A One-Sector Neoclassical Growth Model with Endogenous Retirement

By Kiminori Matsuyama

Final Manuscript

Abstract

This paper extends Diamond's OG model by allowing the agents to make the retirement decision. Earning a higher wage income when young not only enables the agents to save more. It also induces more agents to retire early and gives an additional incentive to save more for retirement. This leads to a higher capital-labor ratio in the following period, and hence the next generation of agents earns a higher wage income when young. Due to this positive feedback mechanism, endogenous retirement magnifies the persistence of growth dynamics and even generates multiple steady states for empirically plausible parameter values.

JEL Classification Numbers: D91, O11

Keywords: Diamond's Overlapping Generations Model, Labor Force Participation of the Elderly, Magnification Effects, Multiple Steady States, Persistence, Poverty Traps

Correspondences:
Kiminori Matsuyama
Department of Economics
Northwestern University
2001 Sheridan Road
Evanston, IL 60208
k-matsuyama@northwestern.edu

1. Introduction.

The labor force participation rate of the elderly declines with economic growth both in time series and cross sections of countries.[1] Although many factors including social security systems undoubtedly have major effects on the retirement behavior, the observed patterns are remarkably similar across countries, suggesting that the trend for early retirement may be a consequence of the rising income.[2] Early retirement in turn affects the growth process of the economy through its effects on the aggregate saving and labor supply.

To study such interactive processes between induced retirement and economic growth, this paper develops a variant of Diamond's (1965) one-sector neoclassical growth model of an overlapping generations (OG) economy, in which each generation lives for two periods. The original Diamond model assumes that the agents work in the first period (when they are young) and retire in the second (when they are old). The present paper endogenizes the labor force participation decision in the second period, while maintaining the original assumption that the agents always work in the first period. In order to generate the patterns consistent with those observed in both the time series and cross sections of countries, we impose restrictions on the parameters of the utility function so that the income effect of the higher wage when young, which encourages retirement, dominates the price effect of the higher wage when old, which discourages retirement.[3]

It turns out that endogenizing the labor force participation of the elderly has significant effects on the mechanics of economic growth. Earning a higher wage income in the first period not only enables the agents to save more, but it also induces more agents to retire in the second period, which provides an additional incentive to save more for retirement. This results in a higher capital-labor ratio in the following period, which implies that the next generation of agents earns a higher wage income in their first period. Due to this positive feedback mechanism,

[1]See, for example, Fuchs (1983) and Gruber & Wise (1999).
[2]See Fuchs (1983) for an earlier survey of the empirical evidence for the income effect on retirement. More recently, Costa (1998) found, using the historical U.S. data *prior to* the introduction of the Social Security system, that the rising income is a major factor responsible for the decline in the elderly's labor force participation rate.
[3]As Fuchs (1983) pointed out, this assumption is not inconsistent with the well-known correlation that, within each cohort, the individuals with lower income tend to retire early, which has more to do with the heterogeneity of the workers. The key mechanism here is that earning a higher wage income when young induces more workers to retire early, *not* that more productive workers retire early. In Section 6, we suggest a way of extending the present model to generate this correlation without changing the main results.

endogenizing the labor force participation of the elderly magnifies the persistence of growth dynamics, thereby slowing down a convergence to the steady state, and even leading to multiple steady states for empirically plausible parameter values.

The result that endogenous retirement magnifies the persistence of growth dynamics suggests that the mechanism discussed in this paper may offer an explanation for the slow convergence puzzle. The present mechanism, however, should not be viewed as an alternative to the other persistence mechanisms previously proposed in the literature.[4] It should be viewed instead as complementary, because endogenizing retirement also amplifies persistence caused by these mechanisms. The possibility of multiple steady states suggests that there are two-way causalities between early retirement and economic development.

Some readers may be surprised by the result that endogenous retirement can generate multiple steady states in a one-sector neoclassical growth model. After all, it is straightforward to show that introducing the consumption-leisure trade-off in the standard one-sector neoclassical growth model with the infinitely-lived representative agent (the RA model for short) does not affect the steady state wage rate.[5] The reason why multiple steady states are possible is that the persistence mechanism in this paper relies on the features of the OG model that are missing in the RA model. First, the labor supply is composed by the inelastic labor supply by the young and the elastic labor supply by the old. It is essential that a high wage reduces the labor participation rate among the old more than among the young. The intergenerational transmission mechanism would be absent if everyone's labor supply were equally elastic, as in the RA model. Second, in the Diamond OG model, no agent's asset holding is constant over time even in the steady state; each agent must first earn the wage income in order to save and own the capital stock, and the individual asset holding increases when young and declines when old. The steady state in the Diamond OG model means that the time profile of the individual asset holding does not change from one generation to another. The steady state in the RA model, on the other hand, means that the agent's asset holding is constant over time. To put it another way, the propensity to save out

[4] See Matsuyama (2005) and the work cited therein for other persistence mechanisms that generate multiple steady states.

[5] To see this, it suffices to note that the wage rate is a function of the capital-labor ratio in a one-sector neoclassical growth model, and that the steady state capital-labor ratio in the RA model is uniquely pinned down by one of the steady state conditions, $\beta f'(k) = 1$, where β is the discount rate. See Blanchard and Fischer (1989, Chapter 2) for a standard reference for the RA model.

of the wage income depends on the agent's age in the Diamond model. A higher wage income by the young increases the aggregate saving, while a higher wage income by the old reduces the aggregate saving. This is not a feature of the RA model. The above discussion should also explain why retirement, not delayed labor force participation, is important for the analysis. If the labor participation rate of the young, instead of the old, were endogenized, the capital stock would move in the opposite direction, causing reversal, rather than persistence.[6]

There are a few related studies. Feldstein (1974) discussed the possibility that social security may end up increasing the aggregate saving through induced retirement, instead of reducing it as the standard analysis might suggest. To explore Feldstein's insight formally, Hu (1979) endogenized the labor supply by the elderly in the Diamond OG model, and examined the general equilibrium effects of social security. However, he did so under the assumptions that ensure the uniqueness of the steady state.[7]

Some readers may think that growth implications of endogenous retirement cannot be important because the elderly accounts for a small share of the labor supply, even when their labor force participation rate is high. Feldstein (1974, p.924), for example, expressed such a view. To respond to such a skepticism within the two-period overlapping generations

[6] Indeed, some previous studies have endogenized the labor supply of the young in the Diamond OG model and demonstrated the possibility of endogenous cycles. See, for example, Reichlin (1986).

