The Good, The Bad, and The Ugly: An Inquiry into the Causes and Nature of Credit Cycles (Credit Traps and Credit Cycles)

By

Kiminori Matsuyama
Northwestern University

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The Goal

A Simple Model to convey the richness of the macroeconomic dynamics in the presence of credit market imperfections.

The Key Element: Heterogeneity of Investment Projects

- **Profitability**
- **Agency Problem**, which gives rise to the borrowing constraint and makes the investment dependent on the wealth of the agents.
- **General Equilibrium Price Effects**, which affect the wealth of the agents that have access to a variety of investment technologies
- **Set-up Costs**

1. Model-I (The Good, The Bad, and The Ugly)

Time: Discrete ($t = 0, 1, 2, \ldots$)

Demography: 2-period lived OG agents as in the standard Diamond model.

- Each generation consists of a continuum of the agents with unit mass.
- Each agent has one unit of the endowment, "labor," in the first period only, supplied inelastically.
- Each consumes only in the second. They save everything.

CRS Production Technology:

$Y_t = F(K_t, L_t) = F(k_t, 1) = f(k_t)$, $K_t \equiv K_t/L_t = k_t$; $f' > 0 > f''$, $f(0) = 0$, $f''(0) = \infty$.

- K_t and $L_t = 1$ are aggregate supplies of the capital good and labor in period t.
- Y_t, the final good, which can be consumed or invested.

Competitive Factor Markets: $\rho_t = f'(k_t) \equiv \Pi(k_t)$. ; $w_t = [f(k_t) - k_t f'(k_t)] \equiv W(k_t)$.

$f'' < 0 \Rightarrow \Pi(k_t)$ is decreasing and $W(k_t)$ is increasing in k_t.

Aggregate Saving: $S_t = W(k_t)$.

Investment Technologies: Two-types of the projects; Type 1 and Type 2.

	Period t	→	Period t+1
Type 1:	m_1 in final good		m_1R_1 in **capital good**
Type 2:	m_2 in final good		m_2R_2 in **final good**

Each project is discrete; the agent can start and manage at most one project.

Young Agents need to borrow $m_j - w_t$ at r_{t+1} to invest in Type j.
Young Agents can also lend w_t at r_{t+1}.

S = I condition:

$W(k_t) = m_1X_{1t} + m_2X_{2t}$, where X_{jt} is the number of type-j projects started in period t.

Capital Accumulation:

$$k_{t+1} = m_1R_1X_{1t}$$

When $X_{jt} > 0$?

Profitability Constraints; the young must be *willing* to finance the project:

For Type 1 projects, $\rho_{t+1}m_1R_1 - r_{t+1}(m_1 - w_t) \geq r_{t+1}w_t$ or

(PC-1) $R_1\Pi(k_{t+1}) \geq r_{t+1}$.

For Type 2 projects,

(PC-2) $R_2 \geq r_{t+1}$.

Borrowing Constraints: The young agents must be *able* to finance the project:

The agents can pledge only a fraction, λ_j, of the revenue to the lender.

(BC-1) $\lambda_1 m_1 R_1 \Pi(k_{t+1}) \geq r_{t+1}(m_1 - W(k_t))$.

(BC-2) $\lambda_2 m_2 R_2 \geq r_{t+1}(m_2 - W(k_t))$.

X_{jt} > 0 iff both (PC-j) and (BC-j) are satisfied.

Equilibrium:

Two Complementarity Slackness Conditions:

$$\text{Max}\{1, (1 - W(k_t)/m_1)/\lambda_1\} \geq R_1\Pi(k_{t+1})/r_{t+1} \qquad X_{1t} \geq 0.$$

$$\text{Max}\{1, (1 - W(k_t)/m_2)/\lambda_2\} \geq R_2/r_{t+1} \qquad X_{2t} \geq 0.$$

S = I condition: $W(k_t) = m_1X_{1t} + m_2X_{2t}$

Capital Stock Adjustment: $k_{t+1} = m_1R_1X_{1t}$

(PC-j) is the binding constraint if $W(k_t) > m_j(1-\lambda_j)$;
(BC-j) is the binding constraint if $W(k_t) < m_j(1-\lambda_j)$.

Example 1: Perfect Credit Market: $m_1(1-\lambda_1) = 0$ and $m_2(1-\lambda_2) = 0$.

Only (PC-j) are binding.

$$r_{t+1} \geq R_1\Pi(k_{t+1})\ ,\ \ X_{1t} \geq 0;$$

$$r_{t+1} \geq R_2\ ,\ \ X_{2t} \geq 0.$$

$$W(k_t) = m_1X_{1t} + m_2X_{2t}$$

$$k_{t+1} = m_1R_1X_{1t}.$$

As long as $R_1\Pi(k_{t+1}) > R_2$, $X_{1t} > 0$ and $X_{2t} = 0$, and

$$k_{t+1} = R_1W(k_t)$$

Otherwise, $R_1\Pi(k_{t+1}) = R_2$.

FIGURE 1-A (The Neoclassical Model a la Diamond); The Case of a small R_2.

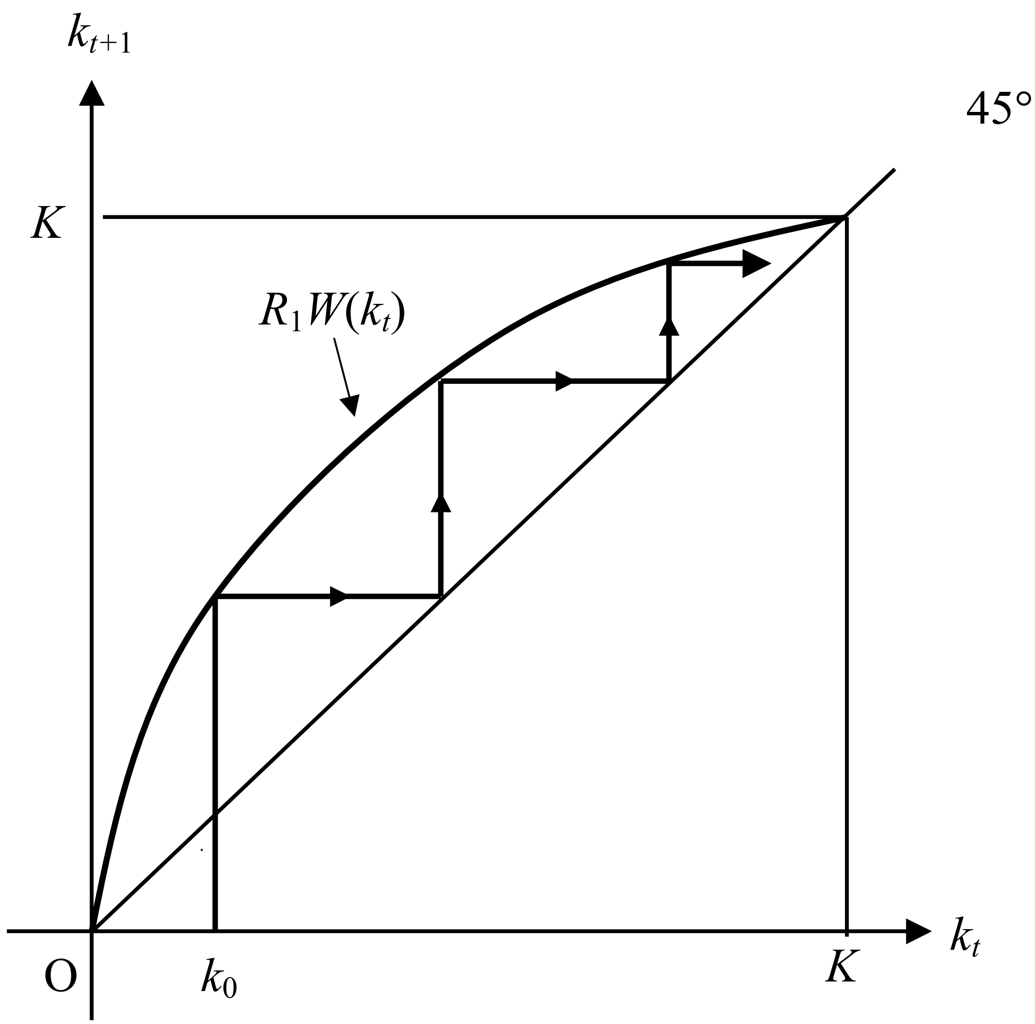
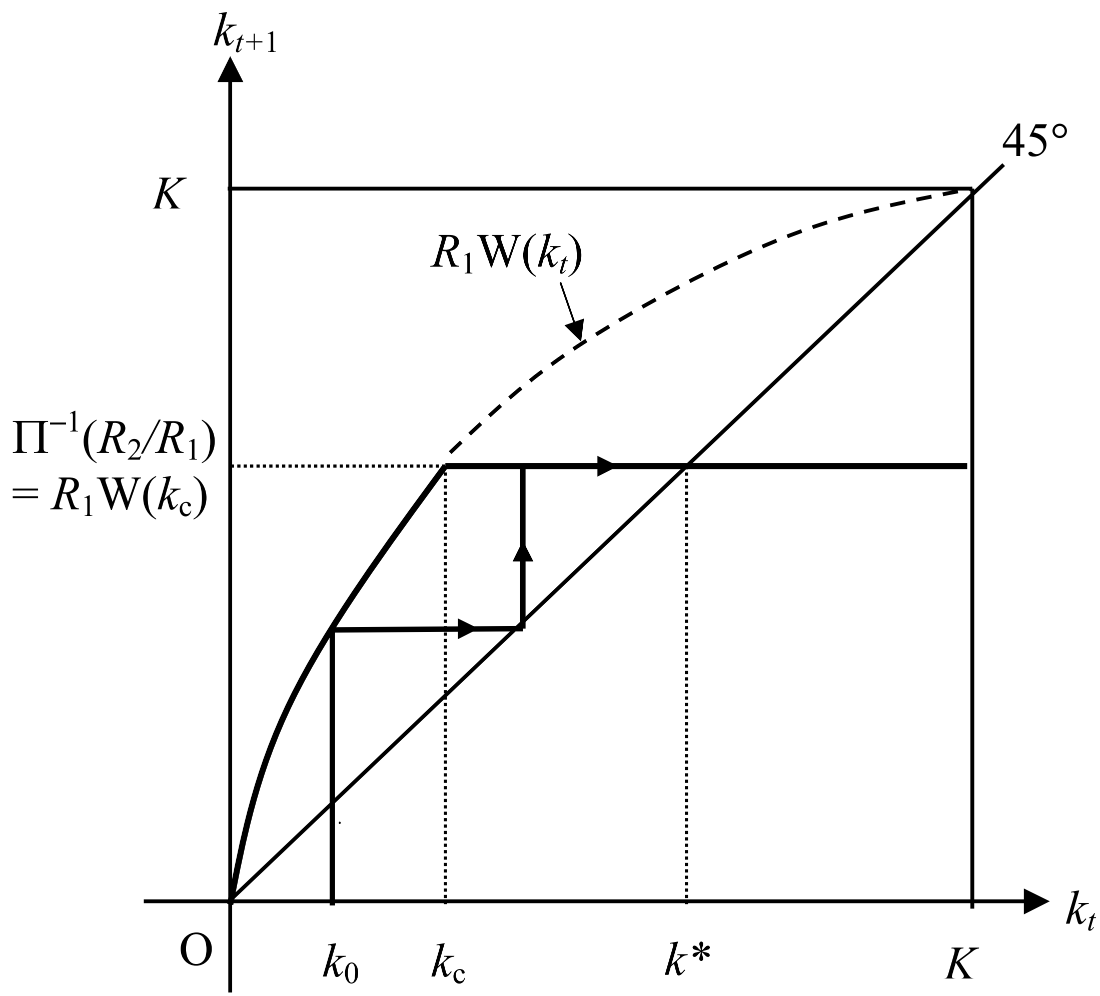


