Aggregate Implications of Credit Market Imperfections

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Abstract:

Credit market imperfections provide the key to understanding many important issues in business cycles, growth and development, and international economics. Recent progress in these areas, however, has left in its wake a bewildering array of individual models with seemingly conflicting results. This paper offers a road map. Using the same single model of credit market imperfections throughout, it brings together a diverse set of results within a unified framework. In so doing, it aims to draw a coherent picture so that one is able to see some close connections between these results, thereby showing how a wide range of aggregate phenomena may be attributed to the common cause. They include, among other things, endogenous investment-specific technical changes, development traps, leapfrogging, persistent recessions, recurring boom-and-bust cycles, reverse international capital flows, the rise and fall of inequality across nations, and the patterns of international trade. The framework is also used to investigate some equilibrium and distributional impacts of improving the efficiency of credit markets. One recurring finding is that the properties of equilibrium often respond non-monotonically to parameter changes, which suggests some cautions for studying aggregate implications of credit market imperfections within a narrow class or a particular family of models.

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Road Map

"All happy families resemble one another. Each unhappy family is unhappy in its own way."

Leo Tolstoy, *Anna Karenina*

1. Introduction

It is widely recognized that the market economy fails to allocate the credit to the most productive investment projects because credit transactions are subject to agency problems. In the presence of such imperfections, the borrower's net worth—also known as the balance sheet condition—plays crucial roles in allocating the credit across entrepreneurs, firms, industries, and nations. A change in the aggregate level of wealth and a change in the distribution of wealth thus affect the equilibrium allocation of the credit and hence patterns of investments. Furthermore, the resulting change in the investments causes a further change in the level and distribution of wealth, which leads to a further change in the equilibrium patterns of investments.

Stimulated in part by the advances made in the microeconomics of financial markets and corporate finance over the last thirty years, the problems of credit market imperfections have recently found many applications in business cycles, growth and development, and international economics. However, this progress has left in its wake a bewildering array of individual models with seemingly conflicting results. For example, do the imperfections add persistence to the macroeconomic dynamics, as suggested by Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and others? Or, do they add volatility, as suggested by Azariadis and Smith (1998), Aghion and Banerjee (2005) and Matsuyama (2004a)? If they are responsible for causing fluctuations, is it because the misallocation of credit creates recessions or because it creates boom-and-bust cycles? Does improving credit markets make fluctuations more or less volatile? Does financial globalization alleviate or exacerbate the credit market imperfections? Do credit transactions between the rich and the poor help to reduce or magnify the inequality among households?, etc.

This paper comes out of my conviction that credit market imperfections offer the key to understanding a wide range of important issues. Yet, aggregate implications of credit market imperfections are so rich and diverse that one should not expect to find a simple answer of any of the above "either-or" questions. It should come as no surprise that they are so rich and diverse because they depend on the manner in which different agents and/or different investments interact with each other, which consequently affects amplification and propagation mechanisms. This suggests that the intuition gained from studying a particular family of models can be misleading because the results may be driven by the specific assumptions made about the set of agents and investment projects that are competing for credit.

This paper offers a road map for understanding aggregate implications of credit market imperfections. By bringing together a diverse set of results (many existing and some new) within a unified framework, it aims to draw a coherent picture so that one is able to see some close connections between seemingly conflicting results across many topics that are ordinarily treated separately or even sometimes viewed as unrelated.[2] To this end, I first develop a simple, highly abstract model of credit market imperfections, which is meant to capture all sorts of agency problems that affect credit transactions. Using this single model throughout, I examine the effects of credit market imperfections in series of relatively simple equilibrium models. The discussion will be organized by the manners in which different models "close the system," i.e., based on the assumptions about the sets of agents and of projects competing for the credit and about the price effects of different investment projects, which determine the amplification and propagation mechanisms in these models. It will be shown how a wide range of aggregate phenomena may be attributed to the common cause. They include, among other things, endogenous investment-specific technical changes[3], development traps, leapfrogging, persistence of inefficient recessions, recurrent boom-and bust cycles, reverse international capital flows, the rise and fall of inequality across nations, and the patterns of international trade. The framework is also used to investigate some equilibrium and distributional impacts of improving the efficiency of credit markets.

One recurring finding is that the properties of equilibrium often respond non-monotonically to parameter changes. For example, increasing borrower's net worth first leads to a higher equilibrium rate of return and then to a lower equilibrium rate of return; improving credit market first leads to increased volatility and then to reduced volatility; productivity improvements first lead to greater inequality and then to reduced inequality. Such non-monotonicity suggests some cautions for studying their effects within a narrow class or a particular family of models. For example, in their attempts to understand the effects of credit market imperfections, many authors study models with no credit market. Yet, there is no reason to believe that the effects of an imperfect credit market are similar to those of no credit market. In their attempts to understand the effects of improving the credit market, many authors compare models with credit market imperfections and models with the perfect credit market. Yet, there is no reason to believe the effects of (partially) improving credit markets are similar to those of eliminating credit market imperfections completely.[4]

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[2]The existing surveys focus on a few specific areas of applications. See Bernake, Gertler, and Gilchrist (1999) for business cycle propagation mechanisms; Banerjee-Duflo (2005) in development economics; Bertola, Foellmi, Zweimueller (2006, Ch.7) for income distributions. Tirole (2005, Part VI, particularly Ch.13) is closest in spirit to this paper, but it does not cover any applications in international economics. Gertler (1988) offers an interesting glimpse on the state of the field before it became a major research topic.

[3]Credit market imperfections could also cause endogenous fluctuations of TFP to the extent they affect financing of working capital. I have chosen to focus on investment-specific technical change because of some recent studies suggesting that investment-specific technical changes perform better than the traditional, neutral (TFP) technical changes; see Greenwood, Hercowitz, Krusell (1997, 2000) and Fisher (2006).

[4]In my view, anyone who believes in the credit market imperfections, at least seriously enough to do research in this area, should never examine the impacts of any policy under the assumption that such a policy could eradicate the imperfections. The most one could hope for in any policy is to improve the credit market.

2. A Simple Model of Credit Market Imperfections: A Single Agent's Perspective

Let us start with a simple and highly abstract model of credit market imperfections, which will be used as a building block in all the equilibrium models discussed below. Here, we will look at the problem faced by a single agent (an entrepreneur or a firm) in isolation, taking its environment as exogenous.

The world lasts for two periods: 0 and 1. The agent is endowed with $\omega < 1$ units of the input in period 0 and consumes only in period 1. There are two ways of converting the period-0 input into the period-1 consumption. First, the agent can run a non-divisible investment project, which converts one unit of the input in period 0 into R units in consumption in period 1 by borrowing $1-\omega$ at the gross market rate of return equal to r. Second, the agent can lend $x \leq \omega$ units of the input in period 0 for rx units of consumption in period 1.

The agent's objective is to maximize the period-1 consumption. By borrowing $1-\omega$ to run the project, the agent could produce R units of the consumption good, from which $r(1-\omega)$ units need to be repaid, so that the period-1 consumption (and the utility) would be equal to $U = R - r(1-\omega) = R - r + r\omega$. This is greater than or equal to $U = r\omega$, the amount that the agent could consume by lending at the gross market rate r, if and only if

(1) $R \geq r$. (PC)

Thus, the project return needs to be higher than (or equal to) the opportunity cost of running the project in order to make the agent *eager* (or *willing*) to borrow and invest. This constraint (as well as the analogous constraints in all the models developed later) shall be called (PC) for *Profitability Constraint*.

Even when (PC) holds with strict inequality and hence the agent is eager to invest, credit market imperfections might keep the agent from investing. To obtain credit, the agent must somehow generate to the lenders the rate of return, r, which is determined by the market. However, *for a variety of reasons*, the entire project output, R, may not be used for the purpose. To capture this in the simplest manner, it is assumed that no more than a fraction, λ, of the project revenue can be pledged to the lenders for the repayment. Thus, the agent can generate the rate of return required by the lenders, if and only if

(2) $\lambda R \geq r(1-\omega)$. (BC)

Only when this constraint is met, the agent is *capable* of borrowing and investing. This constraint (as well as the analogous constraints in all the models developed later) shall be called (BC) for *Borrowing Constraint*.[5] Another way of looking at this constraint is $b = 1-\omega \leq \lambda R/r$, which is to say that that borrowing is limited by the present discounted value of the pledgeable revenue of the project, $\lambda R/r$. This

[5]Those who prefer the language of agency theory may want to refer to the Profitability Constraint as the Borrower's Incentive Compatibility (or Participation) Constraint and to the Borrowing Constraint as the Lenders' Incentive Compatibility Constraint (or Participation) Constraint.

constraint can also be rewritten as $\omega \geq 1 - \lambda R/r$, which states that a sufficiently high net worth would overcome the credit market imperfection.[6]

For the investment to take place, the agent must be both willing *and* capable to borrow, that is, only when both (PC) and (BC) are satisfied. Which of the two is a relevant constraint depends on $\lambda + \omega$. If $\lambda + \omega > 1$, (PC) is more stringent than (BC). That is, the agent can borrow whenever he wants to borrow. In this case, credit market imperfections do not affect the investment decision. If $\lambda + \omega < 1$, on the other hand, (BC) is more stringent than (PC). That is, credit market imperfections may affect investment. Indeed, if $1 \leq R/r < (1-\omega)/\lambda$, (PC) holds but (BC) does not, meaning that the agent cannot initiate the profitable investment project due to the borrowing constraint.

In this simple, highly abstract model of credit market imperfections, the pledgeability, λ, is meant to capture all sorts of agency problems that restrict the agent's ability to finance the profitable investment externally. The severity of these agency problems could depend on the project, the industry, or the institutional factors that determine the general efficiency of credit markets, such as the quality of legal or contractual enforcement, corporate governance, or more broadly the state of financial development of the economy. For this reason, I will later allow the pledgeability parameter, λ, to vary across projects, across industries or across countries in equilibrium models with many projects, many industries, or many countries.

In this simple, highly abstract model of credit market imperfections, the input endowment, ω, is meant to capture the entrepreneur's net worth, the firm's balance sheet condition, or, more broadly, the borrower's credit-worthiness. For this reason, I will later allow the net worth parameter, ω, to vary across agents in equilibrium models with heterogeneous agents. It will also be allowed to depend on the past investment when considering the dynamic implications of credit market imperfections.

Remark 1: The microeconomics of credit markets offers many different agency stories that could be used to justify the assumption that the borrowers cannot fully pledge the project revenue.[7] The simplest story would be that they strategically default, whenever the repayment obligation exceeds the default cost. Alternatively, each project is specific to the agent and requires his services to generate the maximum revenue. Without his services, the revenue would be reduced. Then the borrower, by threatening to

[6]Some authors call the inequality analogous to $\omega \geq 1-\lambda R/r$ "the collateral constraint," while other authors call it "the cash flow constraint," or "the liquidity constraint." In doing so, they assume that the borrower's net worth held only in collateralizeable assets or only in liquid assets could be used to satisfy the constraint. I deliberately avoid the use of the terms "collateral" or "liquidity" because I am primarily concerned with the question of how the borrowing constraint is affected by the (level of) borrower net worth, abstracting from the role of the borrower's portfolio or liquidity holdings. Needless to say, this is an important issue, but its careful treatment would require a much richer framework than the one used in this paper.
[7]See, e.g., Tirole (2005; Ch.3, Supplementary Sections).

withdraw his services, can renegotiate the repayment; see Hart and Moore (1994), Hart (1995), and Kiyotaki and Moore (1997). There is also the costly-state-verification approach of Townsend (1979), used by Bernanke and Gertler (1989) and Boyd and Smith (1997) and others. See also the moral hazard approach used by Holmstrom and Tirole (1997, 1998) and others. A large number of studies are devoted to the issues of the relative merit (conceptual and/or empirical) of different agency stories; see, e.g., Hart (1995, Ch.5, Appendix) and Paulsen, Townsend and Karaivanov (2006). In this paper, however, I will not be concerned with the question of which stories offer most plausible or compelling explanations for the *microeconomic causes* of credit market imperfections. Instead, I will simply treat credit market imperfections as a fact of life, and proceed with investigating their *aggregate* or *equilibrium consequences*, using the highly abstract, reduced form approach, which is meant to encompass all sorts of agency problems discussed in the microeconomic literature.[8]

Remark 2: The careful reader must have undoubtedly noticed that I deliberately avoid the use of the terms such as "debt capacity," "interest rate," and "loan market," and instead use "borrowing constraint," "rate of return," and "credit market." This is because this paper is concerned with aggregate implications of credit market imperfections, arising from broad external financing difficulties. Note that the borrowing constraint arises due to the inability of the borrowers to pledge the entire project revenue to generate the higher rate for the lenders, not because of any arbitrary restriction on the menus of the financial claims that they can issue. The main issues addressed here are general enough that they are independent of the financial structure. Indeed, the model is too abstract to make a meaningful distinction between the equity, the debt, the bonds, or any other forms of financial claims, which should be viewed as an advantage of the model.[9]

3. Partial Equilibrium Models

We have so far looked at only the single agent's problem in isolation, holding all the prices as exogenous, and without worrying about interactions among agents. Let us now start letting many agents interact through equilibrium prices.

[8] Broadly speaking, there are three reasons for this. First, the major causes of credit market imperfections, even if we could identify them in certain specific cases, are likely to vary across investment types, industries, countries and times. Second, at least qualitatively speaking, much of the aggregate and equilibrium implications of credit market imperfections do not depend on the specific nature of the agency problems behind the imperfections. The last, and perhaps the most important, reason is a practical one. This reduced form approach saves space, as well as the time and effort of the reader. For example, this approach enables me to reproduce the key results of Bernanke and Gertler (1989) and of Boyd and Smith (1997), each of which devoted many pages and appendices to explain the optimal contract problem under costly state verification. In contrast, I needed only one short paragraph to describe the borrowing constraint.

[9] See Tirole (2005, p.119) who also argues for the benefits of separating the general issues of credit market imperfections from the questions of the financial structure.

3.1. Homogeneous Agents: The Net Worth (Balance Sheet) Effect

Let us now consider a continuum of homogenous agents with unit mass. As before, each agent is endowed with $\omega < 1$ units of the input at period 0 and consumes only at period 1. In addition to the presence of many agents, the key difference is that the projects run by the agents convert the input into capital. Capital is then used to produce the consumption good in period 1, with the CRS technology, $F(k, \zeta)$, where k is the total supply of capital (which will be determined in equilibrium), and ζ is a vector of "the hidden factors" in fixed supply. Let $f(k) = F(k, \zeta)$, which satisfies $f' > 0 > f''$ and $f'(0) = \infty$. The competitive factor markets reward each unit of capital by $f'(k)$ and the residual, $f(k) - kf'(k) > 0$, goes to the owners of "the hidden factors" in fixed supply.[10]

The agents have two means of converting the input endowments into consumption. First, each agent can run a non-divisible investment project, which converts one unit of the input in period 0 into R units of capital in period 1 by borrowing $1-\omega$ at the market rate, r, which continues to be treated as exogenous.[11] Since each unit of capital earns $f'(k)$ in consumption, each project generates $Rf'(k)$ units of the consumption good. Second, each agent can lend the input endowment in period 0 at the rate of return equal to r, as before. Finally, the total supply of capital is given by $k = Rn$, where n is the number (or the fraction) of the agents who borrow and invest. Both n and $k = Rn$ are determined in equilibrium.

By borrowing $1-\omega$ to run the project, the agent can consume $U = Rf'(k) - r(1-\omega)$. By lending at the market rate, r, the agent can consume $U = r\omega$. By comparing the two, (PC) now becomes

(3) $\quad Rf'(k) \geq r.$ (PC)

On the other hand, (BC) is now given by

(4) $\quad \lambda Rf'(k) \geq r(1-\omega).$ (BC)

Note that the two constraints in this model, (3) and (4), differ from those in the single agent's problem, (1) and (2), only in that the project revenue, R, is now replaced by $Rf'(k)$, and hence become endogenous.

In equilibrium the investment takes place until one of the (BC) and (PC) becomes binding, so that

(5) $$Rf'(k) = Max\left\{1, \frac{1-\omega}{\lambda}\right\}r \qquad \text{(PC) + (BC)}$$

This determines the equilibrium value of k.[12] When $\lambda + \omega < 1$, (5) becomes

[10] The owners of the "hidden factors" play no active role in the economy other than supplying these factors inelastically and absorbing the residual income. The hidden factors are introduced here merely to generate diminishing returns to capital. Later, these "hidden factors" in fixed supply will be given an additional role when this model is embedded in a dynamic setting to endogenize the borrower net worth.