[7] There is an extensive literature in labor economics and public finance, which analyzes the elderly's labor force participation decision. This literature does not address general equilibrium implications of endogenous retirement. Their main objective is an empirical assessment of the impact of social security systems and pension plans on retirement behavior, instead of understanding growth implications of retirement behavior. In an attempt to explain how social security provisions affect the timing of retirement, this literature utilizes increasingly sophisticated models of retirement behavior, including those of stochastic dynamic programming. On the other hand, the present model keeps retirement behavior as simple as possible in order to maintain the tractability of general equilibrium analysis. For example, it is assumed that the agents live only for two periods, that the labor supply when young is exogenous, that there is no uncertainty, and that retirement is a zero-one decision, etc. One may hence be surprised to find that the model of retirement behavior developed below is *not* a special case of those developed in this literature. The reason is that the present model is designed to evaluate the effect of retirement on saving through a change in the time profile of labor income, not through a change in the intertemporal preferences over consumption. This requires that the preferences for retirement be weakly separable from the intertemporal preferences for consumption. (Interestingly, Feldstein (1974, Fig. 1) made this assumption implicitly in his graphic analysis.) It turns out that this condition, when the retirement decision is endogenized, implies that the preferences cannot be intertemporally separable. The recent literature on retirement behavior, on the other hand, imposes the intertemporal separability of preferences so as to make the standard tool of dynamic optimization readily applicable to the problem. It should also be noted that the existing empirical studies impose functional forms that rule out the possibility of nonhomothetic preferences, which are too restrictive for the present analysis. This, too, is a reflection of the difference in objective. To evaluate the incentive effects of social security provisions on the timing of retirement, the income effect may not be important. For the present study, which is concerned with growth implications, it is essential.

framework, the model assumes that the old agent's effective labor supply be a fraction of the young agent's, given by a parameter, θ. It turns out that the magnification effect of endogenous retirement is independent of θ. Of course, this independence is due to the particular functional forms assumed in the paper. However, it suggests that the effect of an increase in the elderly's share in labor supply on the magnification effect is ambiguous in general, which should be sufficient to refute the argument that the elder's share in labor supply must be large enough to have significant effects. The reason why the main result does not have to depend on the elderly's share in the labor supply is that we are dealing with the effect of *endogenous* retirement on growth dynamics. The relevant question is not only how much early retirement causes the wage rate to increase, but also how much increases in the wage rate induce early retirement.

Before proceeding, it is worth clarifying the measure of development adopted in this paper. The standard measure of development, per capita income, is highly misleading when comparing countries that differ significantly in their labor force participation rates. For example, per capita income in Japan may be higher than those in some European countries, in part because the labor force participation rate in Japan is much higher. It is possible that, because of higher output per worker, many people in these European countries can afford to retire early, which make their per capita incomes lower than Japan's. In this case, the output per worker is a better measure of development. The ultimate measure of development, of course, should be the standard of living. In the model developed below, the lifetime utility of the agent is higher if and only if the wage rate, which moves together with the capital/labor ratio and the output per worker, is higher. On the other hand, higher per capita income does not necessarily imply the higher lifetime utility. It is for this reason that the wage rate (or the capital/labor ratio or the output per worker), instead of per capita income, is used as the measure of development.

The rest of the paper is organized as follows. Section 2 sets up the framework. Section 3 considers the special case, in which the agents do not care about consumption when young, so that the only trade-off is between consumption and leisure when old. Under this assumption, the young save all the wage income, independent of the retirement decision. This helps us to explain the key mechanism without worrying about the complications that arise from the joint saving/retirement decision. In this case, however, endogenous retirement does not create large enough persistence to generate multiple steady states, unless the share of capital is implausibly large or unless it interacts with other persistence-enhancing mechanisms. Section 4 allows the agents to also care about consumption when young. This makes the saving and retirement joint decisions and introduces a retirement motive for saving. In this case, endogenous retirement creates persistence large enough to generate multiple steady states for empirically plausible parameter values. Section 5 introduces a form of heterogeneity among agents. Section 6 suggests some directions for future research.

2. The Framework

Time is discrete and extends to infinity. There is a single final good, the numeraire, which can either be consumed or invested. It is produced competitively by a standard constant-return-to-scale technology, $Y_t = F(K_t, L_t)$. Let $k_t \equiv K_t/L_t$ denote the capital-labor ratio, and $f(k_t) \equiv F(k_t, 1)$ denote the production function in its intensive form, which is increasing and concave in k_t. The factor markets are competitive, and both capital and labor earn their marginal values, as follows.

(1) $$r_t = R(k_t) \equiv f'(k_t).$$

(2) $$w_t = W(k_t) \equiv f(k_t) - k_t f'(k_t).$$

The economy is populated by overlapping generations of the equal size, normalized to be one. Each generation lives for two periods. Aside from the fact that they may live in different periods, the agents are homogenous (until section 5). The young in period t−1 supplies one unit of labor inelastically, and earns wage income, w_{t-1}. The agent may consume some of the wage income, c^y_{t-1}, and save the rest, $s_t = w_{t-1} - c^y_{t-1}$, in capital. When the agent becomes old in period t, s/he earns capital income, $r_t s_t$. In addition, the agent may supplement the capital income by continuing to work and earning wage income, equal to $w_t\theta$, where θ is the effective unit of labor supply by the old. Alternatively, the agent may retire. The old agent's labor force participation is a zero-one decision; $e_t = 0$ if s/he retires and $e_t = 1$ if s/he works.[8] The agent's old

[8] The assumption that the retirement is a zero-one decision is made not only because it is realistic for many individuals, but also because it allows for an analytical solution. Gong and Liu (2006) extended the present model to allow for e_t to be a continuous variable, and showed numerically that the results in this paper essentially carry over.

consumption is equal to $c^o{}_t = r_t s_t + w_t\theta e_t$. The agent's choice can thus be described as the solution to the following maximization problem:

(M) Given w_{t-1}, choose $c^y{}_{t-1} \geq 0$, $c^o{}_t \geq 0$, and $e_t \in \{0,1\}$ to maximize $U(c^y{}_{t-1}, c^o{}_t, e_t)$ subject to $c^o{}_t = r_t s_t + w_t\theta e_t = r_t(w_{t-1} - c^y{}_{t-1}) + w_t\theta e_t$.

The parameter, θ, is introduced to allow for the possibility that the elderly accounts for only a small share of the labor supply, even if their labor force participation rate is high. It turns out that the main results obtained below are independent of θ.