Figure 1-B; The Case of a large R_2.



Example 2: $m_1(1-\lambda_1) > 0$ and $m_2(1-\lambda_2) = 0$.

If $W(k_t) < m_1(1-\lambda_1)$, (BC-1) is binding if $X_{1t} > 0$.

If $k_{t+1} < R_1W(k_t)$, $R_2 = r_{t+1}$.

$$\lambda_1 m_1 R_1 \Pi(k_{t+1}) = R_2(m_1 - W(k_t)), \quad \text{if } W(k_t) < m_1(1-\lambda_1) \text{ and } k_{t+1} < R_1W(k_t).$$

Otherwise, it is as in Example 1.

FIGURE 2 (UNDER-INVESTMENT of Type I projects)

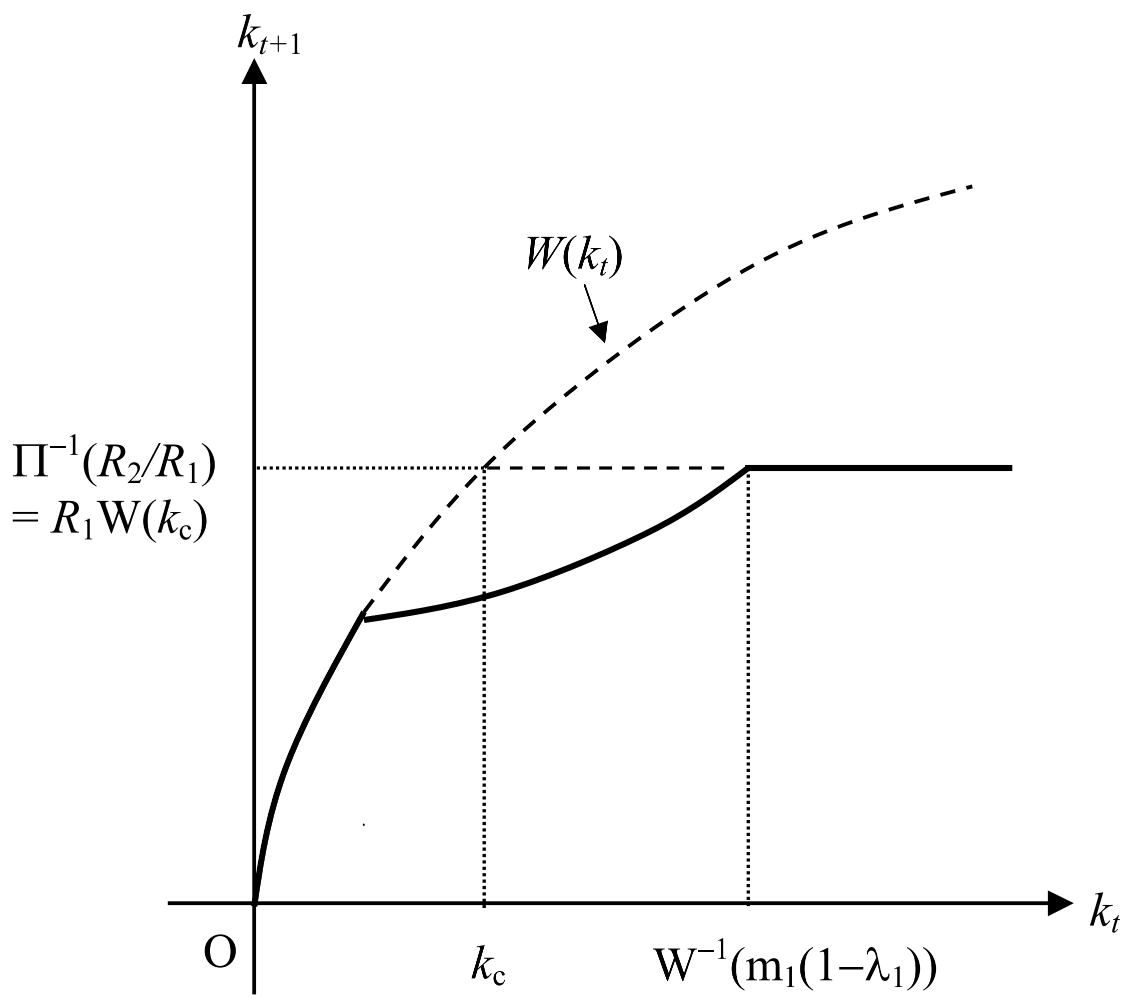


FIGURE 2-A: (Bernanke-Gertler Case)

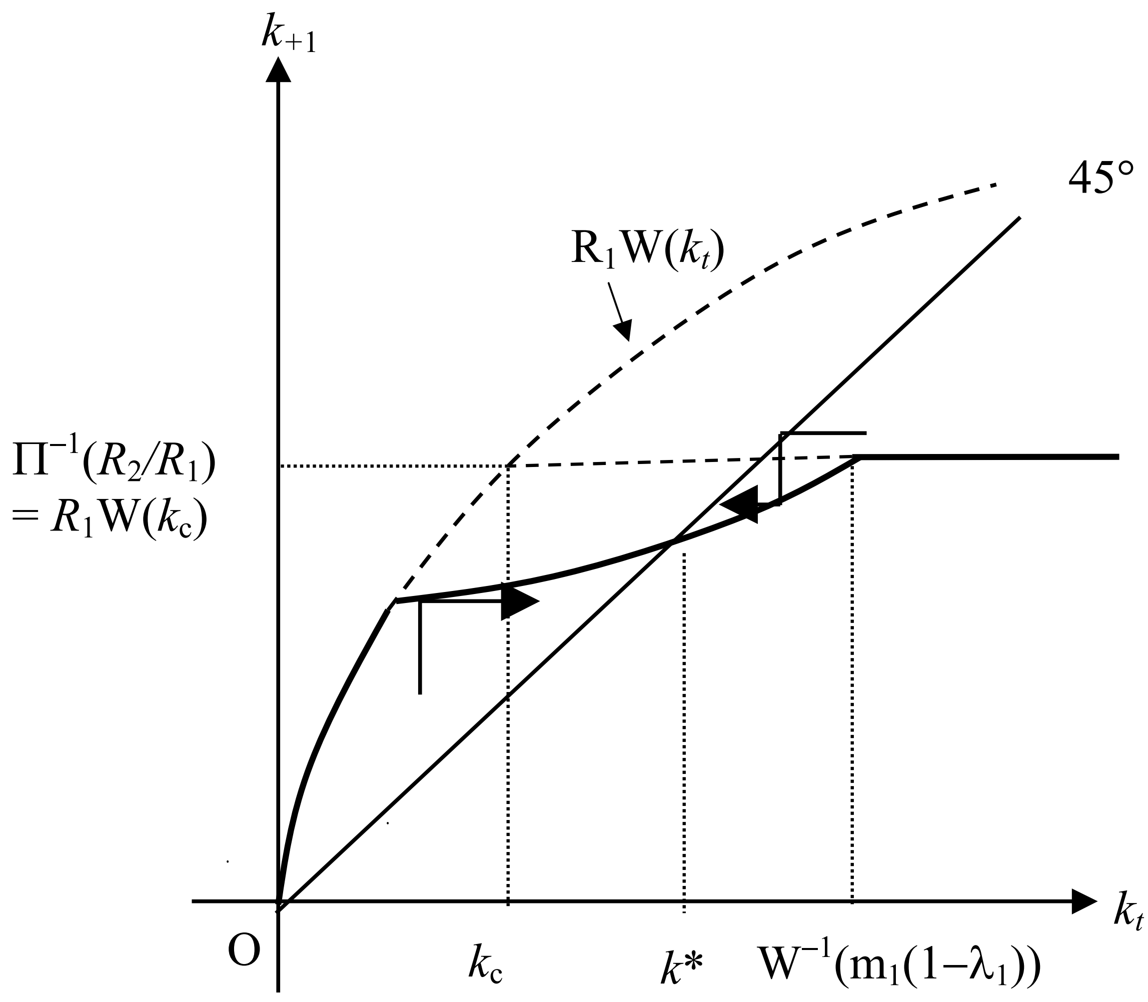


FIGURE 2-B: (A Slow Recovery from the Recession)