[11] One may think that the agents have access to a storage technology of return, r. Alternatively, this may be viewed as a model of a small open economy or of an industry.

[12] Eq. (5) implicitly assumes the interior solution, which can be ensured by imposing that $Rf'(R) < r$. Then, (5) holds with $0 < k < R$, which implies that $0 < n < 1$.

(6) $$Rf'(k) = \left(\frac{1-\omega}{\lambda}\right) r > r$$

Thus, (BC) is binding but (PC) is not; the project return is strictly higher than the opportunity cost of the project. All the agents are eager to invest, but no more agents can borrow and invest, because that would violate (BC). In short, there is *too little* investment. In this case, improving the credit market, an increase in λ, obviously leads to a higher investment. A higher ω also leads to a higher investment. This is *the net worth (or balance sheet) effect*. As the borrower net worth improves, the agents need to borrow less, which eases the borrowing constraint, and hence more investment will be financed. When $\lambda + \omega > 1$, on the other hand, (5) becomes

(7) $$Rf'(k) = r > \left(\frac{1-\omega}{\lambda}\right) r \ .$$

Thus, (PC) is binding and the level of investment is optimal. In this case, there is no net worth effect; a higher ω would not affect the investment.

Remark 3: In this model, only the fraction n of the agents invests in equilibrium, which means that the fraction 1–n of them becomes the lenders. This obviously raises the question, "How can the credit be allocated only to a fraction of *homogeneous* agents?" When $\lambda + \omega > 1$, this is not a problem, because (PC) is binding; $Rf'(k) = r$. Thus, the agents are indifferent between borrowing and lending. However, when $\lambda + \omega < 1$, (PC) is not binding, $Rf'(k) > r$, so that the agents strictly prefer borrowing to lending. There are two possible resolutions for this. First, we may think that the equilibrium allocation necessarily involves credit rationing in this case. The credit is allocated randomly to the fraction n of the agents, while the rest of the agents are denied credit.[13] The latter have no choice but to become the lenders; they would not be able to entice others to switch and become lenders by promising a higher return because that would violate (BC). Second, we may view the homogeneous agent model as the limit case of some heterogeneous agent models. For example, the agents may differ in their endowment, distributed according to G(ω). Then, the equilibrium is given by the threshold level of the endowment, ω_c, such that the agents whose endowments are lower (higher) than ω_c become the lenders (borrowers). Indeed, we will shortly look at such a model with heterogeneous endowments. The above model may be obtained as the limit when G(ω) converges to a single mass point.

Remark 4: Just in case one might suspect that the results here may be driven by the indivisibility of the projects, not by credit market imperfections, the role of the indivisibility assumption here is more subtle than one might think. In the literature, it is often argued that the equilibrium analysis of credit market

[13] For example, imagine the following sequential service constraint: the market approves n credits on the first-come, first-serve basis, and those agents whose credit applications are delayed (for some random reasons) will be denied.

imperfections is fundamentally difficult because it is *necessary* to model heterogeneous agents in order for credit market transactions to take place. This is not true; credit market transactions can take place even among homogeneous agents if there are some indivisibility constraints. In what follows, I find it useful to assume the homogeneous agents within each economy when exploring the effects of heterogeneity across projects or heterogeneity across countries. The indivisibility is assumed only to keep the credit market active even among the homogeneous agents. To ensure that the results are not driven by the indivisibility of the projects *per se*, it is assumed that a continuum of the agents have access to the identical (indivisible) projects. This helps to convexify the aggregate production technologies. Having said this, however, let us now look at some examples with heterogeneous agents.

3.2. Heterogeneous Agents: Distributional Implications

Let us first allow the agents to differ in the input endowment, or the net worth, where ω is distributed according to G(ω). Otherwise, the model is the same as above. In particular, the agents share the same R, that is, they are equally productive as entrepreneurs.

With heterogeneous endowments, different agents face different (BC). For a given level of k, only those with $\omega \geq \omega_c \equiv 1 - \lambda Rf'(k)/r$, can borrow. If (PC) holds strictly, Rf′(k) > r, all of these agents invest. Hence, the total supply of capital is equal to R times the fraction of the agents satisfying (BC), so that

$$k = R\left[1 - G\left(1 - \frac{\lambda R f'(k)}{r}\right)\right]. \tag{8}$$

As the RHS of (8) is decreasing in k, this equation determines k uniquely. As long as Rf′(k) > r holds at this solution, it is indeed the equilibrium value of k. If not, the equilibrium is given by Rf′(k) = r. In what follows, let us assume that the parameters are such that the equilibrium is characterized by (8) with Rf′(k) > r.

Both a lower r and a higher λ increase the RHS for a given k. Hence, both lead to a higher k. The reason is simple. These changes increase the present discounted value of the pledgeable revenue, which raises the borrowing limit. Hence, more agents can finance the project. One can also show that k goes up, when the distribution of the net worth shifts to the right in the First-Order-Stochastic-Dominant manner. This is a generalized version of the net worth effect discussed earlier.

Let us now see the distributional implications of improving the credit market. Let λ go up from λ^- to λ^+. (Superscript "–" denotes the value before the change and superscript, "+" denotes the value after the change.) As noted above, an increase in λ leads to an increase in k from k^- to k^+. This increase in k occurs because a larger fraction of the agents are now able to finance their projects, which means that the threshold level of the net worth has declined from $\omega_c^- = 1-\lambda^- Rf'(k^-)/r$ to $\omega_c^+ = 1-\lambda^+ Rf'(k^+)/r$. Therefore, we need to distinguish three classes of agents. First, those with $\omega < \omega_c^+$ invest neither before nor after the change. Hence, their utility (period-1 consumption) before and after the change are given by $U^-(\omega) = U^+(\omega) = r\omega$. Second, those with $\omega_c^+ < \omega < \omega_c^-$ invest only after the change. Hence, their utility increases from $U^-(\omega) = r\omega$ to $U^+(\omega) = Rf'(k^+) - r(1-\omega)$. Finally, those with $\omega > \omega_c^-$ invest both before and after the change. Hence, their utility *declines* from $U^-(\omega) = Rf'(k^-) - r(1-\omega)$ to $U^+(\omega) = Rf'(k^+) - r(1-\omega)$, because $f'(k)$ declines from $f'(k^-)$ to $f'(k^+)$. Figure 1a illustrates these welfare effects.[14]

Thus, not everyone gains from the credit market improvement. The middle class gains (as well as the owner of the hidden factors, which are complementary inputs to capital in the production of the consumption good), while the rich lose. The reason is that credit market imperfections operate like *entry barriers*. The political economy implications should be clear. If the political power is in the hands of the rich who have easy access to credit, the government has an incentive *not* to improve the credit market.

3.3. Heterogeneous Agents: Replacement Effects

Let us now consider the case where the agents differ also in their productivity. More specifically, each agent is identified by (ω, R) distributed according to $G(\omega, R)$. Figure 1b illustrates the two constraints,

(9) $Rf'(k)/r \geq 1$ (PC)

(10) $\omega \geq \omega_c(k, R) \equiv 1 - \lambda Rf'(k)/r.$ (BC)

The agents located to the right of the vertical line, $R = r/f'(k)$, satisfy (PC), while the agents above the negative-sloped line, $\omega = \omega_c(k, R) \equiv 1 - \lambda Rf'(k)/r$, satisfy (BC). Only the agents satisfying both invest. Thus, the aggregate supply of capital is given by the unique solution to the following equation:

(11) $$k = \int_{\frac{r}{f'(k)}}^{\infty} R\left[\int_{\omega_c(k,R)}^{\infty} g(\omega, R)d\omega\right]dR$$

Again, it is straightforward to show that k goes up in response to a lower r, a higher λ, and a First-Order Stochastic Dominant shift of the net worth distribution to the right.

Let us look at the effects of an improved credit market more closely. An increase in λ from λ^- to λ^+ leads to an increase in k from k^- to k^+, and hence to a decline in $f'(k)$ from $f'(k^-)$ to $f'(k^+)$. These changes move the vertical line to the right, and the negative-sloped line to the left, as shown by arrows in Figure 1b. This means that four classes of the agents may be distinguished. Those in A stop investing. Those in B continue investing. Those in C start investing. The rest never invest. This means that, as a

[14] A change in r would have more complicated welfare effects, but its effects on k and ω_c are straightforward: a rise in r leads to a decline in k by raising ω_c. This is roughly consistent with the evidence found by Gertler and Gilchrist (1994) and others that small manufacturing firms are more sensitive to the tightening of monetary policy.

result of a credit market improvement, the rich but less productive agents in A are replaced by the poor but more productive agents in C.[15] Clearly, those in C are better off because they are now able to borrow and invest in the profitable project, while those in A and in B are worse off because their projects become less profitable due to the entry by the agents in C.

To explore further implications, let us now look at a more specific example. Imagine that there are *only* two types of agents. Their relative productivity and net worth satisfy $R_1 < R_2$, $\omega_1 > \omega_2$, so that type-1 agents are richer but less productive than type-2 agents. Let θ denote the share of Type-1. Suppose furthermore that $1 - \omega_1 < (R_1/R_2)(1 - \omega_2)$ and consider the effects of an increase in λ from λ^- to λ^+, where $1 - \omega_1 < \lambda^- < (R_1/R_2)(1 - \omega_2) < \lambda^+ < 1 - \omega_2$. Then, for Type-1, (PC) is more stringent than (BC) both before and after the change, and, for Type-2, (BC) is more stringent than (PC) both before and after the change. Furthermore, when θ and r are chosen to satisfy the inequalities,

$$\frac{1-\omega_2}{\lambda^- R_2} > \frac{f'(R_1\theta)}{r} \geq \frac{1}{R_1} > \frac{f'(R_2(1-\theta))}{r} \geq \frac{1-\omega_2}{\lambda^+ R_2},$$

one can show that the equilibrium takes the following form:

- Before the change ($\lambda = \lambda^-$), $R_1 f'(k^-) = r$, where $k^- \leq R_1\theta$. That is, (PC) is binding for Type-1 and (BC) is violated for Type-2. Some Type-1 invest, but no Type-2 invest.
- After the change ($\lambda = \lambda^+$), $\lambda^+ R_2 f'(k^+) = r(1-\omega_2)$, where $k^+ \leq R_2(1-\theta)$. That is, (BC) is binding for Type-2. (PC) is violated for Type-1. Some Type-2 invests, but no Type-1 invest.

Thus, with $\lambda = \lambda^-$, only the unproductive but rich agents invest, none of whom is credit-constrained. With $\lambda = \lambda^+$, only the productive but poor agents invest, all of whom are credit-constrained. Furthermore, aggregate investment (the total amount of the inputs going into the projects) may decline as a result of an improvement in the credit market. This is because investment technologies improve endogenously, as the credit shifts from the less productive agents to the more productive agents.

This is by no means a peculiar feature of the above example. More generally, a better credit market does not necessarily mean that there are less credit-constrained among the active firms. Consider the two extreme cases. If $\lambda = 1$, the credit market is perfect so that no firms are credit-constrained. If $\lambda = 0$, on the other hand, the credit market shuts down completely. Hence, only the firms that can self-finance entirely operate so that no active firms are credit-constrained. Only in the intermediate cases should we expect some active firms to be credit-constrained.

[15] Aghion, Fally, and Scarpetta (2007) shows some evidence that, after a financial liberalization, the entry of small firms force larger firms to scale down or to exit completely.

All the models so far have been in partial equilibrium in that the market rate of return required by the lenders, r, is treated as exogenous. It is now endogenized in general equilibrium.

4. General Equilibrium with Endogenous Saving: Capital Deepening vs. Net Worth Effects

Let us go back to the homogeneous case, where all (investing) agents have the same R and ω. In this section, we call them "entrepreneurs," because we also add some agents, "savers", who have no access to the investment projects. The savers are endowed with ω^o units of the input. In addition to the period-1 consumption, they also consume some of the inputs in period 0. More specifically, they maximize $U^o = V(C^o_0) + C^o_1$, subject to the budget constraint, $C^o_1 = r(\omega^o - C^o_0)$, where V is an increasing, concave function. Then they choose their saving, $S^o = \omega^o - C^o_0$, such that $V'(\omega^o - S^o) = r$, which defines their saving function, $S^o(r) \equiv \omega^o - (V')^{-1}(r)$. Since the entrepreneurs save all of their endowment, the aggregate saving of this economy, or the total inputs available to be used in the projects, is given by $S(r) \equiv \omega + S^o(r) = \omega + \omega^o - (V')^{-1}(r)$. Since these inputs are converted into capital at the rate equal to R, the aggregate supply of capital is given by

(12) $$k = RS(r) \equiv R[\omega + \omega^o - (V')^{-1}(r)], \qquad \text{(RC)}$$

where (RC) stands for *the Resource Constraint* of the economy. As before, (PC) and (BC) of the entrepreneurs are given by

(5) $$Rf'(k) = Max\left\{1, \frac{1-\omega}{\lambda}\right\}r \qquad \text{(PC) + (BC)}$$

Eqs. (5) and (12) jointly determine k and r. These equilibrium conditions may be rewritten more compactly as

(13) $$\omega + \omega^o - (V')^{-1}(r) \equiv S(r) = I(r) \equiv \frac{1}{R}(f')^{-1}\left(Max\left\{1, \frac{1-\omega}{\lambda}\right\}\frac{r}{R}\right).$$

Figure 2a depicts eq. (13) by the intersection of the upward sloping aggregate saving schedule, S(r), and the downward-sloping aggregate investment schedule, I(r).[16] Note that S(r) depends on the aggregate endowment, $\omega + \omega^o$, while I(r) depends on the entrepreneur's endowment, ω.

Figure 2b shows the effect of a higher endowment of the savers, which shifts the saving schedule to the right, while keeping the investment schedule intact. An increase in the aggregate saving, which finances the aggregate investment, means that more capital is produced. Due to diminishing returns, the marginal productivity of capital declines, which leads to a low rate of return. In essence, this is the standard neoclassical *capital deepening effect*.

[16] The partial equilibrium model of the previous section may be viewed as a special case, where the saver's preferences are given by $U^o = \rho C^o_0 + C^o_1$, so that the aggregate saving is infinitely elastic. One may also analyze the case without the savers by looking at the special case where the aggregate saving is inelastic at $S(r) = \omega$.

The effect of a higher λ, when (BC) is binding ($\lambda + \omega < 1$), is shown in Figure 2c. The investment schedule shifts to the right, while the saving schedule remains intact. By easing the borrowing constraint, more entrepreneurs could borrow to finance their investment. With the upward-sloping supply of saving, this raises the equilibrium rate of return. Redistributing the wealth from the savers to the entrepreneurs ($\Delta\omega = -\Delta\omega^o > 0$) would have the same effect, through *the pure net worth effect.*

The effect of a higher net worth of the entrepreneurs ($\Delta\omega > 0$), when (BC) is binding ($\lambda + \omega < 1$), without the offsetting change in the saver's wealth, may be viewed as a combination of the two effects discussed above: the capital deepening effect, due to an increase in the aggregate saving, and the pure net worth effect, which increases the aggregate investment. When the latter dominates the former, as shown in Figure 2d, the equilibrium rate of return goes up.[17] However, once the entrepreneur's net worth becomes high enough to make (BC) irrelevant ($\lambda + \omega > 1$), a further increase in ω reduces the rate of return, because only the capital deepening effect is at work. In short, the equilibrium rate of return may respond non-monotonically to the borrower net worth. More generally, a low rate of return in equilibrium could be a sign of either good or bad economic conditions. In Section 6, we will explore further implications of this feature of the model in the context of a global economy.[18]

5. General Equilibrium with Heterogeneous Projects

It has been assumed so far that each agent has access to only one type of project. Let us now look at a model where agents can choose which project to invest.

Again, we consider the world with homogenous entrepreneurs with unit mass, each of whom is endowed with ω units of the input at period 0 and consumes only at period 1. To keep it simple, we assume that there are no outside agents, "savers." Despite that this makes the aggregate saving inelastic, credit market imperfections and the net worth can still affect the equilibrium allocations by changing the composition of credit flows.

