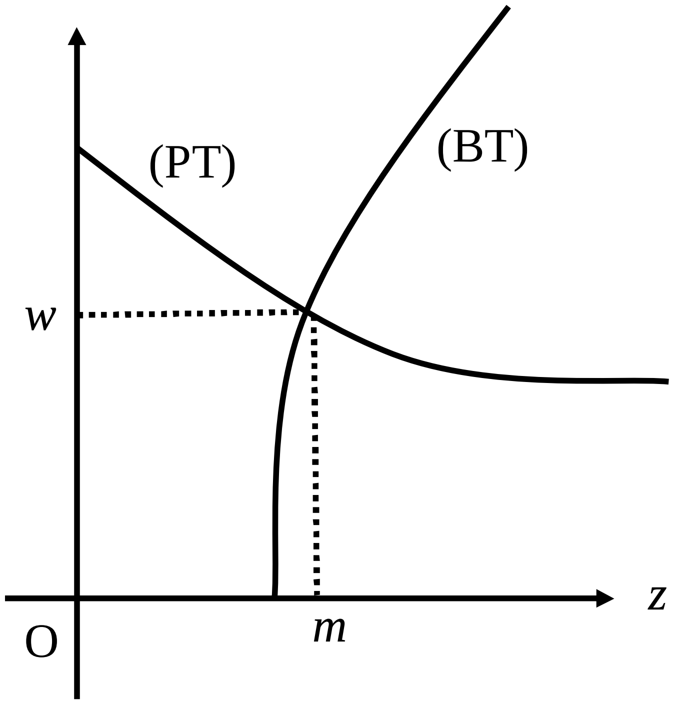
Let us assume:

- $F(h)$ and $F(h^*)$ are nondegenerate.

- Their supports include small h or h^*, such that, in equilibrium, $wh < E(m)$ or $h^* < E(m)$.

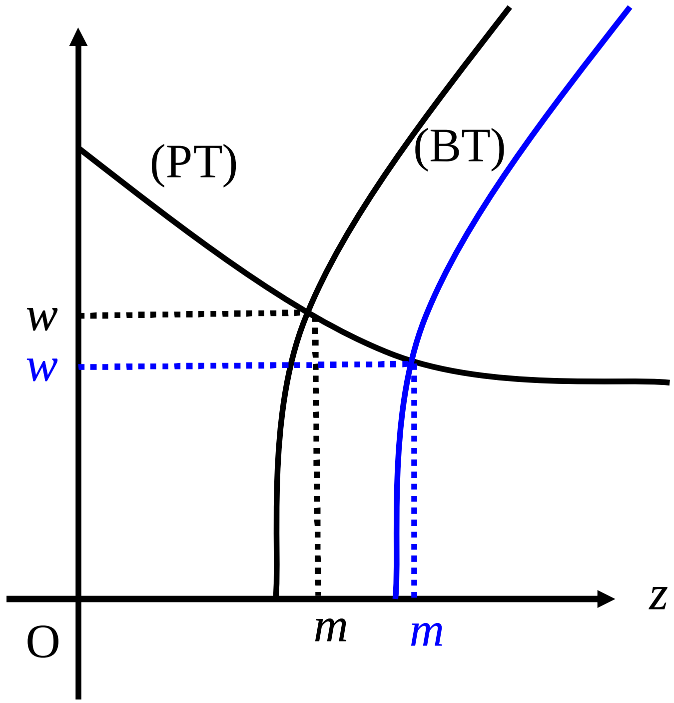
✓Some poor households do not consume goods produced in North: $u(h) < m$ or $u^*(h^*) < m$.
✓(PT) intersects at the upward-sloping part of (BT).

- The world's richest household, which determines the upper end of the North goods, resides in North.