In equilibrium, the agent may be indifferent between working and retiring when old, so that, in spite of the homogeneity, the old generation's labor force participation rate, x_t , may take a value between zero and one, and the labor supply in period t is $L_t = (1+\theta x_t)$. The supply of capital in period t is equal to the total saving made by the young in period t−1. An agent's saving in period t−1 is generally determined jointly with whether s/he retires in period t. If we indicate this dependence by $s_t(e_t)$, then the gross saving by the young generation in period t−1, and hence the capital stock available in period t is given by $K_t = s_t(0)(1-x_t)+s_t(1)x_t$. The capital-labor ratio is therefore $k_t = K_t/L_t = \{s_t(0)(1-x_t)+s_t(1)x_t\}/(1+\theta x_t)$.

Note that this model would be identical to Diamond's original model, if the agent is forced to retire, $e_t = 0$, and hence the labor force participation rate by the old is exogenously equal to zero ($x_t = 0$).

3. Consumption/Retirement Trade-off when Old

Let us consider a special case, where the utility function does not depend on $c^y{}_{t-1}$. That is, the agent cares only consumption and leisure when old. Then the agent saves all the wage income, $s_t = w_{t-1}$, independent of the retirement decision, and hence the total supply of capital in period t is simply $K_t = w_{t-1}$. This simplification has two advantages. It allows us to focus on the retirement/consumption trade-off faced by the old. It also helps us to see how the retirement decision by the current old generation will affect all the future generations without the complication that arises from the joint saving-retirement decision.

The preferences of the old agent in period t are now described by $U(c^o_t, e_t)$, which is strictly increasing in c^o_t, unbounded from above, i.e., $U(c^o_t, e_t) \rightarrow +\infty$, as $c^o_t \rightarrow +\infty$; and satisfies $U(c^o_t, 1) < U(c^o_t, 0)$ for all $c^o_t > 0$. These assumptions imply that one can uniquely define a positive-valued function, ϕ, by

(3) $\quad U(c^o_t + \phi(c^o_t), 1) \equiv U(c^o_t, 0)$.

Conversely, for any positive-valued function, ϕ, $U(c^o_t, e_t) = c^o_t + \phi(c^o_t)(1-e_t)$ satisfies all the above conditions. Therefore, *without further loss of generality*, we can set $U(c^o_t, e_t) = c^o_t + \phi(c^o_t)(1-e_t)$. Or equivalently, we can choose any positive-value function, $\phi(c^o_t)$, as a primitive of the model, instead of the utility function, $U(c^o_t, e_t)$. One can interpret ϕ as the value of leisure (or the compensating differential for working) in the second period.

An agent who has earned w_{t-1} and saved $K_t = w_{t-1}$ when young receives $r_tK_t = r_tw_{t-1}$ when old. If s/he retires, the utility level is $U(r_tK_t, 0) = U(r_tK_t + \phi(r_tK_t), 1)$. On the other hand, if s/he continues to work, it is $U(r_tK_t + w_t\theta, 1)$. Therefore, an old agent in period t chooses to work if $\phi(r_tw_{t-1}) < w_t\theta$; s/he chooses to retire if $\phi(r_tw_{t-1}) > w_t\theta$; and s/he is indifferent $\phi(r_tw_{t-1}) = w_t\theta$.

Given $w_{t-1} = K_t$, the equilibrium in period t is given by eqs. (1) and (2), and

(4) $\quad k_t = K_t/(1+\theta x_t) = w_{t-1}/(1+\theta x_t)$,

(5) $$x_t \begin{cases} = 1 & \text{if} \quad \phi(r_t w_{t-1}) < w_t\theta, \\ \in [0,1] & \text{if} \quad \phi(r_t w_{t-1}) = w_t\theta, \\ = 0 & \text{if} \quad \phi(r_t w_{t-1}) > w_t\theta. \end{cases}$$

These conditions jointly determine the mapping from w_{t-1} to w_t, $w_t = \Psi(w_{t-1})$, which can be applied iteratively to solve for the equilibrium trajectory of the economy, for any initial condition, $w_0 = K_1$.

3.A. Exogenous Retirement

Before proceeding, let us first consider the case where the old generation's labor force participation rate is given exogenously $x_t \in [0,1]$, for all t. That is, a fraction $1-x_t$ of the old generation in period t retires, and the rest stays in the labor force. The Diamond overlapping generations model is a special case where $x_t = 0$ for all t. It is well known that the dynamics in the Diamond model may be complicated unless additional restrictions are imposed on the production function. Since the goal here is to provide a benchmark for the case of endogenous retirement, we impose such restrictions so that, without the endogeneity of retirement, the dynamics is "well-behaved."

More specifically, it is assumed that the production function is a Cobb-Douglas, $f(k) = Ak^{\alpha}$, where $\alpha \in (0,1)$ is the capital share. Then, (2) becomes $w_t = (1-\alpha)A(k_t)^{\alpha}$. From (4) and $w_{t-1} = K_t$, the dynamics is described as

(6) $$w_t = (1-\alpha)A(k_t)^{\alpha} = \frac{(1-\alpha)}{(1+\theta x_t)^{\alpha}}A(w_{t-1})^{\alpha}$$

Figure 1 illustrates the dynamical system, given by (6), under the assumption that the labor force participation rate is constant over time, $x_t = x$. For any x, the mapping is globally concave and the dynamics has a unique steady state. A higher x shifts the mapping down, reducing the wage rate and capital stock in the steady state. The parameter that governs the persistence of the dynamics is equal to the capital share, α, independent of x.

3.B. Endogenous Retirement

We are now ready to examine the effect of endogenous retirement. The dynamics are now described by (5) as well as (6), with $w_t = (1-\alpha)A(k_t)^{\alpha}$ and $r_t = \alpha A(k_t)^{\alpha-1}$. In order to obtain a closed-form solution for the mapping, let us consider $\phi(c^o) = \Lambda(c^o)^{\lambda}$, or equivalently, $U(c^o{}_t, e_t) = c^o{}_t + \Lambda(c^o{}_t)^{\lambda}(1-e_t)$, where $\Lambda > 0$ and $\lambda \in (1, \infty)$. The assumption, $\lambda > 1$, ensures that, as the economy develops, the income effect of a higher wage when young, which encourages retirement, dominates its price effect of a higher wage when old, which discourages retirement. Some algebra yields

