

Aggregate Implications of Credit Market Imperfections (I)

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Road Map:

Introduction:

Lecture 1: Credit Market Imperfections in Static Settings

- A Single Agent's Problem

 - Fixed Investment Case

 - Variable Investment Case

- Partial Equilibrium

 - Homogeneous Agents

 - Heterogeneous Agents; Distributional Implications

 - Heterogeneous Agents; Replacement Effects

- General Equilibrium

 - A Model with Savers

 - A Two-Country Model: Patterns of International Capital Flows

 - A Two-Country Model: Patterns of International Trade

 - A Model of Polarization

Lecture 2: Dynamics with Homogeneous Agents

- The Framework

A Model of Convergence

A Model of Endogenous Investment-Specific (IS) Technical Change

Procyclical Case: Traps and Collapses

Countercyclical Case: Takeover, Cycles, Growth Miracle

A Model with Good and Bad Projects

Under-Investment: Inefficient Recessions and Persistence

Over-Investment: Inefficient Booms and Volatility

A Model with Private Benefits

A Model of Asymmetric Cycles & Intermittent Volatility:

The Good, The Bad, and The Ugly

Lecture 3: Dynamics with Heterogeneous Agents

A Model with Savers (**Unfinished**)

A World Economy Model with International Capital Flows

Dynamics of Household Wealth Distribution

A Single Dynasty's Problem; Individual Poverty Traps

A Model of Interacting Dynasties; Collective Poverty Traps

A Model of Endogenous Heterogeneity: Symmetry-Breaking

“All happy families resemble one another. Each unhappy family is unhappy in its own way.” Leo Tolstoy, *Anna Karenina*

Introduction:

Motivations:

There seems to be broad agreement that, due to credit market imperfections:

- The market economy fails to allocate the credit to its most productive use.
- Borrower Net Worth or Balance Sheet Condition plays crucial roles in allocating the credit.

But, less agreement about the ***aggregate implications:***

- Do the imperfections add *persistence or volatility* to macro dynamics?
- Does improving the credit markets make fluctuations *more or less* volatile?
- Does misallocation of the credit cause *recessions or booms*?
- Does financial globalization *alleviate or exacerbate* the distortions?
- Does the credit market *magnify or reduce* the inequality across households?
etc., etc.

The literature contains a bewildering array of many individual models, hard to make sense of these *seemingly* conflicting results.

In these lectures, I will:

- use *the same, simple model of credit market imperfections throughout*;
- *synthesize* a diverse set of results within a *unified* framework;
- aim to offer a coherent picture of the seemingly conflicting results;
- show how the credit market imperfections can be useful for understanding a wide range of aggregate phenomena, including:
 - Endogenous investment-specific technological changes
 - Development Traps and Leapfrogging
 - Persistent Recessions and Recurrent Boom-and-Bust Cycles
 - Reverse International Capital Flows
 - Rise and Fall of Inequality across Nations
 - New sources of comparative advantage and patterns of international trade

Recurring themes:

- Properties of equilibrium often respond *non-monotonically* to parameter changes. For example,
 - Improving Borrower Net Worth may first lead to a higher market rate of return and then to a lower market rate of return
 - Improving credit market may first lead to an increased volatility and then a reduced volatility.
 - Productivity improvement may first lead to a greater inequality and then a reduced inequality.etc.
- Equilibrium and welfare consequences of the credit market imperfections are *rich and diverse* depending on the general equilibrium feedback mechanisms.

Basic Messages:

(To the outsider of the field):

This is a very exciting field, as credit market imperfections offer a key to understanding many important issues.

(To the insider of the field):

Some cautions for studying the implications within *a narrow class or a particular family* of models and extrapolating from it. In particular,

- Drawing policy implications by comparing a model with credit market imperfections and a model without can be also dangerous, because the effects of improving the credit market could be very different from those of eliminating the credit market imperfections completely.
- The effects of imperfect credit markets could also be very different from the effects of no credit market.

More Remarks before We Begin:

- Macro models with credit market imperfections are not an *alternative* to RBC models. They are *complements*, since credit market imperfections may be viewed as
 - ✓ an *amplification* mechanism for *small exogenous shocks*
 - ✓ a theory of *endogenous* technological change.
- Many topics are not covered, including
 - ✓ Comparing different micro stories behind credit market imperfections
 - ✓ Financial Structure
 - ✓ Financial intermediation
 - ✓ Liquidity, Asset Pricing and Monetary Policy implications
 - ✓ Political Economy
 - ✓ Interactions with other sources of imperfections

We would need an encyclopedia to cover all of this!

This is not a literature survey.

- ✓ I will make no systematic attempt to identify the original contribution for the results discussed below. In fact, no references are given in these slides. If you are interested, consult my NBER 2007 Macro Annual paper.
- ✓ I will deliberately stay away from issues that have already been discussed extensively.
- ✓ I will put more emphasis on issues that, in my view, the literature has not paid attention that they deserve.

Lecture 1: Credit Market Imperfections in Static Models

Lecture 1: Credit Market Imperfections in Static Settings

A Single Agent's Problem

Fixed-Investment Case

Variable-Investment Case

Partial Equilibrium

Homogeneous Agents

Heterogeneous Agents; Distributional Implications

Heterogeneous Agents: Replacement Effects

General Equilibrium

A Model with Savers

A Two-Country Model: Patterns of International Capital Flows

A Two-Country Model: Patterns of International Trade

A Model of Polarization

A Single Agent's Problem: Fixed-Investment Case.

Two Periods: 0 and 1

An Agent (an Entrepreneur or a Firm):

- is endowed with $\omega < 1$ units of the input at Period 0.
- consumes only at Period 1.

Two Means to Convert the Input into Consumption:

- Run a **non-divisible investment project**, which converts one unit of the input in period 0 into R units in **Consumption** in period 1, by **borrowing** $1-\omega$ at the market rate of return equal to r .
- **Lend** $x \leq \omega$ units of the input in period 0 to receive rx units of consumption in period 1. (Or, **Storage** with the rate of return equal to r .)

Agent's Utility = Objective Function = Consumption in Period 1:

$$\begin{aligned} U &= R - r(1-\omega) = R - r + r\omega, && \text{if borrow and run the project,} \\ U &= r\omega && \text{if lend (or put in storage).} \end{aligned}$$

Profitability Constraint: The agent *is willing to* borrow and invest iff

$$(PC) \quad R \geq r$$

Borrowing Constraint: To borrow from the market, the agent must generate the market rate of return, r , per unit to the lenders, yet, *for a variety of reasons*, no more than a fraction, λ , of the project output can be used for this purpose. Thus, the agent *can* borrow and invest iff

$$(BC) \quad \lambda R \geq r(1-\omega).$$

If $\lambda/(1-\omega) < r/R \leq 1$, (PC) holds but not (BC).

- The agent cannot initiate the profitable investment project due to the borrowing constraint.
- A higher ω (and a higher λ) can alleviate this problem.

Broad Interpretations of the Parameters:

λ : the (inversely-related) measure of agency problems affecting credit transactions (may vary across projects or industries).

Institutional quality or the state of financial development (may vary across countries).

ω ; entrepreneur's net worth, the firm's balance sheet, the borrower's credit-worthiness (may vary across agents).

Justifying $\lambda < 1$.

- Strategic Defaults
- Renegotiation
- Moral Hazard (Hidden Action)
- Pure Private Benefit (not necessarily hidden)
- Costly State Verification, etc, etc.

In these lectures,

I will *not* be concerned with the question of which of these mechanisms are most plausible *microeconomic causes* of credit market imperfections.

Instead,

I will simply treat credit market imperfections as a fact of life, and proceed to investigate their *aggregate consequences*.

Why? Three Reasons

- The major causes of credit market imperfections, even if we could identify them in certain specific cases, are likely to vary across investment types, industries, countries and times.
- At least qualitatively speaking, much of the aggregate and equilibrium implications of credit market imperfections do not depend on the specific nature of the agency problems behind the imperfections.
- Perhaps the most importantly, this reduced form approach saves space, as well as the time and effort of the reader. This approach enables me to reproduce many key results in the literature without having to devote many pages to explain the optimal contract problem. I need just a few lines to describe the borrowing constraint.

A Single Agent's Problem: Variable-Investment Case

Let us now allow the agent to choose the scale of project, I , with the revenue equal to $R(I)$, by borrowing $I - \omega$ at the rate, r .

Maximize $U = R(I) - rI + r\omega$ subject to

$$(BC) \quad \lambda R(I) \geq r(I - \omega).$$

Strictly Concave $R(\bullet)$: Let $R'(I^*) \equiv r$. For $\omega < I^* - \lambda R(I^*)/r$, (BC) is binding. The solution, $I(r, \omega) < I^*$, given by $\lambda R(I) = r(I - \omega)$, increases with ω .

$R(I) = RI$: If $R > r$, the agent wants to invest as much as possible. Thus, (BC) is always binding, and $I = \omega(1 - \lambda R/r)^{-1}$ for $r > \lambda R$. The utility is $U = RI - rI + r\omega = R\omega(1 - \lambda)/(1 - \lambda R/r) > R\omega$. (If $r \leq \lambda R$, the agents would invest by infinite amount! If $r > R$, no one would invest.) Later, we will see a GE model, where r is endogenous and satisfies $\mathbf{R} > \mathbf{r} > \lambda \mathbf{R}$ in equilibrium.

So far, we treat only a single agent's problem, holding all the prices as given, and without worrying about interactions among agents.

For the rest, we will gradually let agents interact through equilibrium prices, one by one.

First, let us do so in the factor market.

