Aggregate Implications of Credit Market Imperfections

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Organization of the paper (not this presentation):
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What this paper does:

• By using *the same, simple* abstract model of credit market imperfections throughout,

• *synthesize* a diverse set of results within a *unified* framework.

• show how the credit market imperfections can be a key to understanding a wide range of aggregate phenomena, including:

  ➢ Endogenous investment-specific technological changes
  ➢ Development traps and Leapfrogging
  ➢ Persistent recessions and recurrent boom-and-bust cycles
  ➢ Reverse international capital flows
  ➢ Rise and fall of Inequality across nations
  ➢ New sources of comparative advantage and patterns of international trade

• with the hope of offering a coherent picture across many results that are *seemingly conflicting* and/or *seemingly unrelated*. 
Recurring themes:

- Properties of equilibrium often respond *non-monotonically* to parameter changes. For example,
  
  ➢ Improving borrower net worth or credit market may first lead to a higher market rate of return and then to a lower market rate of return
  ➢ Improving credit market may first lead to an increased volatility and then a reduced volatility.
  ➢ Productivity improvement may first lead to a greater inequality and then a reduced inequality.
  
  etc.

- Equilibrium and welfare consequences of the credit market imperfections are *rich and diverse* depending on the general equilibrium feedback mechanisms.
What are the basic messages?

(To the outsider of the field):
This is an exciting field, as credit market imperfections have such rich implications.

(To the insider of the field):

Non-monotonicity, in particular, suggests
- Drawing policy implications by comparing a model with credit market imperfections and a model without can be also dangerous, because the effects of improving the credit market could be very different from those of eliminating the credit market imperfections completely.
- The effects of imperfect credit markets could also be very different from the effects of no credit market.

More generally,
Some cautions for studying the equilibrium implications within a narrow class or a particular family of models and extrapolating from it.

“All happy families resemble one another. Each unhappy family is unhappy in its own way.”
Leo Tolstoy, Anna Karenina
A Single Agent’s Problem: serve as the building block in all the equilibrium models to come

Two Periods: \( t = 0 \) and \( t = 1 \)

A Single Agent (an Entrepreneur or a Firm):
- is endowed with \( \omega < 1 \) units of the input at period 0.
- consumes only at period 1.

Two Means to Convert the Input into Consumption:
- Run a non-divisible project, which converts one unit of the input in period 0 into \( R \) units in Consumption in period 1, by borrowing \( 1 - \omega \) at the market rate of return equal to \( r \).
- Lend \( x \leq \omega \) units of the input in period 0 for \( rx \) units of consumption in period 1. (Or, Storage with the rate of return equal to \( r \).)

Agent’s Utility = Consumption in period 1:
\[
U = R - r(1 - \omega) = R - r + r\omega, \quad \text{if borrow and run the project,}
\]
\[
U = r\omega \quad \text{if lend (or put in storage).}
\]

Profitability Constraint: The agent is willing to borrow and invest iff
\[
(\text{PC}) \quad R \geq r
\]
**Borrowing Constraint:** To borrow from the market, the agent must generate the market rate of return, \( r \), per unit to the lenders, yet, *for a variety of reasons*, no more than a fraction, \( \lambda \), of the project output can be used for this purpose. Thus, the agent *can* borrow and invest iff

\[
(BC) \quad \lambda R \geq r(1-\omega).
\]

If \( \lambda/(1-\omega) < r/R \leq 1 \), (PC) holds but not (BC).
- The profitable project fails to be financed, due to the borrowing constraint.
- *Necessary Condition: \( \lambda + \omega < 1 \)*
- A higher \( \omega \) (as well as a higher \( \lambda \)) can alleviate the problem

**Broad Interpretations of the Parameters:**

\( \lambda \): agency problems affecting credit transactions (may vary across projects or industries), institutional quality or the state of financial development (may vary across countries)

\( \omega \): entrepreneur’s net worth, the firm’s balance sheet, the borrower’s credit-worthiness (may vary across agents).

*We now start endogenizing \( R, r, \) and \( \omega \) (but not \( \lambda \))*. 
Partial Equilibrium with Homogeneous Agents

Two Departures:
- A Continuum of Homogeneous Agents with Unit Mass
- A Project produces $R$ units of **Capital**, used in the production of the Consumption Good, $f(k) = F(k, \zeta)$, where $F(k, \zeta)$ is CRS but $f(k)$ is subject to **Diminishing Returns**. $\zeta$ is the hidden factors in fixed supply, owned by those who do not have access to the investment technologies.
- $k = Rn$ is Aggregate Supply of Capital; $n$ is the number of agents running the project.