Entrepreneurs can choose one (and only one) of J non-divisible projects ($j = 1,2, \dots J$). A Type-j project converts $m_j > \omega$ units of the inputs in period 0 into m_jR_j units in capital *and* m_jB_j units in the

[17] For example, consider the case without the savers, so that the total saving is equal to ω. Then, $k = R\omega$ and from (5), $r = \lambda Rf'(R\omega)/(1-\omega)$. Simple algebra can show that this is increasing in ω in the range, $\eta/(1+\eta) < \omega < 1-\lambda$, where $\eta \equiv -\log(f')/\log(k) = -kf''/f'$ is the elasticity of the marginal productivity of capital.

[18] Here, the effects of exogenous changes in ω and ω^o (as well as λ) on k are studied. What if we also allow for some feedback from k to ω and ω^o? Imagine that the entrepreneur's net worth and the saver's net worth in period t, ω_t and ω^o_t, jointly determined k_{t+1}, as described above, which in turn determines that $\omega_{t+1} = W(k_{t+1})$ and $\omega^o_{t+1} = W^o(k_{t+1})$. (This can be justified, for example, by embedding our two-period agents into the overlapping agents framework, as discussed later.) The dynamics of this economy then depends on how a change in k affects the distribution of the wealth between the entrepreneurs and the savers. Aghion, Banerjee and Piketty (1999) conducted analysis along this line in a similar setting, and found the case of endogenous cycles, where periods of low investment, during which the wealth distribution is shifted toward the savers, alternate with periods of high investment, during which the wealth distribution is shifted towards the entrepreneurs.

consumption good. Thus, the projects may differ in the set-up cost, productivity, as well as the types of the goods produced (and their compositions). By running a project-j, the agent can consume $m_j[R_jf'(k) + B_j] - r(m_j - \omega) = m_j[R_jf'(k) + B_j - r] + r\omega$ in period 1. Since the agent can always consume $r\omega$ by lending, the *Profitability Constraint for a Type-j Project* (PC-j) is given by

(14) $$R_jf'(k) + B_j \geq r. \qquad \text{(PC-j)}$$

We introduce the credit market imperfections by assuming that only a fraction λ_j of capital and a fraction μ_j of the consumption good are pledgeable to the lenders. Then, the *Borrowing Constraint for a Type-j Project* (BC-j) is given by

(15) $$m_j[\lambda_jR_jf'(k) + \mu_jB_j] \geq r(m_j - \omega), \qquad \text{(BC-j).}$$

Both (PC-j) and (BC-j) need to be satisfied for the credit to flow into type-j projects.

Figure 3 shows the graph of

(16) $$r_j(\omega) \equiv Min\left\{\frac{\lambda_jR_jf'(k) + \mu_jB_j}{1 - \omega/m_j}, R_jf'(k) + B_j\right\}.$$

Both (PC-j) and (BC-j) are satisfied if and only if $r \leq r_j(\omega)$. In other words, this graph shows the maximal rate of return that a type-j project can generate to the lenders without violating (PC-j) nor (BC-j). As shown, the graph is increasing in ω when (BC-j) is the relevant constraint. The reason is that a higher net worth eases the borrowing constraint as the entrepreneurs need to borrow less. This makes it possible for them to promise a higher rate of return to the lenders. The graph is flat when (PC-j) is the relevant constraint.

To describe the equilibrium formally, let n_j denote the measure of type-j projects initiated (and of the agents who invest in type-j projects). Since each type-j projects require m_j units of the input, the aggregate saving equals the aggregate investment if and only if

(17) $$\omega = \Sigma_j(m_jn_j).$$

Since each type-j project produces m_jR_j units of capital, the total supply of capital is given by

(18) $$k = \Sigma_j(m_jR_jn_j).$$

Finally, as the agents compete with one another for credit and they can choose freely among all the projects, credit goes only to the projects which have the highest $r_j(\omega)$ and hence generate the highest rate of return to the lenders, which can be expressed as

(19) $$r = \underset{i=1,2,\ldots J}{Max}\{r_i(\omega)\} \geq r_j(\omega) \equiv Min\left\{\frac{\lambda_jR_jf'(k) + \mu_jB_j}{1 - \omega/m_j}, R_jf'(k) + B_j\right\} \; ; n_j \geq 0 \quad (j = 1, 2, \ldots J),$$

where the two inequalities in (19) are the complementarity slackness conditions. The equilibrium of this economy is fully characterized by (17)-(19).

Let us now look at some special cases.

5.1. A Model with Pure Capital Projects: Endogenous Investment-Specific Technical Change:[19]

Suppose $B_j = 0$ for all j = 1, 2, …, J. Thus, the projects do not differ in the compositions of the output; they all produce homogeneous capital. In this case, (16) is simplified to

$$(20) \qquad \frac{r_j(\omega)}{f'(k)} = Min\left\{\frac{\lambda_j}{1-\omega/m_j}, 1\right\}R_j$$

Note that the projects can be ranked according to the RHS of (20), which is independent of k, and hence independent of the allocation of the credit. This means that the equilibrium allocation of the credit has a bang-bang feature. That is to say, generically, all the credit goes to only one type of project at each given level of the net worth and when a change in the net worth affects the credit composition, the effect is abrupt and drastic: the credit switches completely from one type to another. Needless to say, this is neither realistic nor robust feature of the model, but it greatly helps to simplify the exposition.[20]

5.1.a. Procyclical Productivity Change:

Let us begin with the case where J = 2, and $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$. In words, there are trade-offs between productivity and agency problems. Project 2 is more productive, hence appealing to the borrowers, while Project 1 offers more pledgeable return per unit of investment, which makes them potentially "safer" or "more secure" alternatives for the lenders. Such trade-offs can be important when some advanced projects that use leading edge technologies may be subject to bigger agency problems than some mundane projects that use well-established technologies.

Figures 4a and 4b show two ways in which the graphs of (20) for j = 1 and 2 could intersect with each other. In both cases, there is a critical net worth level, ω_c, below which all the credit flows to type-1 projects ($n_1 = 1 - n_2 = 1$) and above which all the credit flows to type-2 projects ($n_1 = 1 - n_2 = 0$). Then, from (17) and (18), we can show that the equilibrium supply of capital is

$$(21) \qquad k = R_{J(\omega)}\omega, \text{ where } J(\omega) = \begin{cases} 1 & \text{if } \omega < \omega_c, \\ 2 & \text{if } \omega \geq \omega_c, \end{cases}$$

as shown in Figure 4c. Thus, a higher net worth can raise the productivity of the investment technologies *used* from R_1 to R_2. In short, the investment productivity changes procyclically through the credit channel. The intuition should be clear. With a low net worth, the agents have to rely heavily on borrowing. Thus the saving flows into type-1 projects, which generate the higher rate of pledgeable return.[21] When net worth improves, the borrowers need to borrow less, which enables the entrepreneurs

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[19] This case is taken from Matsuyama (2007).

[20] One could remove this feature by letting the endowment be distributed according to G(ω) and by studying the effects of shifts in G(ω). The analysis here may be viewed as the limit case where G(ω) converges to a single mass.

[21] One may call this effect "flight to safety" (as opposed to "flight to quality"), following Barlevy (2003), who also developed a model in which the credit composition shifts toward lower productivity projects during recessions.

to offer the higher return to the lenders with type-2 projects, despite that they generate the lower pledgeable return per unit of investment.

The equilibrium rate of return is now given by

$$(22) \qquad r(\omega) = Min\left\{\frac{\lambda_{J(\omega)}}{1-\omega/m_{J(\omega)}}, 1\right\} R_{J(\omega)} f'(R_{J(\omega)}\omega)$$

Note that a higher net worth affects the equilibrium rate of return through three separate channels. First, it allows the borrowers to pledge more to the lenders per unit of lending. Second, the credit composition may shift toward more productive projects. These two channels work in the direction of a higher rate of return. Offsetting this is the usual capital deepening effect, which works in the direction of a lower return. The overall effect can go either way.

Let us briefly consider the implications of an increase in λ_j. One may think that a better corporate governance or contractual enforcement would always cause the credit to flow into the more productive investment projects. That is certainly the case, if the improvement raises λ_2. But what if it raises λ_1? Look at Figure 4b. In this case, a higher λ_1 leads to a higher ω_c. This offers some cautions. If an attempt to improve corporate governance is more effective for the well-established industries, where the nature of the agency problems are relatively well understood (type-1 projects), it would end up preventing the saving from flowing into new, but more productive technologies, run by small venture capital, where the nature of the agency problems are less understood (type-2 projects).[22]

Dynamic Implications: Credit Traps

We have so far treated ω as exogenous. Let us now explore the dynamic implications of procyclical investment-specific technological changes by allowing some positive feedback from k to ω. To keep it simple, we follow Bernanke and Gertler (1989) and consider the world where the economy consists of a sequence of overlapping generations *à la* Diamond (1965). Time is discrete and extends from zero to infinity (t = 0, 1, 2, …). In each period, a new generation of the homogenous agents arrives and stays active for two periods. For generation-t (those "born" in period t), their "period 0" is period t and their "period 1" is period t+1. They differ from the two-period agents discussed above only in that, instead of being endowed with a fixed ω, they are endowed with ζ, "the hidden factors" in fixed supply, which are used with the capital stock produced by generation-(t−1), k_t, in the production of the final good, $F(k_t, \zeta)$, (where the final good may be used both as the consumption good and the input for the investment project.) They thus earn and save $\omega_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$ during period t. Then, at the end of period t, they enter the credit relationship among themselves and produce k_{t+1}, which will become available in period t+1, and used to produce the final good with "the hidden factors" in fixed supply supplied by

[22] Recall that Figure 4b is applied when $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2 R_2/R_1) < 1$.

generation-(t+1). The dynamics of this overlapping generations economy are described simply by replacing ω by $W(k_t)$ and k by k_{t+1} in eq. (21), which becomes

$$
(23) \qquad k_{t+1} = R_{J(W(k_t))} W(k_t) \text{, where } J(\omega) = \begin{cases} 1 & \text{if } \omega < \omega_c, \\ 2 & \text{if } \omega \geq \omega_c. \end{cases}
$$

For any initial condition, k_0, the entire equilibrium trajectory can be obtained simply by iterating (23) forward.[23]

In what follows, let us assume that $W(0) = 0$, $W(k)/k$ is monotonically decreasing, with $\lim_{k\to 0} W(k)/k = \infty$ and $\lim_{k\to\infty} W(k)/k = 0$, which is the case if $f(k) = k^\alpha$, with $0 < \alpha < 1$. Under these assumptions, the dynamics of the form, $k_{t+1} = R_j W(k_t)$, are characterized by monotone convergence to the unique positive steady state, for any fixed j. In other words, without heterogeneous projects, or without credit market imperfections, the dynamics of the economy look like the standard neoclassical one-sector model *à la* Solow. This may not be the case with (23), because credit market imperfections cause endogenous productivity changes in investment technologies.

Figure 4d illustrates one of three generic cases. It shows the case where $k^* < k_c < k^{**}$, where k^*, k^{**}, and k_c are defined by $k^* \equiv R_1W(k^*)$, $k^{**} \equiv R_2W(k^{**})$, and $W(k_c) \equiv \omega_c$. There are two stable steady states, k^* and k^{**}. The lower one, k^*, may be interpreted as a *credit trap*. In this steady state, the net worth is low so that the saving flows into the projects that generate the higher pledgeable return per unit of investment, although they produce less capital. The resulting lower supply of capital leads to a lower price of the endowment held by the next generation of entrepreneurs, and hence a low net worth. Which steady state the economy will converge to depends entirely on the initial condition. If the economy starts below k_c, it converges monotonically to k^*. If the economy starts above k_c, it converges monotonically to k^{**}. Thus, k_c may be viewed as the critical threshold level for economic development.[24]

5.1.b. Countercyclical Productivity Change

Consider now the case where J = 2 with $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$, and $m_1/m_2 < (1-\lambda_2 R_2/R_1)/(1-\lambda_1) < 1$. Thus, type-1 projects are less productive *and* generate less pledgeable rate of return than type-2 projects. However, the set up cost is much smaller for type-1 projects, so the agents need to borrow much less to invest into these projects, which may give type-1 projects advantage over type-2 projects. For

[23] This conversion to the dynamic framework is simple in part because the "hidden factors" do not include durable assets, such as the land. Otherwise, the borrower net worth in period t would depend on the asset prices in period t, which depends on the future trajectory of the economy, which in turn depends on the investment and the borrower's net worth in period t. Kiyotaki and Moore (1997) and Kiyotaki (1998) argued that such asset price movements make amplification quantitatively significant. This conjecture has been studied by Kocherlakota (2000), Krishnamurthy (2003), Cooley, Marimon and Quadrini (2004), and Cordoba and Ripoll (2004).

[24] The space constraint prevents me from discussing many broad methodological issues associated with poverty trap models; see Azariadis and Stachurski (2005) and Matsuyama (2005b) on these issues.

example, type-1 projects could represent family operated farms or other small businesses, while type-2 projects represent the investments in the corporate sector. Or, type-1 projects represent traditional light industries, such as textile and furniture, which require a relatively small initial expenditure, while type-2 projects represent modern heavy industries, such as steel, industrial equipments, petrochemical, and pharmaceutical industries that require a relatively large initial expenditure.

Figure 5a shows that the graphs of (20) for j = 1 and 2 intersect twice with each other. For an intermediate value of $\omega_c < \omega < \omega_{cc}$, all the credit goes to type-1 projects, $n_1 = 1 - n_2 = 1$. Otherwise, all the credit goes to type-2 projects, $n_1 = 1 - n_2 = 0$. Therefore, the equilibrium supply of capital is now given by

$$(24)\qquad k = R_{J(\omega)}\omega, \text{ where } J(\omega) = \begin{cases} 2 & \text{if } \omega < \omega_c \\ 1 & \text{if } \omega_c < \omega < \omega_{cc}, \\ 2 & \text{if } \omega > \omega_{cc}, \end{cases}$$

as shown in Figure 5b. When the net worth is very low, the entrepreneurs must rely almost entirely on external finance, so that the saving flows into type-2 projects that generate more pledgeable return per unit of investment. As the net worth rises, the entrepreneurs can offer a more attractive rate of return with type-1 projects than with type-2 projects, because they need to borrow little for type-1 projects. Hence, a rise in the net worth leads to a shift of the credit toward less productive projects. If the net worth rises even further, then the borrowing need becomes small enough for type-2 projects that the credit shifts back to more productive type-2 projects.

In this case, a higher λ_1 reduces ω_c and hence expands the range in which the savings flow into the more productive type-2 projects, which offers a caution when thinking about alleviating credit market imperfections targeted to small businesses.

Dynamic Implications: Leapfrogging and Credit Cycles as a Trap

Let us explore the dynamic implications by replacing ω by $W(k_t)$ and k by k_{t+1} in (24) to obtain

$$(25)\qquad k_{t+1} = R_{J(W(k_t))}W(k_t)\text{, where } J(\omega) = \begin{cases} 2 & \text{if } \omega < \omega_c \\ 1 & \text{if } \omega_c < \omega < \omega_{cc}, \\ 2 & \text{if } \omega > \omega_{cc}, \end{cases}$$

Figures 5c and 5d illustrate two possibilities among many. In Figure 5c, $k_c < k^* < k_{cc} < k^{**}$, where k_c and k_{cc} are defined by $W(k_c) \equiv \omega_c$ and $W(k_{cc}) \equiv \omega_{cc}$, and the two stable steady states, k* and k**, are again defined by $k^* \equiv R_1W(k^*)$ and $k^{**} \equiv R_2W(k^{**})$. If $k_c < k_0 < k_{cc}$, the economy converges monotonically to k*. If $k_0 > k_{cc}$, the economy converges monotonically to k**. Hence, as long as we focus our attention to the range above k_c, the dynamics look similar to Figure 4d. However, it can be more complicated if the economy starts below k_c. After the initial phase of growth, the economy will converge to k*, if it falls into the intermediate interval, (k_c, k_{cc}). However, if $R_2W(k_c) > k_{cc}$, the economy could bypass this stage and converge to k^{**}, as indicated by the arrows in Figure 5c. In this case, the long run performance of the economy depends sensitively on the initial condition.[25] Furthermore, it suggests the possibility of *leapfrogging*. That is, an economy that starts at a lower level may take over another economy that starts at a higher level.[26] In Figure 5d, $k^* < k_c < k_{cc} < k^{**}$ and $R_2W(k_c) < k_{cc}$. For $k_0 < k_{cc}$, the economy fluctuates indefinitely.[27] Along these *credit cycles*, an improvement in the current net worth causes a shift in the credit towards the less productive projects that contribute less to the future net worth. The resulting decline in the net worth causes the credit to shift back towards the projects that help more to build the net worth in the following period. For $k_0 > k_{cc}$, on the other hand, the economy converges monotonically to the unique stable steady sate, k^{**}. Thus, this may also be viewed as another example of credit traps except that the traps here take the form of cycles around k_c, instead of the lower steady state, k^*.