Partial Equilibrium with Homogeneous Agents

Two Periods: 0 and 1

A Continuum of Homogeneous Agents with Unit Mass. Each

- is endowed with $\omega < 1$ units of the input at Period 0.
- consumes only at Period 1.

Consumption Good produced at Period 1, with $y = f(k)$, with $f' > 0 > f''$.

- k is the aggregate capital stock available in period 1, which will be determined in equilibrium.
- Each unit of capital earns $f'(k)$ units in the consumption good in Period 1.
- $f(k) - kf'(k)$ is the Ricardian Rent, paid to those who own “the hidden factor” in fixed supply. (e.g., $y = f(k) = F(k, 1)$, where F is CRS and 1 is the total supply of land.) The owners of the hidden factor play no active role in this economy.

Two Means to Convert the Input into Consumption:

- Run a **non-divisible investment project**, which converts one unit of the input in period 0 into R units in **Capital** in period 1, thereby earning $Rf'(k)$ units in consumption, by **borrowing** $1-\omega$ at the market rate equal to r .
- **Lend** $x \leq \omega$ units of the input in period 0 to receive rx units of consumption in period 1. (Or, **Storage** with the rate of return equal to r .)

Aggregate Supply of Capital:

$k = Rx$, where x is the fraction of the agents who invest.

Both x and hence $k = Rx$ are determined in equilibrium.

Note: The return to investing to the project, $Rf'(k) = Rf'(Rx)$, is decreasing in x , due to the diminishing returns.

Agent's Utility:

$$\begin{aligned} U &= Rf'(k) - r(1-\omega) = Rf'(k) - r + r\omega, && \text{if borrow and run the project,} \\ U &= r\omega && \text{if lend (or put in storage).} \end{aligned}$$

Profitability Constraint (PC): $Rf'(k) \geq r$

Borrowing Constraint (BC): $\lambda Rf'(k) \geq r(1-\omega)$.

In equilibrium, k (or x) must adjust in such a way,

- Both (BC) and (PC) must hold.
- One of them must hold with equality.

$$\rightarrow Rf'(k) = \text{Max} \{1, (1-\omega)/\lambda\} r.$$

What is the Aggregate Capital Formation, k , in (partial) equilibrium?

If $\lambda + \omega < 1$, (BC) is tighter than (PC). $\rightarrow \mathbf{Rf'(k) = r(1-\omega)/\lambda > r}$

Too Little Investment.

Net Worth Effect; $\omega \uparrow \rightarrow k \uparrow$

If $\lambda + \omega > 1$, (PC) is tighter than (BC). $\rightarrow \mathbf{Rf'(k) = r > r(1-\omega)/\lambda}$

Optimal Investment.

No Net Worth Effect.

Some Technical Notes:

- 1) Assuming $Rf'(R) < r$ ensures the interior solution, $k = Rx < R$, or $x < 1$.
→ the fraction $1-x$ of agents do not borrow; they instead lend.
- 2) How is the credit allocated among homogeneous agents?
 - If $\lambda + \omega > 1$, (PC) is binding, i.e., the agents are indifferent between borrowing and lending, so it doesn't matter.
 - If $\lambda + \omega < 1$, (PC) is not binding, i.e., the agents strictly prefer borrowing to lending.

Two Possible Resolutions

- ✓ Credit Rationing with some random device (e.g., sequential service constraint)
- ✓ The agents differ in the endowment, distributed according to $G(\omega)$. The model describes the limit where $G(\omega)$ converges to a single mass point.

3) Why Indivisibility? Why (a Continuum of) Homogeneous Agents?

- It is often argued that general equilibrium analysis of credit market imperfections is fundamentally difficult because one must model heterogeneous agents.
- This is not true. Credit market transactions can take place even among homogeneous agents if there are some indivisibility constraints.

In these lectures, I often assume

- **Homogeneous Agents**, in order to keep the analysis simple.
- **Indivisibility of Each Project**, in order to keep credit markets active in spite of the homogeneity of agents
- **A Continuum of Agents**, in order to ensure the convexity of aggregate investment technologies in spite of the indivisibility of each project.

Nevertheless, let us look at some examples with heterogeneous agents.

Partial Equilibrium with Heterogeneous Agents: $\omega \sim G(\omega)$.

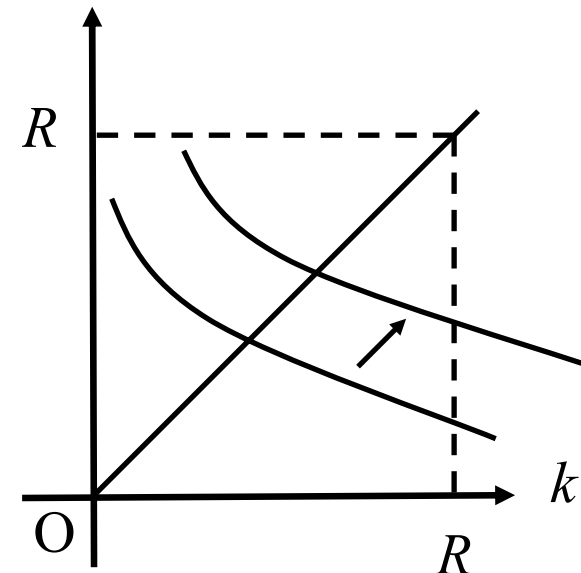
Only those with $\omega \geq \omega_c \equiv 1 - \lambda Rf'(k)/r$ could invest.

$$\rightarrow k = R \left[1 - G \left(1 - \frac{\lambda Rf'(k)}{r} \right) \right]$$

If the unique solution to this equation satisfies $Rf'(k) > r$, then it is the equilibrium.

Comparative Statics:

- $\lambda \uparrow \rightarrow k \uparrow$
- $r \downarrow \rightarrow k \uparrow$
- A first-order-stochastic-dominance shift in $G \rightarrow k \uparrow$ (Net Worth Effect)



Distributional Impacts of Improving Credit Market ($\lambda \uparrow$):

Let λ go up from λ^- to λ^+ . Then,

- k goes up from k^- to k^+
- $\omega_c = 1 - \lambda Rf'(k)/r$ goes down from $\omega_c^- = 1 - \lambda^- Rf'(k^-)/r$ to $\omega_c^+ = 1 - \lambda^+ Rf'(k^+)/r$.

If $\omega < \omega_c^+$,

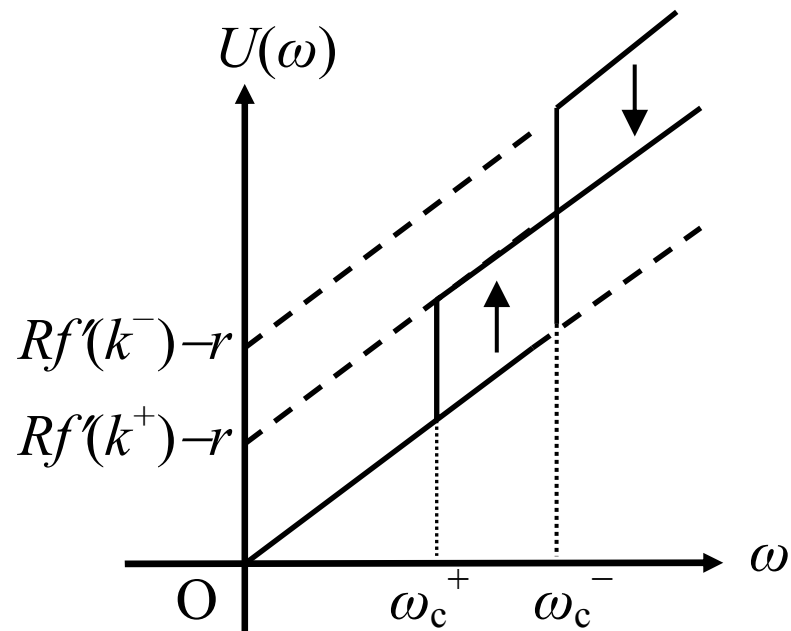
$$U^-(\omega) = U^+(\omega) = r\omega;$$

If $\omega_c^+ < \omega < \omega_c^-$,

$$U^-(\omega) = r\omega < U^+(\omega) = Rf'(k^+) - r(1 - \omega);$$

If $\omega > \omega_c^-$,

$$U^-(\omega) = Rf'(k^-) - r(1 - \omega) > U^+(\omega) = Rf'(k^+) - r(1 - \omega).$$



The middle class (& the owner of the hidden inputs) gains; the rich loses.