**Profitability Constraint (PC):** $Rf'(k) \geq r$

**Borrowing Constraint (BC):** $\lambda Rf'(k) \geq r(1 - \omega)$.

**Equilibrium Condition:** $Rf'(k)/r = \text{Max}\{(1 - \omega)/\lambda, 1\}$

If $\lambda + \omega < 1$, $Rf'(k) = r(1 - \omega)/\lambda > r$; Under-Investment;
Net Worth Effect; $\omega \uparrow \rightarrow k \uparrow$

If $\lambda + \omega > 1$, $Rf'(k) = r > r(1 - \omega)/\lambda$; Optimal Investment;
No Net Worth Effect.
Partial Equilibrium with Heterogeneous Agents: $\omega \sim G(\omega)$ with the same R.

If $Rf'(k) > r$; Only those with $\omega \geq \omega_c$ invest.

\[ \rightarrow k = R[1 - G(\omega_c)] = R\left[1 - G\left(1 - \frac{\lambda Rf'(k)}{r}\right)\right]. \]

Comparative Statics: $\lambda \uparrow \rightarrow \omega_c \downarrow$, $k \uparrow$

Distributional Impacts of $\lambda \uparrow$:

The Middle Class (and those who own the hidden factors) gain; the Rich lose.

Credit Market Imperfections as Barriers to Entry

$\rightarrow$ Political Economy Implications
Partial Equilibrium with Heterogeneous Agents: $(\omega, R) \sim G(\omega, R)$

The investing agents must satisfy both

(PC) \hspace{1cm} Rf'(k)/r \geq 1
and

(BC) \hspace{1cm} \omega \geq \omega_c(k) \equiv 1 - \lambda Rf'(k)/r

\[ k = \int_{f'(k)}^{r} R \left[ \int_{\omega_c(k)}^{\infty} g(\omega, R) \, d\omega \right] dR \]

Composition Effects of Improved Credit Market

The rich, but less productive agents in A replaced by the poor, but more productive agents in C.

Also, with a higher $\lambda$,
- A fraction of the active firms that are credit-constrained first goes up and then goes down.
- Aggregate Investment may decline, as the credit shifts towards the more productive.
A General Equilibrium Model with Endogenous Saving:

- Go back to the homogeneous case, where every (investing) agent has the same $R$ and $\omega$.
- Add some “savers”, with no access to the investment technology, who choose to maximize $U^o = V(C^o_0) + C^o_1$ subject to $C^o_1 = r(\omega^o - C^o_0)$.

$\Rightarrow$ Saving by the Savers: $V'(\omega^o - S^o(r)) \equiv r \Rightarrow S^o(r) \equiv \omega^o - (V')^{-1}(r)$.

**Resource Constraint (RC):** $k = R[\omega + S^o(r)] = R[\omega + \omega^o - (V')^{-1}(r)]$.

$\Rightarrow \quad \frac{k}{R} = S(r) \equiv \omega + \omega^o - (V')^{-1}(r)$.

***(PC)+ (BC):** $Rf'(k) = \text{Max}\{1, (1-\omega)/\lambda\} r$.

$\Rightarrow \quad \frac{k}{R} = I(r) \equiv \frac{1}{R} \left( f' \right)^{-1} \left( \text{Max}\left\{ 1, \frac{1-\omega}{\lambda} \right\} \frac{r}{R} \right)$.

which jointly determines $k$ and $r$.

- $S(r)$ depends on $\omega + \omega^o$;
- $I(r)$ depends only on $\omega$. 

![Graph](image-url)
Capital Deepening Effect: 
\[ \Delta \omega^0 > 0 \]

Net Worth Effect: 
\[ \Delta \omega = -\Delta \omega^0 > 0 \text{ (and } \Delta \lambda > 0) \]
when \( \lambda + \omega < 1 \).

Combined Effects: 
\[ \Delta \omega > 0 \]
when \( \lambda + \omega < 1 \).

The equilibrium rate of return is *non-monotonic in* \( \lambda \) (and \( \omega \));
A Two-Country Model: Patterns of International Capital Flows

Two Countries: **North and South** of the kind described above

North and South share the same $f(k)$ and $R$, but may differ in $\lambda$, $\omega$, and $\omega^o$.