(7) $$w_t = \Psi(w_{t-1}) \equiv \begin{cases} \dfrac{(1-\alpha)}{(1+\theta)^{\alpha}}A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in (0, w^{-}], \\ (1-\alpha)\Omega^{(\mu-1)}\left[A(w_{t-1})^{\alpha}\right]^{\mu} & \text{if} \quad w_{t-1} \in (w^{-}, w^{+}), \\ (1-\alpha)A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in [w^{+}, \infty), \end{cases}$$

where

(8) $$\mu \equiv \frac{\lambda}{\alpha + \lambda(1-\alpha)} > 1,$$

(9) $$\Omega \equiv \left(\frac{\Lambda \alpha^{\lambda}}{\theta(1-\alpha)}\right)^{1/(\lambda-1)}$$

and

(10) $$w^{-} \equiv \frac{(1+\theta)^{1/(1-\mu)}}{(A\Omega)^{1/\alpha}} < w^{+} \equiv \frac{1}{(A\Omega)^{1/\alpha}}.$$

The choice of parameterization in (7)-(10) was made so that the reader can see how the map, given by (7), depends on A and Λ, by inspection. The map is also illustrated in Figures 2a and 2b, which assume that the map intersects with the 45° line in the interval, (w^{-}, w^{+}). (It is easy to find a set of parameter values that ensures the existence of such an intersection by adjusting, say, Λ.)

If $w_{t-1} \in (0, w^{-}]$, the old generation, having earned and saved little when young, does not retire: the labor force participation rate is $x_t = 1$. If $w_{t-1} \in [w^{+}, \infty)$, the old generation, having earned and saved enough when young, chooses to retire: $x_t = 0$. Thus, the dynamics of the economy in both the lower and higher ranges is similar to the case of an exogenous labor force participation rate. In particular, the persistence parameter is equal to α.

In the middle range, (w^{-}, w^{+}), the labor force participation rate changes with w_{t-1}. Some algebra shows that the capital-labor ratio and the labor force participation rate in this range change with $K_t = w_{t-1}$, as follows.

(11) $$k_t = (A\Omega)^{\mu - 1/\alpha}(w_{t-1})^{\mu} = (w^{+})^{1-\mu}(w_{t-1})^{\mu},$$

(12) $$x_t = \frac{1}{\theta}\left[\left((A\Omega)^{1/\alpha} w_{t-1}\right)^{1-\mu} - 1\right] = \frac{(w_{t-1})^{1-\mu} - (w^{+})^{1-\mu}}{(w^{-})^{1-\mu} - (w^{+})^{1-\mu}}.$$

An increase in the wage rate leads to a decline in the participation rate, and to a more-than-proportionate increase in the capital-labor ratio.

The homogeneity of the agents makes it simple to derive the map in the middle range, where $x_t \in (0,1)$. It is given by the condition that the agents are indifferent between working and retiring when old. A large Λ (a higher value of retirement), by increasing Ω, reduces the equilibrium labor force participation rate and increases the capital-labor ratio, as seen in (11) and (12). Therefore, it shifts the map, Ψ, upwards in the middle range, while it has no effect on the map in both the low and high ranges. The middle range itself moves to the left in response.

An exogenous increase in A, the total factor productivity, shifts the map upward everywhere. However, the shift is bigger in the middle range, and the range itself moves to the left, because the old generation earns more capital income out of saving, which reduces the labor force participation rate and the capital-labor ratio, as seen in (11) and (12).

3.C. Persistence

Note that, in the middle range, the elasticity of the map is equal to $\alpha\mu > \alpha$, i.e., higher than the case with a constant x. In other words, the persistence of the dynamics is magnified by the factor to $\mu > 1$. Eq. (8) shows that the magnification factor μ is increasing in λ. When the value of retirement rises sharply with economic growth, a higher wage rate would be needed to keep the old generation in the labor force. Note also that μ is independent of θ.

This feature of endogenous retirement, the magnification of persistence, has important implications for the growth process. If $\alpha\mu < 1$, the case depicted in Figure 2a, the economy still converges to the unique steady state. However, the speed of convergence is slower, as the economy traverses through the middle range. If $\alpha\mu = 1$, the map generates a unit-root dynamics in the middle range. If $\alpha\mu > 1$, the map becomes convex in the middle, so that growth accelerates. Furthermore, there may exist multiple steady states, two stable and one unstable, as depicted in Figure 2b. In the lower stable steady state, the wage is low and people do not retire. In the higher stable steady state, the wage is high and people retire. Two otherwise identical economies, if their initial positions are separated by the unstable steady state, converge to different steady states.

When an economy is trapped in the lower of the two stable steady states, a variety of the government policies can be used to lift the economy out of the trap and to move toward the higher stable steady state. This can be done by making the elderly retire either by force or by subsidy. (Indeed, the subsidy does not need to be conditioned on retirement; even a simple, unconditional transfer to the elderly can induce the elderly to retire because retirement is a normal good.) It should be pointed out, however, that the above mentioned policies are not Pareto-improving. These policies must reduce the welfare of either the current old generation (if they are forced to retire) or the future generations (if the taxes are imposed on them to finance the subsidies). The reason is simple. The model has neither externality (because all the intertemporal linkages operate through markets) nor dynamic inefficiencies (because the steady state interest rate is positive, and hence is higher than the steady state growth rate of the economy, which is equal to zero). Therefore, the equilibrium allocation of the economy described above, even in the case where the economy is trapped in the lower stable steady state, is Pareto-efficient and thus cannot be Pareto improved by means of taxes, subsidies, transfers, or other standard corrective policy measures.

How big is the magnification effect of endogenous retirement? As it stands, the model does not generate quantitatively significant effects. Even when λ is taken arbitrarily large, $\alpha\mu$ cannot be greater than $\alpha/(1-\alpha)$, which is equal to 0.5 for $\alpha = 1/3$ and to 2/3 for $\alpha = 0.4$. In order for the map to become convex in the middle to generate multiple steady states, it is necessary to have $\alpha > 1/2$.

This merely suggests that this simple model cannot explain a quantitatively large persistence. The next section will consider the case where the agents also care about consumption when young. This makes saving and retirement joint decisions, and introduces a retirement motive for saving. It will be shown that, by affecting the saving decision, endogenous retirement can increase persistence enough to generate multiple steady states for an empirically plausible value of α.

Alternatively, endogenous retirement can be combined with other mechanisms for persistence. It is worth noting that endogenous retirement not only supplements other mechanisms for persistence, but also enhances their power of generating large persistence. As an illustration, let us suppose that the total factor productivity, A, is now endogenous, and evolves according to

(13) $A_t = A_0(K_t)^\gamma$.

The idea is that the level of aggregate capital stock can also be viewed as a proxy for knowledge capital. Through knowledge spillovers, the capital stock affects the total factor productivity of the economy, and γ measures the externality effect of knowledge spillovers. Since this effect is purely external, the agent does not take it into account when making decisions. Aggregate externalities of this kind have been suggested by Romer (1986) and others in the endogenous growth literature as a way of generating persistence in dynamics.