The rich may benefit from the credit market imperfections, which acts as an entry barrier. → Political Economy Implications

Heterogeneous Agents: $(\omega, R) \sim G(\omega, R)$

Only the agents satisfying both

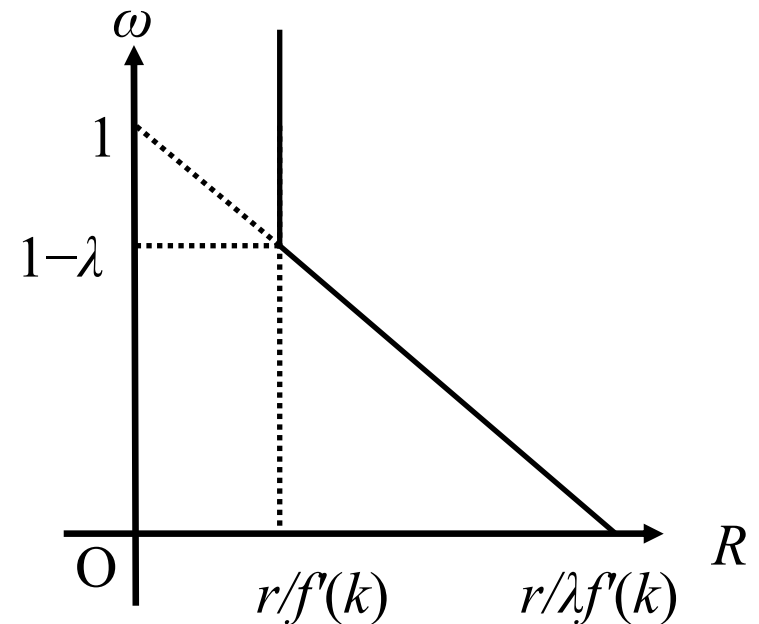
$$(PC) \quad Rf'(k)/r \geq 1$$

and

$$(BC) \quad \omega \geq \omega_c(k) \equiv 1 - \lambda Rf'(k)/r$$

invest.

$$\rightarrow k = \int_{\frac{r}{f'(k)}}^{\infty} R \left[\int_{\omega_c(k)}^{\infty} g(\omega, R) d\omega \right] dR$$



Comparative Statics:

- $\lambda \uparrow \rightarrow k \uparrow$
- $r \downarrow \rightarrow k \uparrow$

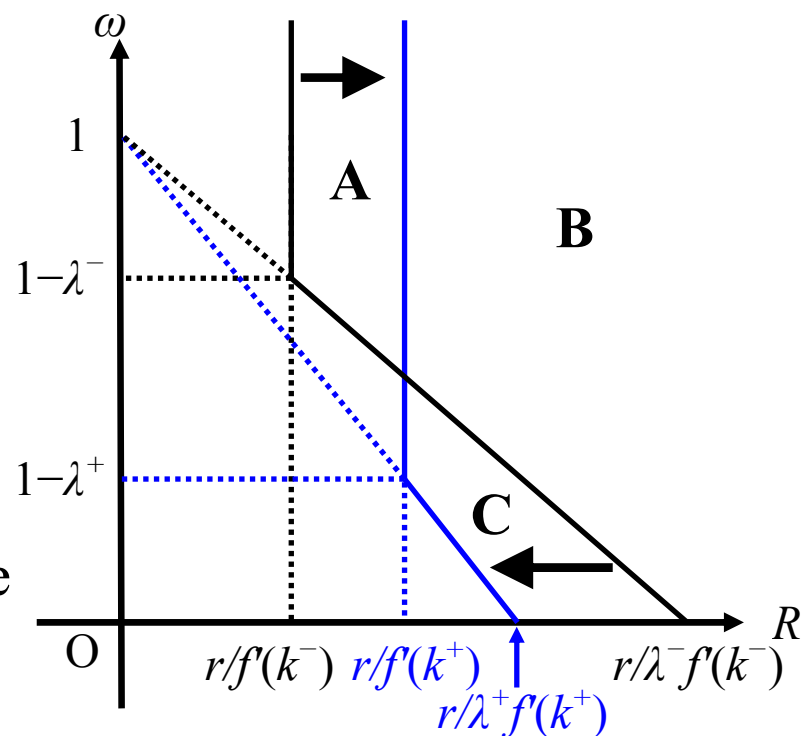
Composition Effects of Improved Credit Market

With a higher λ , which increases k and hence reduces $f'(k)$,

- those in A stop investing
- those in B continue investing
- those in C start investing
- the rest never invest.

The rich, but less productive agents in A are replaced by the poor, but more productive agents in C.

- A&B are worse off.
- C are better off.



Two-Types of Agents: $R_1 < R_2$, $\omega_1 > \omega_2$ (θ ; the share of Type-1).

Suppose $1 - \omega_1 < (R_1/R_2)(1 - \omega_2)$.

Let λ go up from λ^- to λ^+ , where $1 - \omega_1 < \lambda^- < (R_1/R_2)(1 - \omega_2) < \lambda^+ < 1 - \omega_2$.

Then,

- (PC) is always tighter than (BC) for Type-1.
- (BC) is always tighter than (PC) for Type-2.

Furthermore,

let θ and r be such that $\frac{1 - \omega_2}{\lambda^- R_2} > \frac{f'(R_1 \theta)}{r} \geq \frac{1}{R_1} > \frac{f'(R_2(1 - \theta))}{r} \geq \frac{1 - \omega_2}{\lambda^+ R_2}$,

Then, in equilibrium,

- Before the change, only some Type-1 but no Type-2 invest, with $R_1 f'(k^-) = r$, where $k^- \leq R_1 \theta$. ***(PC) is binding for Type-1; (BC) is violated for Type-2.***
- After the change, only some Type-2, but no Type-1 invest, with $\lambda^+ R_2 f'(k^+) = r(1 - \omega_2)$, where $k^+ \leq R_2 (1 - \theta)$. ***(BC) is binding for Type-2. (PC) is violated for Type-1.***

The unproductive but rich agents, who are never credit-constrained, are replaced by the productive but poor agents who are always credit-constrained.

Furthermore,

- Aggregate Investment (the total amount of the inputs going into the projects) may go down.

More generally, improving the credit market might lead to:

- An increase in the fraction of the credit-constrained among the active firms. → You cannot use the extent to which the firms are credit-constrained as a measure of the credit market imperfections.
- A decline in the aggregate investment, due to endogenous improvement in the investment technologies.

In the extreme case of $\lambda = 0$, the only self-financing firms can operate, thus no active firms are credit-constrained. In the other extreme, $\lambda = 1$, nobody is constrained. In the intermediate cases, some active firms may be constrained.

Non-monotonicity!!

All the models so far are in partial equilibrium in that they all treat, r , the market rate of return required by the lenders, as exogenously given.

Let us now endogenize it.

General Equilibrium: Endogenizing r .

A Model with Savers

- Go back to the homogeneous case, where every (investing) agent has the same R and ω .
- Add some outside agents, “savers”, with no access to the investment technology, who choose to

Maximize $U^0 = V(C^0_0) + C^0_1$ subject to $C^0_1 = r(\omega^0 - C^0_0)$.

→ Saving by the Savers: $V'(\omega^0 - S^0(r)) \equiv r \rightarrow S^0(r) \equiv \omega^0 - (V')^{-1}(r)$.

If the storage technology does not exist (or its rate of return is sufficiently low to make it irrelevant),

Resource Constraint (RC):

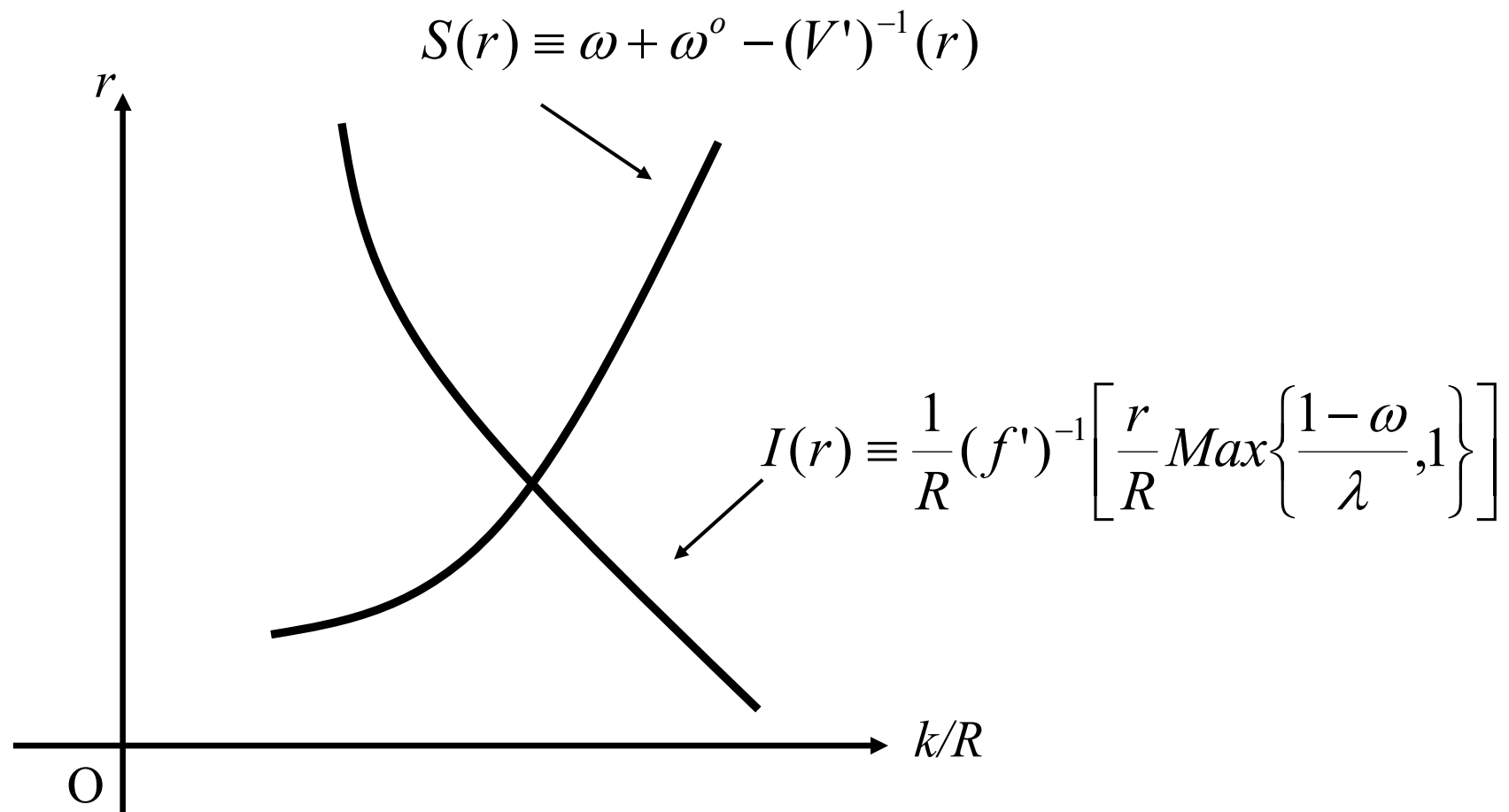
$$k = R[\omega + S^0(r)] = R[\omega + \omega^0 - (V')^{-1}(r)] \leftrightarrow \frac{k}{R} = S(r) \equiv \omega + \omega^0 - (V')^{-1}(r).$$