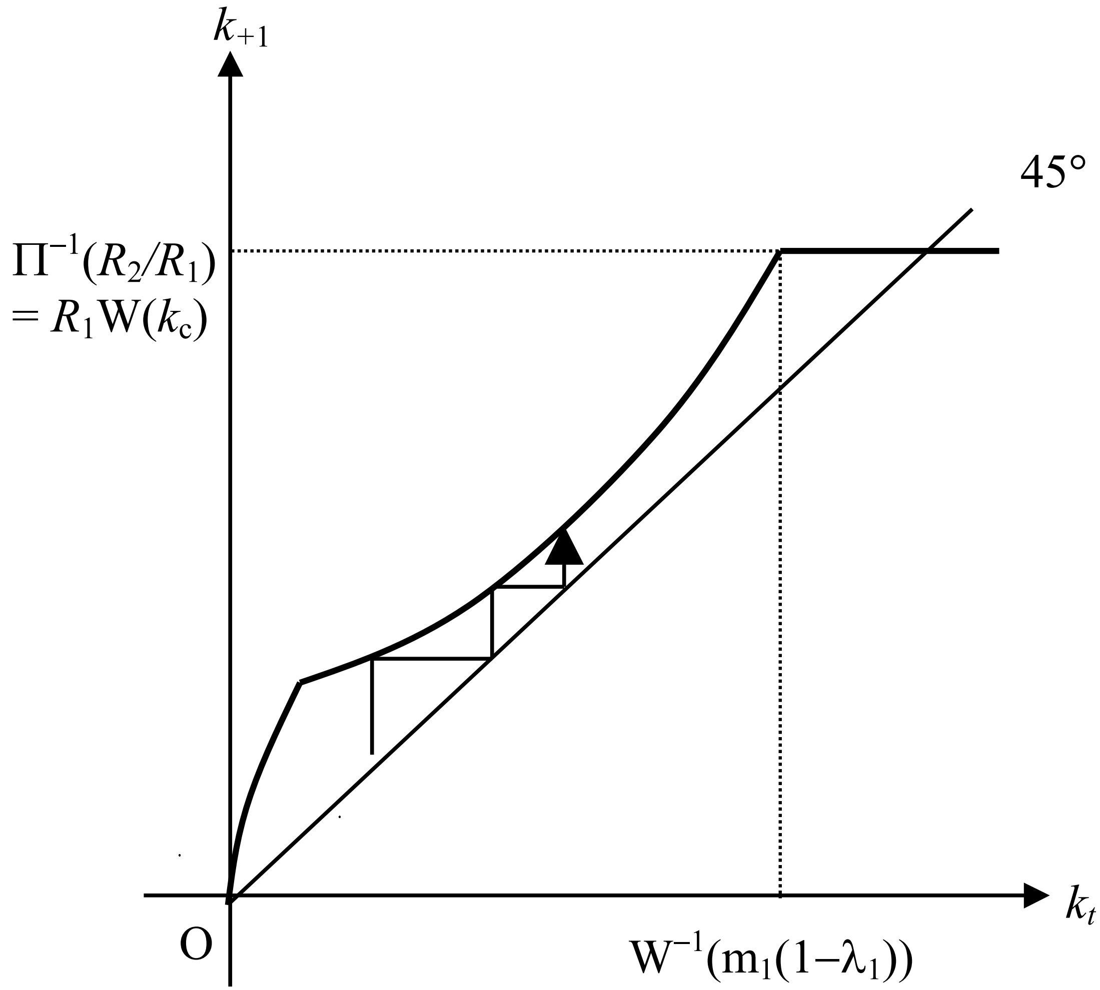
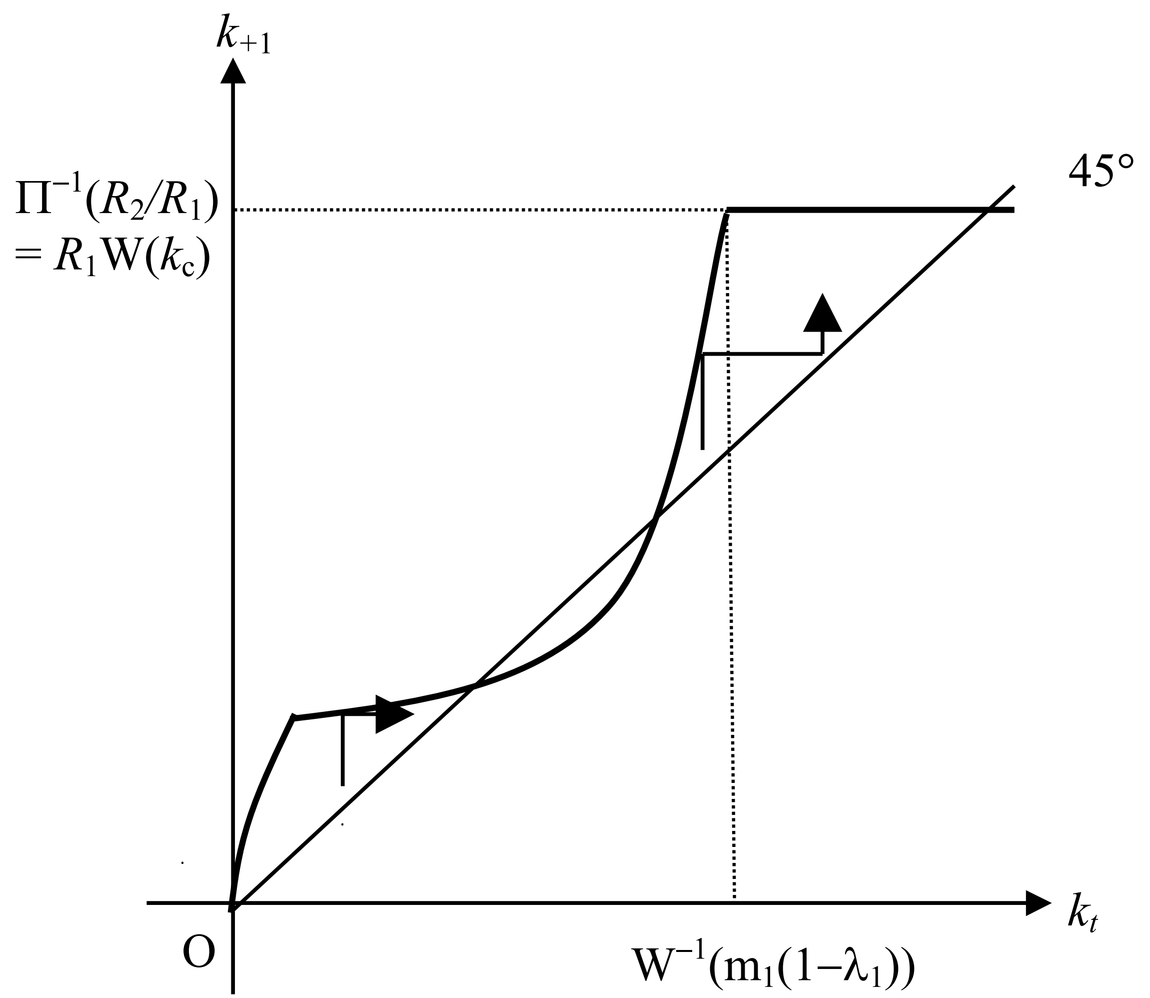


FIGURE 2-C (Multiple Steady States)



Example 3: $\mathbf{m_1(1-\lambda_1) = 0}$ **and** $\mathbf{m_2(1-\lambda_2) > 0}$**.**

(PC-1) is binding.

For $W(k_t) < m_2(1-\lambda_2)$, (BC-2) is binding if $X_{2t} > 0$ i.e., $k_{t+1} < R_1W(k_t)$.