Further Assumptions:
- The Input and the Consumption Good are *tradeable*. $\rightarrow$ This allows the agents to lend and borrow and make the repayment across the borders.
- Physical Capital and the “hidden inputs” is *nontradeable*. We later relax this assumption.
- Only the agents in North (South) can produce Physical Capital in North (South), effectively ruling out FDI. We later relax this assumption.

**Experiment:**

Suppose the agents in North can pledge $\varphi \lambda_N$ to the lenders in the South, and the agents in South can pledge $\varphi \lambda_S$ to the lenders in the North.

Now let $\varphi$ change from $\varphi = 0$ (Financial Autarky) to $\varphi = 1$ (Full Financial Integration).
Neoclassical View
\[ \lambda_N = \lambda_S, \; \omega_N = \omega_S, \; \omega^o_N > \omega^o_S; \]

Capital Flight:
\[ \lambda_N > \lambda_S, \; \omega_N = \omega_S, \; \omega^o_N = \omega^o_S; \] or
\[ \lambda_N = \lambda_S, \; \omega_N - \omega_S = \omega^o_S - \omega^o_N > 0. \]
Dynamic Implications:

- Let us introduce a dynamic feedback from $k_N$ to $\omega_N$ (and from $k_S$ to $\omega_S$).
- We can do this by embedding the above structure into an OG framework; so that a higher investment by the current generation leads to a higher demand for the endowment of the next generation, which leads to a higher net worth, $\omega$.
- This could lead to Endogenous Inequality across countries from an intermediate value of $R$.
- Going from a low value of $R$ to a higher value of $R$ could generate Inverted U-curve patterns of Endogenous Inequality.

Schematically…
Some Other Extensions:

- Allowing the agents in the North to run the project in the South with reduced productivity could lead to Two-Way Flow of Financial Capital and FDI.
  - Savers in the South lends to Firms in the North, which invest in the South.
  - FDI can be used to bypass the external capital market in the South.

- Introducing Trade in Inputs, which are subject to some trade costs.
  - This could lead to positive spillovers in neighboring countries; Regional contagions (East Asian booms and Latin American stagnations)

- Endogenous Investment Technologies
  - Two-Way Causality between Productivity Differences vs. Credit Market Imperfections
  - Financial Capital may flow into countries with worse credit markets; A solution to the allocation puzzle??
General Equilibrium Model with Heterogeneous Projects (with Homogeneous Agents)

- A Continuum of Homogeneous Agents with Unit Mass (No Savers)
- Each Agent can choose one (and only one) of $J$ non-divisible projects.

<table>
<thead>
<tr>
<th></th>
<th>Period 0</th>
<th>Period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Type-j Project:</em></td>
<td>$m_j$ units of the input</td>
<td>$m_jR_j$ units in capital &amp; $m_jB_j$ units in consumption</td>
</tr>
</tbody>
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$m_j$: the (fixed) set-up cost,
$R_j$: project productivity in capital
$B_j$: project productivity in final good

**Profitability Constraint (PC-\(j\))**: $R_jf'(k) + B_j \geq r$

**Borrowing Constraint (BC-\(j\))**: $m_j[\lambda_jR_jf'(k) + \mu_j B_j] \geq r(m_j - \omega)$,

$\lambda_j$: pledgeability of capital produced by project-\(j\)
$\mu_j$: pledgeability of the final good produced by project-\(j\)
Equilibrium Conditions;

(1) \[ \omega = \sum_j (m_j n_j). \]

(2) \[ k = \sum_j (m_j R_j n_j). \]

(3) \[ r \geq \min \left\{ \frac{\lambda_j R_j f'(k) + \mu_j B_j}{1 - \omega / m_j}, R_j f'(k) + B_j \right\}; n_j \geq 0 \quad (j = 1, 2, \ldots J) \]

where \( n_j \) is the measure of type-\( j \) projects initiated.
Example 1: $J = 2; R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$. $B_1 = B_2 = 0$.

**Key Trade-offs: Productivity vs. Agency Problems;**

Project-2 is more productive, but comes with bigger agency problems than Project-1.