5.2. A Model with Private Benefits

A higher net worth might also shift the composition of credit toward less "socially productive" projects, when the agents are attracted to running some "socially unproductive" projects, because they generate more "private benefits," "personal satisfaction" or some other consumption values, which mean little to the lenders. To capture this idea, let $R_1 < R_2$, and $B_1 > B_2 = 0$ with $\lambda_1 = \lambda_2 = 1$ and $\mu_1 = \mu_2 = 0$. Thus, capital is fully pledgeable, but the consumption good is not pledgeable at all. Type-1 projects are less "socially productive" than type-2, but it is a lot of fun to run type-1 projects. Let $\Delta R \equiv R_2 - R_1 > 0$. From (17)-(19), one can show that $k = R_2\omega$ if $\omega < \omega_c \equiv (\Delta R/R_2)m_1$ or $\Delta Rf'(R_2\omega) \geq B_1$; $k = R_1\omega$ if $\omega \geq \omega_c$ and $\Delta Rf'(R_1\omega) \leq B_1$. If $\omega \geq \omega_c$ and $\Delta Rf'(R_1\omega) > B_1 > \Delta Rf'(R_2\omega)$, then $\Delta Rf'(k) = B_1$, which means $R_1\omega < k < R_2\omega$. In words, all the credit goes to type-2 projects *either* when the agents cannot borrow for type-1 projects *or* the private benefits of type-1 projects are not big enough to compensate its low productivity in capital when everybody else invests in type-2; all the credit goes to type-1 projects when the agents can borrow for type-1 projects *and* the private benefits of type-1 projects are big enough when everybody else invests in type-1. Otherwise, the credit goes to both types so that the total productivity (i.e., including the private benefit) are equalized between type-1 and type-2.

Figure 6a illustrates the case where $B_1 > \Delta Rf'((R_1/R_2)\Delta Rm_1)$. Then,

[25] Mathematically, for any $\varepsilon > 0$, there exist open intervals, I^* and $I^{**} \subset (0, \varepsilon)$, such that, as $t \rightarrow \infty$, $k_t \rightarrow k^*$ for $k_0 \in I^*$ and $k_t \rightarrow k^{**}$ for $k_0 \in I^{**}$.

[26] For example, imagine that only type-1 projects, textile and other industries that emerged at the time of the first industrial revolution are available initially and some countries, say Britain, have succeeded in reaching the steady state, k^*. Then, the second industrial revolution arrives and type-2 projects, some new technologies like chemical and steel industries, are born. Britain, located in k^*, is unable to switch to the new technologies, while some, but not all, latecomers, say Germany, come from behind and take over the technology leadership by successfully adopting the new technologies.

[27] Although these figures depict period-2 cycles, the fluctuations can take a more complicated form.

(26) $k = R_{J(\omega)}\omega$, where $J(\omega) = \begin{cases} 2 & \text{if } \omega < \omega_c, \\ 1 & \text{if } \omega \geq \omega_c. \end{cases}$

In this case, the agents enjoy running type-1 projects so much that they will do so whenever they are rich enough to borrow, that is, $\omega \geq \omega_c \equiv (\Delta R/R_2)m_1$.

Dynamic Implications: Credit Cycles

Again, we can look at the dynamic implications by setting $\omega = W(k_t)$ and $k = k_{t+1}$ in eq. (26). Figure 6b shows the possibility of credit cycles. During booms, a high net worth allows the agents to pursue projects that generate personal satisfaction but less capital, which slows down the economy. During recessions, the agents cannot pursue such projects, hence the credit goes to projects that generate more capital, which leads to the next boom. Note that the welfare implications of these credit cycles are very different from those shown in Figure 5d. Here the booms occur as a result of the misallocation of credit, and they end when a sufficiently high net worth eventually corrects the misallocation. If the credit markets were perfect, and the agents could fully pledge their "private benefits" to the lenders, the booms would not occur and without the booms the economy would never experience slowdowns. In contrast, the recessions along the cycles shown in Figure 5d occur as a result of the misallocation of credit.

5.3. A Model with Pure Capital and Consumption Projects:

Now let us look at the case where J =2 with $R_1 = R > R_2 = 0$ and $B_1 = 0 < B_2 = B$. Thus, type-1 projects produce only capital, while type-2 projects produce only the consumption good.

The equilibrium conditions, (17) through (19), now become

(27) $\omega = m_1n_1 + m_2n_2$

(28) $k = m_1Rn_1$

(29) $$Min\left\{\frac{\lambda_1}{1-\omega/m_1}, 1\right\}Rf'(k) = r \geq Min\left\{\frac{\mu_2}{1-\omega/m_2}, 1\right\}B\ ;\ n_2 \geq 0.$$

Note that (29) contains the complementary slackness condition only for type-2. Since only type-1 projects produce capital, $f'(0) = \infty$ ensures $n_1 > 0$, hence the first equality in (29).

Figure 7 shows the equilibrium in the absence of credit market imperfection ($\lambda_1 = 1$, $\mu_2 = 1$), which is given by

(30) $k = \begin{cases} R\omega & \text{if } \omega < \omega_c, \\ R\omega_c, & \text{if } \omega \geq \omega_c, \end{cases}$

where ω_c is now defined by $Rf'(R\omega_c) \equiv B$. Thus, all the credit goes to the capital-generating project until its return becomes equal to the return of the consumption-generating project, which absorbs all additional credit.

5.3.a. Persistence of Inefficient Recessions: Financial Accelerator

Starting from this benchmark, let us introduce the credit market imperfection to the capital-generating type-1 projects ($\lambda_1 < 1$ and $\mu_2 = 1$). With a sufficiently small λ_1, there is an interval of ω, in which some of the credit flows into type-2 projects ($k < R\omega$), despite that type-1 generates a higher return than type-2 projects, $Rf'(k) > B$, as shown in Figure 8a. This under-investment to type-1 projects occurs because (BC-1) is binding: $Rf'(k) > \lambda_1 Rf'(k)/(1-\omega/m_1) = B$. The graph is upward-sloping over this interval, because a higher net worth shifts the credit flows from the consumption-generating type-2 to the capital-generating type-1 projects by easing (BC-1).

By setting $\omega = W(k_t)$ and $k = k_{t+1}$, one can easily see how a credit market imperfection of this kind introduces *persistence* into the dynamics. Figures 8b through 8d show three possibilities.[28] Figure 8b replicates the key result of Bernanke and Gertler (1989). There is a unique steady state, k*, which is characterized by the under-investment. Now imagine that the economy is hit by a one-time shock, which temporarily reduces the productivity of the final goods production. Without the credit market imperfection, the economy would go back to its steady state, $RW(k_c)$, after one period. With the credit market imperfection, however, the economy goes back only gradually towards its steady state, k*, as indicated by the arrow. Even though the shock itself is temporary, it reduces the current net worth, which tightens the borrowing constraint, reducing the future investment. This in turn reduces future net worth, and so on. In short, the credit multiplier or financial accelerator mechanism creates an echo effect, transforming the i.i.d. shocks into positive serial correlations. In Figure 8c, the unique steady state is $k^* = RW(k_c)$, in which the marginal productivity is equalized across projects and there is no under-investment. However, the financial accelerator is at work at lower ranges. Thus, when the economy starts with a low capital stock, the credit market imperfection slows down the recovery process, prolonging the inefficient recessions. In Figure 8d, this mechanism is so strong that it creates two stable steady states, the lower of which is characterized by the under-investment, and the economy may be permanently trapped into a recession. All of these cases imply persistence because the type of investment that helps to enhance the future borrower net worth is subject to credit market imperfections.

5.3.b. Inefficient Booms and Volatility:[29]

[28] As shown in all these figures, the graph intersects with the 45° line no more than twice. This can be proved in the same way as the proof in Matsuyama (2004, p.865; Lemma).

[29] This and the next cases are based on Matsuyama (2004a).

Let us now introduce the credit market imperfection to type-2 projects instead ($\lambda_1 = 1$ and $\mu_2 < 1$). With a sufficiently small μ_2, there is an interval of ω for which the credit continues to flow into the capital-generating type-1 projects, even after that the return of type-1 projects becomes lower than type-2 projects ($k > R\omega_c$), as shown in Figure 9a. This *over-investment* to type-1 projects occurs because (BC-2) is binding, that is, $\mu_2B/(1-\omega/m_2) \leq Rf'(k) < B$. Note that the graph is non-monotonic. It is initially upward-sloping, because all the additional credit go to type-1 projects because net worth is too low for type-2 projects to be financed: $\mu_2B/(1-\omega/m_2) < Rf'(R\omega) < B$. Eventually, net worth becomes sufficiently high so that some credit flows into type-2 projects: $\mu_2B/(1-\omega/m_2) = Rf'(k) < B$. In this range, the graph is downward-sloping because further increase in net worth shifts the credit flow from the capital-generating type-1 to the consumption-generating type-2 projects by easing (BC-2).

The non-monotonicity of the graph carries over to the dynamics. Instead of putting persistence into the dynamics, a credit market imperfection of this kind puts *volatility* into the dynamics. It may generate over-shooting, oscillatory convergence, or endogenous fluctuations. In Figure 9b, its unique steady state is unstable and the economy fluctuates indefinitely within the interval I. One can show that the two conditions are necessary for endogenous fluctuations (as well as oscillatory convergence and over-shooting) to occur. First, B needs to be sufficiently high; otherwise, credit would never flow into the type-2 projects. Second, μ_2 can be neither too high nor too low. The intuition is simple. If type-2 projects suffer from major agency problems (a small μ_2), they are never financed. (Just think of the case $\mu_2 = 0$, which completely shuts down the credit for type-2.) Hence, credit always goes only to type-1 projects. If type-2 projects are subject to minor agency problems (a large μ_2), they are financed as soon as they become more productive than type-1 projects. (Just think of the case $\mu_2 = 1$, which brings us back to the perfect credit market case.) Endogenous fluctuations occur only for *intermediate* values of μ_2. That is, the condition requires that the agency problems associated with the consumption-generating type-2 projects are too big to be financed when the net worth is low, but small enough to be financed when the net worth is high.[30] Again, the welfare implications of these fluctuations are similar to the case of Figure 6b and opposite of Figure 5d. That is, the misallocation of credit causes booms, which collapse when a sufficiently high borrower net worth corrects the misallocation of the credit.

An interesting extension is to add some exogenous sources of fluctuations to this model. For example, suppose that B may change over time. Recall that B needs to be big enough for the graph to

[30]Aghion, Banerjee and Piketty (1999) also showed that endogenous cycles occur when the parameter representing the degree of credit market imperfection has an intermediate value. They interpreted it as saying that countries at an intermediate level of financial development are subject to volatility. This may be an appropriate interpretation in the context of their model, but not here. Recall that we are looking at situations where the agents have access to many investment opportunities and face no borrowing constraint when financing the capital-generating projects, and seeing what might happen when we change the imperfections that affect the financing of alternative projects which could divert the credit flow away from the capital producing projects. One could argue that a better credit market might be more prone to financing such alternative projects, thereby diverting the credit flow away.

look as in Figure 9b. If it is not big enough, the downward-sloping part of the graph is located far to the right so that the $RW(k_t)$ intersects with the 45 ° line at k**. If B permanently stays small, then the economy converges to k**. However, imagine that every once in a while B becomes big enough to make the graph look as in Figure 9b. With occasional arrivals of alternative investment opportunities, which divert the credit away from the capital-generating projects, the economy fluctuates around k*, below k**, at least until B becomes small again.

5.3.b. Hybrid Cases: Asymmetric Cycles and Intermittent Volatility

The two previous cases offer seemingly conflicting views of credit market imperfections, one suggesting persistence, the other suggesting volatility. However, they are not actually conflicting. Indeed, each might capture different phases of business cycles as the following hybrid model illustrates.

Let $J = 3$ with $R_1 = R > R_2 = R_3 = 0$, $B_1 = 0$, $B_2 > B_3 > 0$, and λ_1, $\mu_2 < 1$, $\mu_3 = 1$. Thus, type-1 is the only capital-generating projects, while there are now two different types of consumption-generating projects (type-2 and type-3). Between the two, type-2 is more productive than type-3, but type-3 is not subject to the borrowing constraint. One could show, under certain parameter values,

- Type-2 projects become irrelevant for a small ω (because they cannot satisfy the borrowing constraints) so that type-1 projects effectively compete with type-3.
- Type-3 projects become irrelevant for a large ω (because more productive type-2 projects can be financed) so that type-1 effectively compete with type-2.

In other words, the model looks like the "persistence of inefficient recessions" model within the lower range, and the "inefficient booms and volatility" model within the higher range.

The dynamics may now look like Figure 10a, combining the features of Figure 8c and Figure 9b. In this case, there is no stable steady state. The equilibrium path is characterized by *asymmetric cycles*, along which the economy goes through a slow recovery from recessions, and, once in booms, experiences a period of high volatility, and then, plunges into recessions.[31] Alternatively, the dynamics may look like Figure 10b, combining the features of Figure 8d and Figure 9b. In this case, there is a unique steady state, k*.

Now consider the following thought experiment. Imagine that the economy is regularly hit by some i.i.d. shocks, shaking the graph up and down. Figure 10b represents the situation when the size of a shock is below a certain threshold level, while Figure 10a represents the situation when the size of a shock slightly exceeds the threshold level. Then, for most of the time, the economy fluctuates around k*, exhibiting the financial accelerator mechanism *à la* Bernanke and Gertler (1989). However, the

[31]For the empirical evidence for the business cycle asymmetry, see, for example, Falk (1986), Sichel (1993), and Acemoglu and Scott (1997).

economy encounters intermittently bubble-like asymmetric boom-and-bust cycles, during which it experiences volatility much larger than the shock that triggers it.

6. General Equilibrium with Heterogeneous Agents and Capital: Patterns of International Capital Flows

We have so far assumed that capital produced is homogeneous. Let us now look at a model with heterogeneous capital, where different agents produce different types of capital. One context in which this problem arises naturally is the case where the agents differ in their countries of residence and capital they produce are nontradeable.[32]

Imagine the world economy, consisting of two countries; North and South. The structure of each country is given by the model with endogenous saving discussed in section 4. The two countries share the identical technologies and preferences, but they may differ in λ, ω, and ω^o. To avoid a taxonomical analysis, let us assume $1 > \lambda_N \geq \lambda_S > 0$, $1 > \omega_N \geq \omega_S > 0$, and $1 > \omega_N^o \geq \omega_S^o > 0$. Both the input endowment and the consumption good can be traded between the two countries. This allows the agents to lend and borrow across the borders. On the other hand, it is assumed that capital (as well as the hidden factors) is nontradeable. Let us also assume that only the entrepreneurs in the North (South) know how to produce capital used in the production of the consumption good in the North (South).[33] We use this model to explore the implications of credit market imperfections on the patterns of international capital flows and economic development.[34]

The autarky equilibrium of each country is obtained from (5) and (12) by adding the subscripts, j = N or S, as follows:

(31) $$k_j = R[S_j(r_j)] = R[\omega_j + \omega^o_j - (V')^{-1}(r_j)]. \qquad \text{(RC-j)}$$

(32) $$Rf'(k_j) = Max\left\{1, \frac{1-\omega_j}{\lambda_j}\right\} r_j \qquad \text{(PC-j)+(RC-j)}$$

Or, from (13),

(33) $$S_j(r_j) = \frac{k_j}{R} = I_j(r_j)$$

where $S_j(r_j) \equiv \omega_j + \omega^o_j - (V')^{-1}(r_j)$ and $I_j(r) \equiv \frac{1}{R}(f')^{-1}\left(Max\left\{1, \frac{1-\omega_j}{\lambda_j}\right\}\frac{r}{R}\right)$ (j = N or S).