$R_1\Pi(k_{t+1}) = r_{t+1} = \lambda_2 m_2 R_2/(m_2 - W(k_t))$,	if $W(k_t) < m_2(1-\lambda_2)$ and $k_{t+1} < R_1W(k_t)$.
$R_1\Pi(k_{t+1}) = r_{t+1} = R_2$,	if $W(k_t) > m_2(1-\lambda_2)$
$k_{t+1} = R_1W(k_t)$.	Otherwise

If $R_1\Pi(R_1m_2(1-\lambda_2)) < R_2$, then the map has a downward-sloping segment.

(To simplify the notations, $R_1 = 1$, $R_2 = R$, $\lambda_2 = \lambda$, $m_2 = m$.)

FIGURE 3 (Over-Investment of Type I projects)

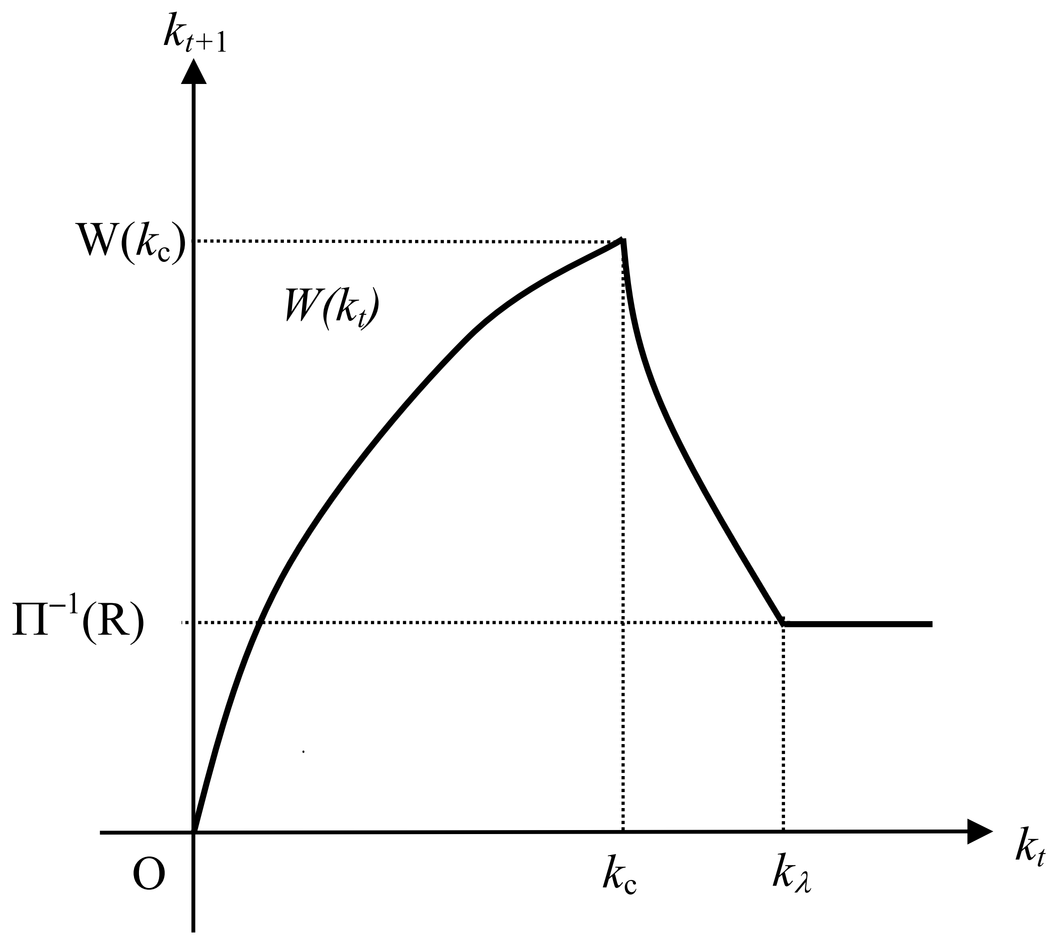


Figure 3-A (Overshooting)