By inserting (13) into (6) and by recalling $K_t = w_{t-1}$, it can be shown that the dynamics would follow

(14) $$\ln(w_t) = \text{const.} + (\alpha + \gamma)\ln(w_{t-1}),$$

when the old generation's labor force participation rate is exogenous and constant. To generate accelerating growth and the possibility of multiple steady states, γ has to be much larger than α for any plausible value of α. For example, $\alpha = 1/3$ implies that γ must be more than twice as large as α. Even with $\alpha = 0.4$, γ must be more than 50% larger than α.

With endogenous retirement, on the other hand, the dynamics follow, from (7) and (13),

(15) $$\ln(w_t) = \text{const.} + \mu(\alpha + \gamma)\ln(w_{t-1}) = \text{const.} + \frac{\lambda(\alpha + \gamma)}{\alpha + \lambda(1-\alpha)}\ln(w_{t-1}),$$

in the middle range. Note that endogenous retirement not only generates persistence in addition to the externality effect, but also enhances the externality effect. Hence, multiple steady states are possible for a plausible value of α, and a much smaller γ. (For example, for $\alpha = 1/3$, $\gamma > 1/3$ would suffice for a sufficiently large λ. For $\alpha = 0.4$, $\gamma > 0.2$ would suffice.)

4. Introducing a Joint Saving-Retirement Decision

Let us now introduce the retirement motive for saving. As stated before, the problem of the agent who becomes old in period t can be described as:

(M) Given w_{t-1}, choose $c^y_{t-1} \geq 0$, $c^o_t \geq 0$, and $e_t \in \{0,1\}$ to maximize $U(c^y_{t-1}, c^o_t, e_t)$ subject to $c^o_t = r_t s_t + w_t \theta e_t = r_t(w_{t-1} - c^y_{t-1}) + w_t \theta e_t$.

For the rest of the analysis, two additional restrictions will be imposed on the utility function.

First, c^y_{t-1} and c^o_t are assumed to be weakly separable from e_t, so that the intertemporal preferences for consumption are independent of the retirement decision. Without such a restriction, the effect of the retirement decision on saving can be arbitrary. The result can change according to how the marginal rate of substitution between consumption in two periods depends on e_t. Retirement may reduce marginal utility of some consumption items, such as business suits, while raising that of other items, such as books. Such introspection, however, may not be useful for the level of aggregation that we are dealing with. Although retirement may increase marginal utility of books, reading books may reduce the retired person's need for other items, thereby reducing marginal utility of consumption in general. The assumption of weak separability, while restrictive, seems to offer the most natural benchmark.[9] Note that the weak separability assumption does not eliminate the retirement motive for saving, because the retirement decision affects the time-profile of labor income. Rather, it means that retirement cannot affect saving by changing the intertemporal preferences over consumption.

Second, in order to focus on the magnification effect of endogenous retirement, it is useful to impose the restriction on the preferences in such a way that when the labor force participation rate is constant, $x_t = x$, the persistence parameter is equal to α, independent of x. This is satisfied if and only if intertemporal preferences are Cobb-Douglas.

These two restrictions jointly imply that the utility function can be written in the form,

$$U(c^y_{t-1}, c^o_t, e_t) \equiv U(z_t, e_t), \quad \text{with } z_t \equiv Z(c^y_{t-1}, c^o_t) = \left(\frac{c^y_{t-1}}{1-\beta}\right)^{1-\beta}\left(\frac{c^o_t}{\beta}\right)^{\beta},$$

where $\beta \in (0,1]$ is a constant, independent of e_t. Note that the preferences here are not intertemporally separable.

Given the preferences assumed, the utility maximization yields,

$$(16) \qquad s_t = w_{t-1} - c^y_{t-1} = \beta w_{t-1} - (1-\beta)\left(\frac{w_t\theta}{r_t}\right)e_t \ .$$

Eq. (16) shows that the agent's saving is contingent on the retirement decision. Those who decide to retire save more than those who decide not to. The difference, "the retirement motive

[9] At least, the results under this assumption should be considered "neutral." Interestingly enough, the assumption of weak separability between the retirement decision and two period consumptions was made implicitly in Feldstein's

for saving," would be larger if β is smaller. Eq. (16) also shows the positive correlation between the asset holding of the agents at the beginning of their second period and their retirement decision. This should not be interpreted as saying that the wealthy retires early, because the wealth and the retirement are jointly determined in this model.

The value of z_t also depends on the retirement decision, as follows.

(17) $$z_t = \left[w_{t-1} + \left(\frac{w_t \theta}{r_t} \right) e_t \right] (r_t)^{\beta} .$$

Hence, the opportunity cost of retirement is equal to $w_t \theta (r_t)^{1-\beta}$. Therefore, the retirement decision is given by

(18) $$e_t \begin{cases} = 1 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1} \right) < w_t \theta (r_t)^{1-\beta} , \\ \in \{0,1\} & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1} \right) = w_t \theta (r_t)^{1-\beta} , \\ = 0 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1} \right) > w_t \theta (r_t)^{1-\beta} , \end{cases}$$

where ϕ, the value-of-retirement function, is defined in a manner similar to (3). That is, it is a positive-valued function, uniquely defined by $U(z+\phi(z), 1) \equiv U(z, 0)$, for any $U(z, e)$, which is strictly increasing and unbounded from above in z, and $U(z, 1) < U(z, 0)$ for all $z > 0$. Or equivalently, we can set $U(z, e) = z + \phi(z)(1-e)$, without further loss of generality.

The aggregate saving by the young in period t−1, which is equal to K_t, can be obtained from (16) by aggregating across all the agents, as follows.

(19) $$K_t = \beta w_{t-1} - (1-\beta) \left(\frac{w_t \theta}{r_t} \right) x_t .$$

Since the labor supply is $L_t = (1+\theta x_t)$, this implies that

(20) $$k_t = \frac{\beta w_{t-1} - (1-\beta) \theta (w_t / r_t) x_t}{1 + \theta x_t} .$$

Note that the retirement decision by the old affects the capital-labor ratio through two different channels, even after controlling for its effects on the factor prices. One channel is the labor supply effect, which appears in the denominator of (20). The other is the saving for retirement

(1974) graphic analysis, when he drew the indifference curves defined over the space, (C_1, C_2) in his notation, independent of the agent's retirement behavior.

effect, captured in the second term of the numerator. Both effects work in the same direction. A higher labor force participation rate by the elderly thus implies a lower capital-labor ratio, and hence a lower wage for the next generation.