[32] Another context in which this problem arises naturally is the case where different agents have expertise in different industries and/or technologies and capital are highly specialized in a specific industry or technology.

[33] Or, the entrepreneur's productivity, R, declines substantially when operating abroad. This assumption effectively rules out the foreign direct investment. Later, some implications of relaxing this assumption will be discussed.

[34] Prasad, Rajan, and Subramanian (2006) and Kose, Prasad, Rogoff, and Wei (forthcoming) offer overviews of the empirical patterns. The model here extends Matsuyama (2005a, Section 2) by adding the savers.

Suppose now that the two countries become fully financially integrated so that the agents from both countries can lend and borrow their input endowments across the borders and repay in the consumption good without additional costs. By "without additional costs," is meant, among other things, that the pledgeability in each country, λ_j, is independent of the location of the lenders. Of course, one could think more generally that the borrowers can pledge the fraction $\varphi\lambda_j$ ($0 \leq \varphi \leq 1$), when borrowing from abroad. Here, however, the analysis is restricted to the two extreme cases of the autarky $\varphi = 0$ and the full financial integration, $\varphi = 1$.[35]

Full financial integration leads to a Rate of Return Equalization (RRE) across the two countries,

$$(34) \qquad Min\left\{\frac{\lambda_N}{1-\omega_N},1\right\}Rf'(k_N) = r = Min\left\{\frac{\lambda_S}{1-\omega_S},1\right\}Rf'(k_S), \qquad \text{(RRE)}$$

where $f'(0) = \infty$ ensures the interior solution, $k_N, k_S > 0$. The world-wide resource constraint (WRC) is given by

$$(35) \qquad k_N + k_S = R[S_N(r) + S_S(r)] = R[\omega_N + \omega^o_N + \omega_S + \omega^o_S - (V')^{-1}(r)]. \quad \text{(WRC)}^{36}$$

The world equilibrium is determined by (34)-(35), which can also be rewritten as

$$(36) \qquad S_N(r) + S_S(r) = \frac{k_N + k_S}{R} = I_N(r) + I_S(r).$$

When $\lambda_S/(1-\omega_S) \geq 1$, which also implies $\lambda_N/(1-\omega_N) > 1$, (RRE) becomes simply $f'(k_N) = r = f'(k_S)$, or equivalently, $k_N = k_S$. In this case, (BC) is not binding in either country, so that the movement of international capital flows is entirely dictated by the difference in marginal productivity. As a result of financial integration, the investment in South is financed by the lending from North, and capital flows until the difference in marginal productivity is eliminated.

Even when $\lambda_S/(1-\omega_S) < 1$, so that (BC) is binding in South, the lending flows from North to South, if the two countries differ mostly in the saver's wealth. This is illustrated by Figure 11a, which assumes $\lambda_N = \lambda_S$, $\omega_N = \omega_S$, $\omega^o_N > \omega^o_S$. Then, the two countries share the same investment schedule, while North's investment schedule is located to the right of South's. Hence, the autarky rate of return is lower in North than in South ($r_N < r_S$). With financial integration, the rates of return are equalized. The equilibrium rate of return is now given by $[S_N(r) + S_S(r)]/2 = I_N(r) = I_S(r)$, as shown by the intersection of the (common) investment schedule and the average saving schedule (depicted by the upward-sloping dotted curve). North experiences a rise in its rate of return, which increases its saving and reduces its

[35]Of course, a priori, there is no reason to believe that the effect is monotone in φ so that the following results should be interpreted with great caution. However, dealing with the intermediate cases would substantially complicate the analysis, as one would have to take into account two separate borrowing constraints, one for the domestic and one for the international borrowings. Caballero and Krishnamurthy (2001) and Aoki, Benigo, and Kiyotaki (2006) both studied this issue in small open economy models.
[36]It is assumed here that the two countries are of the equal size to minimize the notation, even though allowing for different country sizes is straightforward.

investment, and hence run a current account surplus, while South experiences a fall in its rate of return, which reduces its saving and increases its investment, and hence run a current account deficit. In short, North's saving flows to South to finance its development. This captures the standard neoclassical view of the global financial integration.

The reverse flows occur, however, if North's autarky rate of return is higher than South's. In Figure 11b, the two countries share the same saving schedule, while North's investment schedule is located to the right of South's, hence, $r_N > r_S$. With financial integration, the rate of return is equalized at the level given by the intersection of the (common) saving schedule and the average investment schedule (depicted by the downward-sloping dotted curve). North (South) witnesses its rate of return to fall (rise), its saving to fall (rise), and its investment to rise (fall), and hence its current account to turn into a deficit (surplus). In short, the "capital flight" from South finances North's investment. One way in which the situation depicted in Figure 11b can occur is $\lambda_N > \lambda_S$, $\omega_N = \omega_S$, $\omega^o_N = \omega^o_S$. This case captures the view that weak corporate governance and any other institutional factors contribute to financial insecurity in South and hence capital flight from South to North. Another way in which the situation depicted in Figure 11b can occur is $\lambda_N = \lambda_S$, $\omega_N - \omega_S = \omega^o_S - \omega^o_N > 0$. In this case, a larger share of the wealth is in the hand of the savers in the South than in the North. Then, even though the two countries do not differ in the other dimensions, the "capital flight" occurs from South to North. This is because the firms in the South have weaker balance sheet conditions than the firms in the North. This makes the former more dependent on external finance, which in turn makes them less credit-worthy. A financial integration forces the firms in the South to compete with those in the North when financing their investments, which put the former in disadvantage. As a result, South's saving flows to finance North's investment.

Figure 11c illustrates the case where $\lambda_N = \lambda_S$, $\omega_N > \omega_S$, $\omega^o_N = \omega^o_S$. In this case, North's saving and investment schedules are both located to the right of South's. If the pure net worth effect dominates the capital deepening effect, we have $r_N > r_S$.[37] Again, this implies that, with financial integration, the saving flows from South to North, because the firms in the South have the weaker financial position than those in the North.[38]

Note that, in many of these cases, $k_N > k_S$ continues to hold after the full financial integration, so that the marginal productivity of investment remains higher in the South: $Rf'(k_N) < Rf'(k_S)$. Obviously, the assumption that only the local firms can produce the capital stock used in the production of the final

[37] Again, for the case without the saver, $r_N > r_S$ if $\eta/(1+\eta) < \omega_S < \omega_N < 1 - \lambda_N = 1 - \lambda_S$. See Matsuyama (2005a, section 2).

[38] In their moral hazard model, Gertler and Rogoff (1990) demonstrated that capital flows from the rich to the poor is muted in the imperfect information case, compared to the perfect information case. It is not clear whether the reverse capital flows occur in their model, unless the net worth of the poor is negative. In the present model, the reverse capital flows occurs even if the net worth of the Southern entrepreneurs is slightly less than that of the Northern entrepreneurs.

good in each country, plays an important role in the analysis. If any firm from any country could operate anywhere at the same productivity, the difference in marginal productivity would be eliminated by two-way flows, in which some FDI flows from North to South (some agents in North produce capital in South), and at the same time, the saving flows from South to North. Even if their productivity declines when operating abroad, the two-way flows may occur, as long as their financial advantage is more than enough to offset the productivity disadvantage of operating abroad.[39]

So far, it has been assumed that the two countries share the identical technologies. Obviously, if North is more productive than South, the reverse capital flows could occur.[40] For example, if $R_N > R_S$, the capital flows from South to North even without credit market imperfections. However, it is difficult to draw a sharp distinction between the two theories, one based on credit market imperfections and one based on technological differences, because the technological differences may be caused by credit market imperfections. Recall that credit market imperfections may often prevent the credit from flowing into the most productive agents (in the model of section 3.3) or the most productive projects (in some models of section 5). To the extent that credit market imperfections cause endogenous changes in investing technologies, it would be a challenge to tell the two theories apart empirically.

Dynamic Implications: Symmetry-Breaking and Endogenous Inequality

In the above analysis, it is shown how the cross-country net worth distribution (ω_N, ω_S) affects the cross-country distribution of the capital stock (k_N, k_S). Let us now introduce positive feedback from (k_N, k_S) to (ω_N, ω_S). To keep it simple, let us remove the savers from the model. By replacing ω_j by $W_j(k_{jt})$ and k_j by k_{jt+1} in eq. (31), we obtain the dynamics of each country in autarky, as follows:

(37) $$k_{jt+1} = RW(k_{jt}) \qquad (j = N \text{ or } S),$$

which implies that each country converges monotonically to k*, where $k^* \equiv RW(k^*)$.

From eq. (34)-(35), the dynamics of the world economy under financial integration are given by:

(38) $$Min\left\{\frac{\lambda_N}{1-W(k_{Nt})},1\right\}f'(k_{Nt+1}) = Min\left\{\frac{\lambda_S}{1-W(k_{St})},1\right\}f'(k_{St+1}) \qquad \text{(RRE)}$$

(39) $$k_{Nt+1} + k_{St+1} = R[W(k_{Nt}) + W(k_{St})], \qquad \text{(WRC)}$$

which jointly determine (k_{Nt+1}, k_{St+1}) as a function of (k_{Nt}, k_{St}). Hence, from any initial condition, (k_{N0}, k_{S0}), the equilibrium trajectory can be solved for by iterating (38) and (39) forward.

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[39]See Ju and Wei (2006, 2007) for some related analysis of the two-way flows of FDI and the lending. Similar mechanisms might be at work at regional levels within a country. Savings in rural areas, instead of financing the local businesses, may flow into big city financial centers, which finance the investment into the rural areas by big businesses whose headquarters are located in metropolitan areas.

[40]Lucas (1990), for example, argued that human capital externalities might be the reason why the saving does not flow from the North to the South.

Let us look at the steady states. In what follows, let us restrict ourselves to the case where $\lambda_N = \lambda_S = \lambda$, which means that the only possible source of heterogeneity across countries is in the initial capital stocks.[41] *If* the borrowing constraints are binding in both countries in steady state, the steady state conditions are given by

$$\frac{f'(k_N)}{1-W(k_N)} = \frac{f'(k_S)}{1-W(k_S)} \qquad \text{(RRE)} \tag{40}$$

$$k_N + k_S = R[W(k_N) + W(k_S)] \qquad \text{(WRC):} \tag{41}$$

Figure 12 illustrates these conditions for an intermediate value of R. It shows that there are three steady states. One of them, (*SS*), is symmetric, given by $(k_N, k_S) = (k^*, k^*)$. The other two are asymmetric, (AS_N) and (AS_S), given by $(k_N, k_S) = (k_H, k_L)$ and $(k_N, k_S) = (k_L, k_H)$, where $k_H > k^* > k_L$. Furthermore, (*SS*) is unstable because $\lambda f'(k)/[1-W(k)]$, is increasing at $k = k^*$, so that the pledgeable rate of return in each country is increasing in the steady state capital stock. The instability of (*SS*) seems to suggest that (AS_N) and (AS_S) are stable. *If* this is the case, the two-country world economy develops unevenly under financial integration. Thus, this captures the structuralist view that poor countries are unable to compete in integrated capital markets against rich countries, which can offer financial security to the lenders and that the global capital market contributes to uneven development of the world economy, creating the core-periphery patterns or and the International Economic Order, or the World-System of the Rich and the Poor.[42]

While the above analysis is suggestive, verifying analytically the two "ifs" above is difficult.[43] Instead of the two-country case, Matsuyama (2004b) studied the above model with a continuum of countries and showed analytically,

- In autarky, the world economy as a whole converges to the symmetric steady state, regardless of the initial distribution of capital stocks across countries.
- For a sufficiently small λ, and for an intermediate range of R, financial integration causes a symmetry-breaking.[44] That is, the symmetric steady state loses its stability and many asymmetric stable steady states to emerge. In any stable steady state, some countries become richer than in autarky, while other

[41]Sakuragawa and Hamada (2001) studied the case where only one country (South) suffers from the credit market imperfections in a similar model.
[42]The intellectual origin of this view can be traced back to the structuralism of Nurkse (1953), Myrdal (1957), and Lewis (1977).
[43]Incidentally, Boyd and Smith (1997) obtained the exactly same dynamics, (38)-(39), in their two-country model of the credit market imperfection based on the costly state verification problem. They found numerical examples with one unstable symmetry steady state and two stable asymmetric steady states. See also Kikuchi (2006), who considered the case of two countries with unequal population sizes. His simulation shows that, if the country sizes are similar, the asymmetric steady states are stable. However, he also found endogenous fluctuations around the asymmetric steady states, when the countries sizes are sufficiently different.
[44] Matsuyama (2005c) discusses the notion of symmetry-breaking and its applications to economics.

countries become poorer than in autarky. Thus, the world economy is endogenously divided into the rich and the poor.

Two implications of these results deserve emphasis. First, this example demonstrates how a *partial* improvement in the credit market (a move from $\varphi = 0$ to $\varphi = 1$, while keeping λ less than one) could have dramatic distributional consequences that are perhaps surprising to many; Financial integration alleviates the credit market imperfections in some countries and exacerbates the credit market imperfections in other countries. Second, the instability of the symmetric steady state and the existence of asymmetric steady states occur only for an intermediate value of R. This suggests *the rise and fall of inequality across nations*. That is, as productivity R improves over time, the world economy may first experience divergence, as some countries start taking off, and then follow by convergence, as other countries start catching up, thereby generating the inverted U-curve patterns of inequality across nations.

` One key assumption above is that "hidden factors" are nontradeable. This means that the investment in one country would improve the future net worth of the entrepreneurs in the same country, but not elsewhere. If these factors were freely tradeable, then the investment in one country would have the same effect on the net worth in any country, which would eliminate the persistence of inequality across countries. The interesting case would be when some of these factors are tradable at some positive costs. Then, the investment demand would have bigger spillovers in the neighboring countries, which might lead to some regional contagion effects as well as divergence at the global scale.

7. General Equilibrium with Heterogeneous Agents with Heterogeneous Projects: Patterns of International Trade

In all the models with heterogeneous agents above, it has been assumed that each agent has access to only one type of projects. Let us now discuss a model with heterogeneous agents, where each agent has access to a diverse set of projects, in the context of international trade.

Consider a variation of the Ricardian model with a continuum of tradeable goods, indexed by $z \in [0,1]$, à la Dornbusch, Fischer, and Samuelson (1977). The economy is populated by a continuum of homogeneous agents, each of whom is endowed with $\omega < 1$ units of the input. Let us now call this input labor, following the tradition of the trade literature. The preferences are given by symmetric Cobb-Douglas, so that demand for good z is $D(z) = E/p(z)$, where $p(z)$ is the price of good z and E is the aggregate expenditure in this economy. To produce any tradeable good, the agents must run a project. Each project in sector z requires one unit of labor and generates R units of good z. Each agent may run one project or may simply become a worker, by supplying the labor endowment to other agents.

Since any project requires one unit of labor, and the labor endowment of any agent is $\omega < 1$, each agent who runs the project must employ $1-\omega$ units of labor supplied by those who do not run the project.

Let w be the wage rate, which the employers can pledge to pay to the workers after the project has been completed and the output has been sold. By running a project in sector z, the entrepreneur earns p(z)R, out of which they pay the wage bill, $w(1-\omega)$, so that they consume $p(z)R - w(1-\omega)$. By not running the project and supplying labor, they consume $w\omega$. Hence, any agent is willing to run the project in sector z if and only if $p(z)R - w(1-\omega) \geq w\omega$, and equivalently,

(42) $\quad p(z)R \geq w, \quad$ (PC-z)

where (PC-z) stands for the *Profitability Constraint for Sector z*. This constraint may not be binding, because the employers can pledge only a fraction of the project revenue for the wage payment. The employers in sector z can pledge only $\lambda(z)p(z)R$, where $\lambda(z)$ is continuous and strictly increasing with the range from zero to one. Because of the partial pledgeability, the projects in sector z take place if and only if they satisfy

(43) $\quad \lambda(z)p(z)R \geq w(1-\omega), \quad$ (BC-z)

where (BC-z) stands for the *Borrowing Constraint for Sector z*. Note that the pledgeable fraction of the project revenue, $\lambda(z)$, is now sector-specific. The assumption that it is strictly increasing means that the sectors are indexed such that the agency problems underlying the borrowing constraint are bigger in lower indexed sectors.