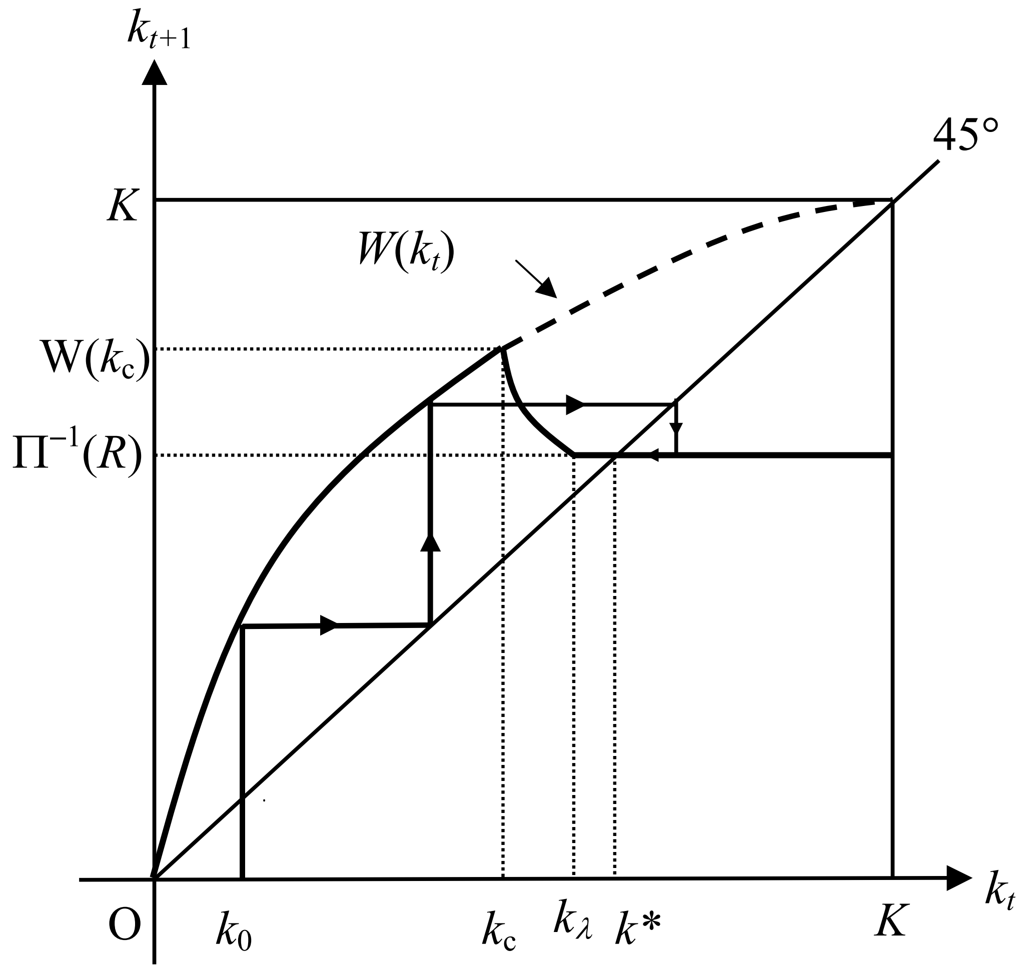


Figure 3-B: Oscillatory Convergence

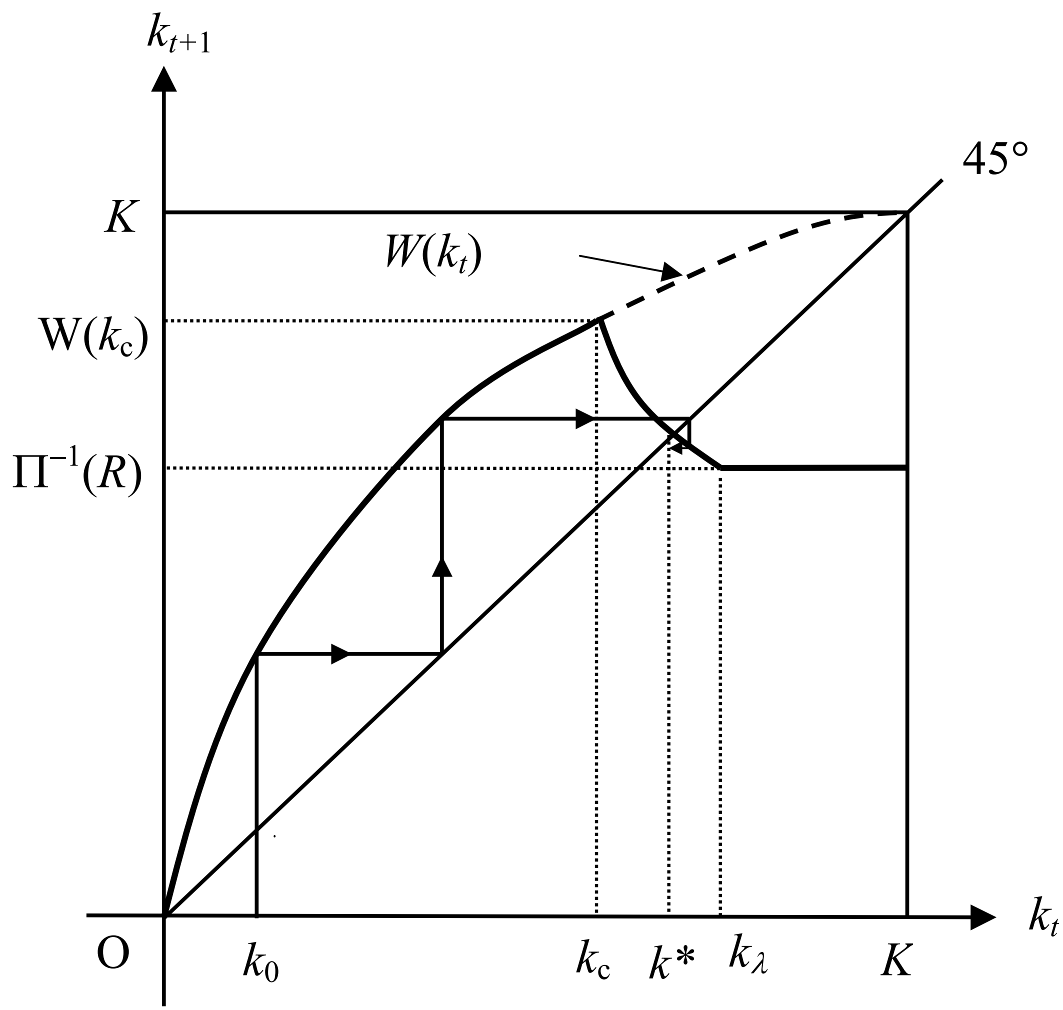


Figure 3-C: Endogenous Fluctuations

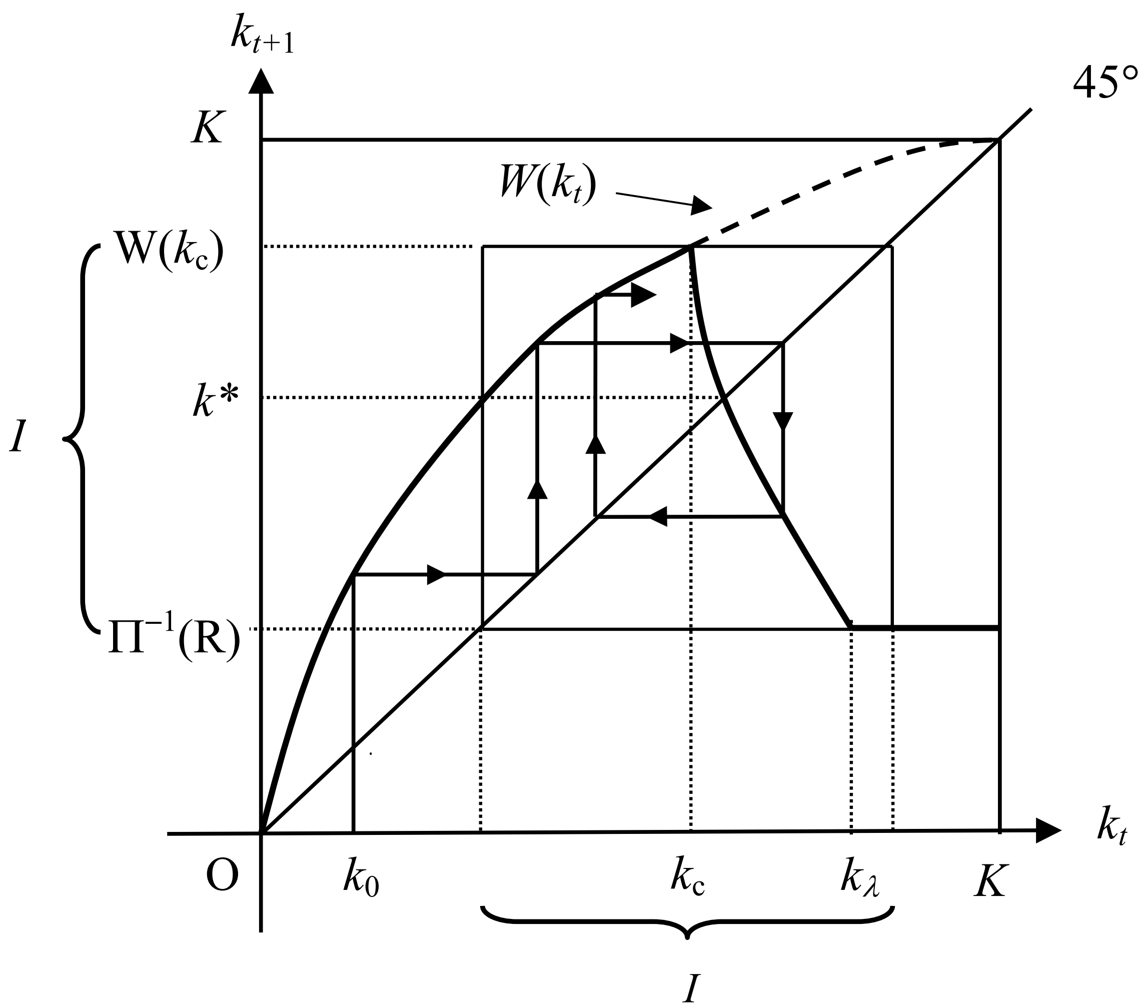
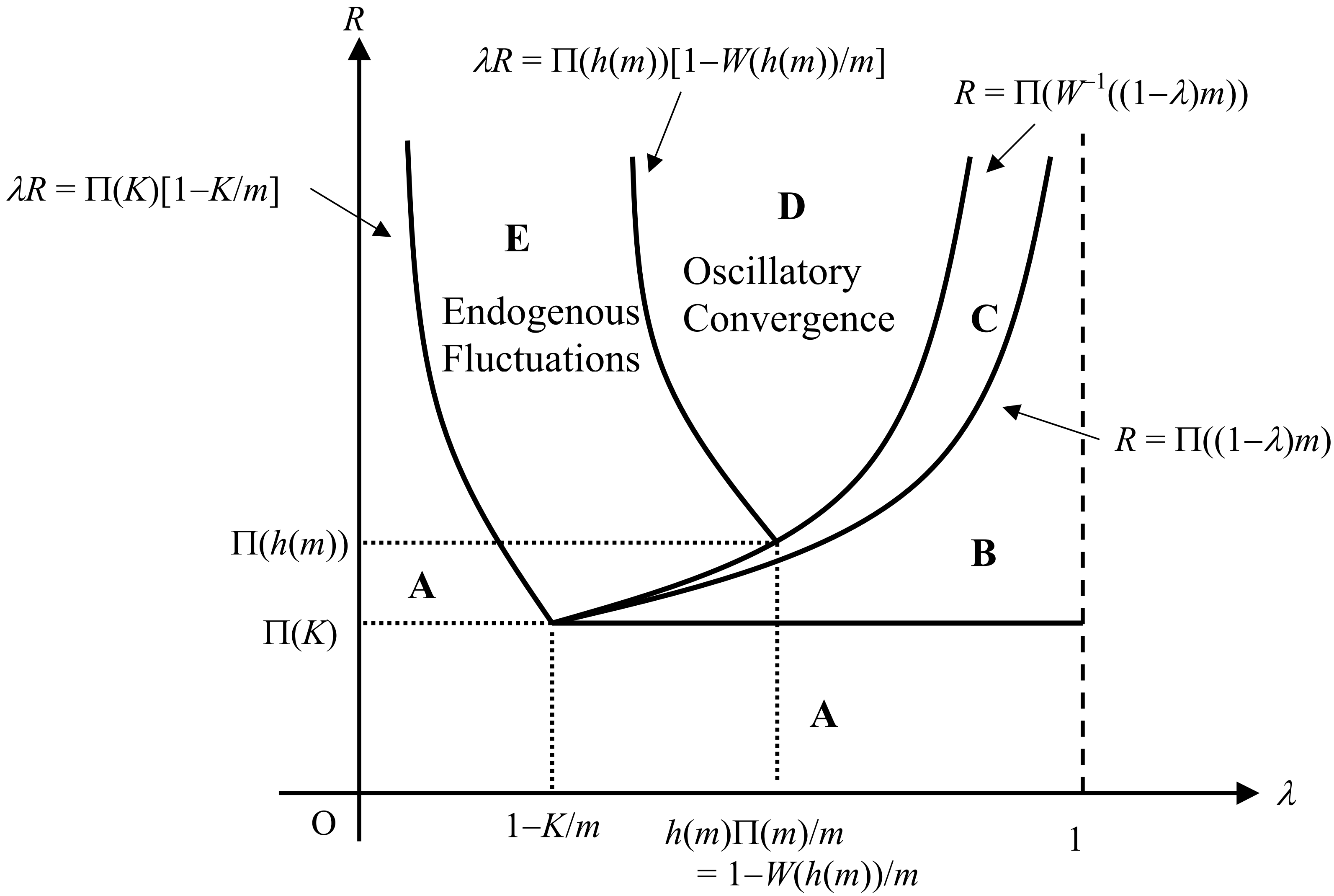


Figure 3-D


R
λR = Π(h(m))[1−W(h(m))/m]
R = Π(W⁻¹((1−λ)m))
λR = Π(K)[1−K/m]
D
E
Oscillatory
Convergence
Endogenous
Fluctuations
C
R = Π((1−λ)m)
Π(h(m))
B
A
Π(K)
A
O
1−K/m
h(m)Π(m)/m
= 1−W(h(m))/m
1
λ

Example 4: (A Hybrid of Example 2 and Example 3)

	Period t	→	**Period t+1**
Type 1 (Good)	m_1 units in final good		m_1R_1 units in **capital good**
Type 2 (Bad)	m_2 units in final good		m_2R_2 units in **final good**
Type 3 (Ugly)	m_3 units in final good		m_3R_3 units in **final good**

As before, let λ_j be the pleageable fraction of the project revenue for Type j investment**.**

Suppose $\mathbf{m_2(1-\lambda_2) > m_1(1-\lambda_1) > m_3(1-\lambda_3) = 0.}$

With a high R_2/R_1 and a small R_3/R_1, one could make

Type-2 projects are irrelevant for a sufficiently low k_t

Type-3 projects are irrelevant for a sufficiently high k_t.