The labor force participation rate satisfies in equilibrium,

$$
(21)\qquad x_t \begin{cases} = 1 & \text{if} \quad \phi\big((r_t)^{\beta} w_{t-1}\big) < w_t\theta(r_t)^{1-\beta}, \\ \in [0,1] & \text{if} \quad \phi\big((r_t)^{\beta} w_{t-1}\big) = w_t\theta(r_t)^{1-\beta}, \\ = 0 & \text{if} \quad \phi\big((r_t)^{\beta} w_{t-1}\big) > w_t\theta(r_t)^{1-\beta}. \end{cases}
$$

Given w_{t-1}, the equilibrium in period t is given by eqs. (1), (2), (20) and (21).

4.A. Exogenous Retirement

Before proceeding, let us take a brief look at the case of exogenous retirement. Under the assumption of the Cobb-Douglas production function, (20) implies that the wage rate in period t is given by,

$$
(22)\qquad w_t = (1-\alpha)A(k_t)^{\alpha} = (1-\alpha)\left[\frac{\beta}{1+\{\theta+(1-\beta)\theta(1-\alpha)/\alpha\}x_t}\right]^{\alpha} A\big(w_{t-1}\big)^{\alpha}.
$$

When $x_t = x$, the map is globally concave, and the economy converges to the unique steady state, as depicted in Figure 1. Note also that the persistence of the dynamics is equal to α, independent of x.

4.B Endogenous Retirement

Let us now look at the case where the labor force participation rate is endogenous. Again, let us assume that $f(k) = A(k)^{\alpha}$, $\alpha \in (0,1)$, and $\phi(z) = \Lambda(z)^{\lambda}$, or equivalently, $U(z, e) = z + \Lambda(z)^{\lambda}(1-e)$, where $\Lambda > 0$, $\lambda \in (1, \infty)$. Then, eqs. (21) and (22) imply

$$
(23)\qquad w_t = \Psi(w_{t-1}) \equiv \begin{cases} (1-\alpha)(\beta^*)^{\alpha} A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in (0,w^-], \\ (1-\alpha)\left[\Omega^{\alpha} A^{\beta-1}\right]^{\frac{\mu-1}{1-\beta(1-\alpha)}}\left[A(w_{t-1})^{\alpha}\right]^{\mu} & \text{if} \quad w_{t-1} \in (w^-,w^+), \\ (1-\alpha)\beta^{\alpha} A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in [w^+,\infty), \end{cases}
$$

where

(24) $$\mu \equiv \frac{\lambda}{\alpha + (\lambda\beta + 1 - \beta)(1-\alpha)} > 1,$$

(25) $$\beta^* \equiv \frac{\beta}{1 + \theta + (1-\beta)\theta(1-\alpha)/\alpha} < \beta,$$

(26) $$\Omega \equiv \left(\frac{\Lambda \alpha^{\lambda\beta + (1-\beta)}}{\theta(1-\alpha)} \right)^{1/(\lambda-1)},$$

(27) $$w^- \equiv \frac{(\beta^*)^{1/\mu-1}}{\left(A^\beta \Omega\right)^{1/1-\beta(1-\alpha)}} < w^+ \equiv \frac{\beta^{1/\mu-1}}{\left(A^\beta \Omega\right)^{1/1-\beta(1-\alpha)}} .$$

It is easy to verify that (23) is an extension of (7), by letting $\beta = 1$. This extension keeps the qualitative features of (7). In particular, there is a middle range in which the labor force participation rate changes and the persistence parameter is greater than α. Here the magnification factor is given by (24). In this range, the capital-labor ratio and the labor force participation rate change as follows.

(28) $$k_t = \left(A^\beta \Omega\right)^{\mu-1/1-\beta(1-\alpha)} \left(w_{t-1}\right)^\mu = \beta\left(w^+\right)^{1-\mu} \left(w_{t-1}\right)^\mu,$$

(29) $$x_t = \frac{\alpha}{\alpha\theta + (1-\alpha)(1-\beta)\theta} \left[\beta\left(\left(A^\beta \Omega\right)^{1/1-\beta(1-\alpha)} w_{t-1} \right)^{1-\mu} - 1 \right] = \frac{\left(w_{t-1}\right)^{1-\mu} - \left(w^+\right)^{1-\mu}}{\left(w^-\right)^{1-\mu} - \left(w^+\right)^{1-\mu}} .$$

It can also easily be seen that the effects of A and Λ are similar to before.

The effect of a change in β, on the other hand, is difficult to evaluate; as seen in (23)-(27), β appears everywhere. Nevertheless, what interests us most is how the magnification effect of endogenous retirement changes with the introduction of the retirement motive for saving. Eq. (24) shows that a smaller β makes μ larger. For example, let $\beta = 1/2$, so that the agent puts equal weight on each period. Then, for a sufficiently large λ, the map becomes convex in the middle, for any $\alpha > 1/3$. If $\beta = 1/3$, which implies the agent's discount rate is about 2.2% per year, if the period length is 30 years, then for $\alpha = 1/3$, $\lambda > 1.4$ is enough for the convexity. Note, again, that μ is independent of θ.

5. Heterogeneity of Agents

In the model presented above, the homogeneity of the agents and the zero-one nature of the retirement decision ensured that the price elasticity of the labor supply by the old generation (as a group) is infinite. This implies that, if the old generation's labor force participation rate is between zero and one, the agents must be indifferent regarding retirement. This greatly simplifies the analysis. This supply-side condition alone determines the map, $w_t = \Psi(w_{t-1})$, in the middle range. The demand factors for labor need to be evoked only to pin down the value of x. Generally speaking, the presence of heterogeneity or allowing for a partial retirement would make the analysis more difficult because both the supply and demand sides would then need to be taken into account to derive the map. This section discusses one relatively simple way of making the price elasticitity of the elderly's labor supply finite without losing the tractability of the model, by introducing a form of heterogeneity into the analysis.