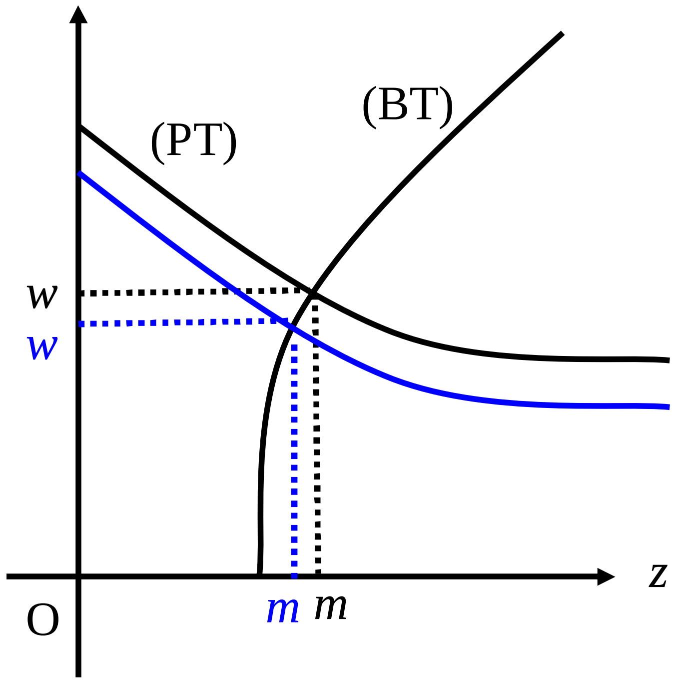
Relative Population Size: A faster population growth in South (dn > 0) shifts the (BT) curve to the right, hence dm > 0 and dw < 0.

- All households in North, the rich and the poor, are better off, as the ToT improves.
- New industries are born, as dw < 0 implies that the world's richest becomes richer, and these industries are in North
- Some old industries migrate from North to South (dm > 0): ***Product Cycles.***
- The rich in South, those with $u(h) > m$, are worse off, as the ToT moves against them.
- The poor in South, those with $u(h) < m$, are unaffected, as they essentially live in autarky, and hence are insulated from the ToT change.



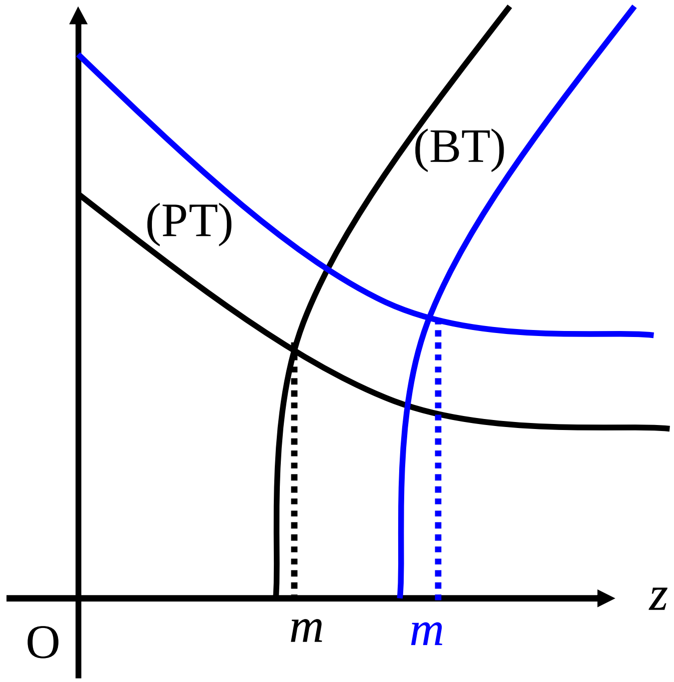
Productivity growth in North ($g(z) = 0$, $g^*(z) > 0$) shifts (PT) down, hence $dw < 0$.

- All households in North are better off.
- With (BT) upward-sloping, $dm < 0$, and $-g^*(m) < dw/w < 0$. As the ToT improves, the poor in North, those with $u^*(h^*) < m$, consume more South goods, increasing demand for South's labor. To keep its labor market in balance, South specializes in a narrower set of goods, abandoning the upper end of industries, which move to North.
- With $-g^*(m) < dw/w$, the rich in South, those with $u(h) > m$, are better off if $g^*(z)$ is constant over $(m, u(h)]$. If not, they can be worse off.
- The poor in South, those with $u(h) < m$, are unaffected.