For a lower value of k_t, the dynamics are link Example 2.
For a higher value of k_t, the dynamics are link Example 3.

Figure 4-A

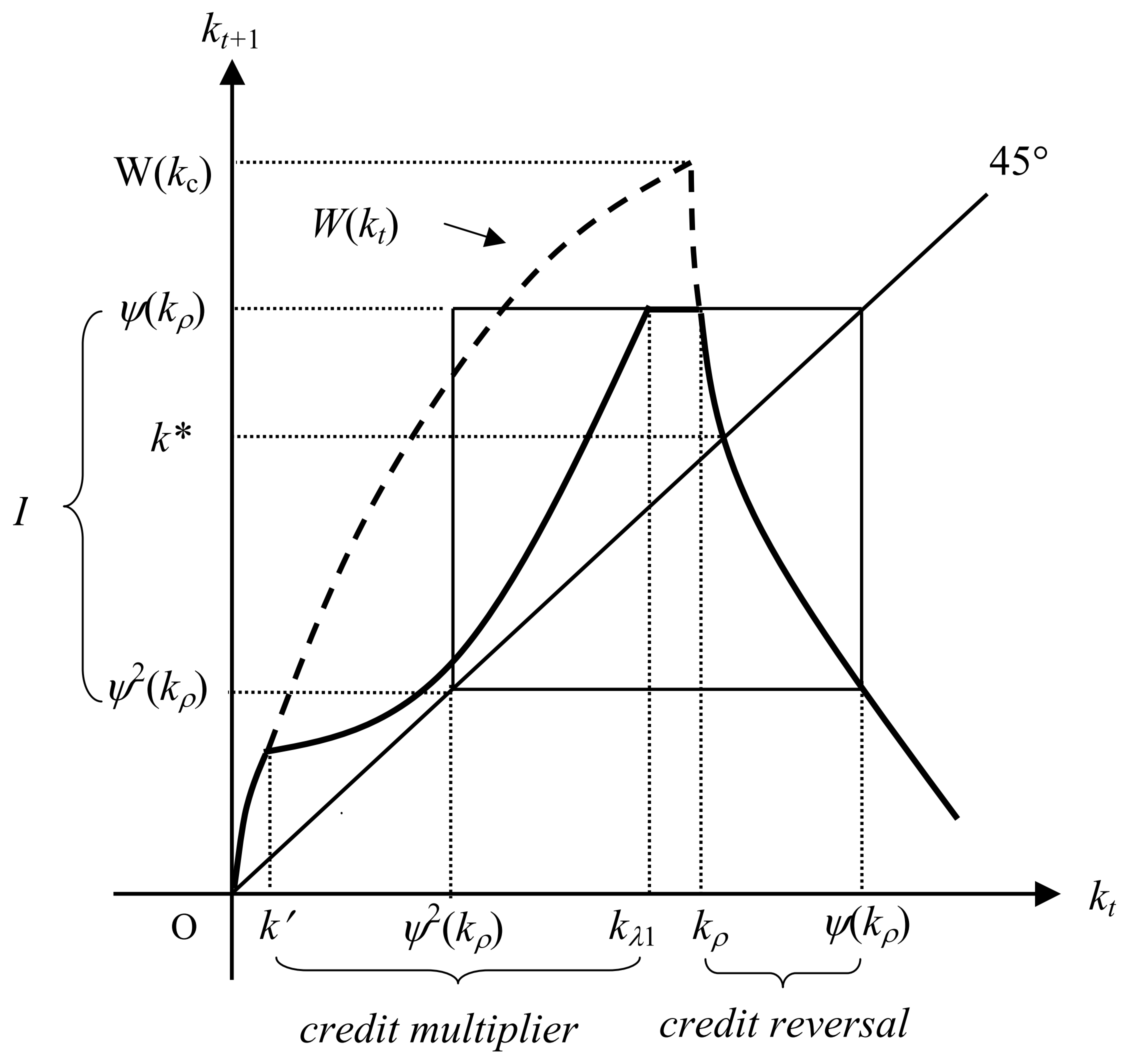

k_{t+1}
W(k_c)
$W(k_t)$
45°
$\psi(k_\rho)$
k^*
I
$\psi^2(k_\rho)$
O
k'
$\psi^2(k_\rho)$
$k_{\lambda 1}$
k_ρ
$\psi(k_\rho)$
k_t
credit multiplier
credit reversal

Figure 4-B

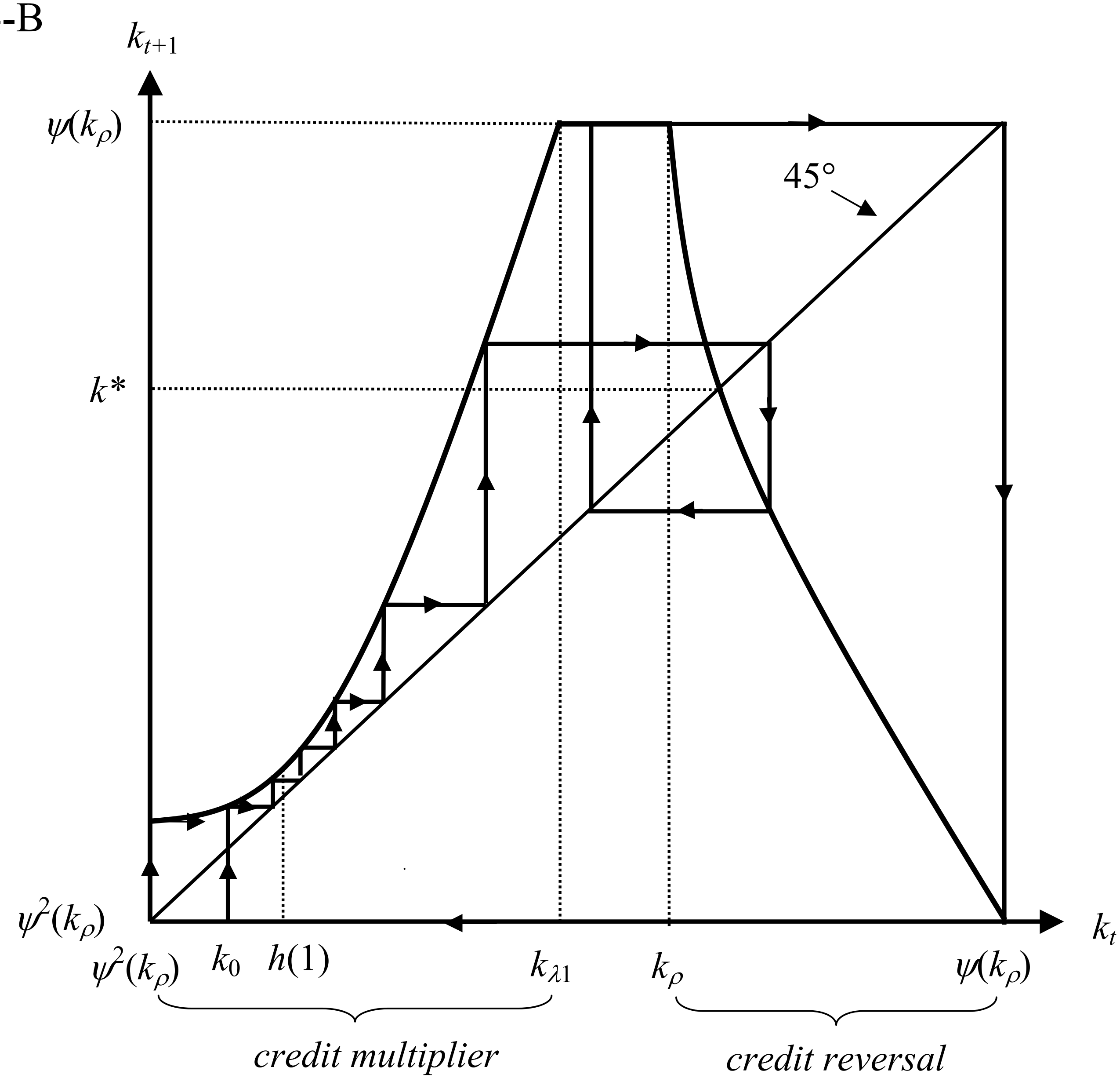
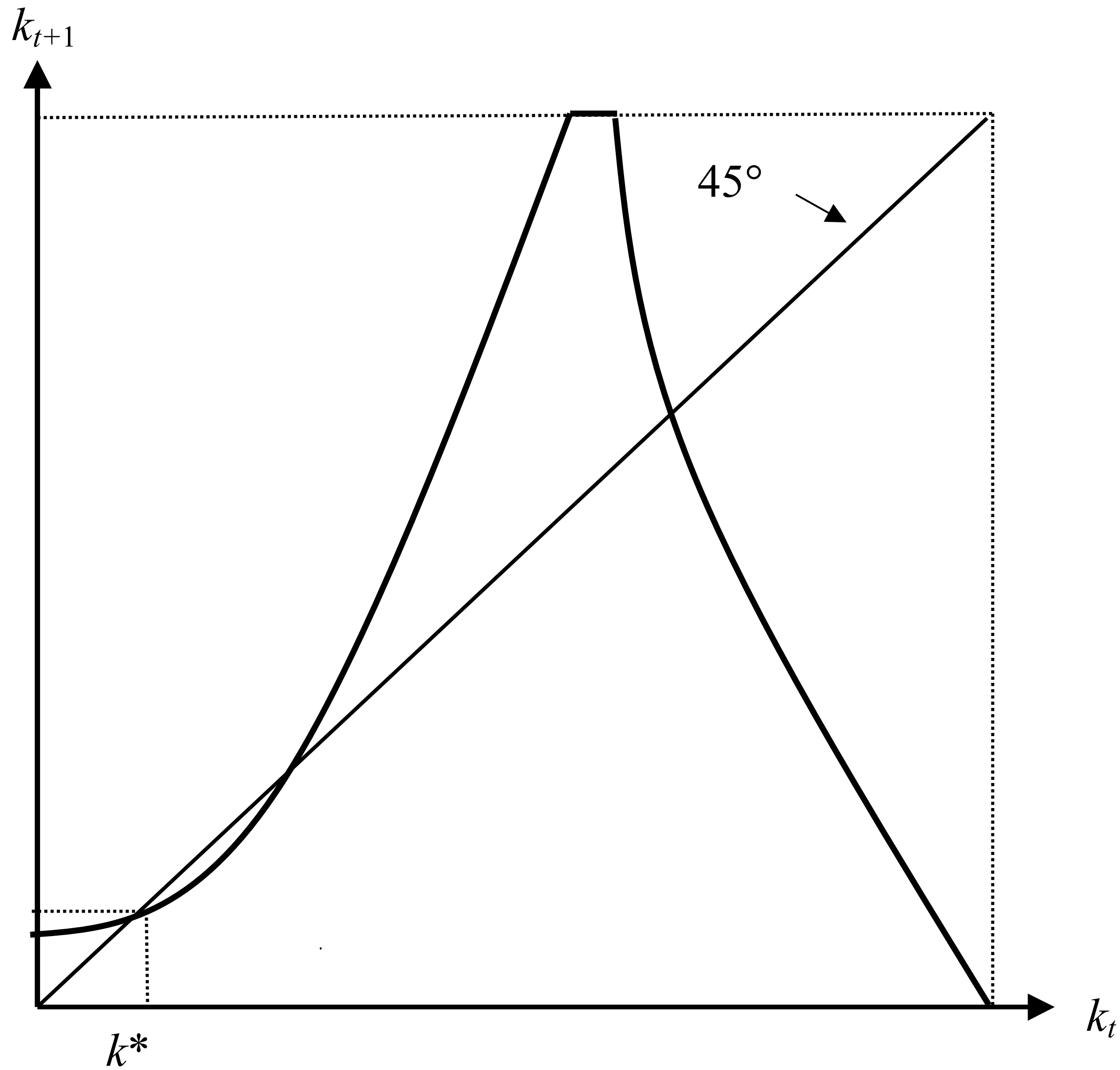