The Cobb-Douglas preferences ensure that, in autarky, the economy produces in all the sectors. Thus, both (PC-z) and (BC-z) must be satisfied for all z. Furthermore, for each z, one of them must be binding; otherwise, no agent would become workers. Therefore,

(44) $\quad p(z)/w = \max\{1, (1-\omega)/\lambda(z)\}/R.$

It is decreasing in $\lambda(z) < 1-\omega$ and constant for $\lambda(z) > 1-\omega$. Note that, for $\lambda(z) < 1-\omega$, (BC-z) is binding and $p(z)R > w$. In the sectors plagued by big agency problems, each project must earn higher revenues in order to assure the workers for their wage payment. The higher prices and higher project revenues in these sectors are due to the difficulty of obtaining the credit, which restricts the entry in these sectors.[45] To see this, let n(z) denote the number of projects run in sector z. Then, the total output in sector z is n(z)R, which must be equal to D(z) in autarky. Thus, $E = p(z)D(z) = p(z)n(z)R$. Hence, (44) becomes

(45) $\quad n(z) = \min\{1, \lambda(z)/(1-\omega)\}E/w,$

which is increasing in $\lambda(z) < 1-\omega$ and constant for $\lambda(z) > 1-\omega$. Since each project requires one unit of labor, and the aggregate labor endowment is equal to ω, the resource constraint in this economy is given by

(46) $\quad \int_0^1 n(z)dz = \omega.$

[45]This means that the entrepreneurs are not indifferent between the sectors. They prefer running the project in lower-indexed sectors. See Remark 3 for how to allocate the credit when the agents are not indifferent.

Summing up (45) for all z and using (46) yields

$$(47) \qquad n(z) = \frac{\min\{1, \lambda(z)/(1-\omega)\}}{\int_0^1 \min\{1, \lambda(s)/(1-\omega)\}ds}\omega,$$

which implies $n(z) < \omega$ for low z and $n(z) > \omega$ for high z.[46] This restricted entry and the resulting excess profits enable the incumbent firms to satisfy their borrowing constraints in low-indexed sectors.

Now, suppose that the world economy consists of two countries of the kind analyzed above, North and South. They have identical parameters except $\lambda(z)$ and ω. Furthermore, it is assumed that $\lambda_N(z) = \lambda_N\Lambda(z)$ and $\lambda_S(z) = \lambda_S\Lambda(z)$, where $\Lambda(z)$ is continuous and increasing in z with the range from zero to one, and $0 < \lambda_N, \lambda_S < 1$. This means that the agency problems underlying the borrowing constraint have two components; $\Lambda(z)$ depends on the technologies and other sector-specific factors, and λ_N, and λ_S depend on corporate governance, legal enforcement and other country-specific factors that determine the overall level of financial development in these economies. In what follows, let us assume $(1-\omega_N)/\lambda_N < (1-\omega_S)/\lambda_S$.

From (44), the autarky prices in North and South, $p_N(z)$ and $p_S(z)$, are now given by

$$(48) \qquad p_j(z)/w_j = \max\{1, (1-\omega_j)/\lambda_j\Lambda(z)\}/R \qquad (j = N, S).$$

Since $(1-\omega_N)/\lambda_N < (1-\omega_S)/\lambda_S$, eq. (48) implies that $p_N(z)/w_N \leq p_S(z)/w_S$ for all z and $p_N(z)/w_N < p_S(z)/w_S$ for z such that $\Lambda(z) < (1-\omega_S)/\lambda_S$, as shown in Figure 13a. This means that the credit market imperfections effectively become the source of North's absolute advantage over South.

Hence, when North and South trade with each other, the equilibrium relative wage must satisfy $w_N > w_S$, so that South gains comparative advantage in high indexed sectors. Figure 13b shows the patterns of comparative advantage. North, whose credit market functions better and whose entrepreneurs are richer and hence more credit-worthy, specializes and exports in the lower indexed sectors that suffer from bigger agency problems. South specializes and exports in higher indexed sectors, which are subject to smaller agency problems. The relative wage rate and the marginal sector, $\Lambda(z_c) = \Lambda_c$, are determined by the balanced trade condition.[47]

8. General Equilibrium with Pure Price Effects: A Model of Polarization

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[46]Note that the binding borrowing constraints in low-indexed sectors give rise to positive profits. The total profit in sector z is equal to $E - wn(z)$, which is positive for $\lambda(z) < 1 - \omega$ and zero for $\lambda(z) > 1 - \omega$. Summing it up across all the sectors and using (46) verifies that the aggregate profit Π is given by $\Pi = E - w\omega$. Hence, the aggregate income Y satisfies $Y = w\omega + \Pi = E$.

[47]This section is taken from Matsuyama (2005a, section 3). Earlier studies that looked at credit-based explanations of the patterns of trade include Kletzer and Bardhan (1087) and Becker (2002). See Manova (2006a, 2006b) and Wynn (2005) for more recent examples. This is a part of the growing literature that seeks the institutional origins of comparative advantage, such as Acemoglu, Antras, and Helpman (forthcoming), Costinot (2006), Levchenko (forthcoming), Nunn (forthcoming), and Vogel (forthcoming).

In all the models we have looked at so far, credit market imperfections distort the allocation of resources. In the following model, credit market imperfections do not distort the allocation of resources, and yet, they have distributional implications through their effects of prices. The model is clearly very special, but it helps to highlight how the net worth effect could operate through prices rather quantities.

Consider a continuum of agents with unit mass, whose input endowment in period 0 is distributed according to G(ω). In addition to lending x ≤ ω units of the input in period 0 for rx units of consumption in period 1, each agent now has access to an investment project with the *variable* scale $I \geq m$, which converts I units of the input into RI units in consumption in period 1. To operate this project at the scale equal to I, the agent needs to borrow I–ω at the market rate equal to r. Here, m is the minimum investment requirement, i.e., investing I < m generates nothing. As before, each agent maximizes the period-1 consumption. By running this project at the scale, $I \geq m$, the agent can consume U = RI – r(I–ω) = (R – r)I + rω. By lending, the agent can consume U = rω. Therefore, if R < r, the agent prefers lending; if R = r, the agent is indifferent; and if R > r, the agent *wants to* borrow and invest *as much as possible.*

However, the agent can pledge only the fraction λ of the project revenue, hence facing the following borrowing constraint:

(49) λRI ≥ r(I–ω) (BC).

If r ≤ λR, the agent would borrow and invest by infinite amount, which would never occur in equilibrium. However, for λR < r < R, the agent would borrow and invest up to its borrowing limit, as long as it also satisfies the minimum investment requirement, m. This means that, for λR < r < R, the investment demand schedule by an agent with the input endowment, ω, is given by

$$I(\omega) = \left(1-\frac{\lambda R}{r}\right)^{-1}\omega \quad \text{if } \omega \geq \omega_c \equiv m\left(1-\frac{\lambda R}{r}\right),$$

and zero otherwise. Therefore, the credit market equilibrium is given by

$$\text{(50)} \quad \text{Aggregate Saving} = \int_o^\infty \omega dG(\omega) = \left(1-\frac{\lambda R}{r}\right)^{-1}\int_{m(1-\lambda R/r)}^\infty \omega dG(\omega) = \text{Aggregate Investment}$$

for λR < r < R. Figure 14a illustrates this condition. The vertical line represents the LHS of (50), while the downward-sloping curve represents the RHS of (50). For a sufficiently small λ, i.e., if $\lambda < \int_0^{m(1-\lambda)} \omega dG(\omega) \Big/ \int_0^\infty \omega dG(\omega)$, the vertical line intersects with the downward-sloping part of the aggregate investment schedule, ensuring that λR < r < R holds in equilibrium. In this equilibrium, the *relatively* rich become the borrowers; they borrow as much as possible from the *relatively* poor, who have no choice but to lend to the rich. In this model, what separates the rich from the poor is their relative position in the wealth distribution. They do not have to be rich by any absolute standard, because the equilibrium rate of return always adjusts to make sure that some agents would have to become lenders, while others would become borrowers.

Now suppose that λ is reduced further. This shifts down the aggregate investment schedule. However, the aggregate investment does not change, due to the inelastic aggregate saving. The overall effect is hence a reduction in r such that λ/r remains constant, which also means that ω_c remains intact. Thus, a change in λ has no effects on the allocation of resources, as r moves endogenously to offset any effect that λ might have.

However, it has distributional effects, as seen by calculating the period-1 consumption for each agent as follows:

$$(51)\qquad U(\omega) = \begin{cases} \dfrac{(1-\lambda)R}{1-\lambda R/r}\omega & \text{if } \omega \geq \omega_c \equiv m\left(1-\dfrac{\lambda R}{r}\right) \\ \\ r\omega & \text{if } \omega < \omega_c \equiv m\left(1-\dfrac{\lambda R}{r}\right) \end{cases}$$

which is illustrated by Figure 14b. Note that the marginal return of having an additional unit of the input differs across the agents. For the poor, it is equal to r, which is strictly lower than the project return, R, because the credit market imperfection prevents the poor from borrowing to invest. For the rich, on the other hand, it is equal to $(1-\lambda)R/(1-\lambda R/r)$, which is strictly higher than R, because of *the leverage effect*. That is, the credit market imperfections enable them to borrow at the market rate strictly lower than the project return, R. It is precisely due to the leverage effect that makes the rich wanting to borrow as much as possible, which is precisely the reason why their (BC) is binding, i.e., eq. (49) holds with equality for the rich. The arrows depict the effects of a lower λ, which reduces r. By moving the terms of trade against the poor lenders and in favor of the rich borrowers, this further magnifies the disparity of the marginal returns on wealth between the rich and the poor.

Long Run Implications on Wealth Distribution*:* What would happen to the wealth distribution if we allow for some feedback from $U(\omega)$ to ω? Following Banerjee and Newman (1993) and Galor and Zeira (1993), imagine that each agent has an offspring, to whom he leaves the bequest, which is an increasing function of $U(\omega)$. Since the shape of $U(\omega)$, including the threshold level of wealth, ω_c, is a function of $G(\omega)$, this determines the dynamic evolution of wealth distribution, $G_{t+1}(\bullet) = \Phi(G_t(\bullet))$, which can be iterated to solve for the long run wealth distribution from any initial distribution. In some cases, the long run distribution converges to a single mass point, regardless of the initial distribution. This occurs if a fast wealth accumulation by the rich and their strong investment demand drives up the equilibrium rate of return so much that the poor lenders could also accumulate their wealth by lending, which helps them to cross over the threshold level of wealth. This is the case where the rich's wealth "trickles down" to the poor through the credit market. In other cases, the long run distribution converges to a two-point distribution, regardless of the initial distribution. The credit market causes an endogenous polarization of the society between the rich and the poor. The rich maintain a high level of wealth in part because of the cheap credit offered by the poor, who have no choice but to lend their small saving to the rich. In some other cases, the long run distribution depends on the initial distribution, exhibiting the history dependence.[48]

9. Concluding Remarks

Credit market imperfections provide the key to understanding many important issues in business cycles, growth and development, and international economics. Recent progress in these areas, however, has left in its wake a bewildering array of individual models with seemingly conflicting results. Using the same single model of credit market imperfections throughout, this paper brought together a diverse set of results within a unified framework. In so doing, it showed how a wide range of aggregate phenomena may be attributed to credit market imperfections. They include, among other things, endogenous investment-specific technical changes, development traps, leapfrogging, persistent recessions, recurring boom-and-bust cycles, reverse international capital flows, the rise and fall of inequality across nations, and the patterns of international trade. The framework is also used to investigate some equilibrium and distributional impacts of improving the efficiency of credit markets. One recurring finding is that the properties of equilibrium often respond non-monotonically to parameter changes, which suggests some cautions for studying aggregate implications of credit market imperfections within a narrow class or a particular family of models.

Although the simple framework used in this paper enabled me to discuss many issues within the limited space, it has some limitations. First, it is highly restrictive in the dynamic feedback mechanisms. For example, it rules out endogenous savings by the investing agents, and hence the possibility that they may accumulate the net worth in anticipation of their future financing needs, the issue addressed by Greenwood and Jovanovic (1990), Buera (2006), and others. The model also rules out the possibility that the borrower's net worth might depend on the future allocation of credit through the equilibrium determination of durable assets owned by the borrowers, the issue addressed by Shleifer and Vishny (1992), Kiyotaki and Moore (1997), and Kiyotaki (1998). The model also assumes that all the projects are completed in one period. This rules out any issues associated with multistage financing, such as project

[48] In essence, this is what is shown by Matsuyama (2000). The literature on the evolution of household wealth distributions under credit market imperfections is vast. In addition to the three studies already mentioned, see Aghion and Bolton (1997), Freeman (1996), Matsuyama (2006), Mookherjee and Ray (2002), Piketty (1997). Just as in the macro dynamics, the implications of the credit market imperfections on the long run wealth distribution depend sensitively on the assumptions about the way different households interact with each other, which cannot be explained here due to the space constraint. A proper exposition of this literature would require a whole new paper.

terminations and refinancing, as addressed by Clementi and Hopenhayn (2006), DeMarzo and Fishman (2006) and Gertler (1992). More importantly, allowing for such multi-period projects is essential for understanding the liquidity implications of credit market imperfections, as shown by Holmstrom and Tirole (1997, 1998) and Kiyotaki and Moore (2002, 2005a, 2005b). Second, the pledgeablity λ, which measures (inversely) the severity of agency problems behind the credit market imperfections, has been treated as exogenous. To the extent that it reflects the state of financial development, we would like to introduce some feedback mechanisms from the investments to the credit market efficiency in order to address the two-way causality between economic growth and financial development, the issue addressed by Acemoglu and Zilibotti (1997), Greenwood and Smith (1997), Martin and Rey (2004), and Saint-Paul (1992). To the extent that it reflects the quality of legal or contractual enforcement and other institutional factors, we would like to endogenize it in order to address some political economy issues. Finally, aggregate implications of credit market imperfections have been examined in the otherwise neoclassical competitive framework. While this is useful for isolating the effects of credit market imperfections, it would be interesting to examine how credit market imperfections might interact with other departures from the neoclassical framework. For example, introducing credit market imperfections into the monopolistic competitive framework, also rich and diverse in its aggregate implications as pointed out by Matsuyama (1995, 1997), would be essential for understanding how credit market imperfections affect the process of product innovation, firm entry dynamics, as well as agglomeration economies.

I believe that incorporating these additional elements into the present framework would only strengthen the basic message of the paper. Credit market imperfections are rich and diverse in the aggregate implications and they provide the key to understanding a wide range of important issues. What has been discussed here is merely the tip of the iceberg.