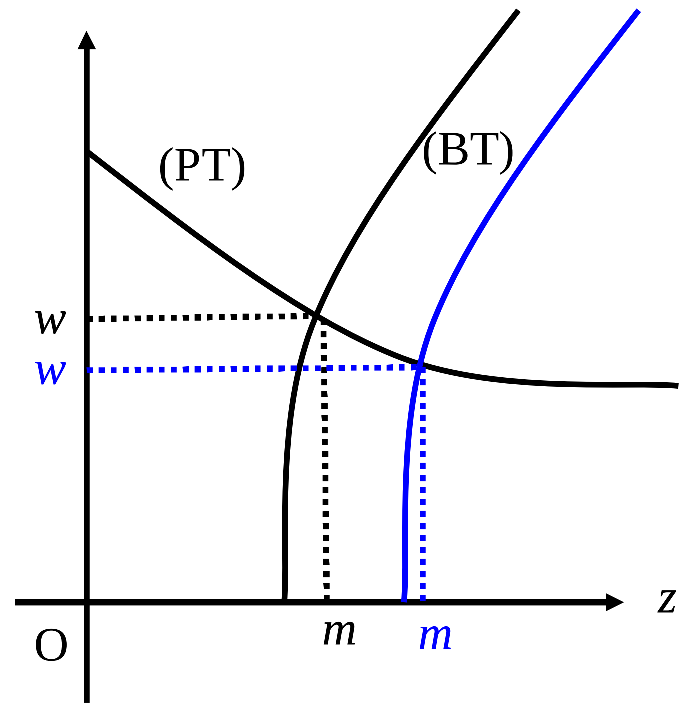
Productivity growth in South ($g(z) > 0$, $g^*(z) = 0$) shifts (PT) up and (BT) to the right, hence $dm > 0$, and $-\infty < dw/w < g(m)$.

- If $g(z)$ is constant over $[0,m]$, all households in North are better off; New industries are born in North. ***Product Cycles, again***.
- If $g(z)$ is faster at m than $[0,m)$, North can be worse off.
- The effect on the rich in South, those with $u(h) > m$, is ambiguous even with the uniform change. They can be worse off if the ToT moves against them.
- The poor in South, those with $u(h) < m$, insulated from the ToT change, are better off.



Global productivity growth ($g(z) = g^*(z) > 0$) shifts (BT) to the right, while (PT) unchanged, hence $dm > 0$ and $dw < 0$.

- All households in North are better off, and new industries are born. With $dm > 0$, ***Product Cycles*** again.
- The poor in South, those with $u(h) < m$, are better off.
- The effect on the rich in South, those with $u(h) > m$, is ambiguous.

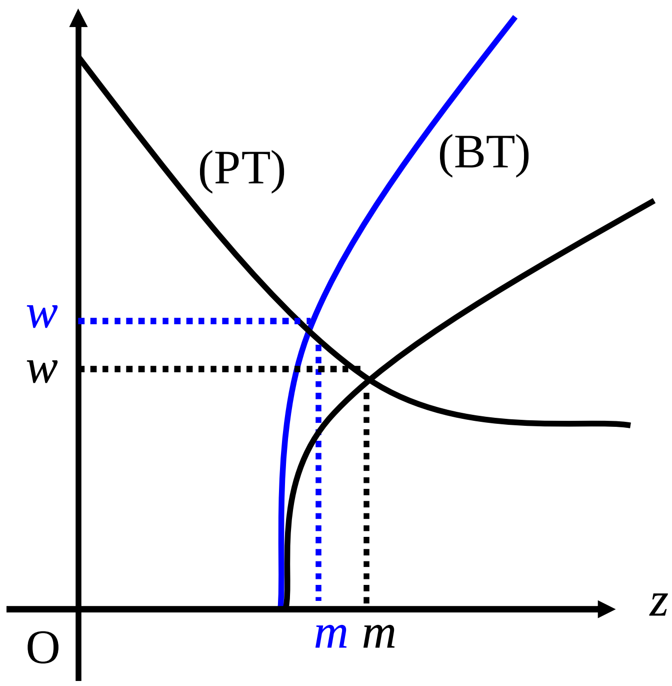


Income Transfers:

Consider South's *domestic* transfer policy, which redistributes income from the Rich, those with $u(h) > m$, who spend their additional income on imports from North, to the Poor, those with $u(h) < m$, who spend their additional income on the South goods.