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**Procyclical Investment Specific Tech Change**

**Dynamic Implications: Credit Traps**
Example 2: \( J = 2 \) and \( R_2 > R_1 > \lambda_2R_2 > \lambda_1R_1, \) \( m_2/m_1 > (1-\lambda_1)/(1-\lambda_2R_2/R_1) > 1. \) \( B_1 = B_2 = 0. \)

The less productive and less “secure” project-1 have advantage of smaller set-up costs
Example 3: \( J = 2; \lambda_1 = \lambda_2 = 1, \mu_1 = \mu_2 = 0, \Delta R \equiv R_2 - R_1 > 0, B_1 > B_2 = 0 \)

Project-1 is less “socially productive” but generates more “private benefits” or “personal satisfaction” than Project-2.

- Project-1 cannot be financed if \( \omega < (\Delta R/R_2)m_1 \).
- If \( B_1 > \Delta Rf'(R_1(\Delta R/R_2)m_1) \), the agents invest to Project-1 whenever \( \omega > (\Delta R/R_2)m_1 \).

- In boom, the entrepreneurs can finance the self-indulgent project.
- In recession, they cannot.

Along these cycles, the booms occur due to the misallocation of the credit.
Example 4: $J = 2; R_1 > R_2 = 0, B_1 = 0 < B_2$ and $\lambda_1 < 1, \mu_2 = 1$,

**Persistence of Inefficient Recessions: Financial Accelerator Models**

Under-investment of Capital-Generating Project

A Temporary Shock has an Echo Effect

```
\[ \lambda_1 R f'(k) = B(1 - \omega/m_1) \]

```

Slow Recovery from Recession

```
R\omega_c
\]

```

Permanent Recession

```
RW(k_c)
\]

```

```
\[ RW(k_t) \]

```

```
45^\circ
\]

```

```
W^{-1}(m_1(1-\lambda_1))
\]

```

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Example 5: \( J = 2; \ R_1 > R_2 = 0, \ B_1 = 0 < B_2 \) and \( \lambda_1 = 1, \ \mu_2 < 1, \)

**Inefficient Booms and Volatility:**

Over-Investment to Capital-Generating Project

Dynamic Implications: Endogenous Cycles

Again, non-monotonicity; Endogenous Fluctuations Occur for an intermediate value of \( \mu_2 \)
Example 6: Hybrid of “Persistence of Inefficient Recessions” & “Inefficient Booms and Volatility” Models

Asymmetric Cycles and Intermittent Volatility
A Two-Country Model: Patterns of International Trade:

Two Countries: **North and South** ( j = N or S)

A **Continuum of Tradeable Consumption Goods**, \( z \in [0,1] \)
Symmetric Cobb-Douglas preferences.

**Homogeneous Agents** with Unit Mass, each endowed with \( \omega < 1 \) units of the Input (**Labor**)

**Tradeable Consumption Goods** produced by the projects run by agents
- Each agent can run at most one project.
- Each project in sector \( z \) converts one unit of labor to \( R \) units of good \( z \).
  \( \rightarrow \) To run the project, one must hire \( 1 - \omega \) units of labor at the market wage rate, \( w \), from those who don’t run the project.

**Profitability Constraint (PC-\( z \)):** \( p(z)R \geq w \)

**Borrowing Constraint (BC-\( z \)):** \( \lambda\Lambda(z)p(z)R \geq w(1-\omega) \),
  \( 0 \leq \lambda \leq 1 \): country-specific factors
  \( 0 \leq \Lambda(z) \leq 1 \): sector-specific factors, continuous and increasing in \( z \).
Under $\omega_N > \omega_S$ and/or $\lambda_N > \lambda_S$.

**Autarky Equilibrium:**
- (PC-\(z\)) is binding for $\Lambda(z) > (1 - \omega)/\lambda$.
- (BC-\(z\)) is binding for $\Lambda(z) < (1 - \omega)/\lambda$.

- The credit market imperfection restricts entry to the low-indexed sectors.
- The rent created by the limited entry makes the lenders happy to finance the firms in these sectors.

$\rightarrow$ North has *absolute* advantage.

**World Equilibrium:** A higher wage in North.

- North’s *comparative* advantage in low-indexed sectors.
- South’s *comparative* advantage in high-indexed sectors.

North, with the better contractual environment, specializes in the sectors that are more subject to agency problems.
A Model of Polarization:

Two Periods: 0 and 1

A Continuum of Agents with Unit Mass:
• The input endowment at period 0, $\omega$, is distributed as $\omega \sim G(\omega)$.
• Consumes only at period 1.