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Figure 1a: Distributional Impacts

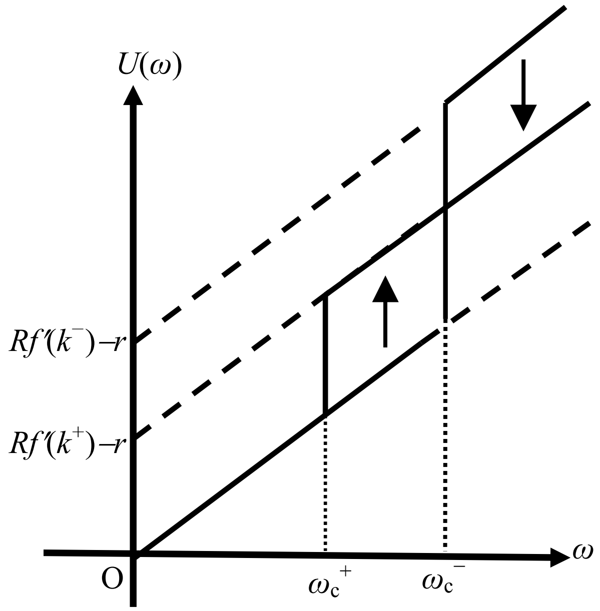


Figure 1b: Replacement Effects

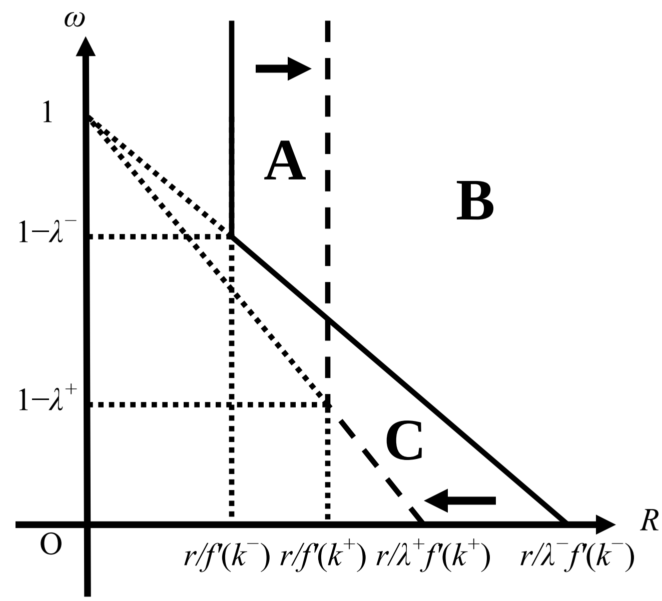


Figure 2a: General Equilibrium with Endogenous Saving

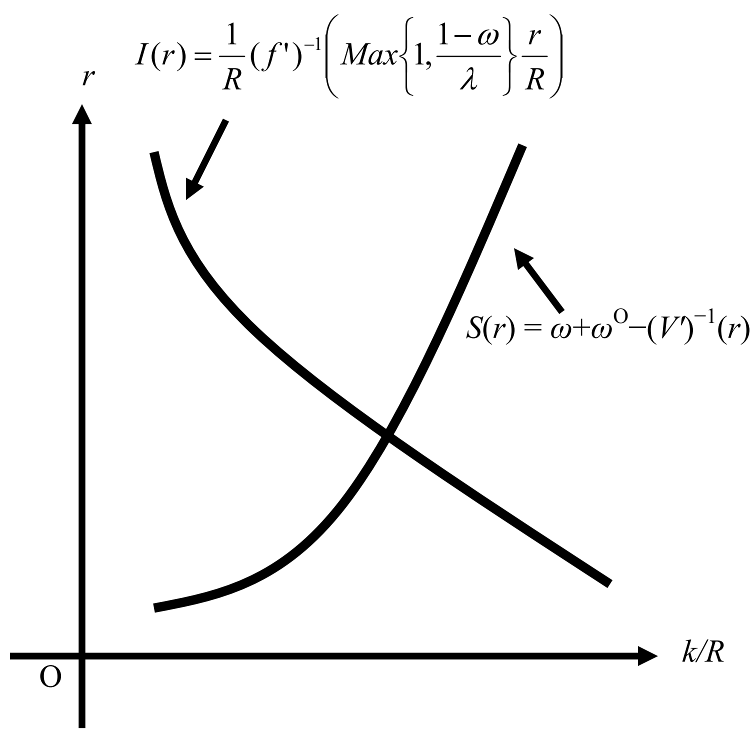


Figure 2b: Capital Deepening Effect: $\Delta\omega^0 > 0$

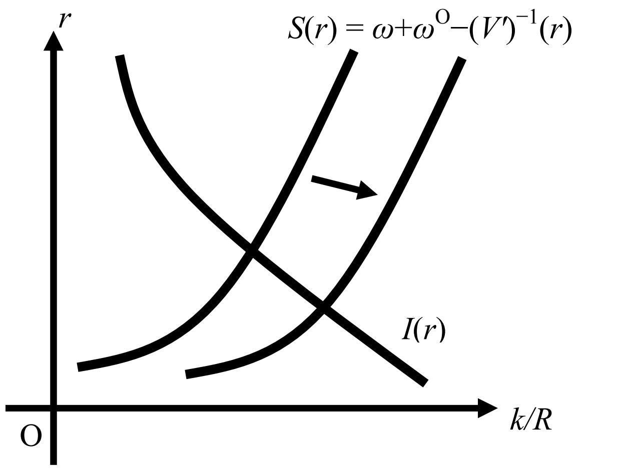


Figure 2c: Net Worth Effect: $\Delta\omega = -\Delta\omega^0 > 0$ (and $\Delta\lambda > 0$) for $\lambda + \omega < 1$.

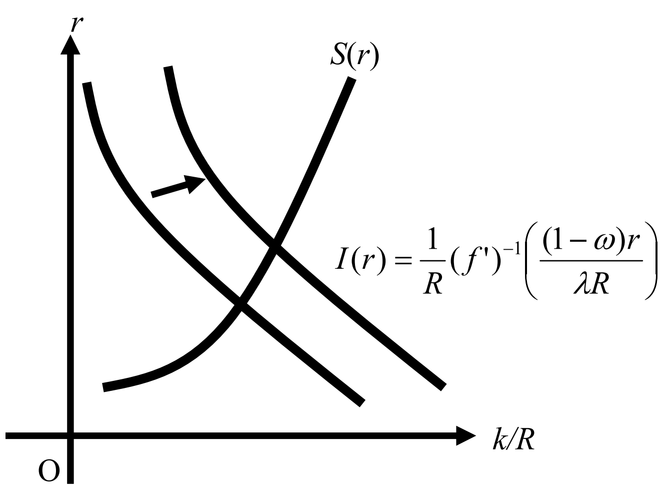


Figure 2d: Combined Effects: $\Delta\omega > 0$, for $\lambda + \omega < 1$,

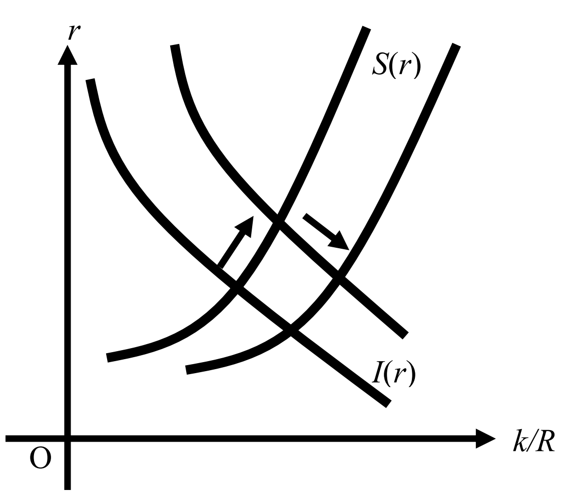


Figure 3:

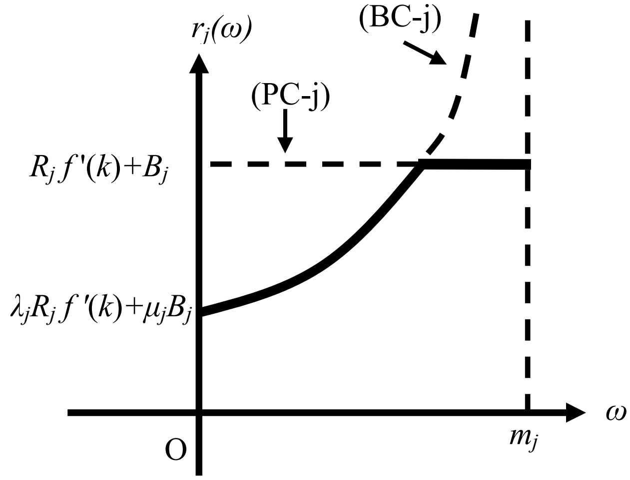


Figure 4a

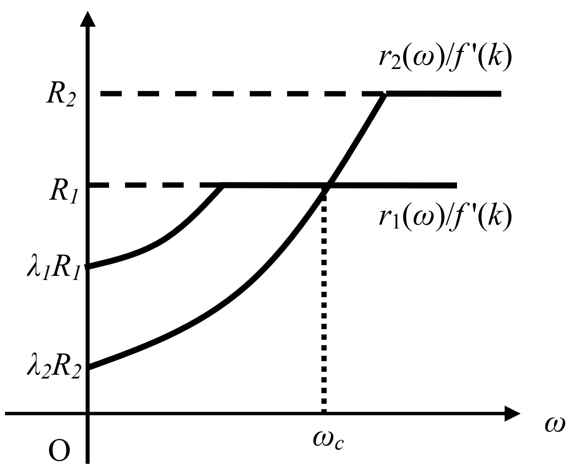


Figure 4b

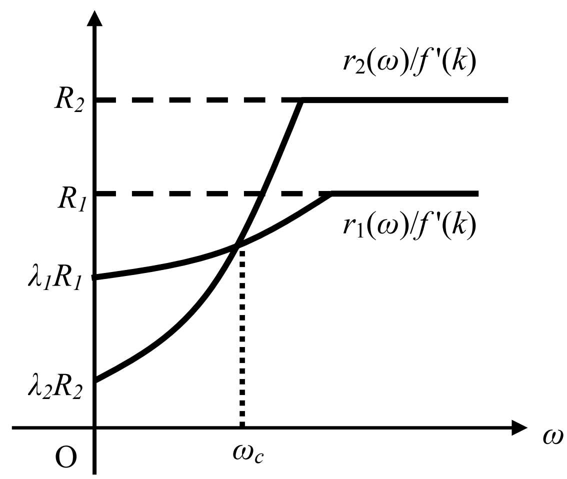


Figure 4c: Procyclical Productivity Change

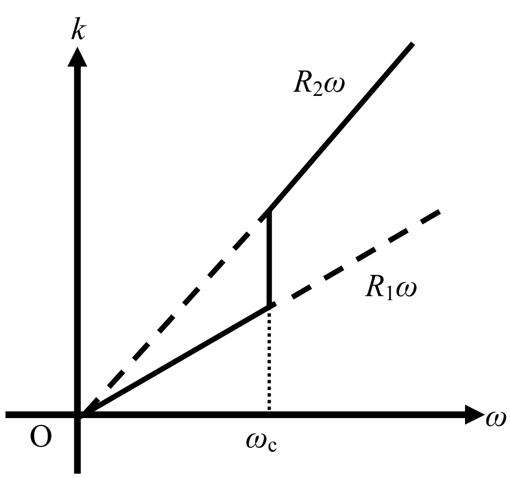


Figure 4d: Credit Traps

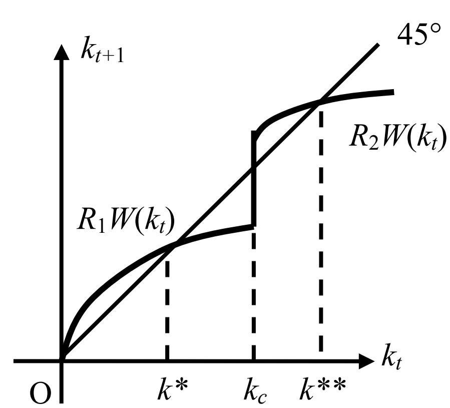


Figure 5a

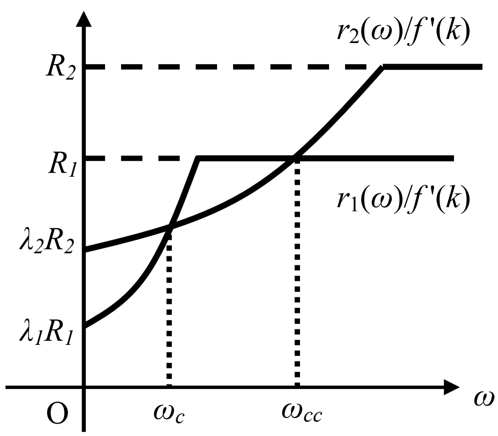


Figure 5b

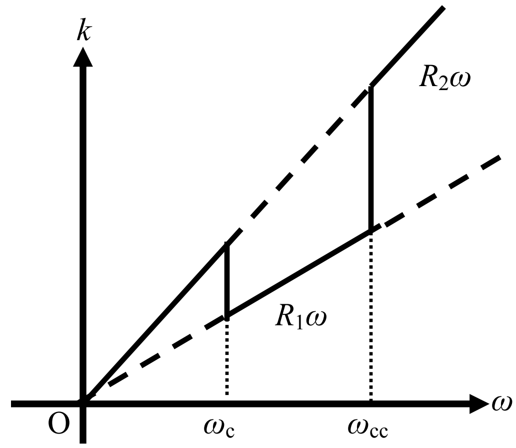


Figure 5c: Leapfrogging

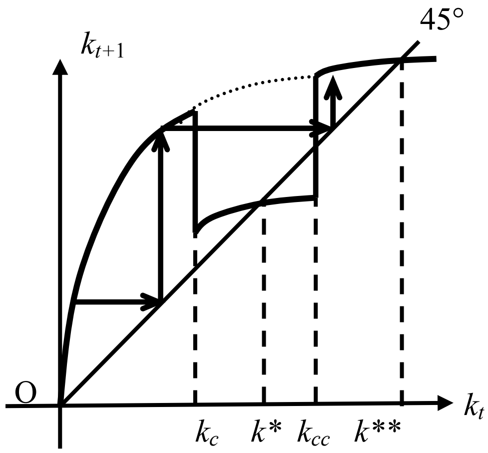


Figure 5d: Credit Cycles as a Trap

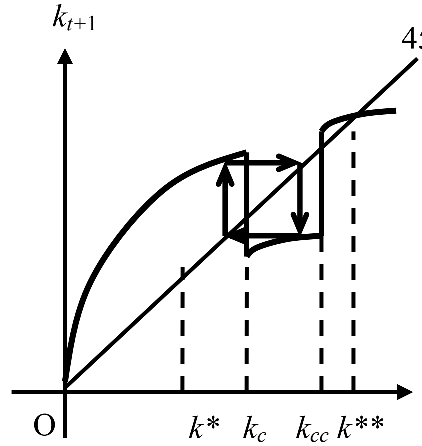


Figure 6a

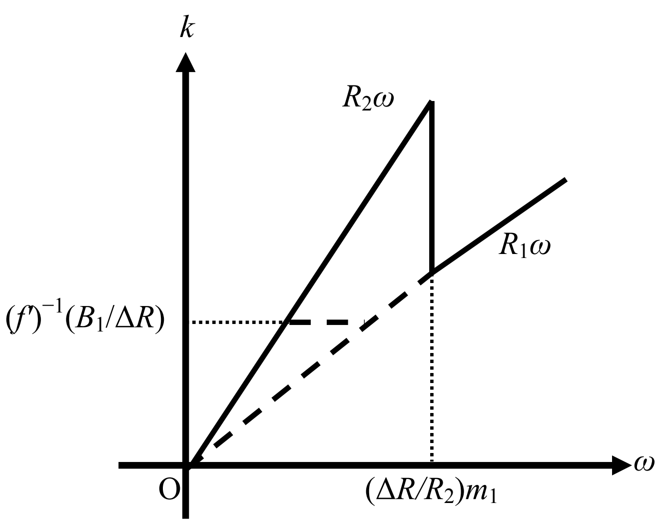


Figure 6b: Credit Cycles

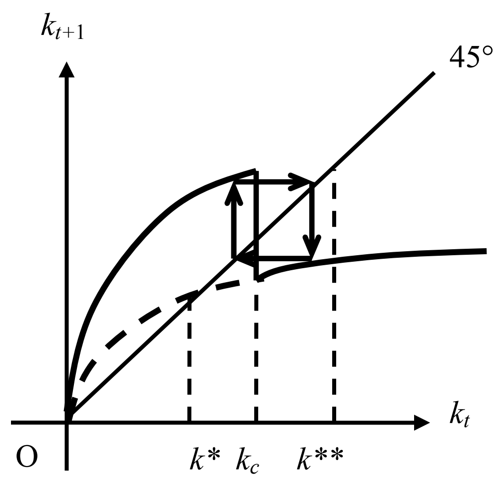


Figure 7: Perfect Credit Case

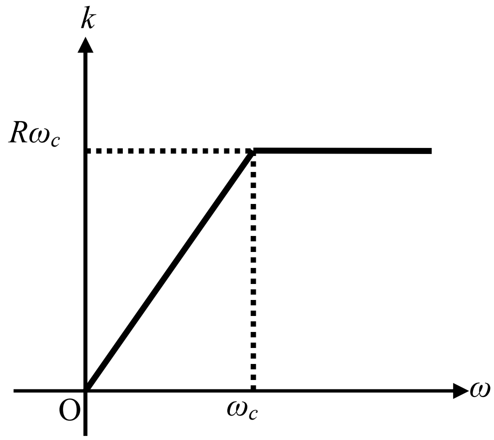


Figure 8a: Under-investment of Type-1

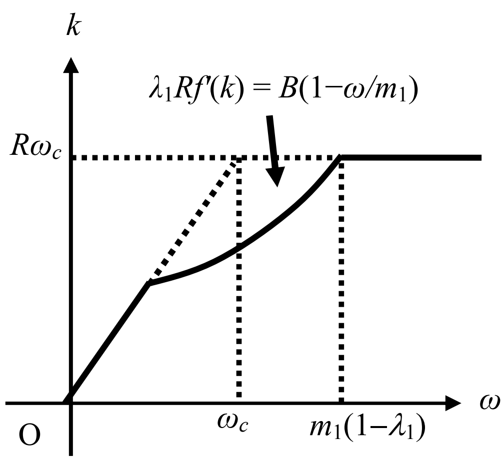


Figure 8b: Financial Accelerator

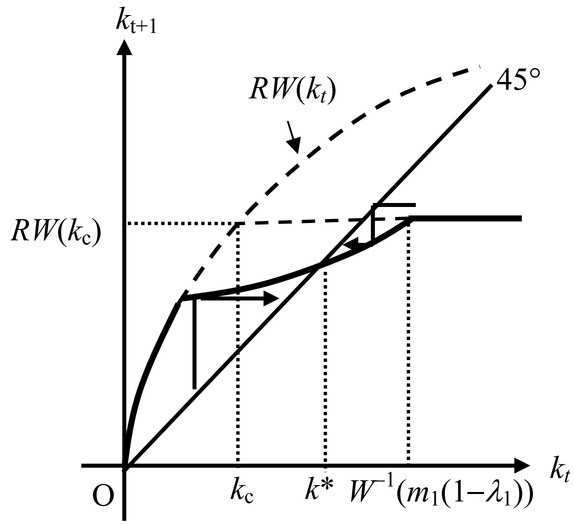


Figure 8c: Slow Recovery

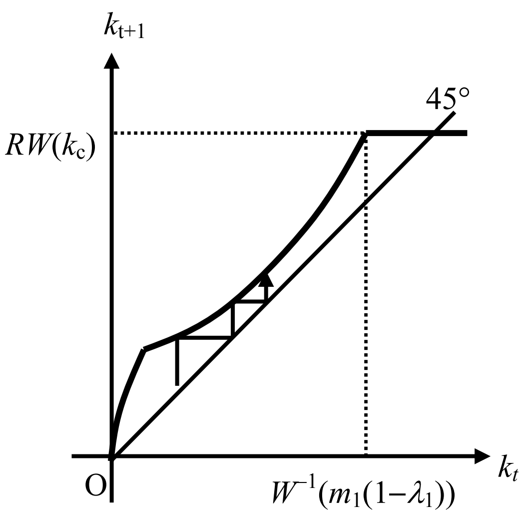


Figure 8d: Multiple Steady States

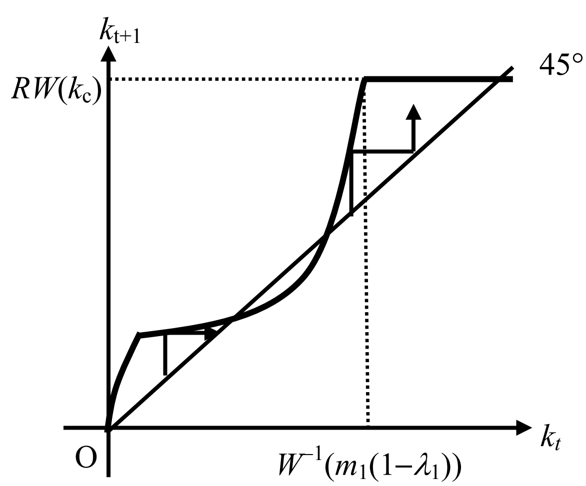


Figure 9a: Over-Investment to Type-1

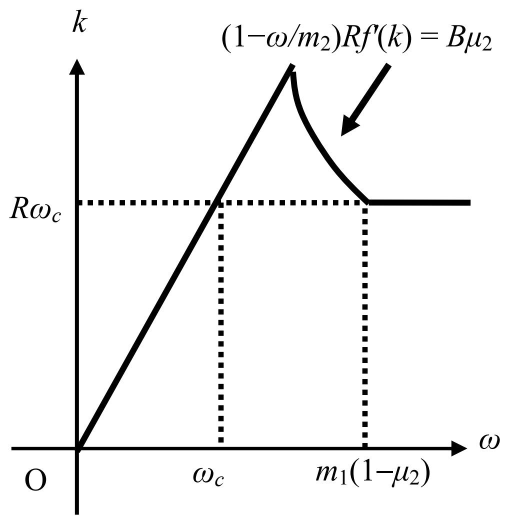


Figure 9b: Inefficient Booms and Volatility

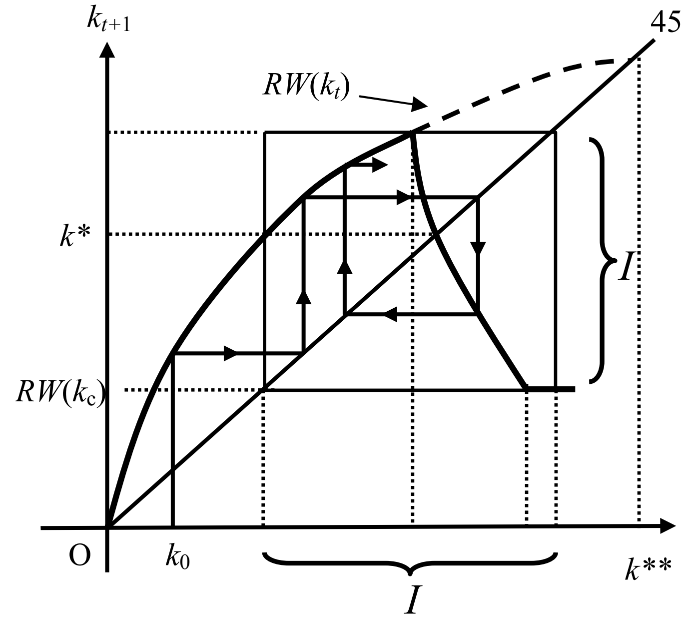


Figure 10a

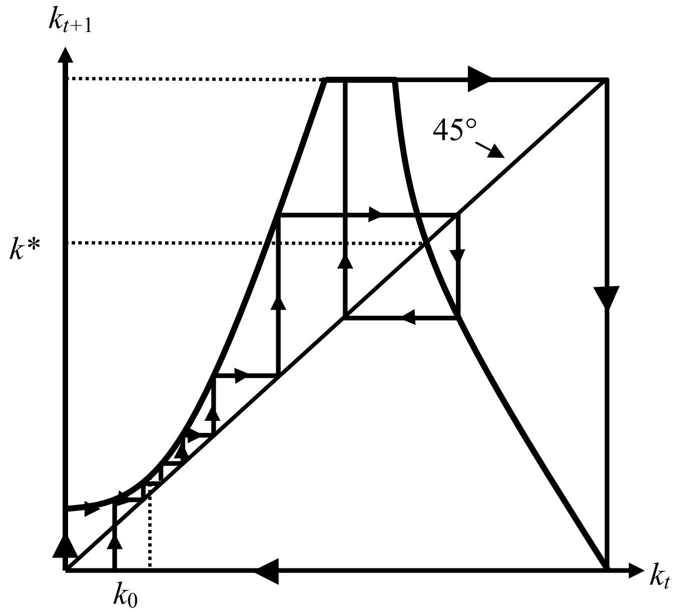


Figure 10b

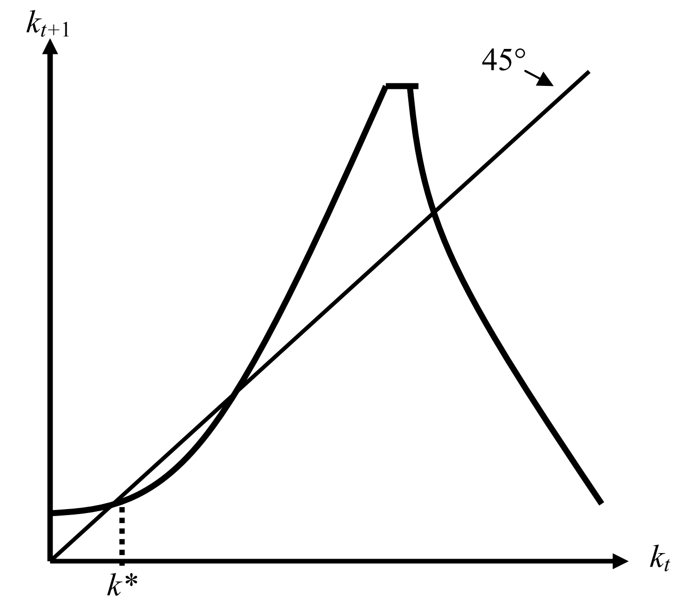


Figure 11a: Neoclassical View of Financial Integration ($\lambda_N = \lambda_S$, $\omega_N = \omega_S$, $\omega^o_N > \omega^o_S$)

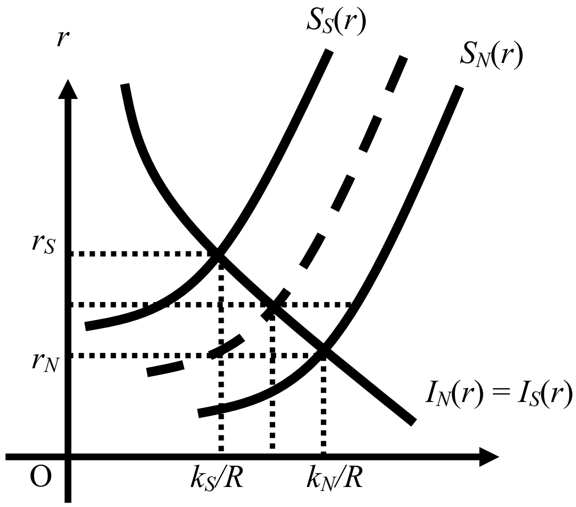


Figure 11b: Capital Flight (I): $\lambda_N > \lambda_S$, $\omega_N = \omega_S$, $\omega^o_N = \omega^o_S$; OR
Capital Flight (II); $\lambda_N = \lambda_S$, $\omega_N - \omega_S = \omega^o_S - \omega^o_N > 0$.

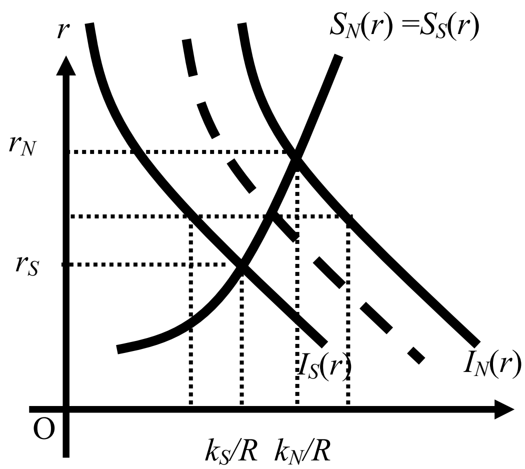


Figure 11c: Capital Flight (III): $\lambda_N = \lambda_S$, $\omega_N > \omega_S$ $\omega^o_N = \omega^o_S$.

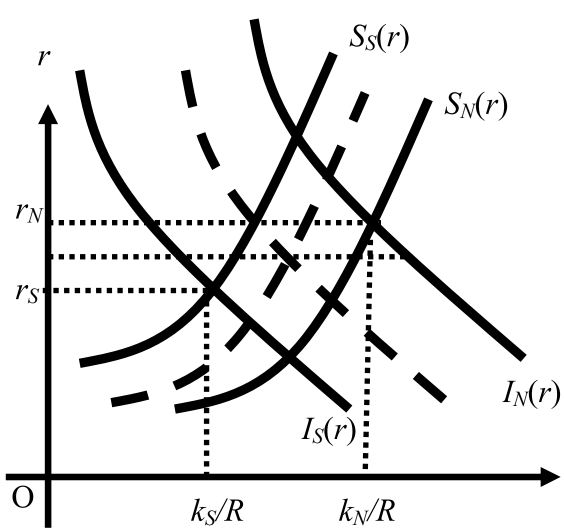


Figure 12: Symmetry-Breaking and the Emergence of Core-Periphery Patterns

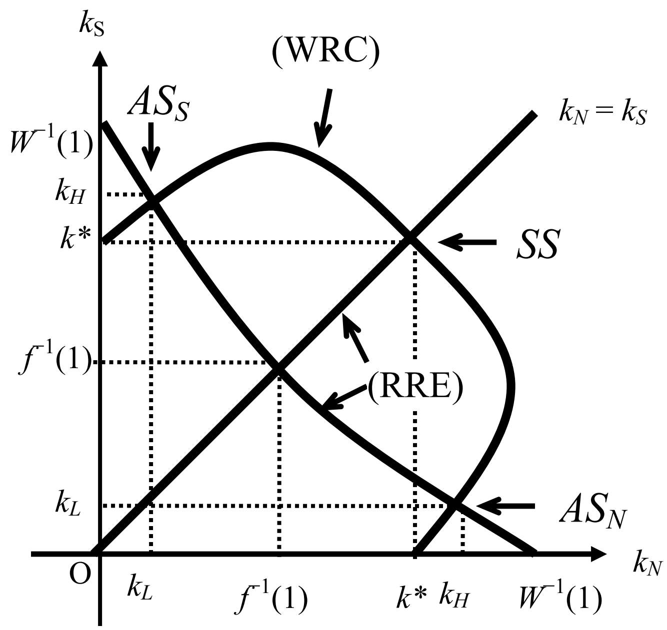
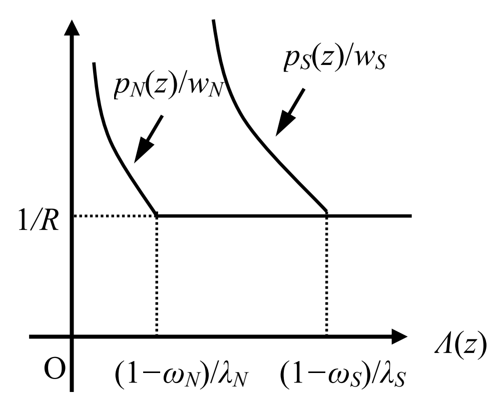


Figure 13a: Patterns of Absolute Advantage

Figure 13b: Patterns of Comparative Advantage



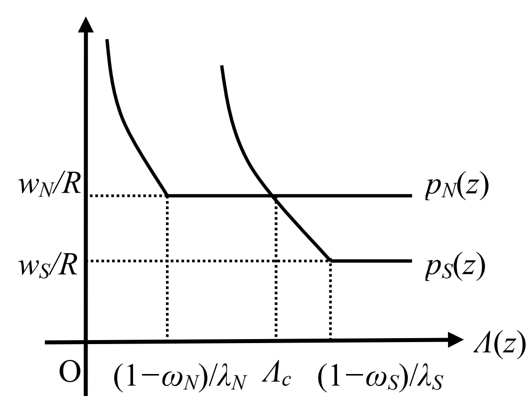


Figure 14a

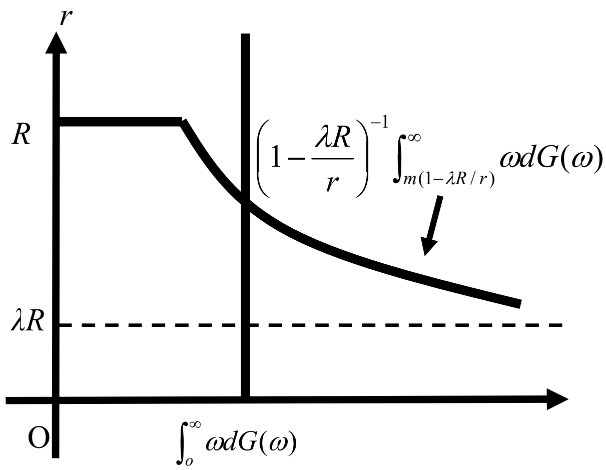


Figure 14b