(BT) shifts up, hence $dm < 0$ and $dw > 0$.

- All households in North are worse off, as the ToT moves against North.
- The poor in South are better off, due to the transfer; no effect from the ToT change.
- The rich in South: their income is taken away, but the ToT moves in favor. Perhaps, paradoxically, they may be better off.



Example:

- Homogenous households in North: $h^* = 1$.
- Two-types of households in South: 50% with h_L and 50% with h_H, where $h_L < h_H$.

With a transfer per household, T, measured in Home labor,

South's Labor Market: $w(h_L+h_H)N/2 = \{w(h_L+T)+ E(m)\}N/2 + N^*E(m)$,

→ $\int_0^m a(s)ds = (h_H - T)(2-n)/n$

→ $dw = A'(m)dm = -\frac{A'(m)}{a(m)}\left(\frac{2-n}{n}\right)dT > 0$, evaluated at $T = 0$,

Rich South Household's Budget Constraint: $w\int_0^m a(s)ds + \int_m^{u_H} a^*(s)ds = w(h_H - T)$.

→ $a^*(u_H)du_H = -wdT + \left[\int_m^{u_H} a^*(s)ds\right]dw.$

$$= \left\{-\frac{A'(m)}{a(m)}\left(\frac{2-n}{n}\right)\left[\int_m^{u_H} a^*(s)ds\right] - w\right\}dT$$

With a sufficiently large $|A'(m)|$, the positive ToT effect offsets more than the primary effect of transfer.

All the households in South may be better off by adopting a "domestic" policy of redistributing from the rich to the poor (at the expense of North).

Likewise,

- If the rich in South steal income from the poor in South, all the southern households can be made worse off, including the rich who exploit the poor. (North benefits)

- If North adopts a domestic policy of redistributing income from the rich to the poor, the resulting ToT deterioration can make all the households in North can be worse off, including the poor, who receives the transfer. (South benefits.)

Notes:

- This may be viewed as an example of 3-Agent Transfer Paradox: see Bhagwati-Brecher-Hatta (1983). Indeed, one may also interpret this example as a 3-country model with High-income North, Middle-income South, and Low-income South, where the population is homogeneous within each country, and Middle-income South and Low-income South differ only in their labor endowments.

- A close connection between this result and the earlier result of Immiserizing Growth, which states,

✓The poor South, who nevertheless is rich enough to buy goods from North, may lose from its own productivity growth, as this could cause a large ToT deterioration.
✓The flip side of Immiserizing Growth is that they could gain from throwing away some of their income.
✓Here, instead of throwing away, they give it to the poorest who do not affect the ToT.

- An extension of this model to a multi-country setting is just a short-step from the above example: see Matsuyama (2000, Section V).

- Stibora-de Vaal (2007) studied the effects of trade policies in this model.

Flam-Helpman (1987): Vertical Differentiation & North-South Trade

Like my (2000) model,

- A continuum of goods; the set of goods produced is endogenous.
- Only the rich demand for higher-indexed goods. As the households become richer, new goods are introduced at the upper end.
- North (South) has comparative advantage in higher (lower)-indexed goods.

Unlike my (2000) model,

- Goods are indexed according to product quality, and high-quality and low-quality goods are gross substitutes.
- A reduction in the prices of a low quality good induces the households to *switch* from high quality to low quality good.
- Some goods at the bottom end are not produced.

Interpretation: The goods are *vertically differentiated* products within an industry, and the model is used to address the issues of *intra-industry* trade.

Note: Nondegenerate income distributions are essential to generate intra-industry trade, as we need some poor households in North, who buys low-quality southern goods, and some rich households in South, who buy high-quality northern goods.