Profitability Constraint (PC) + Borrowing Constraint (BC):

$$Rf'(k) = \text{Max}\{1, (1-\omega)/\lambda\}r \leftrightarrow \frac{k}{R} = I(r) \equiv \frac{1}{R}(f')^{-1}\left[\frac{r}{R}\text{Max}\left\{\frac{1-\omega}{\lambda}, 1\right\}\right]$$

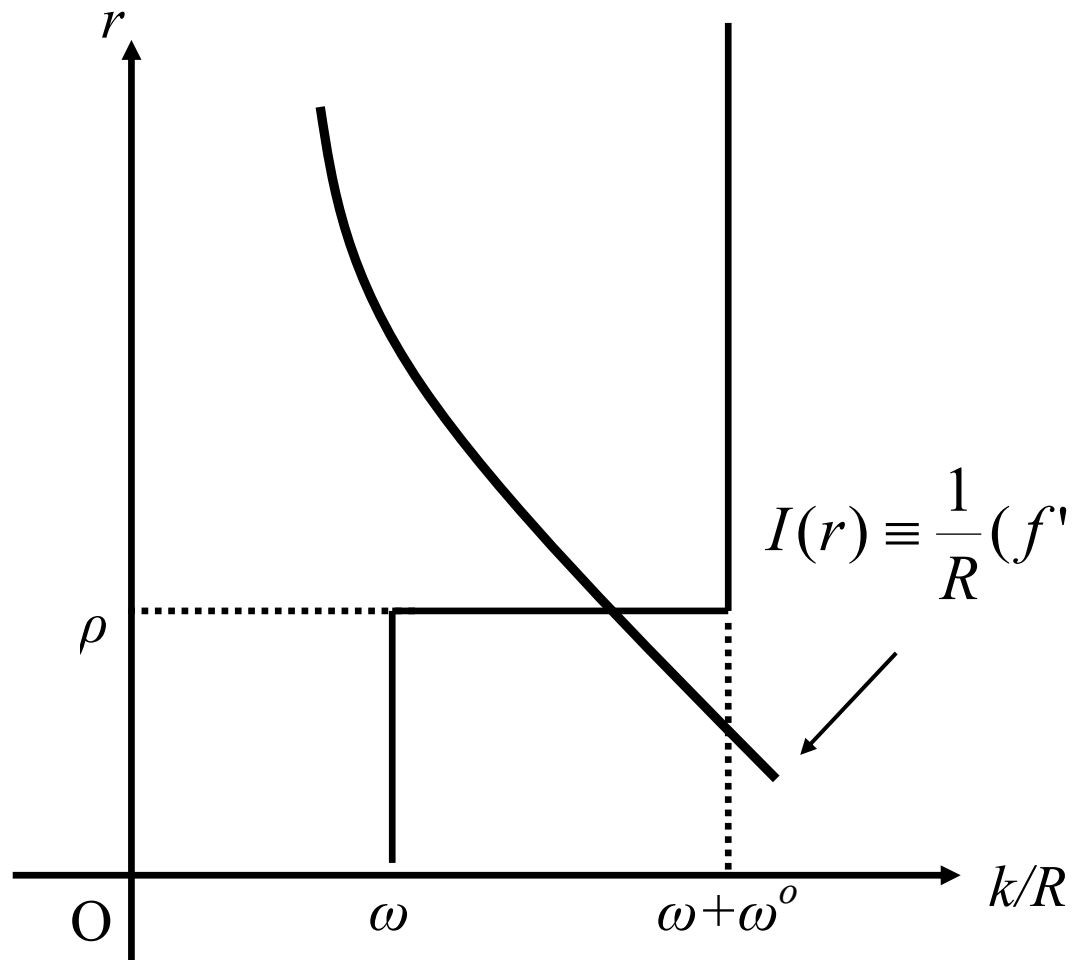
These two conditions jointly determine k and r.

A Graphical Illustration: Aggregate Saving = Aggregate Investment;

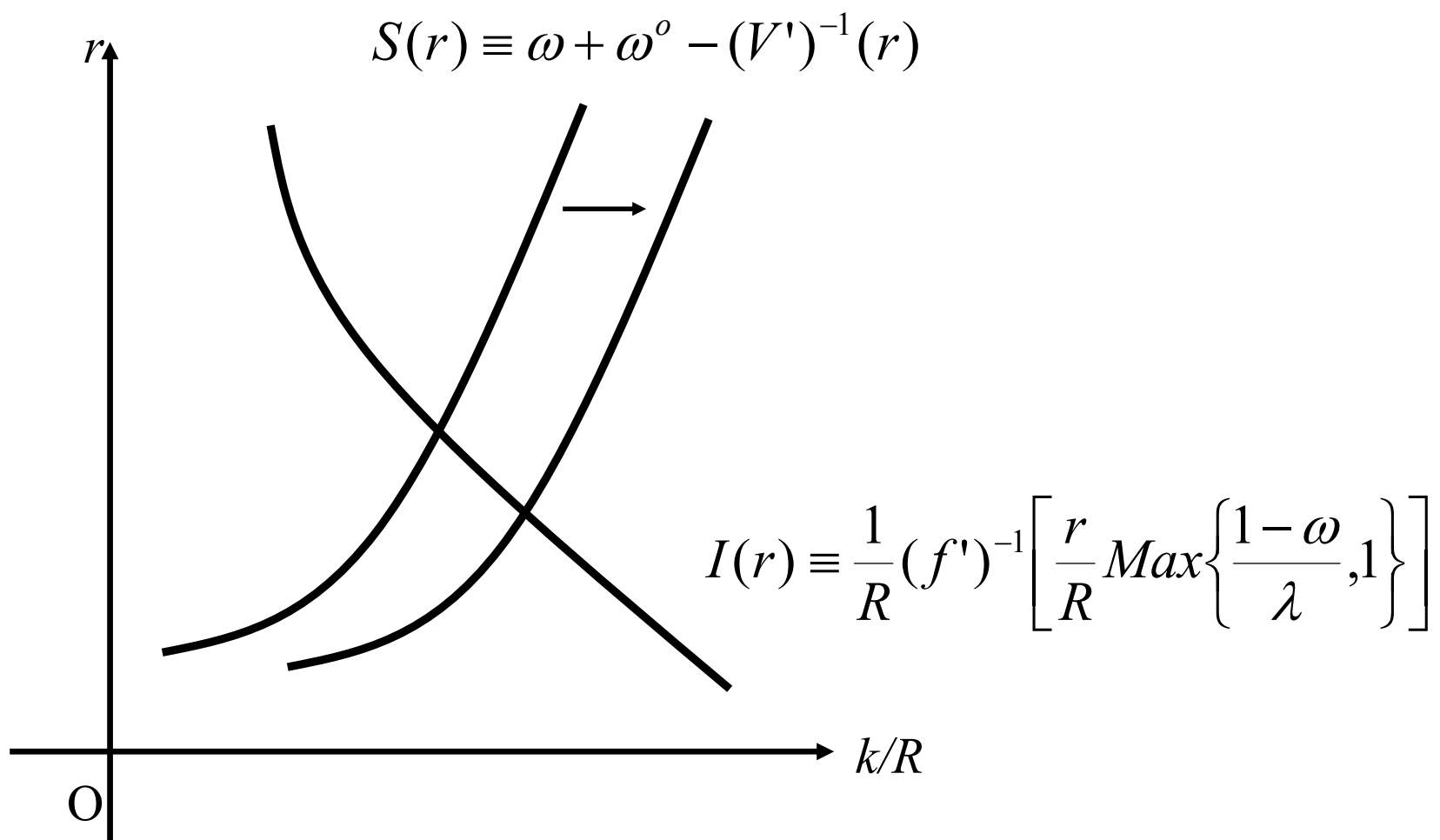


Note: $S(r)$ depends on $\omega + \omega^0$, while $I(r)$ depends only on ω .

Special Case, $U^0 = \rho C^0_0 + C^0_1$, can replicate the partial equilibrium model.