Two Ways to Convert the Input into Consumption.
• Can run an investment project with the variable scale $I \geq m$, which converts I units of the input into RI units in consumption in period 1, by borrowing $I - \omega$ at the rate equal to $r$. (m is the minimum investment requirement, i.e., investing $I < m$ generates nothing.)
• Lending $x \leq \omega$ units of the endowment in period 0 for $rx$ units of consumption in period 1.

Agent’s Utility = Objective Function = Consumption in Period 1:
$$U = RI - r(I - \omega) = (R - r)I + r\omega, \quad \text{if borrow and run the project,}$$
$$U = r\omega \quad \text{if lend (or put in storage).}$$

If $r > R$, the agent does not want to invest.
If $r = R$, the agent is indifferent.
If $r < R$, the agent wants to borrow and invest as much as possible.
**Borrowing Constraint:** The agent *can* borrow and invest iff

\[(BC) \quad \lambda RI \geq r(I-\omega).\]

If \(r \leq \lambda R < R\), the agent could borrow and invest by infinite amount. Never happens in equilibrium!

For \(\lambda R < r < R\), the agent borrows as much as possible and invest, if it can satisfies the minimum investment requirement.

**Agent’s Investment Demand** for \(\lambda R < r < R\):

\[I(\omega) = \left(1 - \frac{\lambda R}{r}\right)^{-1} \omega \text{ if } \omega \geq m\left(1 - \frac{\lambda R}{r}\right); \quad I(\omega) = 0; \text{ otherwise.}\]
Credit Market Equilibrium:

Total Supply \(= \int_{0}^{\infty} \omega dG(\omega) = \left(1 - \frac{\lambda R}{r}\right)^{-1} \int_{m(1-\lambda R/r)}^{\infty} \omega dG(\omega) = \text{Total Demand}\)

\(\lambda R < r < R\)

if \(\int_{0}^{m(1-\lambda)} \omega dG(\omega) > \lambda\).

In this range,

a lower \(\lambda\) reduces \(r\), keeping \(\lambda/r\) constant.
\[
U(\omega) = \begin{cases} 
\frac{1-\lambda}{1-\lambda R/r} R\omega & \text{if } \omega \geq m\left(1-\frac{\lambda R}{r}\right) \\
r\omega & \text{if } \omega < m\left(1-\frac{\lambda R}{r}\right).
\end{cases}
\]

Note that \( r < R < \frac{1-\lambda}{1-\lambda R/r} R \).

The marginal value of having an additional unit of the input is strictly
- lower than \( R \) for the poor, unless it would push them above the threshold.
- higher than \( R \) for the rich, because it would enable them to invest more by borrowing more at the market rate strictly lower than the project return \( R \). (The Leverage Effect)
In this model,

- credit market imperfections have no effect on the quantity, or any aggregate variables.
- For any wealth distribution, the relatively rich become investors, and the relatively poor are prevented from investing.
- A lower $\lambda$ makes, by reducing $r$, enrich the rich who borrow to invest, and impoverish the poor who has no choice but to lend.

$\rightarrow$ **A Polarization!** (not necessarily a greater inequality)

*Dynamic Implications:* What if we allow for some feedback from $U(\omega)$ to $\omega$?

- The Poor may benefit from the credit demand by the rich (Trickle Down Effect)
- Endogenous Inequality

**Interactions between the Rich and the Poor may also take place through Labor Markets.**

A proper discussion of this requires entirely a whole new paper.
**Concluding Remarks:**

- Credit Market Imperfections are *rich and diverse* in the aggregate implications.
  - It is so rich that they are useful for understanding a wide range of important issues.
  - It is so diverse that properties of equilibrium often respond *non-monotonically* to parameter changes, suggesting some cautions for studying the aggregate implications of within a narrow class or a particular family of models

- Although this paper synthesizes a diverse set of results with a unified framework, it is far from comprehensive. A large number of issues have not been discussed.
  - Multi-stage financing and liquidity implications
  - Net worth revaluation through asset price changes,
  - Endogenous net worth accumulation by borrowers
  - Endogenous growth, financial intermediation, development of financial markets
  - Asset pricing and monetary policy implications
  - Political economy implications
  - Interacting with other sources of inefficiency such as product market imperfections

→ This is merely the tip of the iceberg: more work needs to be done.