Some Main Results:

- Technical progress and population growth brings the introduction of high quality goods and the disappearances of low quality goods.
- Goods in the middle are not produced.
- A shift that causes a continuing deterioration of South's terms of trade, which makes South goods cheaper, causes some goods to disappear from North and reemerge in South, but only with some delay.
- A deterioration of South's terms of trade also discourages North from producing the upper end of the spectrum.

The Model

Two Countries: Home (North) and Foreign (South)*. Foreign Labor as the *numeraire.*

Two Types of Goods:

- *Outside Good*, y, which may be produced and consumed in any quantity
- *Vertically Differentiated Products*, $z \in [0,\infty)$, which comes in discrete units

Technologies:

For the outside good, $a = a^* = 1$. For the vertically differentiated products,

(A1) $A(z) \equiv a^*(z)/a(z) > 1$ is continuous and strictly increasing in $z \in [0,\infty)$.

$$p(z) = a^*(z),\ z \in [0,m];\ \ p(z) = wa(z),\ z \in [m,\infty), \text{ where}$$

(PT) $w = A(m) > 1$,

ensuring that South produces the outside good, whose price is equal to one.

Households: N household at Home; N^* households at Foreign

- A Home household with h units of labor earns wh; h is distributed as $F(h)$.
- A Foreign household with h^* units of labor earns h^*; h^* is distributed as $F^*(h^*)$.

Preferences: A Household with income I chooses y, the *quantity* of the outside good, and z, the *quality* of the vertically differentiated product, to

$$\text{Max } u(y, z) \quad \text{subject to } y + p(z) \leq I.$$

Note: The desired quantity of the vertically differentiated product is assumed to be one.

We want to ensure that high income households choose a higher z, which can be interpreted as high-quality, so that, when, combined with (A1),

- South has comparative advantage in low-quality products, chosen by the poor.
- North has comparative advantage in high-quality products, chosen by the rich.

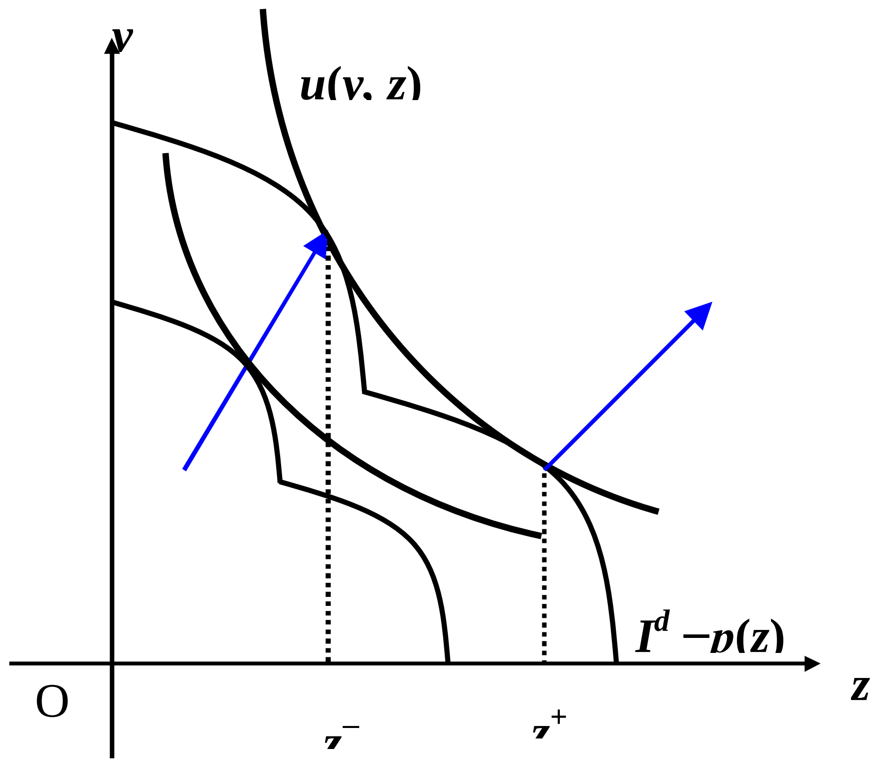
Flam-Helpman work with the specific functional forms:

$$u(y, z) = ye^{\alpha z}; \quad a(z) = e^{\gamma z} / A; a^*(z) = e^{\gamma^* z} / A^*, \quad \text{with} \quad \alpha > 0; 0 < \gamma < \gamma^*.$$