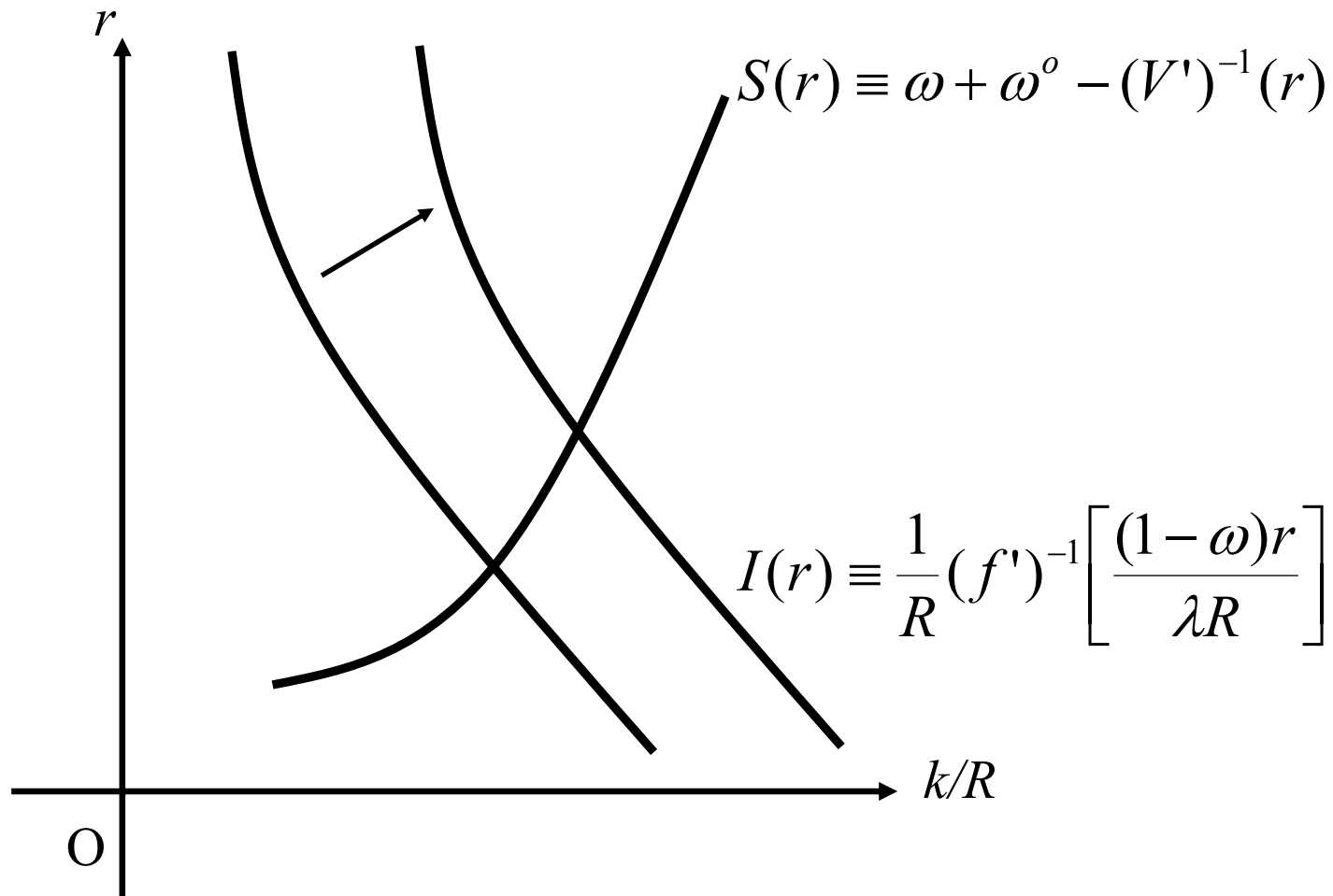


A higher wealth of the savers, $\Delta\omega^0 > 0$, leads to more capital formation and a lower rate of return. (**Capital Deepening Effect**)

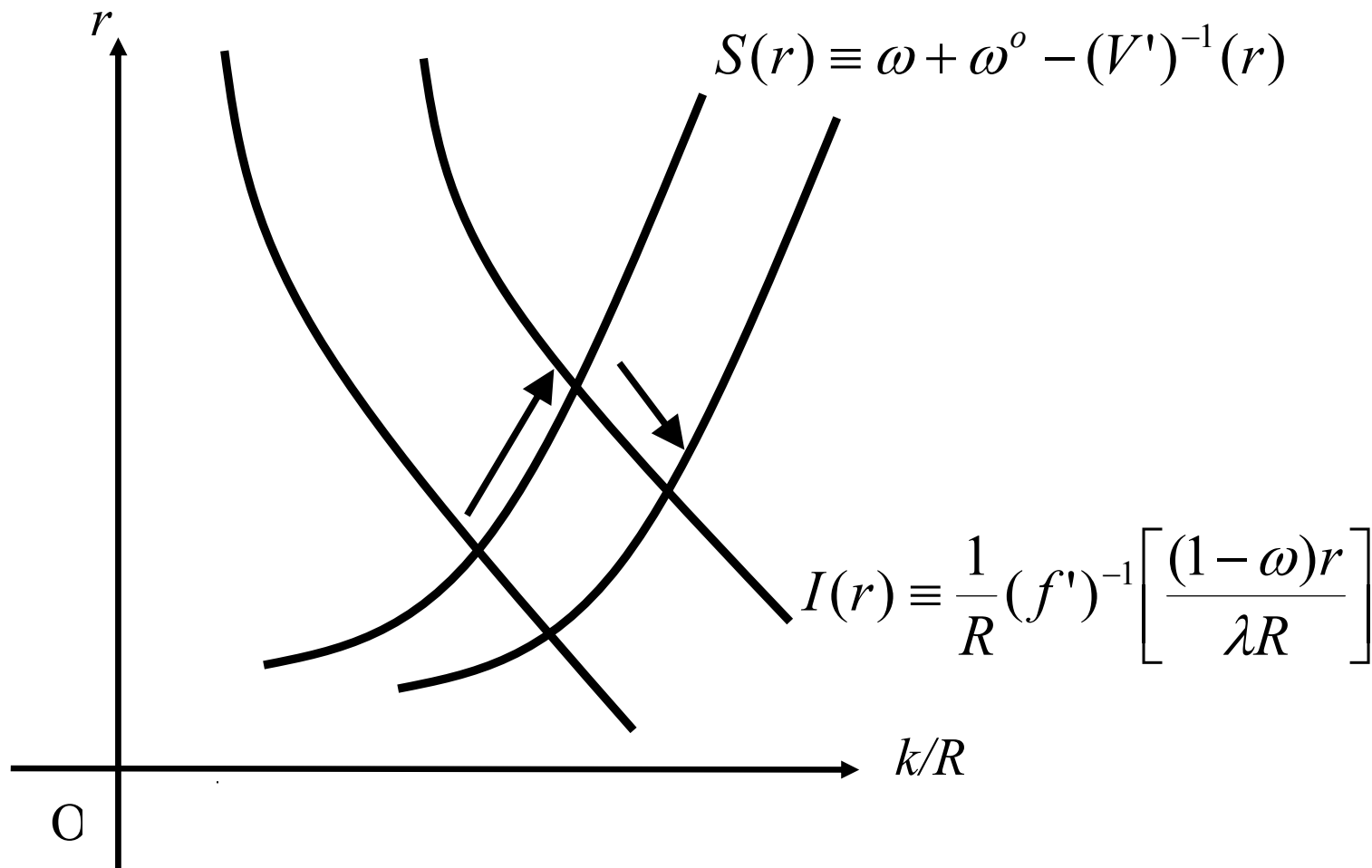


When $\lambda + \omega < 1$,

- Redistributing Wealth from “Savers” to “Investors,” $\Delta\omega = -\Delta\omega^0 > 0$, leads to more k and a higher rate of return. (**Net Worth Effect**)
- A credit market improvement, $\Delta\lambda > 0$, has the same effect.



When $\lambda + \omega < 1$, a higher endowment of “Investors,” $\Delta\omega > 0$, leads to more capital formation and *may* lead to a higher rate of return, as the **Net Worth Effect** may dominate the **Capital Deepening Effect**.



The above analysis looked at how ω^0 , ω and λ affect k .

What would be dynamic implications if we allow for some feedback from k to ω^0 or to ω , or to λ ? (Wait until Lecture 3).

So far, we have assumed the homogeneity of capital produced.

Let us now look at an example with heterogeneous capital, where different types of agents produce different types of capital.

A Two-Country Model: Patterns of International Capital Flows

Two Countries: North & South of the kind described above. They have the identical $f(k)$ and R , but may differ in λ , ω , and ω^0 .

Further Assumptions:

- The Input and the Consumption Good are *tradeable*. \rightarrow This allows the agents to lend and borrow across the borders.
- Physical Capital and the hidden inputs are *nontradeable*.
- Only the agents in North (South) can produce Physical Capital in North (South). Alternatively, the agent's productivity, R , is substantially lower when operating abroad. \rightarrow This effectively rules out Foreign Direct Investment.

Experiment: Suppose the agents in North can pledge $\varphi\lambda_N$ to the lenders in the South, and the agents in South can pledge $\varphi\lambda_S$ to the lenders in the North. Let φ change from $\varphi = 0$ (Autarky) to $\varphi = 1$ (Full Integration).