Suppose now that the agents differ in their preferences. More specifically, $\phi(z)$ has the following simple form: $\phi(z) = m$ if $z < \eta$, and $\phi(z) = M$ if $z \geq \eta$, where m is a sufficiently small positive number and M is a sufficiently large, but finite number. The agents differ in their values of η, and let $G(\eta)$ be the distribution function. Then, from (18), $e_t = 1$ if and only if $(r_t)^\beta w_{t-1} <$ η, and hence the labor force participation rate is

$$(30) \qquad x_t = 1 - G\left((r_t)^\beta w_{t-1}\right) = 1 - G\left(\left(\alpha^\alpha (1-\alpha)^{1-\alpha} A\right)^{\frac{\beta}{\alpha}} (w_t)^{\frac{\beta(\alpha-1)}{\alpha}} w_{t-1}\right).$$

The equilibrium dynamics are now determined jointly by (22) and (30). Clearly, the case of an exogenous retirement is a special case, where G is constant. In this case, the persistence parameter is α. When G is degenerate and has a mass on a single point, eq. (30) shows that $x_t \in$ (0,1) requires that $(w_t)^{\beta(\alpha-1)}(w_{t-1})^\alpha$ must be constant. This implies that the persistence parameter is equal to $\alpha/\{\beta(1-\alpha)\}$. (Note that the magnification factor in this case is $1/\{\beta(1-\alpha)\}$, which coincides with (24) with $\lambda = +\infty$.)

Except these two extreme cases, one cannot solve for the map in a closed form (unless G is allowed to depend on parameters, such as α, β, and θ). Nevertheless, some qualitative effects of endogenous retirement can be seen from (22) and (30), which jointly imply the dynamics in the following form:

(31) $w_t = H\left((w_t)^{\beta(\alpha-1)}(w_{t-1})^{\alpha}\right)(w_{t-1})^{\alpha}$,

where H is a function, that is increasing if and only if G is increasing. A total differentiation of (31) yields

(32) $$\frac{d\log(w_t)}{d\log(w_{t-1})} = \alpha\left[\frac{\alpha+\xi}{\alpha+\beta(1-\alpha)\xi}\right],$$

where ξ is the elasticity of H. Since $\beta(1-\alpha) < 1$, the magnification factor, given in the bracket, is greater than one whenever $\xi > 0$. Hence, in any range where G is increasing, the map is steeper and the persistence parameter is bigger than the case of an exogenous retirement, as illustrated in Figure 3. A large amount of heterogeneity (i.e., a smaller ξ) makes the magnification effect smaller, but expands the range in which the magnification effect operates. It should also obvious that, depending on the shape of G, there can be any number of steady states.

6. Concluding Remarks

This paper endogenized the retirement decision in Diamond's overlapping generations model and studied the interdependence between the labor force participation by the elderly and economic growth. Earning a higher wage income in the first period not only enables the agents to save more, but it also induces more agents to retire in the second period, which provides an additional incentive to save more for retirement. This results in a higher capital-labor ratio in the following period, which implies that the next generation of the agents earns a higher wage income in their first period. Due to such positive feedback mechanism, the endogeneity of retirement magnifies the persistence of growth dynamics, thereby slowing down a convergence to the steady state, and even generating multiple steady states for empirically plausible parameter values.

Obviously, there are many ways in which the model can be extended. Only a few will be suggested below. First, while section 5 discusses the case where the agents differ in their reservation wages, differences in earning capacity might also be important as a source of heterogeneity that affects the retirement decision. Earning capacity differences can be modeled by endowing agents with different effective units of labor. Such an extension would generate some interesting predictions. For example, if the ratio of the effective units endowed when young and when old is the same across agents, those with higher ability would are more likely to retire than those with less ability. On the other hand, if those with lower earnings when young happen to be those whose earning capacity depreciates faster as they age, (such an assumption may be a reasonable way of capturing the situation that the job held by unskilled may be more physically demanding), then this result can be reversed, and the poor may retire early, a prediction consistent with the evidence that the better educated tends to retire later (see, for example, Fuchs 1983). Yet these changes will not alter the basic feature of the model, i.e., earning a higher wage income when young induces more workers to retire early.

Second, the model can be extended to allow stochastic shocks. For example, suppose that the total factor productivity, A, is subject to i.i.d. shocks. If there is a negative shock in period t, the old generation may find themselves short of the retirement income, and be forced to work. The negative shock would thus reduce the young generation's wage income in period t not only through the direct effect of lower productivity but also through the indirect effect of higher labor force participation by the old generation. The lower wage income when young in turn force this generation to work in period t+1, which depresses the next generation's wage income. Thus, a temporary technology shock propagates across periods. Another possibility is that population growth may be subject to shocks. A baby boom generation might have to work longer, which affects the next generation. Similarly, the impacts of a large inflow of migration in one period may affect not only the current generation, but also the future generations. Likewise, wars, plagues and other shocks that decimate one generation could boost the wage income for the next generation so much that the economy may escape from the lower steady state, and all the future generations may enjoy higher standard of living. (For example, some historians suggest that Black Death set up the stage for the future European Miracle, by boosting the wage income.) It should be pointed out that the present model has neither externalities nor dynamic inefficiencies. The above argument merely suggests that the loss in one generation is accompanied by the gains in the future generations, and does not imply any Pareto improvement.

Third, the model can be applied to the analysis of social security. The previous contributions on this subject, such as Hu (1979), made assumptions that ensure the uniqueness of the steady state, which would imply that policy could only change the steady state locally, thereby imposing substantial restrictions on potential impacts of social security.

Fourth, the preferences used in the above analysis assume that the income effect of a higher wage, which encourages early retirement, dominates the price effect of higher wage, which discourages early retirement. This feature may be useful for capturing some political economy aspects of the social security system, because it suggests that there are stronger political demands for social security in more developed countries.

It is hoped that the model presented in this paper will stimulate further research along these lines.

References:

Blanchard, O.J., and S. Fischer (1989) *Lectures on Macroeconomics*, Cambridge, MIT Press.

Costa, D. L. (1998) *The Evolution of Retirement: An American Economic History, 1880-1990*, Chicago, University of Chicago Press.

Diamond, P. A. (1965) "National Debt in a Neoclassical Growth Model," *American Economic Review*, Vol. 55, pp. 1026-1050.

Feldstein, M. S. (1974) "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy*, Vol. 82, pp. 905-926.

Fuchs, V. R. (1983) *How We Live: An Economic Perspective on Americans from Birth to Death*, Cambridge, Harvard University Press.

Gong, L., and N. Liu (2006), "One Sector Neoclassical Growth Model with Endogenous Retirement: A Continuous Case," unpublished typescript, Guanghua School of Managemetn, Peking University.

Gruber, J., and D. Wise, eds. (1999) *Social Security and Retirement Around the World,* Chicago, University of Chicago Press.

Hu, S. C. (1979) "Social Security, the Supply of Labor, and Capital Accumulation," *American Economic Review*, Vol. 69, pp. 274-283.