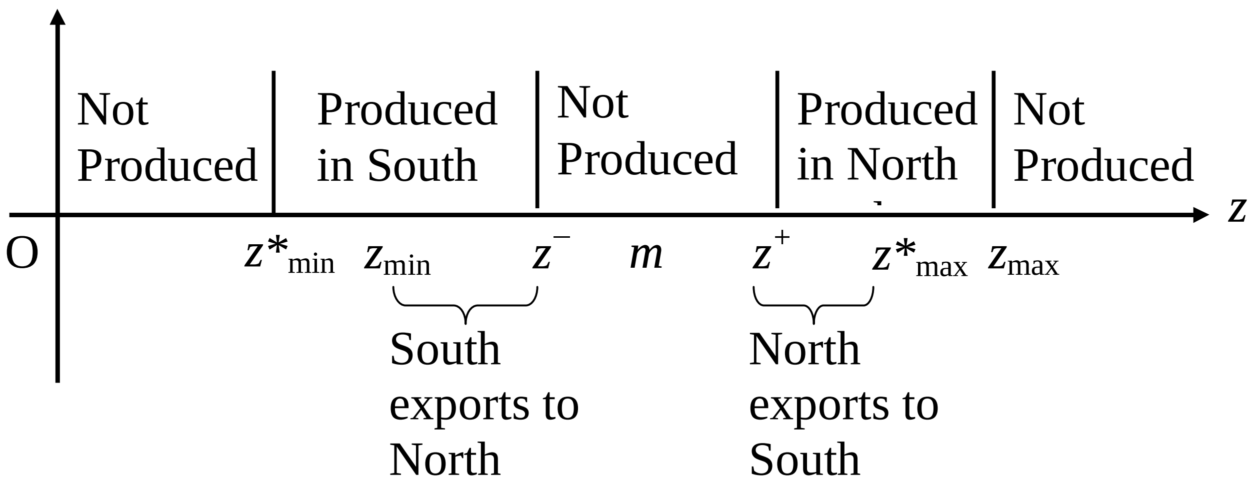
The solution to the maximization yields an income level, I^d, such that

- Households with $I = I^d$ are indifferent between $z^- < m$, and $z^+ > m$.
- Those with $I < I^d$ buy low-quality South goods $z < z^-$.
- Those with $I > I^d$ buy high-quality North goods, $z > z^+$.

Figure 1 of Flam-Helpman



Equilibrium Patterns of Production and Trade:



z_{min} (z^*_{min}): chosen by the poorest households in North (South)
z_{max} (z^*_{max}): chosen by the richest households in North (South).

Intra-industry trade takes place if there are some households in North with $I < I^d$ (hence, $z_{min} < z^-$) and some households in South with $I > I^d$ (hence, $z^*_{max} > z^+$).

Flam-Helpman (1987) conducted comparative statics on income distributions, relative population sizes, productivity growth, etc.

Stokey's (1991) Model of Vertical Differentiation and North-South Trade

Flam-Helpman has the properties that

- Each household must choose only one from a continuum of vertically differentiated goods. The rich who wear expensive evening gowns will not wear T-Shirts.
- Unless the supports of income distributions overlap, no intra-industry trade between North and South.

Stokey (1991) applied her (1988) model of vertical differentiation to North-South trade.