k_{t+1}
$\psi(k_\rho)$
45°
k^*
$\psi^2(k_\rho)$
k_t
$\psi^2(k_\rho)$
k_0
$h(1)$
$k_{\lambda 1}$
k_ρ
$\psi(k_\rho)$
credit multiplier
credit reversal

Figure 4-C



2. Model-II (Credit Traps and Credit Cycles)

Investment Technologies:

	Period t	→	Period t+1
Type j:	m_j in final good		m_jR_j in **capital good**

Profitability Constraint: $R_1\Pi(k_{t+1}) \geq r_{t+1}$, (PC-j)

Borrowing Constraint: $\lambda_j m_j R_j \Pi(k_{t+1}) \geq r_{t+1}(m_j - W(k_t))$, (BC-j)

λ_j: the fraction of the pledgeale fraction of the project revenue

Equilibrium

X_{jt}: the measure of type-j projects initiated in period t.

(1) $W(k_t) = \sum_j(m_j X_{jt})$.

(2) $k_{t+1} = \sum_j(m_j R_j X_{jt})$.

(3) $r_{t+1} \geq R_j\Pi(k_{t+1})/\max\{1, (1 - W(k_t)/m_j)/\lambda_j\}$ (j = 1, 2,…J)

where $X_{jt} > 0$ (j = 1, 2,…J) implies $r_{t+1} = R_j\Pi(k_{t+1})/\max\{1, (1 - W(k_t)/m_j)/\lambda_j\}$.

We also need $k_0 > 0$.

Three Examples.

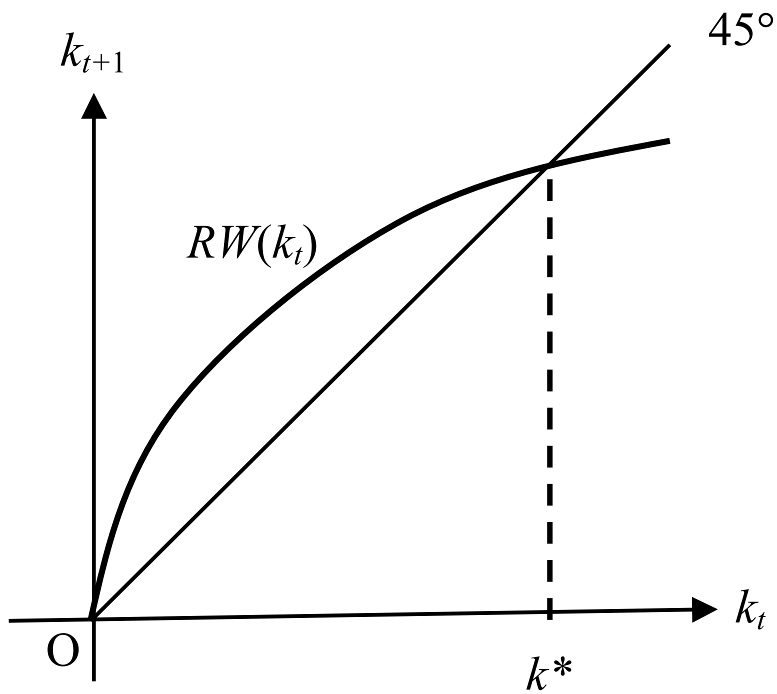
Example 1: Homogenous Projects (J = 1): The Neoclassical Convergence

(4) $0 < X_t = W(k_t)/m < 1$,

(5) $k_{t+1} = RW(k_t)$,

(6) $r_{t+1} = R\Pi(RW(k_t))/\max\{1, (1 - W(k_t)/m)/\lambda\}$

Figure 1



Example 2: Credit Traps and Credit Collapses ($R_1 < R_2$ and $\lambda_1 R_1 > \lambda_2 R_2$)