Matsuyama, K. (2005) "Poverty Traps," forthcoming in L. Blume and S. Durlauf eds., *The New Palgrave Dictionary of Economics*, 2nd Edition, Macmillan.

Reichlin, P. (1986) "Equilibrium Cycles in an Overlapping Generations Economy with Production," Vol. 40, pp. 89-102.

Romer, P. M. (1986) "Increasing Returns and Long Run Growth," *Journal of Political Economy*, Vol. 94, pp. 1002-1037.

Figure 1: Growth Dynamics with Exogenous Retirement

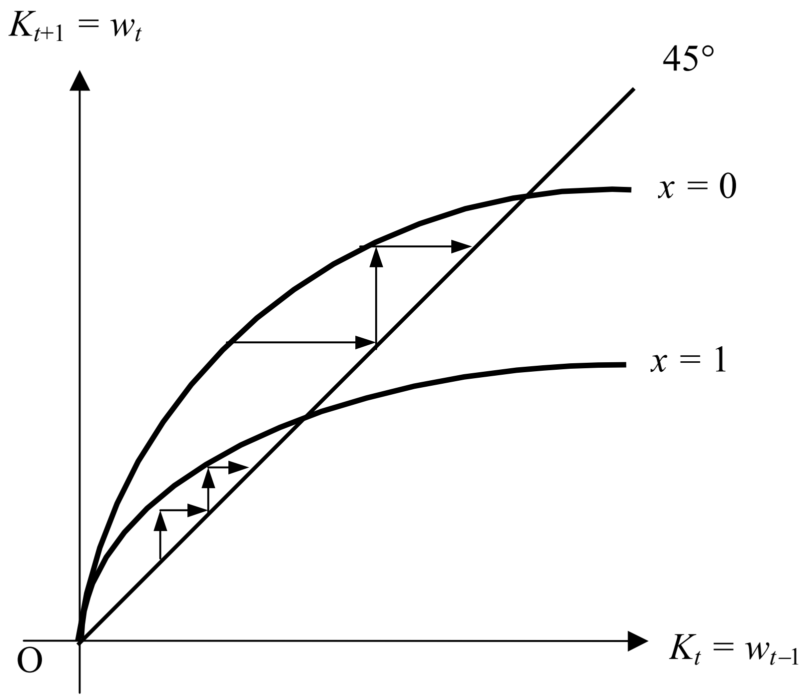

Figure 2: Growth Dynamics with Endogenous Retirement

Figure 2a:

($\alpha\mu < 1$)

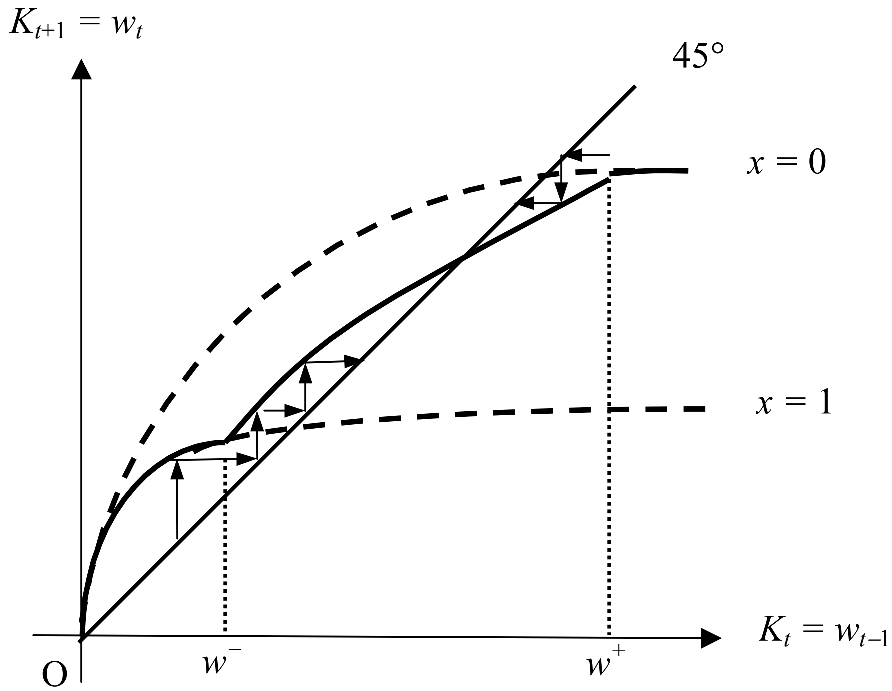

Figure 2b:

($\alpha\mu > 1$)

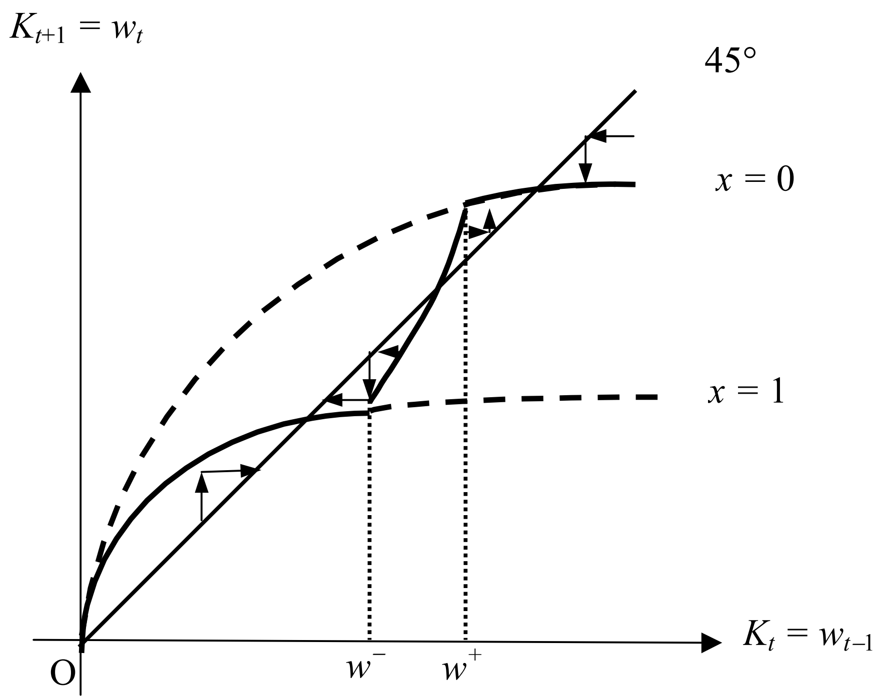

Figure 3: Growth Dynamics with Endogenous Retirement
A Case of Heterogeneous Agents