Autarky Equilibrium for $j = N$ or S :

(RC): $k_j = R[\omega_j + S^0(r_j)] = R[\omega_j + \omega_j^0 - (V')^{-1}(r_j)].$

(PC) + (BC): $Rf'(k_j) = \text{Max}\{1, (1-\omega_j)/\lambda_j\}r_j.$

World Equilibrium (with Full Financial Integration):

World Resource Constraint (WRC):

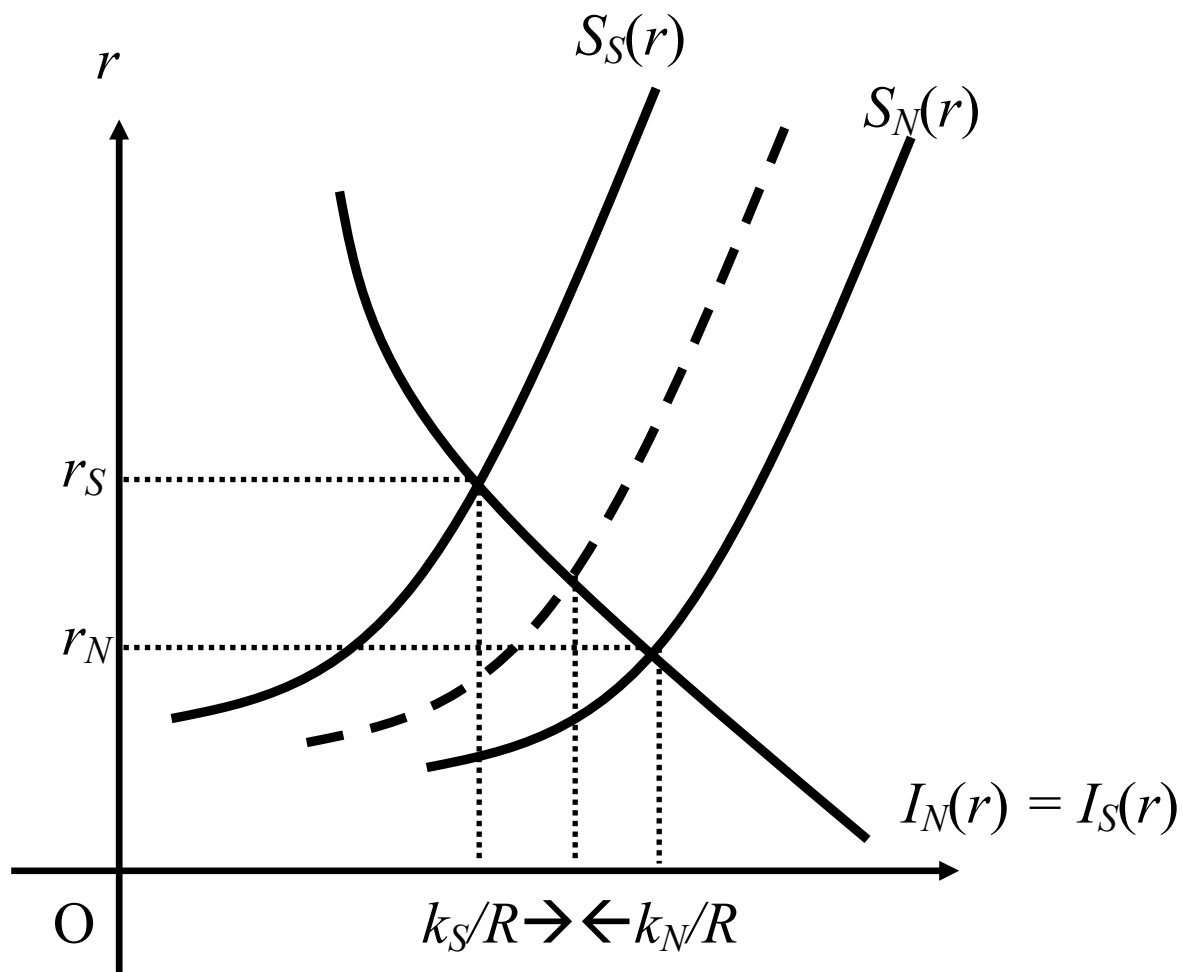
$$k_N + k_S = R[\omega_N + \omega_N^0 + \omega_S + \omega_S^0 - (V')^{-1}(r)].$$

Rate of Return Equalization (RRE):

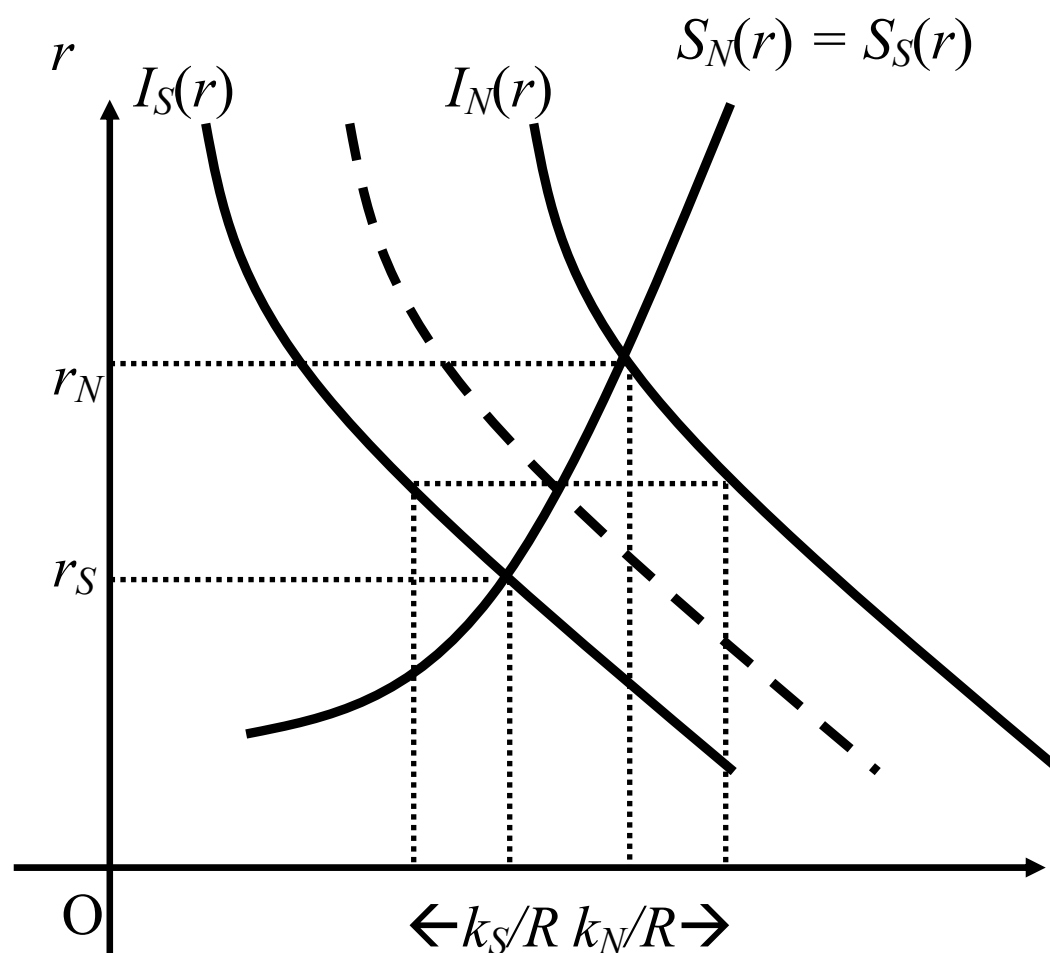
$$\text{Min}\{1, \lambda_N/(1-\omega_N)\}Rf'(k_N) = r = \text{Min}\{1, \lambda_S/(1-\omega_S)\}Rf'(k_S)$$

(WRC) + (RRE) \rightarrow $S_N(r) + S_S(r) = I_N(r) + I_S(r).$

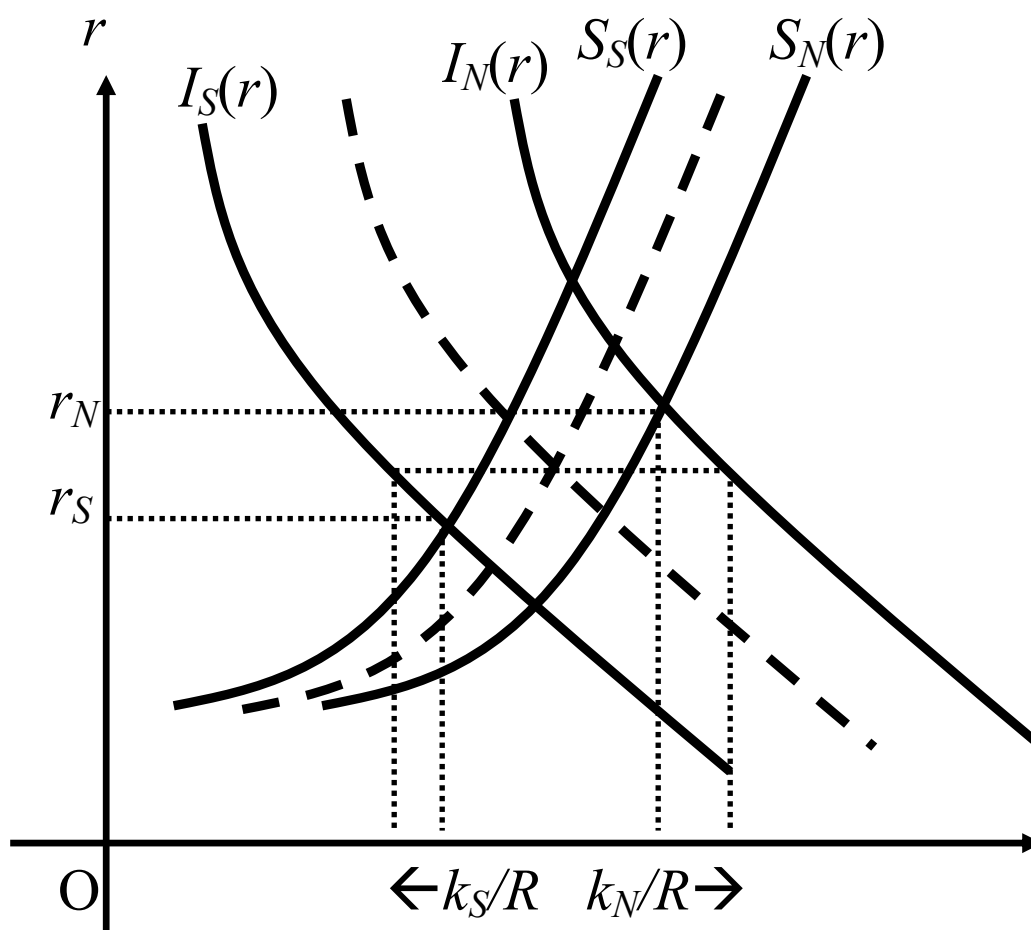
Neoclassical View: North's saving helps to finance South's development. ($\lambda_N = \lambda_S$, $\omega_N = \omega_S$, $\omega_N^0 > \omega_S^0$)



Reverse Capital Flows (I): South's saving leaves for North, and South's capital formation declines, if $\lambda_N > \lambda_S$, $\omega_N = \omega_S$, $\omega_N^0 = \omega_S^0$ (South has poorer financial institutions), or if $\lambda_N = \lambda_S$, $\omega_N - \omega_S = \omega_S^0 - \omega_N^0$ (The savers account for a larger share of the wealth in the South than in the North).



Reverse Capital Flows (II): South's saving leaves for North, and South's capital formation declines, because North's entrepreneurs are more credit-worthy (and if the net worth effect dominates the capital deepening effect): $\lambda_N = \lambda_S$, $\omega_N > \omega_S$ $\omega_N^0 = \omega_S^0$.



The above analysis looked at how the distribution of borrower net worth across countries, (ω_N, ω_S) , or (λ_N, λ_S) , affect the distribution of capital (k_N, k_S)

What would be dynamic implications if we allow for some feedback from (k_N, k_S) to (ω_N, ω_S) or (λ_N, λ_S) ? (Again, wait until Lecture 3).

So far, we have assumed that each agent has access to only one type of investment project.

Let us now look at an example, where an agent can choose which project to invest.

A Two-Country Model: Patterns of International Trade:

Two Countries: **North and South** ($j = N$ or S)

A Continuum of Tradeable Consumption Goods, $z \in [0,1]$
Symmetric Cobb-Douglas preferences.

Homogeneous Agents with Unit Mass, each endowed with $\omega < 1$ units of **Labor**

Tradeable Consumption Goods produced by the projects run by agents

- Each agent can run at most one project.
- Each project in sector z converts one unit of labor to R units of good z .

→ To run the project, one must hire $1 - \omega$ units of labor at the market wage rate, w , from those who don't run the project.

Agent's Income:

$I = p(z)R - w(1-\omega)$ by running the project in sector z .

$I = w\omega$ by not running any project and working for others

Profitability Constraint: The agent *is willing to* run the project in sector z iff

$$(PC-z) \quad p(z)R \geq w$$

Borrowing Constraint: The agent is unable to pledge to the lenders/workers more than a fraction, $\lambda\Lambda(z)$, of the project revenue for the wage repayment. Thus, the agent *can* borrow and start the project in sector z iff

$$(BC-z) \quad \lambda\Lambda(z)p(z)R \geq w(1-\omega),$$

$0 \leq \lambda \leq 1$: country-specific factors

$0 \leq \Lambda(z) \leq 1$: sector-specific factors, continuous and increasing in z .

The Closed Economy Case:

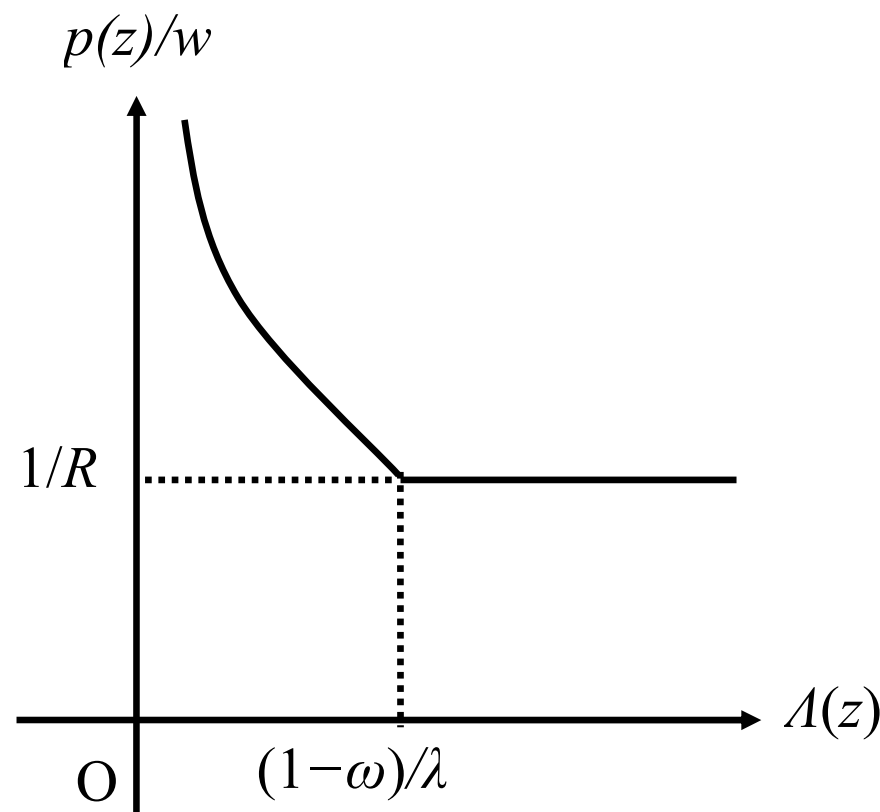
- Both (PC-z) and (BC-z) hold, because the economy produces all the goods.
- One of them is binding in each z , because there would be no workers otherwise.

$$p(z)/w = \max \{1, (1 - \omega)/\lambda \Lambda(z)\} / R$$

➤ (PC-z) is binding in $\Lambda(z) > (1-\omega)/\lambda$

➤ (BC-z) is binding in $\Lambda(z) < (1-\omega)/\lambda$.

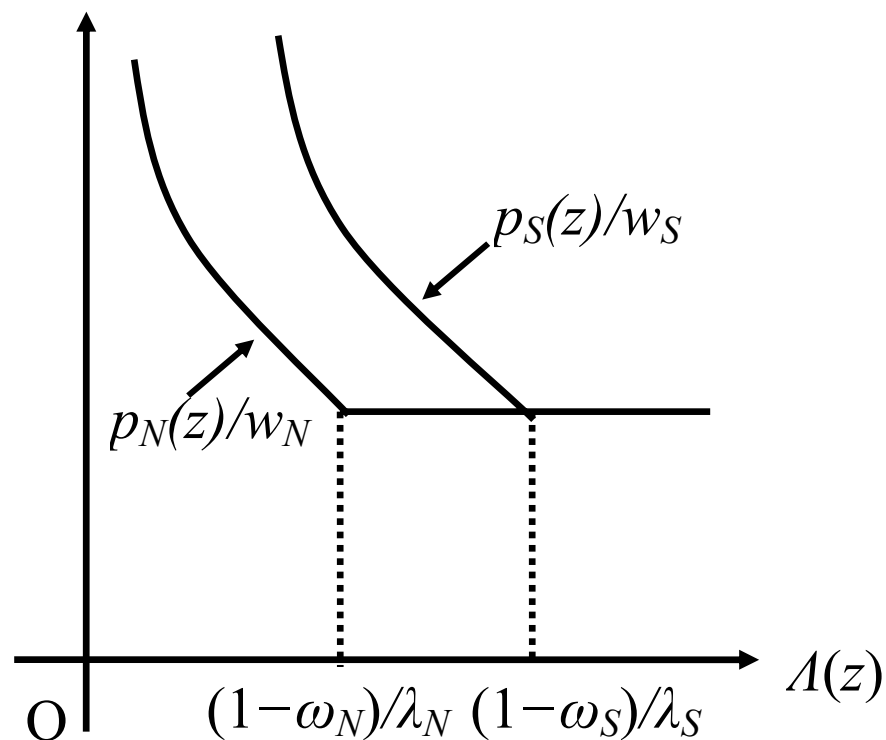
The credit market imperfection restricts entry to the lower-indexed sectors, and the rent created by the limited entry makes the lenders happy to finance the firms in these sectors.



World Economy with North and South: ($\omega_N > \omega_S$, $\lambda_N \geq \lambda_S$).

In Autarky: $p_j(z)/w_j = \max \{1, (1 - \omega_j)/\lambda_j \Lambda(z)\} / R$ ($j = N, S$)

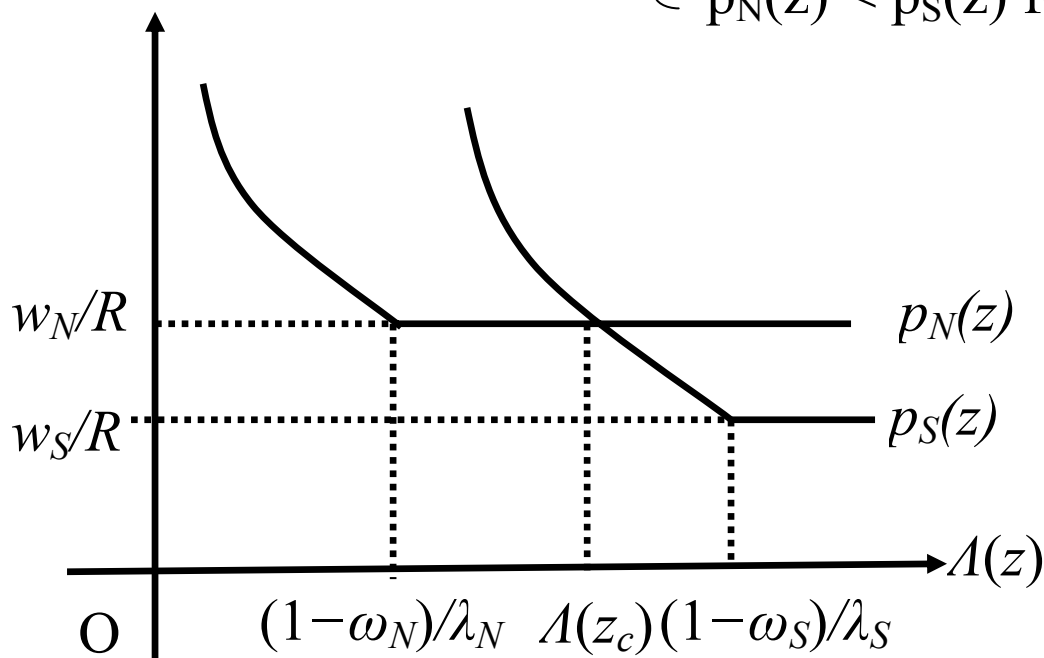
North (South) has *absolute* advantage (disadvantage) in low-indexed goods.



In World Trade Equilibrium:

North's *absolute* advantage translates into a higher wage in North, which implies North's (South's) *comparative* advantage in low (high)-indexed sectors.

$$w_N > w_S \leftrightarrow \begin{cases} p_N(z) > p_S(z) \text{ for } z < z_c. \\ p_N(z) < p_S(z) \text{ for } z > z_c. \end{cases}$$



In all the models we have looked at so far, credit market imperfections distort the allocation of resources.

In the next model, credit market imperfections do not distort the allocation of resources, and yet, have distributional implications through their effects of prices.

A Model of Polarization:

Two Periods: 0 and 1

A Continuum of Agents with Unit Mass:

- The input endowment at period 0, ω , is distributed as $\omega \sim G(\omega)$.
- Consumes only at period 1.

Two Ways of Converting the Input into Consumption.

- Can run **an investment project** with **the variable scale** $I \geq m$, which converts I units of the input into RI units in consumption in period 1, by borrowing $I - \omega$ at the market rate equal to r . (m is **the minimum investment requirement**, i.e., investing $I < m$ generates nothing.)
- **Lending** $x \leq \omega$ units of the endowment in period 0 to receive rx units of consumption in period 1.

Agent's Utility = Objective Function = Consumption in Period 1:

$$U = RI - r(I-\omega) = (R - r)I + r\omega, \text{ if borrow and run the project,}$$
$$U = r\omega \text{ if lend (or put in storage).}$$

If $r > R$, the agent does not want to invest.

If $r = R$, the agent is indifferent.

If $r < R$, the agent *wants to* borrow and invest **as much as possible**.

Borrowing Constraint: The agent *can* borrow and invest iff

$$(BC) \quad \lambda RI \geq r(I-\omega).$$

If $r \leq \lambda R < R$, the agent could borrow and invest by infinite amount. Never happens in equilibrium!

For $\lambda R < r < R$, the agent borrows as much as possible and invest, if it can satisfies the minimum investment requirement.

Agent's Investment Demand for $\lambda R < r < R$:

$$I(\omega) = \begin{cases} \left(1 - \frac{\lambda R}{r}\right)^{-1} \omega & \text{if } \omega \geq m \left(1 - \frac{\lambda R}{r}\right), \\ 0 & \text{if } \omega < m \left(1 - \frac{\lambda R}{r}\right) \end{cases}$$

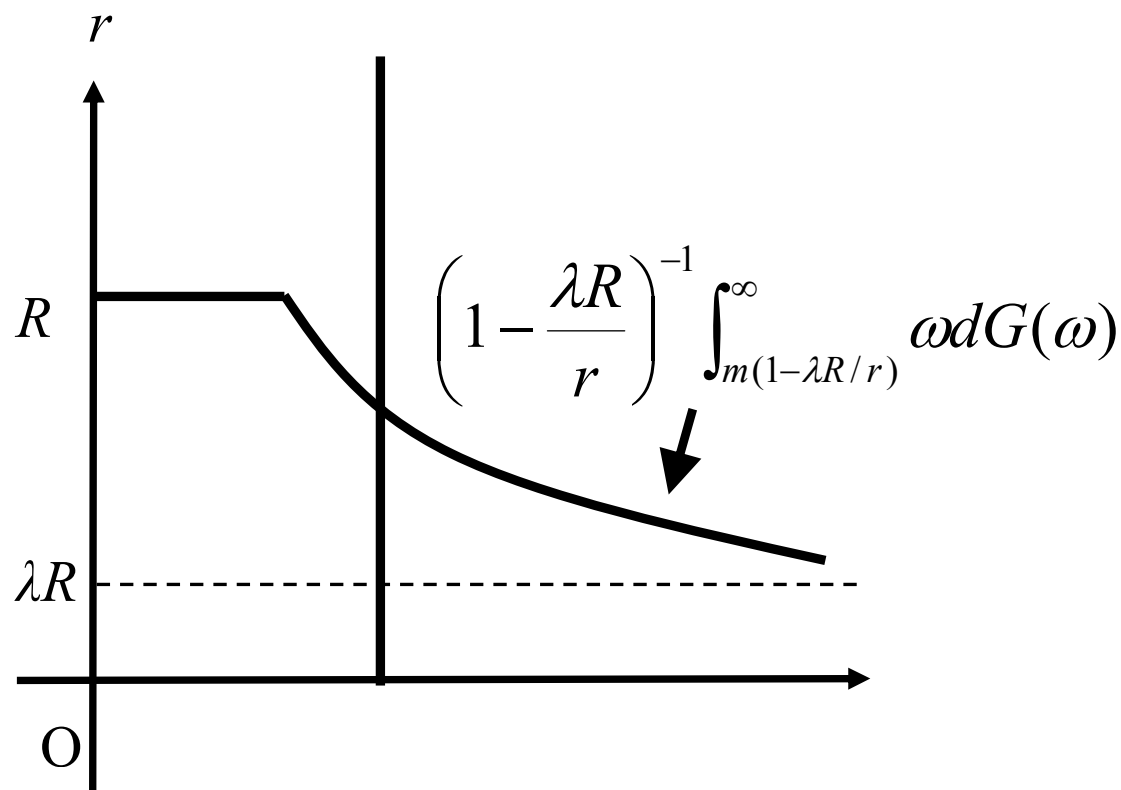
Credit Market Equilibrium:

$$\text{Total Supply} = \int_0^{\infty} \omega dG(\omega) = \left(1 - \frac{\lambda R}{r}\right)^{-1} \int_{m(1-\lambda R/r)}^{\infty} \omega dG(\omega) = \text{Total Demand}$$

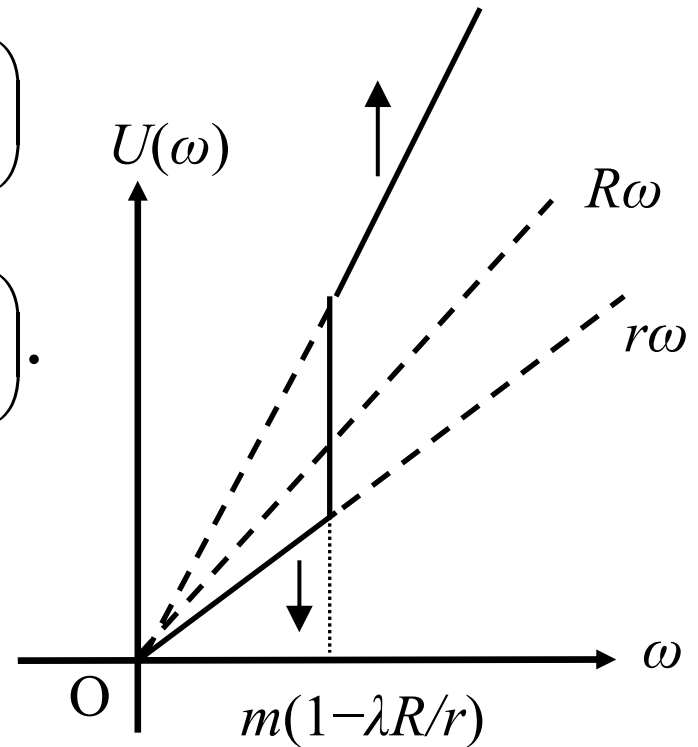
$\lambda R < r < R$ in equilibrium if λ is small enough to satisfy

$$\frac{\int_0^{m(1-\lambda)} \omega dG(\omega)}{\int_0^{\infty} \omega dG(\omega)} > \lambda.$$

In this range, a lower λ reduces r , keeping λ/r constant.



$$U(\omega) = \begin{cases} \frac{1-\lambda}{1-\lambda R/r} R\omega & \text{if } \omega \geq m \left(1 - \frac{\lambda R}{r}\right) \\ r\omega & \text{if } \omega < m \left(1 - \frac{\lambda R}{r}\right). \end{cases}$$



Note that $r < R < \frac{1-\lambda}{1-\lambda R/r} R$.

The marginal return of having an additional unit of the input is strictly

- lower than R for the poor, unless it would push them above the threshold.
- higher than R for the rich, because it would enable them to invest more by borrowing more at the market rate strictly lower than the project return R .

(The Leverage Effect)

In this model,

- credit market imperfections have no effect on the aggregate.
- For any wealth distribution, the *relatively* rich become investors, and the *relatively* poor are prevented from investing.
- A lower λ makes, by reducing r , enrich the rich who borrow to invest, and impoverish the poor who has no choice but to lend.

→ **A Polarization!** (not necessarily a greater inequality)

What if we allow for some feedback from $U(\omega)$ to ω ? Wait until Lecture 3.

Throughout Lecture 1, the net worth of the agents, ω , is treated as exogenous.

The next two lectures introduce the dynamic feedback from the investment to the net worth.

Lecture 2 will look at some models with homogeneous agents, where ω is a single scalar variable.

Lecture 3 will look at some models with heterogeneous agents, with the dynamics of the net worth distributions.