$$R_j / \max\{1, (1 - W(k_t)/m_j)/\lambda_j\} \qquad (j = 1, 2)$$

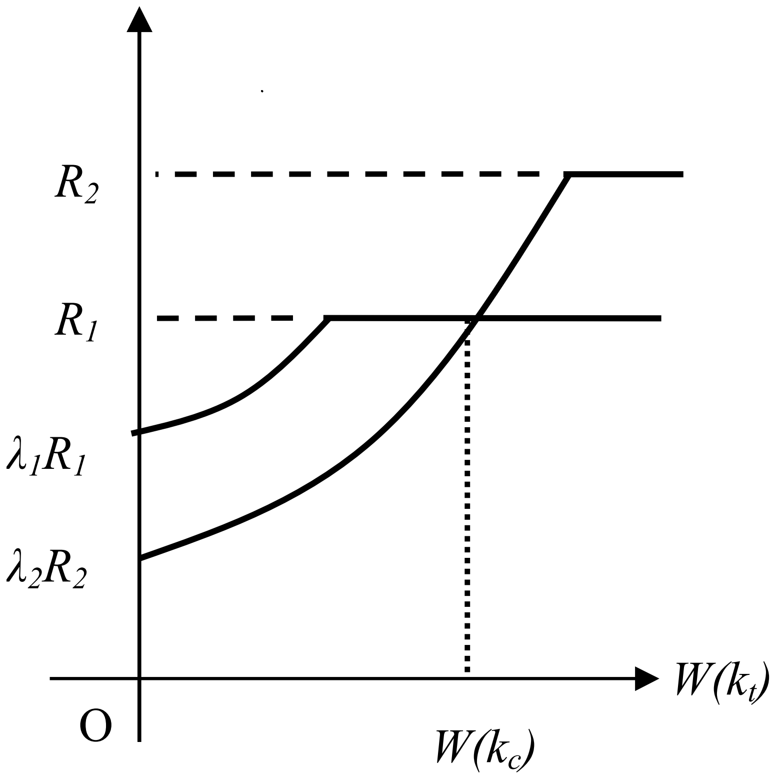


Figure 2a

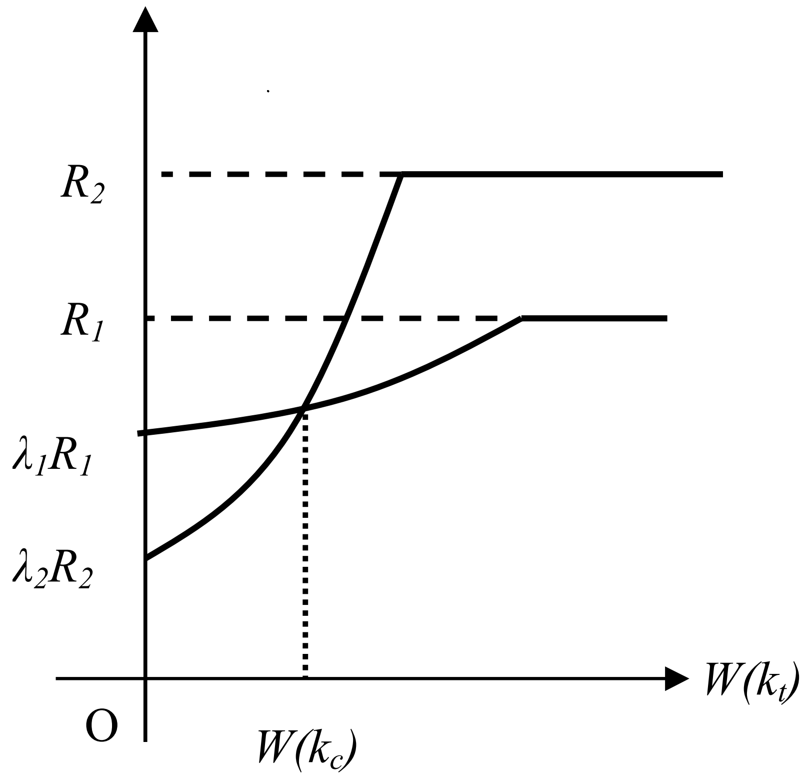
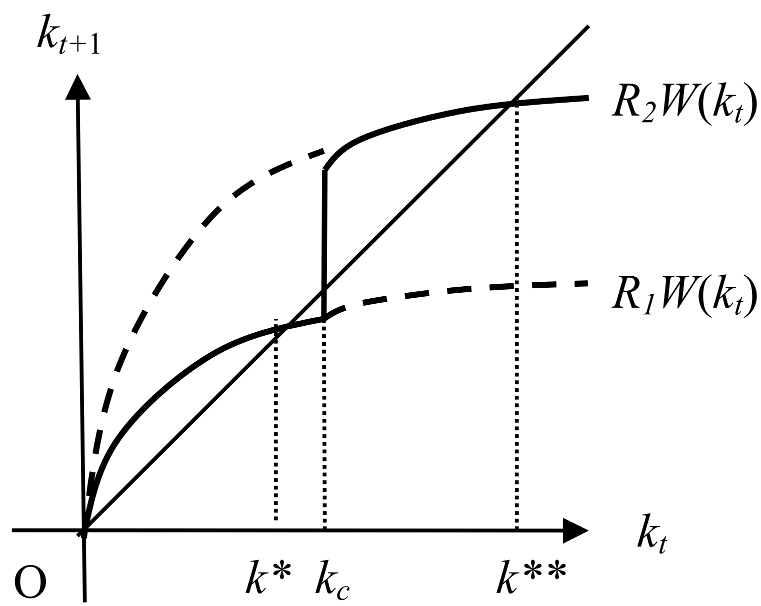


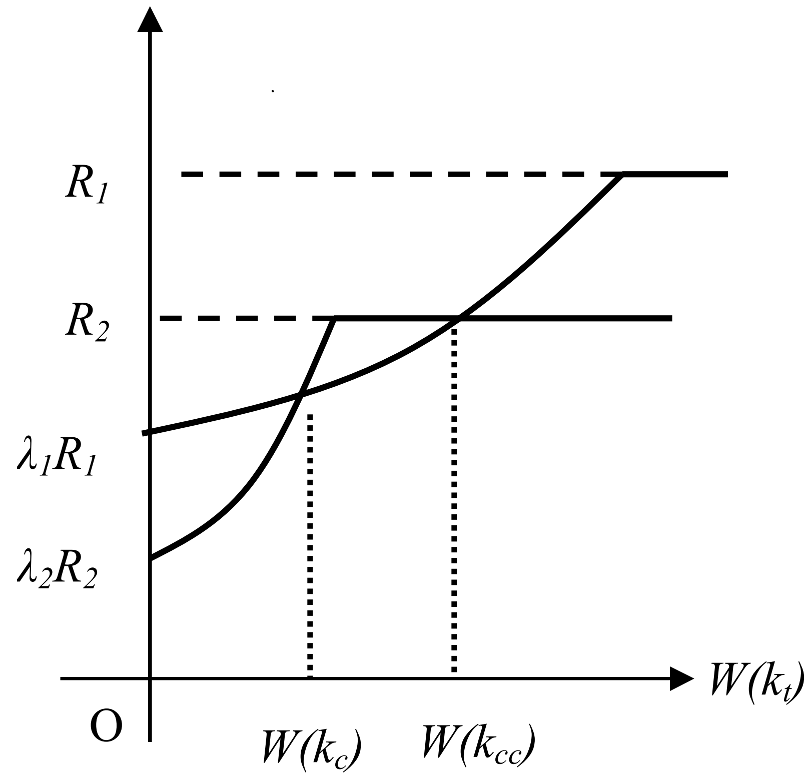
Figure 2b

$$k_{t+1} = \begin{cases} R_1W(k_t) & \text{if} \quad k_t < k_c, \\ R_2W(k_t) & \text{if} \quad k_t > k_c. \end{cases} \tag{7}$$

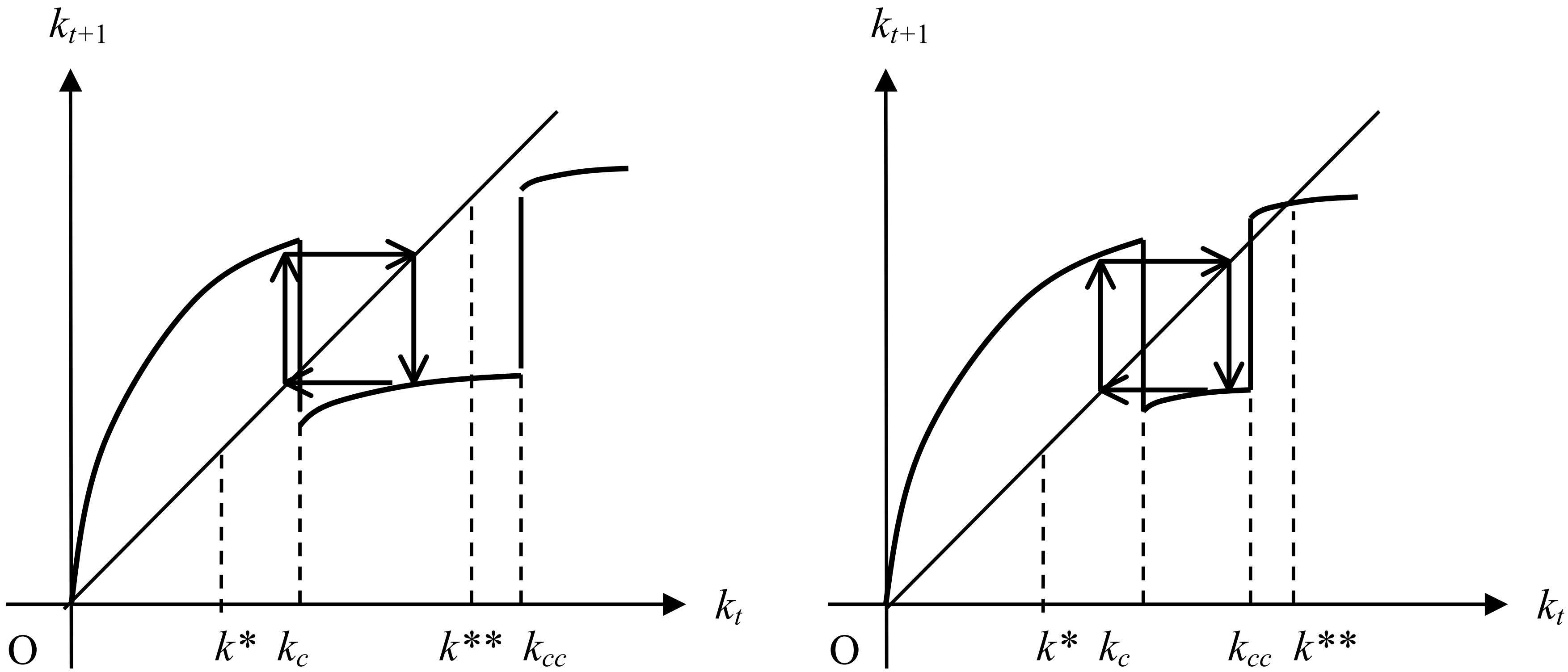


Example 3: Credit Cycles and Growth Miracles (J = 2 with $R_1 > R_2 > \lambda_1 R_1 > \lambda_2 R_2$, and $m_2/m_1 < (1-\lambda_1 R_1/R_2)/(1-\lambda_2)$.

Figure 4



$$k_{t+1} = \begin{cases} R_1 W(k_t) & \text{if} \quad k_t < k_c \text{ or } k_t > k_{cc} \\ R_2 W(k_t) & \text{if} \quad k_c < k_t < k_{cc}. \end{cases} \tag{8}$$



Several Extensions.

Heterogeneity of Agents,
Heterogeneity of Capital; e.g., Matsuyama (2004, Ecta).