- Higher-quality goods offer more desirable features than lower-quality goods. (Cheap clothes only help you stay warm. Expensive cloths not only help you stay warm but also help you look good.) More specifically,

A continuum of features, $\xi \in [0,\infty)$ over which preferences are defined.
A continuum of goods z $\in [0,\infty)$; Good z offers one unit of all the features, $\xi \in [0,z]$.
A household with income I maximizes, given the prices of good z, $p(z)$,

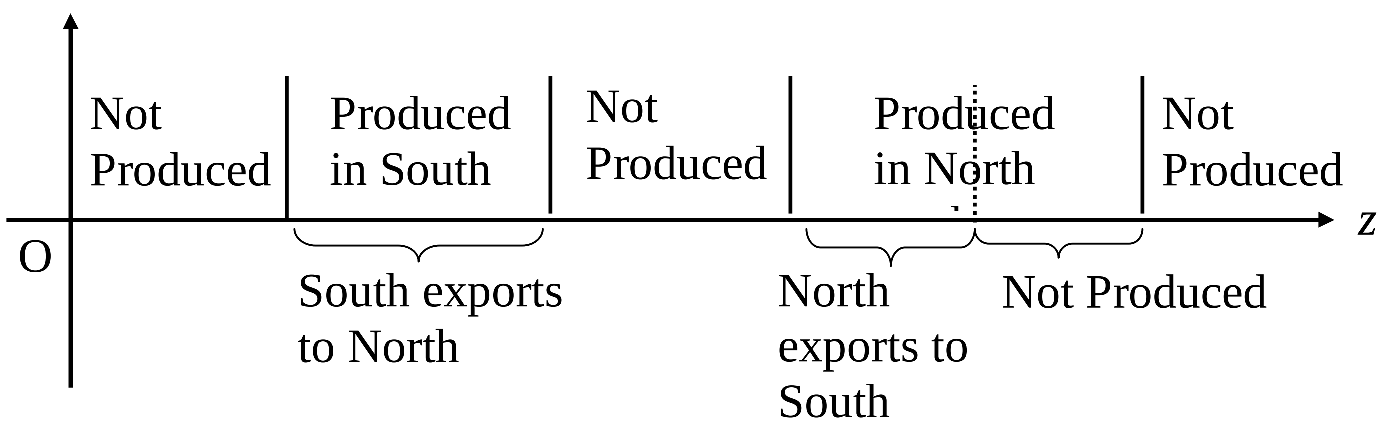
$$V = \int_0^\infty u(q(\xi))d\xi \text{, s.t. } q(\xi) = \int_\xi^\infty c(z)dz \text{ and } \int_0^\infty p(z)c(z)dz \le I \text{,}$$

where $q(\xi)$ is the units of feature ξ consumed and $c(z)$ is the units of good z purchased.

- The rich may want to buy a range of vertically differentiated goods, both high and low quality, while the poor may be able to afford only low-quality goods (unlike Flam-Helpman, more similar to my 2000 model).

- Even if the population is homogeneous within each country (so every household in North is strictly richer than every household in South), which she assumes, intra-industry trade may occur (unlike Flam-Helpman, more similar to my 2000 model).

Equilibrium Patterns of Production and Trade



In spite of these differences between Flam-Helpman and Stokey, the two models share many similar properties.

- Technical progress and population growth brings the introduction of high quality goods and the disappearances of low quality goods.
- Goods in the middle are not produced.
- A shift that causes a continuing deterioration of South's terms of trade, which makes South goods cheaper, causes some goods to disappear from North and reemerge in South, but only with some delay.
- A deterioration of South's terms of trade also discourages North from producing the upper end of the spectrum.

Personal Note: I do not know what to make of the feature of the Flam-Helpman-Stokey models that there is always a gap in the middle.

Some Research Ideas:

- What if there are many industries that are vertically differentiated as in the Flam-Helpman or Stokey models?
- Would it be feasible (and interesting) to consider a hybrid of models similar to Flam-Helpman-Stokey and one similar to my 2000 model?

Non-Homothetic Preferences in Monopolistic Competitive Models of North-South Trade

Markusen

Foellmi-Hepenstrick-Zweimueller

Behrens-Murata

Fajgelbaum-Grossman-Helpman

Non-Homothetic Intertemporal Preferences:

Fertility Transition: