Revisiting the Model of Credit Cycles with Good and Bad Projects

By

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1. Introduction

Macro dynamics of borrower net worth (BNW) and investment under financial frictions

- **Low BNW as a cause of Slow Recovery:** much studied in the literature
  Low BNW makes it hard to finance projects with positive pecuniary externalities, which could help to improve NW of other borrowers.
  → persistence of low BNW; prolonged recessions

- **High BNW as a cause of Crises:** here
  High BNW makes it easy to finance projects with less pecuniary externalities, diverting credit flow away from those with more pecuniary externalities, which makes it hard to sustain high BNW → Macro volatility; boom-bust cycles
  - ➢ Fundamental Instability of Market Economy *a la* Goodwin
  - ➢ **"Success breeds crises"** *a la* Minsky-Kindleberger;
  - ➢ "**Credit booms gone bust,**" Mendoza-Terrones (2008); Schularick-Taylor (2012)

This paper builds on my model of credit cycles with "Good" and "Bad" projects

The Good, The Bad, and The Ugly (Theoretical Economics 2013)

- Overlapping generations of agents who live for two periods a la Diamond
- In their 1st period, the agents sell their endowments to build up their NW, which are used to finance their 2nd period consumption.
- Agents have access to heterogeneous investment projects that differ in:
  - **Profitability**
  - **Pledgeability (Financial Frictions or Borrowing Constraint)**
  - **General Equilibrium Effects (Pecuniary Externalities)**
- As BNW changes, composition of credit flows shifts across different investment projects, which in turn affect BNW of the next generation.

Obviously, dynamics depend on the set of investment projects that are competing for credit.

Three cases

1. The Good and The Bad
2. The Good and The Ugly
3. The Good, The Bad, and The Ugly

The Good and The Bad (TE 2013; Sections 2-4)

The Good

- Require the use of inputs, "labor," supplied by the next generation of the entrepreneurs, thus improving their NW
- *May be* subject to (small) financial frictions

The Bad

- Independently profitable. No pecuniary externalities to future borrowers
- Subject to financial friction, can be financed only with a sufficiently high BNW.

Key Mechanism: the Good breed the Bad; the Bad destroy the Good

- With a low BNW, only the Good are financed, generating high demand for the inputs supplied by future borrowers, improving their net worth, creating a boom.
- With a high BNW, the Bad are also financed, diverting the credit flow away from the Good, reducing demand for the inputs supplied by future borrowers and hence their net worth, creating a bust

Conditions for Endogenous Credit Cycles, or Volatility

- The Bad are sufficiently profitable.
- The Bad are subject to an *intermediate* degree of financial frictions.

Reducing (but not eliminating) financial frictions may cause volatility!

The Good and The Ugly (similar to Bernanke-Gertler; not in TE)

The Good

- Require the use of inputs, "labor," supplied by the next generation of the entrepreneurs, thus improving their NW.
- Subject to financial friction, need some NW to invest.

The Ugly (Think of storage, or running a parking lot, instead of building a factory)

- Not so profitable
- Little or no need for inputs, hence no spillover effects on the future borrowers
- No financial friction.

Key Mechanism:

- With a low NW, some credits go to the Ugly, instead of the Good.
- By slowing down a NW improvement, the Ugly acts as a drag to recovery.

Key Results:

- Persistence; One-time shock has an echo effect.
- Slow recovery, prolonged recession, or even a trap

The Good, The Bad, and The Ugly: (TE 2013; Section 5)

The Good

- Require the use of inputs, "labor," supplied by the next generation of the entrepreneurs, thus improving their NW.
- Subject to financial friction, need some NW to invest.

The Bad

- Independently profitable. No spillover effects to future borrowers
- Subject to financial friction, can be financed only with a sufficiently high NW.

The Ugly

- Not so profitable
- No need for inputs, hence no spillover effects on the future borrowers
- Not financial friction.

Key Mechanism:

- With a low NW, the Good compete with the Ugly, which acts as a drag on the Good.
- With a high NW, the Good compete with the Bad, which destroys the Good.

Key Results: Asymmetric Fluctuations and Intermittent Volatility

This Paper revisits the model of credit cycles with Good and Bad projects:

1) A Reformulation; Deriving the same nonlinear piecewise-smooth 1D-map governing the equilibrium path under a much simpler set of assumptions.

2) Detailed analysis of the nature of fluctuations under Cobb-Douglas

- Subcritical flip bifurcation of the Steady State (SS)
  - Co-existence of the stable SS with other attractors (cyclic or chaotic)
  - *Corridor Stability*: SS stable against small shocks, unstable against large shocks
  - *Catastrophic and Irreversible* transitions

  Caution for studying nonlinear dynamic models by linearizing around SS.
- Border Collision Bifurcations
  - An *immediate* transition from stable SS to an *asymmetric* stable n-cycle (n≥3), with n–1 consecutive periods of "up" followed by one period of "down"
  - An immediate transition from stable SS to chaotic *attractors*
  - *Robust* chaos (persistent under parameter perturbation)

  These are features unique to "Regime-switching," piecewise smooth models.
  - In most examples of chaos in economics, a transition is NOT immediate.
  - Furthermore, many of them are NOT attractors (i.e., "unobservable")
  - Most examples of chaotic attractors are NOT robust.

2. Reformulating the Model of Credit Cycles with Good and Bad Projects

Time: Discrete (t = 0, 1, 2,…)

Final Good can be consumed or invested:

CRS technology, $Y_t = F(K_t,L_t)$ with physical capital, K_t and "labor", $L_t = 1$

$y_t \equiv Y_t/L_t = F(K_t/L_t,1) \equiv f(k_t)$, where $k_t \equiv K_t/L_t$;

$f'(k) > 0 > f''(k)$, $f(0) = 0$, $f''(0) = \infty$.

Competitive Factor Markets:

$\rho_t = f'(k_t) > 0$, decreasing; $\quad w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t) > 0$, increasing.

Demography: A variant of Diamond's Overlapping Generations model.

- A continuum of identical agents with unit mass in each generation.
- Each agent has one unit of the endowment, "inputs" or "labor," only in the 1st period (when "young"), supplied inelastically at w_t
- Each consumes only in the 2nd (when "old"). → They save everything when young.

Aggregate Saving (Credit Supply): $S_t = W(k_t)$

Three Means of Converting w_t into c_{t+1}

- **Lending:** They can always lend w_t at r_{t+1}.

- **The Good** projects: convert one unit of the final good in t to one unit of **capital** in t+1. with the gross rate of return, $f'(k_{t+1})$.

- **The Bad** projects: convert m units of the final good in t to mB units of **final good** in t+1. with the gross rate of return, B.
  - Indivisible, $m > 0$ is a fixed investment requirement, a parameter.
  - Each young agent can run at most one Bad project.
  - Need to borrow $m - w_t$.
  - limited pledgeability. Only a fraction, μ of the revenue can be pledged to the lender.

In TE (2013, Sections 2-4)

- Both the Good and the Bad are indivisible and subject to limited pledgeability.
- Heterogeneous agents:
  - Some, "entrepreneurs," have access only to the Good, which create "jobs".
  - Some, "traders," have access only to the Bad, which create no jobs.
  - The rest, "lenders," have access to neither the Good nor the Bad.

For the credit to flow to the Bad, the young must be both *willing and able* to finance it.

(1) **Profitability Constraint (PC):** $B \geq r_{t+1}$

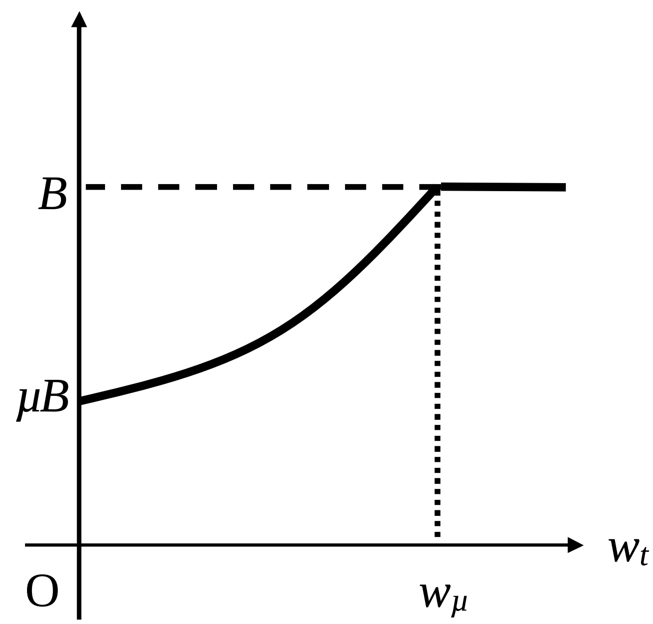
(2) **Borrowing Constraint (BC):** $\mu mB \geq r_{t+1}(m - w_t.)$.

(BC) is the tighter constraint than (PC) iff $w_t < m(1-\mu) \equiv w_\mu$.

Define the maximal rate of return that the agent can pay by running the Bad project without violating (BC) and (PC):

(3) $R(w_t) \equiv BMin\left\{\dfrac{\mu}{1-w_t/m}, 1\right\}$

$$= \begin{cases} \dfrac{\mu B}{1-w_t/m} & \text{if } w_t < w_\mu \\ B & \text{if } w_t > w_\mu. \end{cases}$$



Equilibrium Conditions: X_t : the measure of the Bad projects started in t.

(4) Rate of Return: $f'(k_{t+1}) = r_{t+1} \geq R(w_t)$; $X_t \geq 0$; $[f'(k_{t+1}) - R(w_t)]X_t = 0$

(5) S = I: $W(k_t) = k_{t+1} + mX_t$

Note: $w_{t+1} = W(k_{t+1})$ is increasing in k_{t+1} but not in X_t.

- The Good improve BNW of the next cohort. In this sense, they are "Good."
- The Bad generate no benefit to the next cohort. In this sense, they are "Bad."

If $X_t > 0$, $f'(k_{t+1}) = R(W(k_t))$ from (4). If $X_t = 0$, $k_{t+1} = W(k_t)$ from (5). Thus,

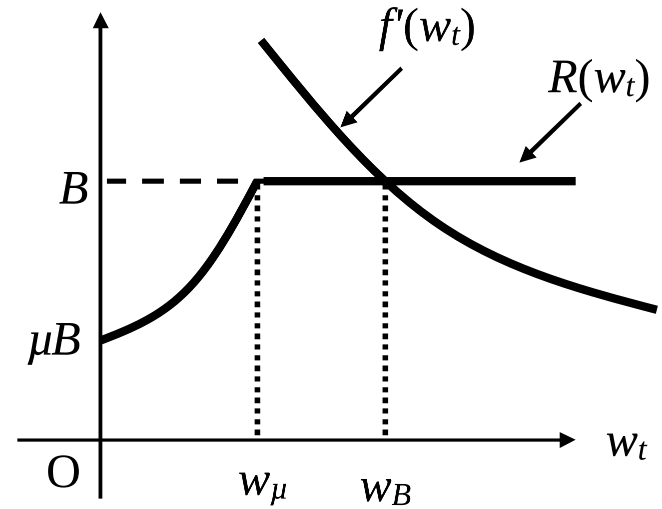
Equilibrium Trajectory:

$$(6) \quad k_{t+1} = \Psi(k_t) \equiv \begin{cases} W(k_t) & \text{if } k_t < k_c \\ \\ (f')^{-1}\big(R(W(k_t))\big) & \text{if } k_t > k_c, \end{cases}$$

where k_c is uniquely defined by $f'\big(W(k_c)\big) \equiv R(W(k_c))$.

No Distortion Case: $f'(w_\mu) > B$

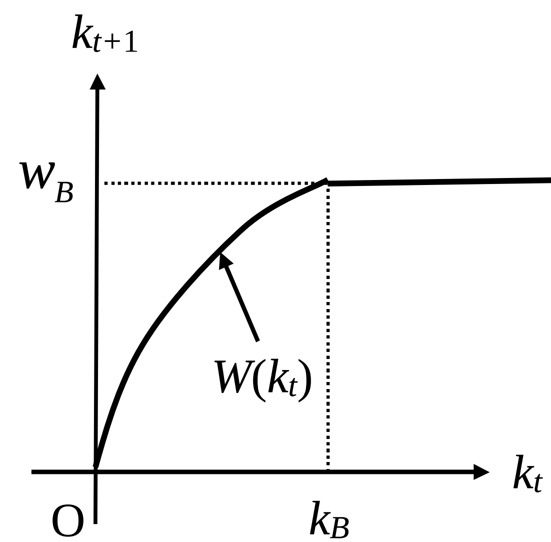
$\Leftrightarrow W(k_\mu) \equiv w_\mu < (f')^{-1}(B) \equiv w_B \equiv W(k_B) \Leftrightarrow k_\mu < k_B$



The Bad is as profitable as the Good at $w_B \equiv W(k_B)$.
(BC) is non-binding at $w_\mu \equiv W(k_\mu)$.

With $w_B \equiv W(k_B) > w_\mu \equiv W(k_\mu)$,
(BC) is never binding.

$$(7) \quad k_{t+1} = \begin{cases} \Psi_L(k_t) \equiv W(k_t) & \text{if } k_t < k_B \\ \Psi_R(k_t) \equiv w_B & \text{if } k_t > k_B \end{cases}$$

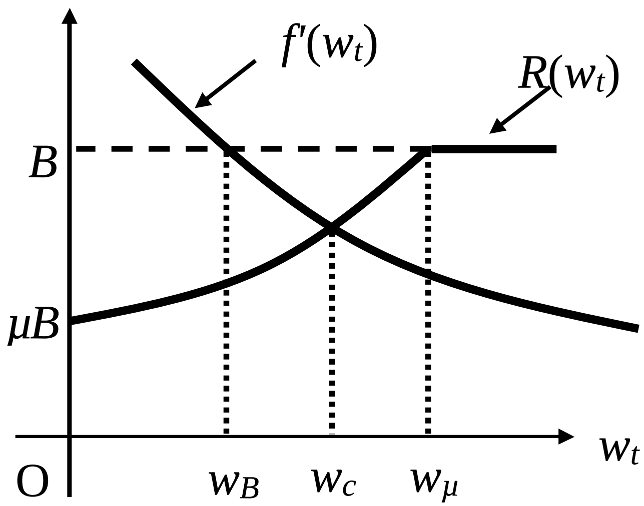


Distortionary Case: $f'(w_\mu) < B$

$\Leftrightarrow w_\mu \equiv W(k_\mu) > w_B \equiv W(k_B)$

For $w_t \in (w_B, w_\mu)$,

the Bad are more profitable but BC is binding.
Over-Investment of the Good
(Under-investment of the Bad).



$$(8) \quad k_{t+1} = \Psi(k_t) = \begin{cases} \Psi_L(k_t) \equiv W(k_t) & \text{if } k_t < k_c \\ \Psi_M(k_t) \equiv (f')^{-1}\left(\dfrac{\mu B}{1 - W(k_t)/m}\right) & \text{if } k_c < k_t < k_\mu \\ \Psi_R(k_t) \equiv w_B & \text{if } k_t > k_\mu, \end{cases}$$

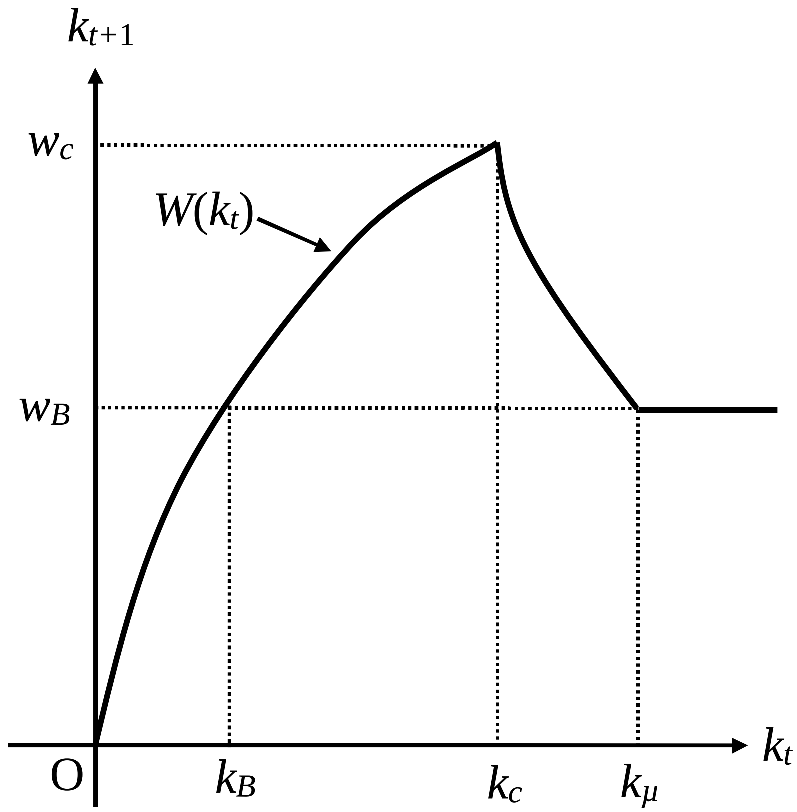
where k_c satisfies $f'(W(k_c)) = \mu B / [1 - W(k_c)/m] < B$.

The map has *three* branches, with a hump in the middle.

Left (Upward) branch: All the credit goes to the Good

Middle (Downward) branch: **Some credit goes to the Bad, but BC is binding.** Downward-sloping, because a higher NW relaxes BC, driving up the rate of return, diverting the credit flows away from the Good.

Right (Flat) branch: BC is no longer binding.



In what follows, assume

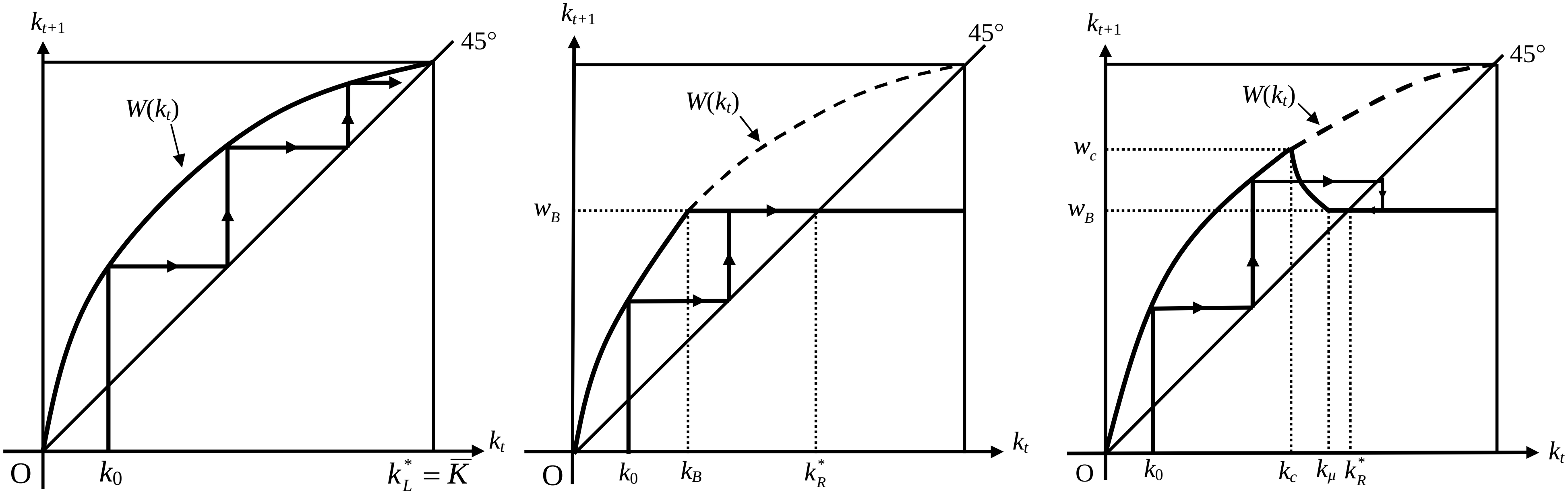
(A1) There exists $\overline{K} > 0$ s.t. $W(\overline{K}) = \overline{K}$ and $W(k) > k$ for all $k \in (0, \overline{K})$

This ensures that Ψ maps $(0, \overline{K}]$ into itself and has a unique steady state, $k^* \in (0, \overline{K}]$

(A2) $\overline{K} < m$. This ensures $w_t < m$.

3. Dynamic Analysis: General Case

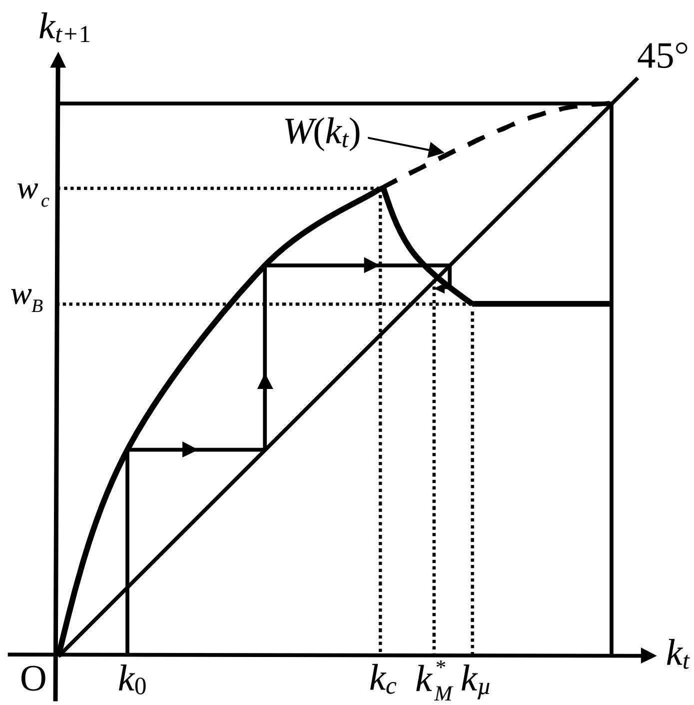
Denote the unique steady state by k_L^*, k_M^*, or k_R^*, depending on which branch it exists.



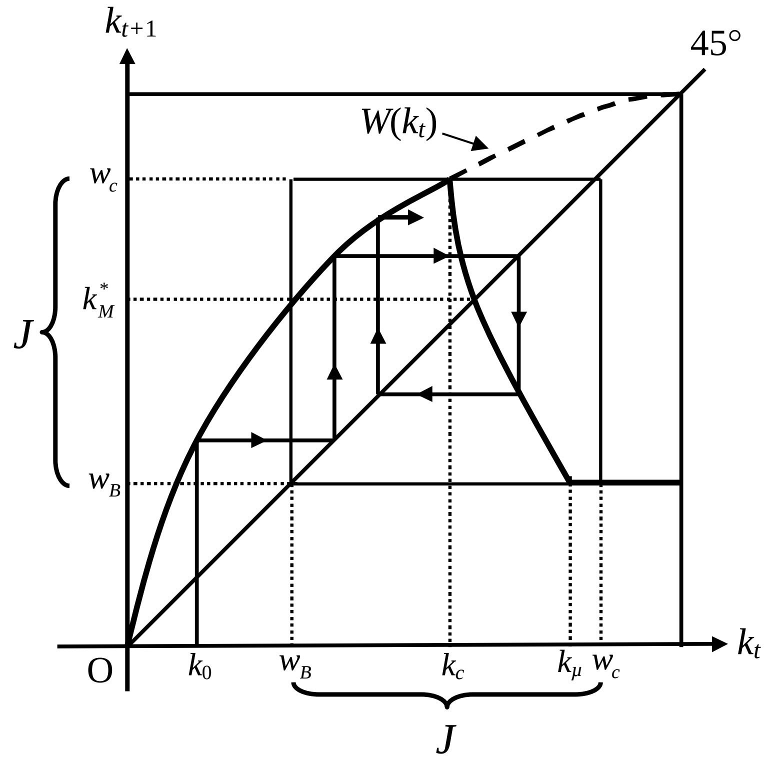
A: Global monotone converging to $k_L^* = \overline{K}$

B: Globally mapped into $k_R^* = w_B$ monotonically

C. Globally mapped into $k_R^* = w_B < w_c$ with over-shooting



D: (Locally) Oscillatory Converging to k_M^*



E: Unstable Steady State, k_M^*. Endogenous Fluctuations

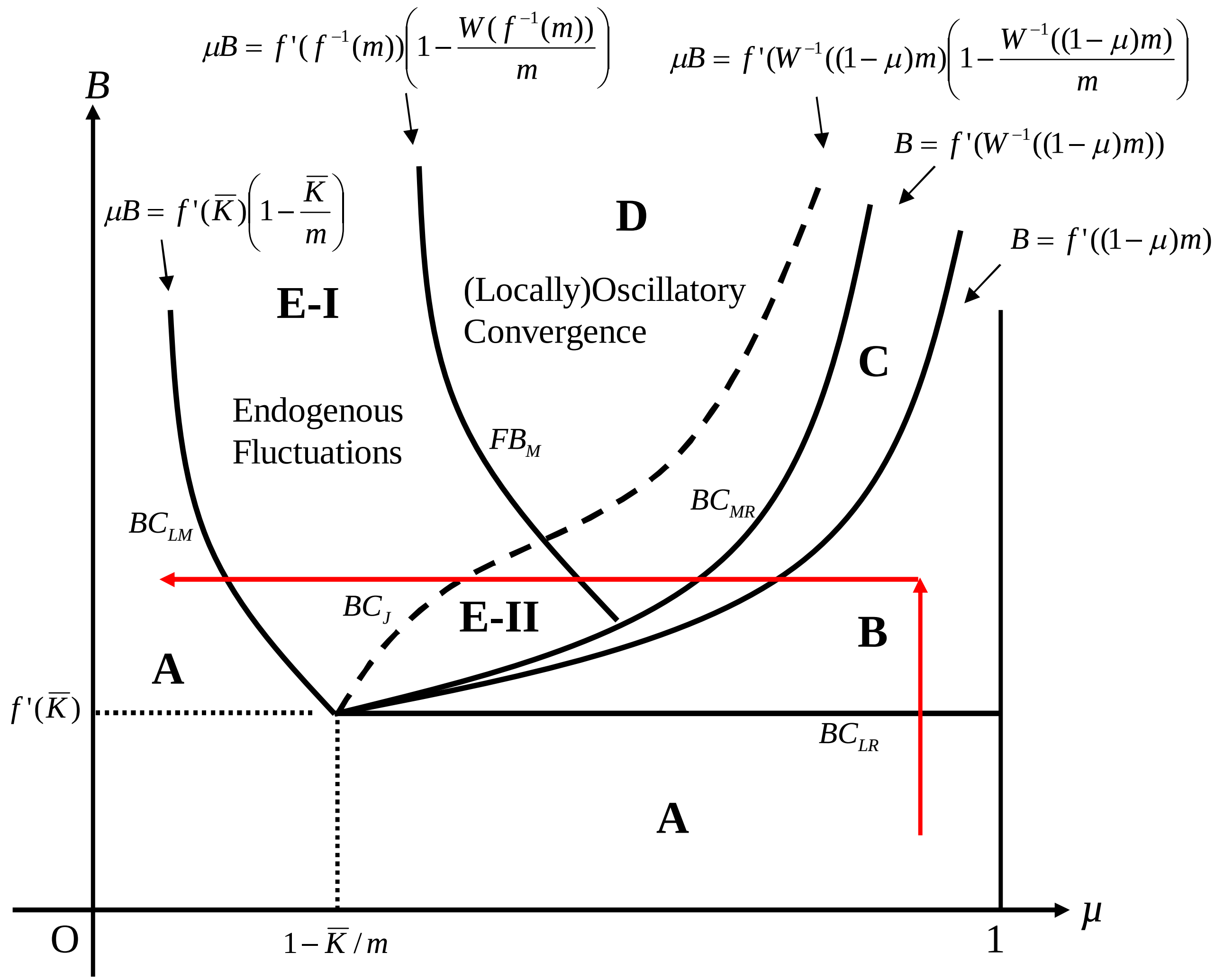
Depending on whether the absorbing interval, *J*, includes the flat *R* branch,

- E-I: Only *L* and *M* branches are involved in *J*.
- E-II: All three (*L*, *M*, and *R*) branches are involved in *J*, (as shown here).

Parameter Configuration in (μ,B) for a fixed $m \in (\overline{K}, f(\overline{K}))$.

Volatility in D & E, which requires
- A large B
- An intermed. value of μ

i.e., in the presence of some projects
- with less pecuniary externalities
- profitable so the agents are eager to invest
- able to invest *with and only with* high NW, due to the frictions that are large but not prohibitively large.



A First Tour at Bifurcations: Border-Collision (BCB) and Flip (FB)

Following the red arrows,

From A to B by crossing
BC_LR (the BCB of the fixed point in *L* and of the fixed point in *R*) $\Rightarrow w_B = k_R^* = k_L^* = \overline{K}$

From B to C and to D by crossing
BC_MR (the BCB of the fixed point in *M* and of the fixed point in *R*) $\Rightarrow w_B = k_R^* = k_M^* = k_\mu$

From D to E-II by crossing
FB_M (the flip bifurcation of the fixed point in *M*) $\Rightarrow \Psi'(k_M^*) = -1$.

From E-II to E-I by crossing
BC_J (the contact of the absorbing interval, *J*, with w_μ) $\Rightarrow \Psi(k_c) = w_c = k_\mu$.

From E-I to A by crossing
BC_LM (the BCB of the fixed point in *L* and of the fixed point in *M*) $\Rightarrow k_M^* = k_L^* = \overline{K}$

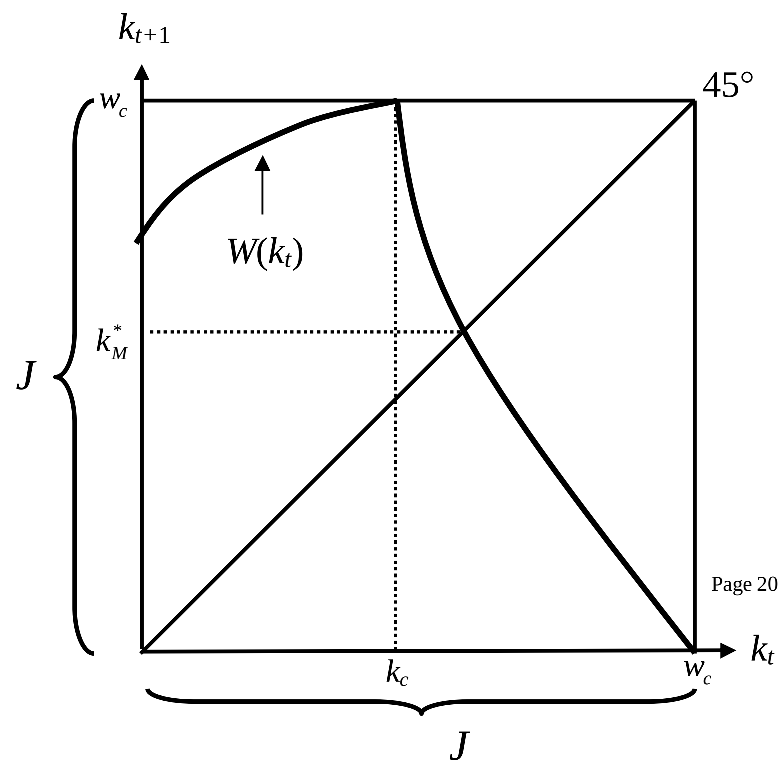
If we cross *BC_MR* from C to E-II, bypassing D, we observe a border-flip bifurcation, where $w_B = k_R^* = k_M^* = k_\mu$ and $\Psi'(k_M^*) < -1$.

Restricting Ψ on the Absorbing Interval above BC_J.

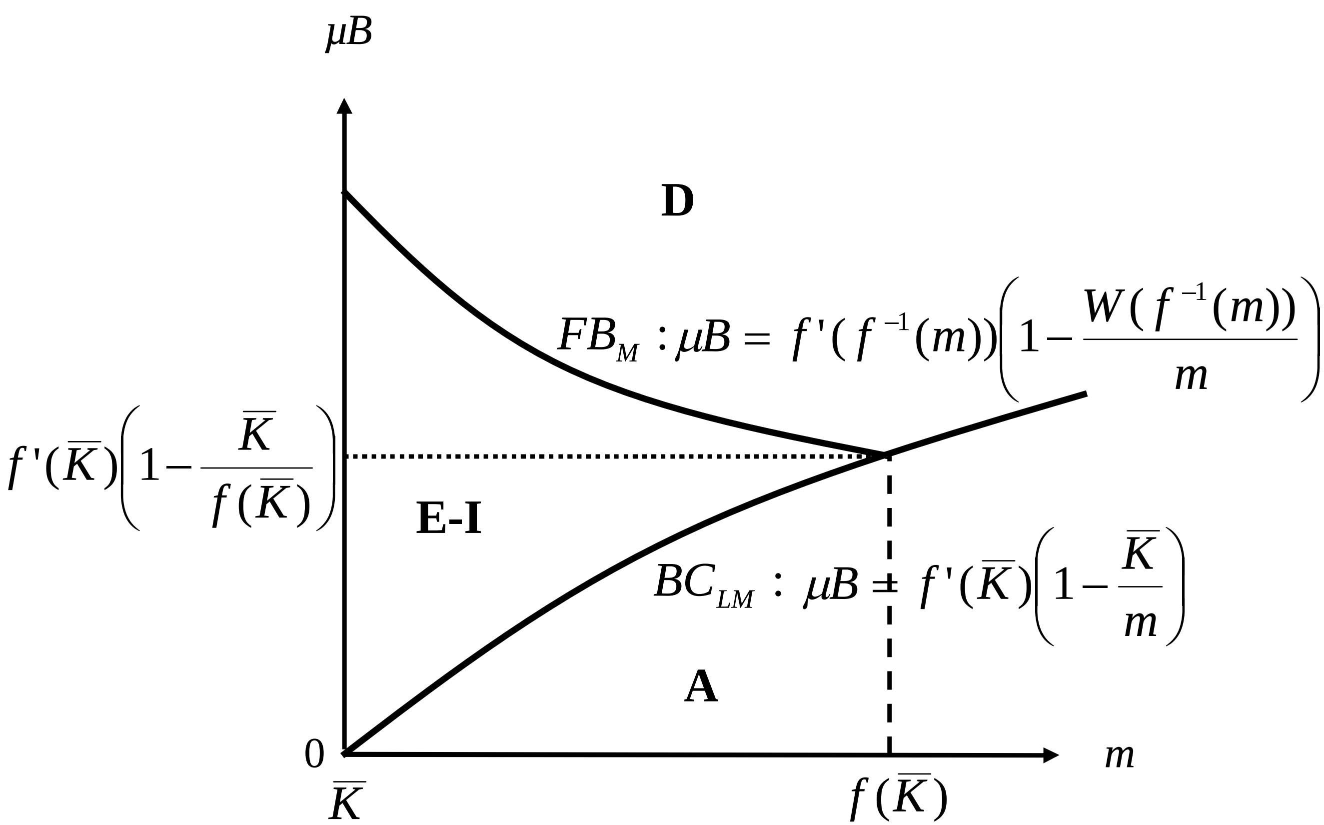
Ψ has only L and M branches on $J = [\Psi_M(w_c), w_c]$ above BC_J,

$$k_{t+1} = \Psi(k_t) = \begin{cases} \Psi_L(k_t) \equiv W(k_t) & \text{if } \Psi_M(w_c) \le k_t < k_c \\ \Psi_M(k_t) \equiv (f')^{-1}\left(\dfrac{\mu B}{1 - W(k_t)/m}\right) & \text{if } k_c < k_t \le w_c \end{cases}$$

Notice that
μ and B enter the system only through μB.



Parameter Configuration in (*m*, *μB*) above BC_J.



Main Results for the General Case: To Summarize

Proposition 1 (Effects of μ)

For any $B > f'(\bar{K})$, endogenous fluctuations occur for an intermediate value of μ.

Proposition 2 (Effects of B)

For any $\mu \in (0,1)$, a sufficiently high B creates the steady state, k^*_M, around which the dynamics is oscillatory.

Proposition 3 (Effects of μB)

For a sufficiently high B, μ and B only affect the dynamics through its product, μB. As μB rises, **A → E-I → D**, and endogenous fluctuations occur for an intermediate value of μB.

4. Dynamic Analysis: Cobb-Douglas Case $f(k) = A(k)^{\alpha},\ 0 < \alpha < 1$

Rewrite (6) in $w_t = W(k_t)$ with the normalization, $(1-\alpha)A = 1$. Then,

$$(10)\quad w_{t+1} = T(w_t) \equiv \begin{cases} T_L(w_t) \equiv (w_t)^{\alpha} & \text{if } w_t < w_c \\ T_M(w_t) \equiv \left[\dfrac{1}{\mu\beta}\left(1 - \dfrac{w_t}{m}\right)\right]^{\frac{\alpha}{1-\alpha}} & \text{if } w_c < w_t < w_\mu \\ T_R(w_t) \equiv \left[\dfrac{1}{\beta}\right]^{\frac{\alpha}{1-\alpha}} & \text{if } w_t \geq \max\{w_c, w_\mu\} \end{cases}$$

where $$(w_c)^{1-\alpha} \equiv \frac{1}{\mu\beta} Max\left\{1 - \frac{w_c}{m}, \mu\right\}; \qquad w_\mu \equiv m(1-\mu)$$

and $$0 < \alpha, \mu < 1; \quad \beta \equiv \frac{1-\alpha}{\alpha} B > 0; \quad (1-\alpha)m < 1 < m.$$

This maps (0,1] into itself.

A: $w_c > W(\bar{K}) = 1 \Leftrightarrow \mu\beta < Max\left\{1 - \frac{1}{m}, \mu\right\}.$

B: $m(1-\mu) \equiv w_\mu < w_c < 1 \quad \Leftrightarrow \quad 1 < \beta < [m(1-\mu)]^{\alpha-1}$

C: $w_c < m(1-\mu) \equiv w_\mu < (\beta)^{(1-\alpha)/\alpha} < 1 \Leftrightarrow \quad [m(1-\mu)]^{\alpha-1} < \beta < [m(1-\mu)]^{1-1/\alpha}$

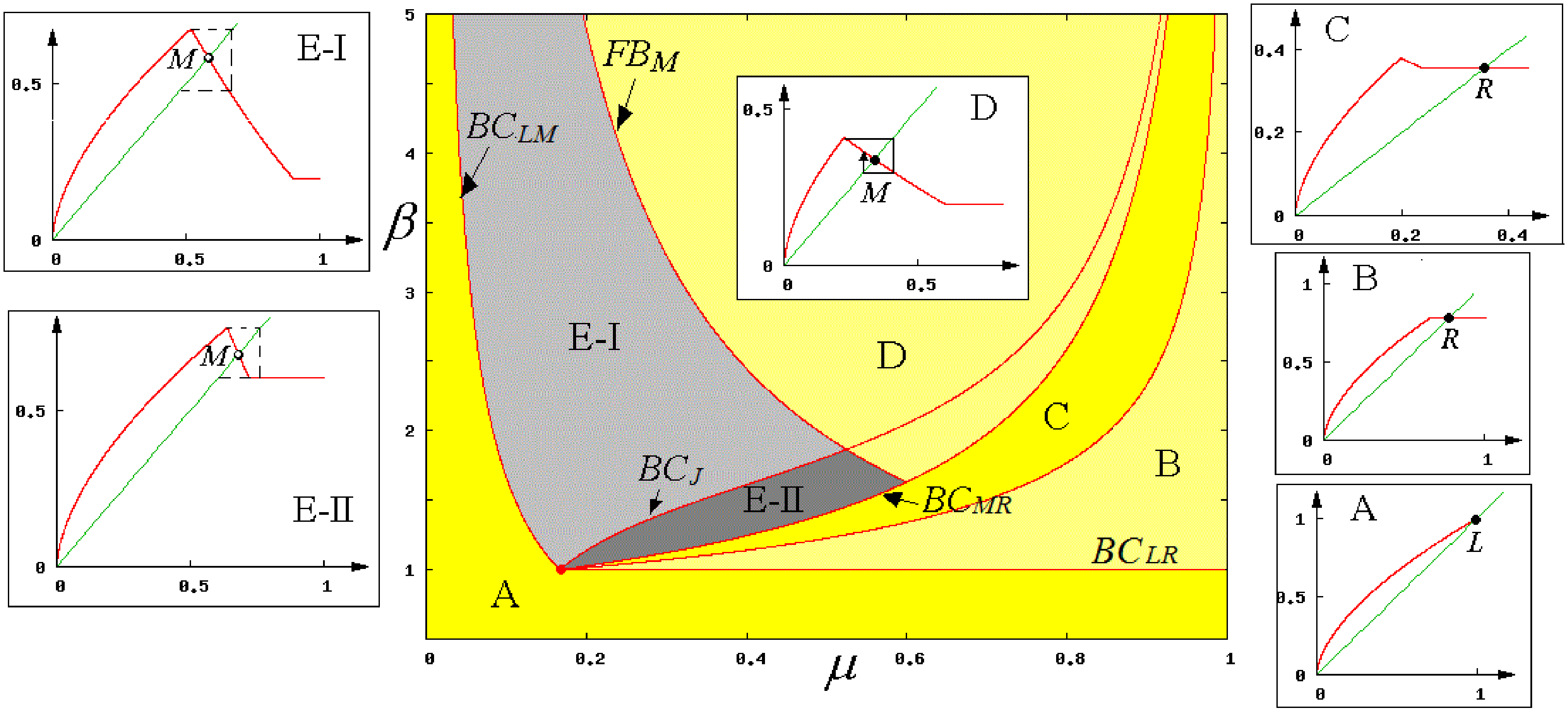
D: $\mu\beta > \alpha[(1-\alpha)m]^{1-1/\alpha} \quad \& \quad \beta > [m(1-\mu)]^{1-1/\alpha}$

E: $\left(1 - \frac{1}{m}\right) < \mu\beta < \alpha[(1-\alpha)m]^{1-1/\alpha} \quad \& \quad \beta > [m(1-\mu)]^{1-1/\alpha}$

E-I: $w_c = W(k_c) < k_\mu \Leftrightarrow (w_c)^\alpha = W(w_c) < W(k_\mu) = m(1-\mu) \equiv w_\mu$

E-II: $w_c = W(k_c) > k_\mu \Leftrightarrow (w_c)^\alpha > m(1-\mu) \equiv w_\mu$

Parameter Configuration- (μ, β)

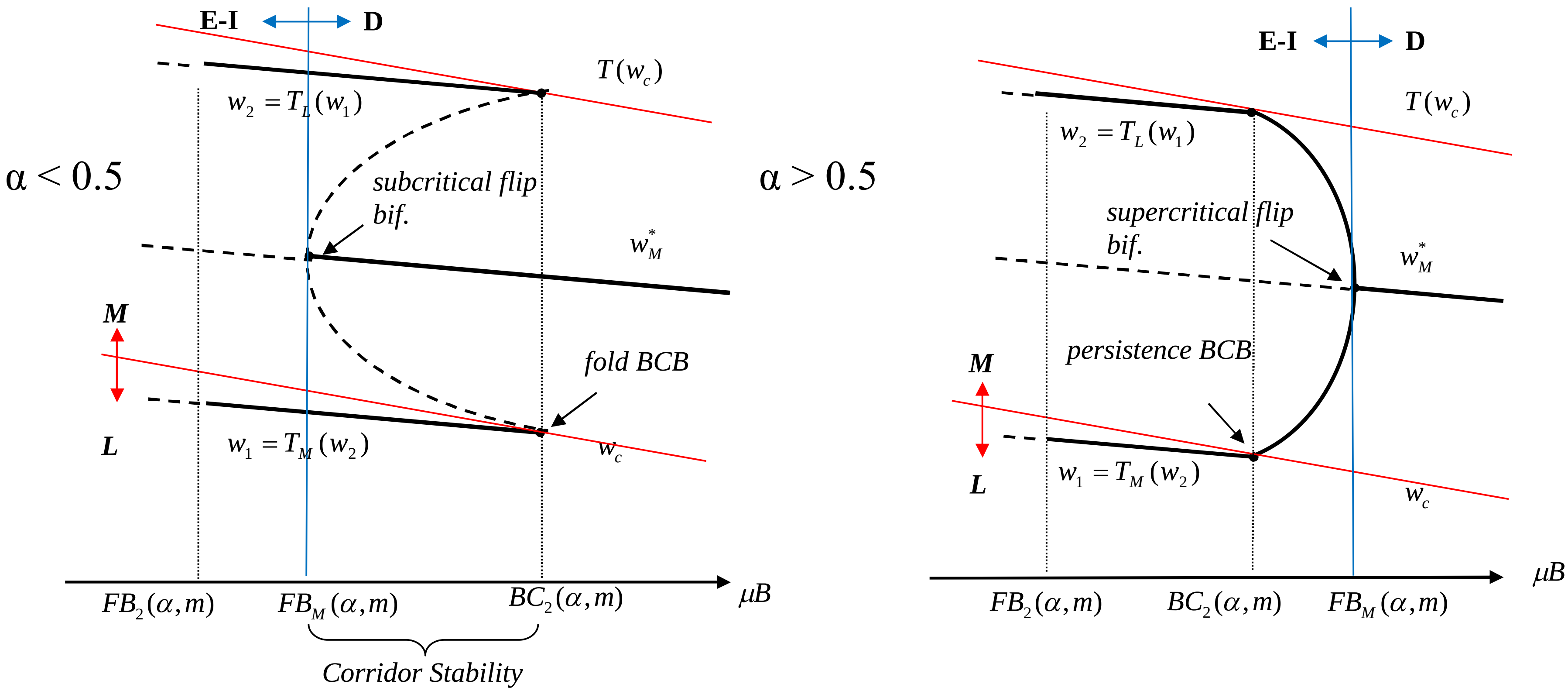


Boundaries of E-I are:

- BC_J (the contact of the absorbing interval, J, with w_μ) $\Rightarrow T(w_c) = w_\mu$.
- FB_M (the flip bifurcation of the fixed point in M) $\Rightarrow T'(w_M^*) = -1$.
- BC_{LM} (the BCB of the fixed point in L and of the fixed point in M) $\Rightarrow w_c = 1$.

Crossing the FB_M curve: Flip Bifurcation of the Steady State

- If α < 0.5, **subcritical; Corridor Stability**
- If α = 0.5, **degenerate**; a continuum of 2-cycles, (not asymptotically) stable.
- If α > 0.5, **supercritical**; a stable 2-cycle coexisting with an unstable ss in E.



More on the *Subcritical* Flip of Steady State (SS)

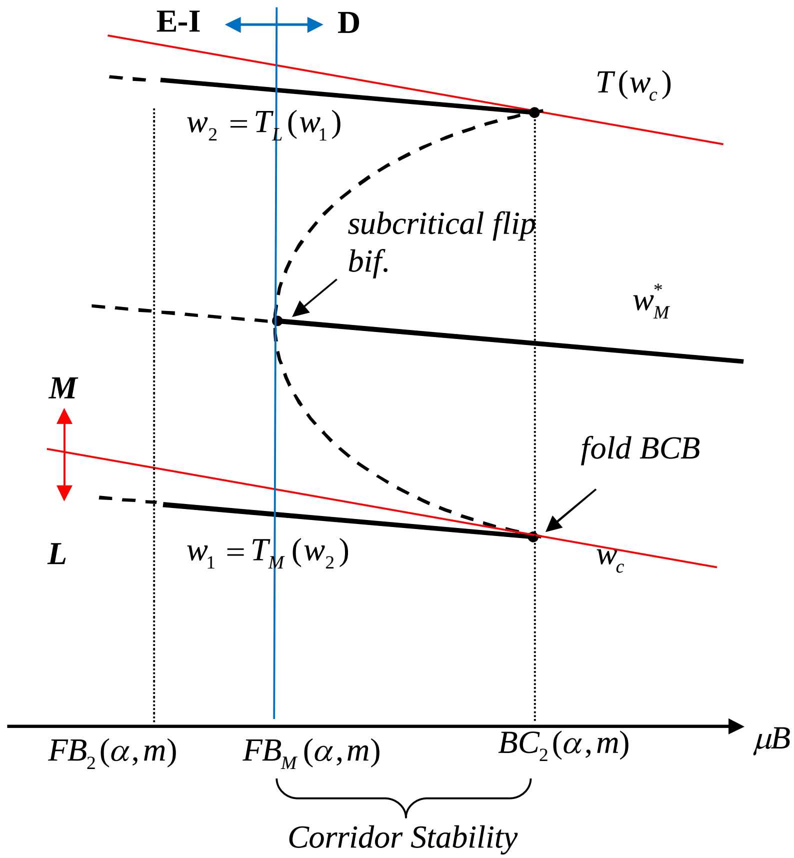
Between the Flip of SS and the Fold BCB of the 2-cycle

- ***Co-existence*** of a locally stable SS and a locally stable 2-cycle, with their basins of attraction separated by an unstable 2-cycle, in D
- ***Corridor Stability*** (*a la* Leijonhufvud)**:** SS is stable against small shocks, unstable against large shocks.

After the Flip, the effects are

- ***Catastrophic***
- ***Irreversible***

Main Message:
Caution for studying nonlinear dynamic models by linearizing around the unique steady state.

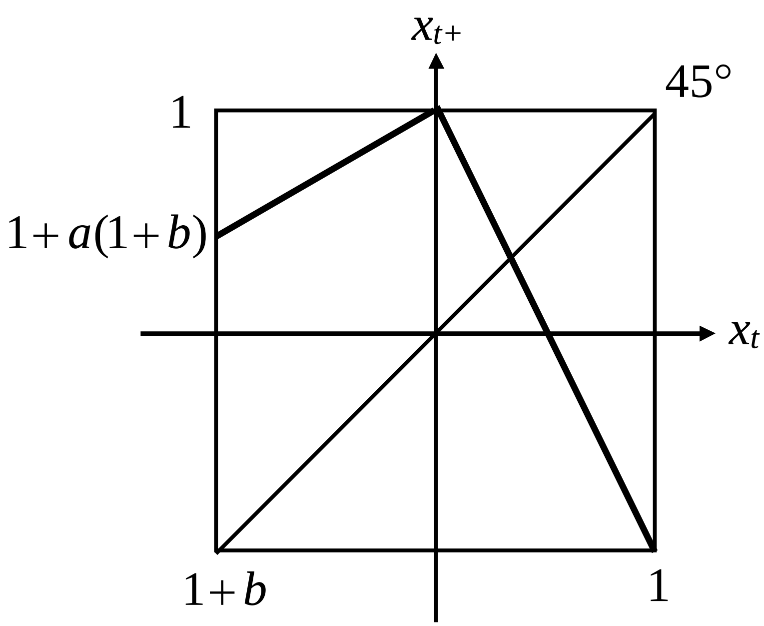


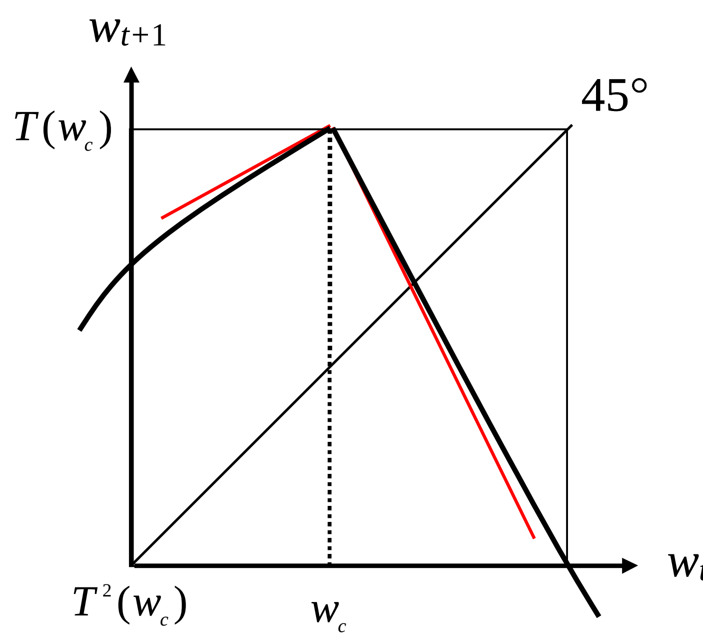
Crossing the BC_{LM} curve:

This can be analyzed by using the **skew-tent map as a linear approximation:**

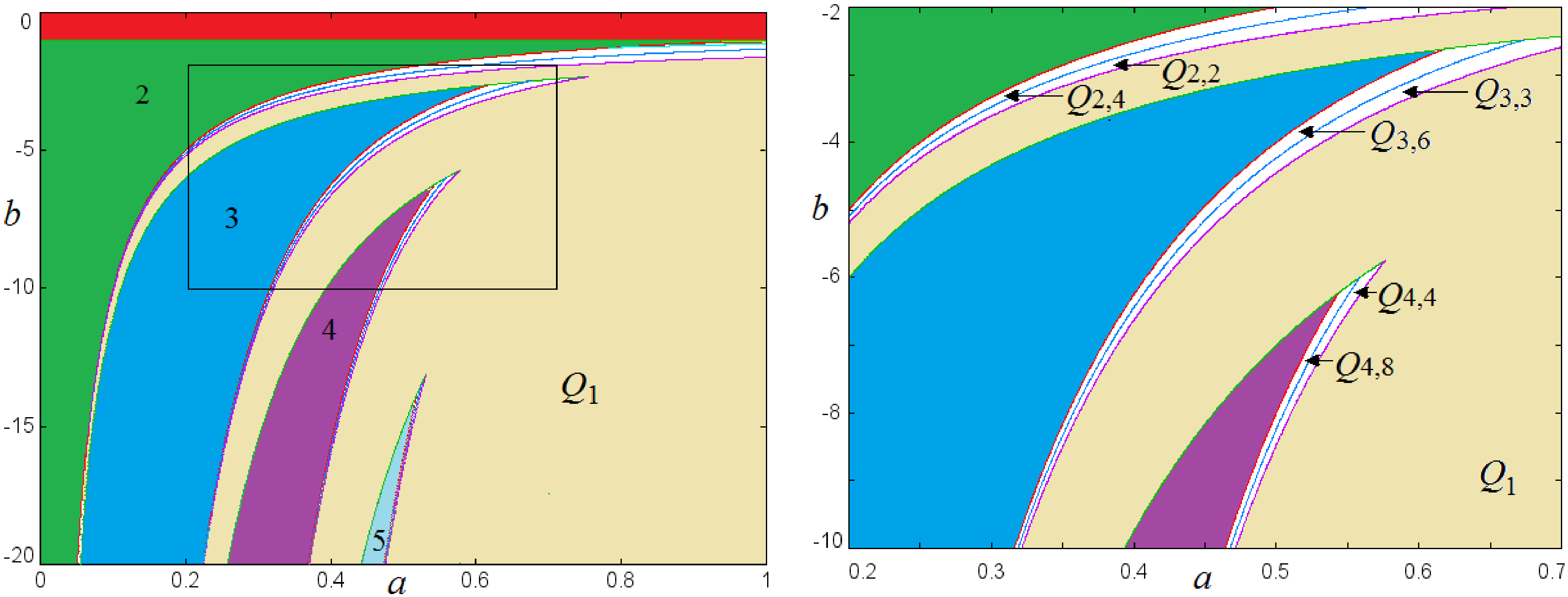
$$\begin{array}{lcll} t_L(x) & = & ax+1 & \text{if} \quad x<0 \\ t_R(x) & = & bx+1 & \text{if} \quad x \geq 0 \end{array}, \qquad (0 < a < 1 \text{ and } b < 0),$$

with $a = \lim_{w \uparrow 1} T'(w) = \alpha$ and $b = \lim_{w \downarrow 1} T'(w) = -\dfrac{\alpha}{(1-\alpha)(m-1)} \equiv B_m(\alpha) < -1.$



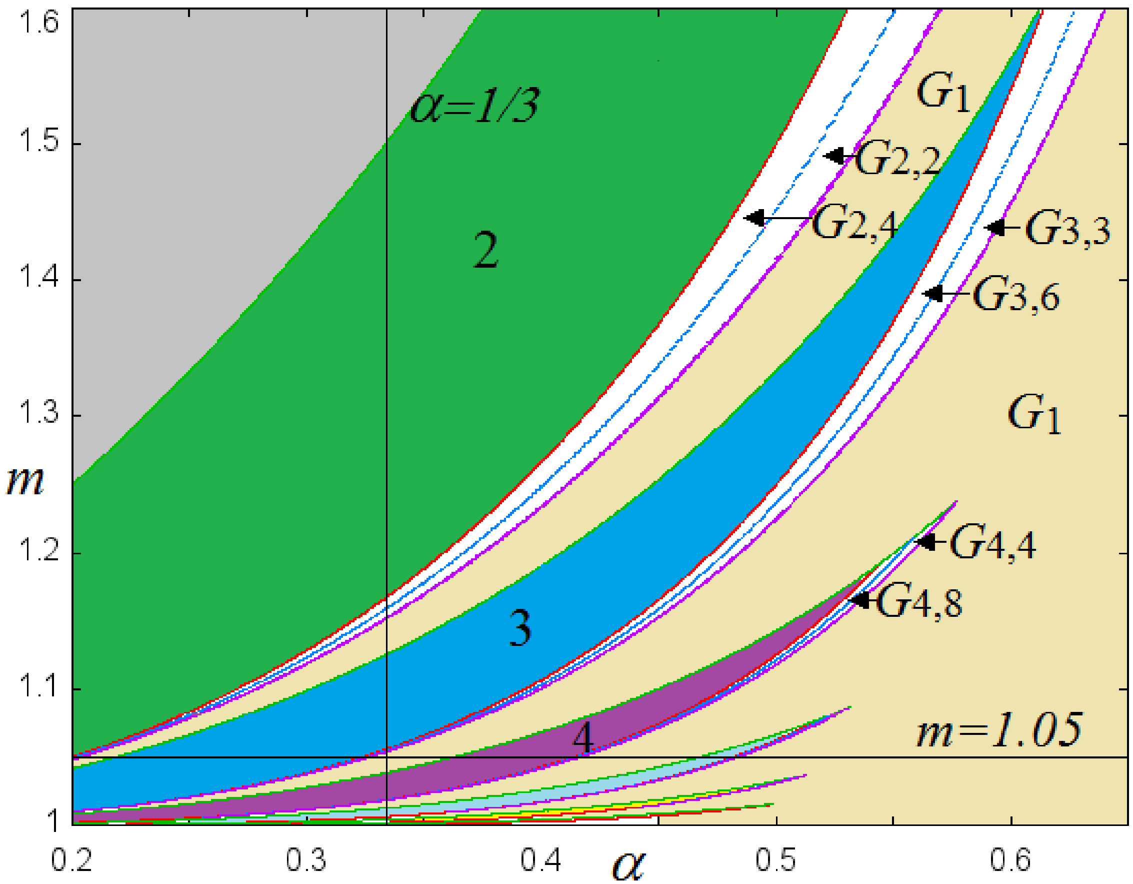


Bifurcation diagram for Skew Tent Map



Numbers: the periodicity of stable cycles; (For n ≥ 3, n–1≥ 2 consecutive periods of "up" followed by one period of "down")
Yellow; chaotic attractor with one interval
White: chaotic attractor with multiple intervals. (The second subscript indicates the number of intervals.)

Bifurcation diagram for Our Map Upon Crossing the BC_{LM} curve



The numbers: the periodicity of stable cycles;

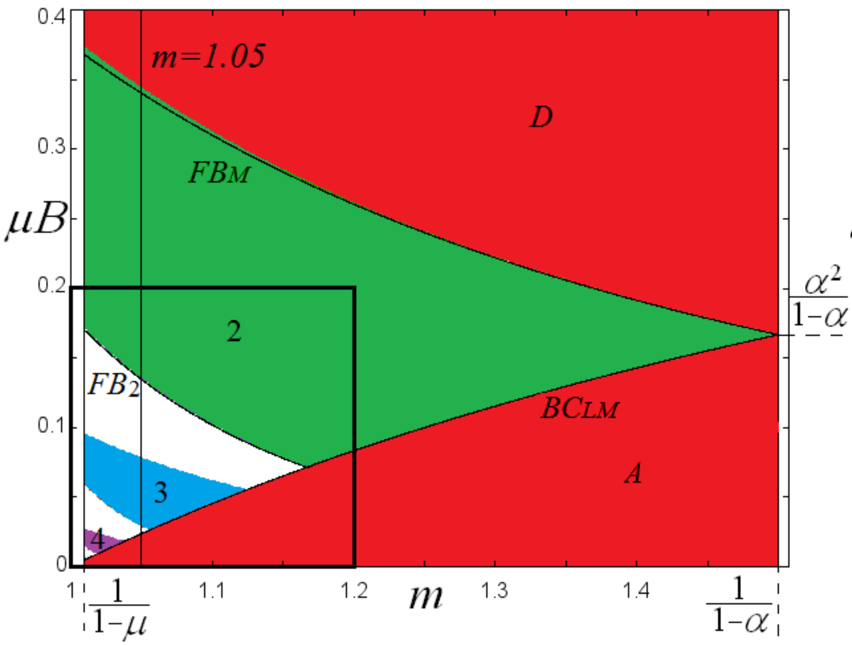
Yellow; chaotic attractor with one interval

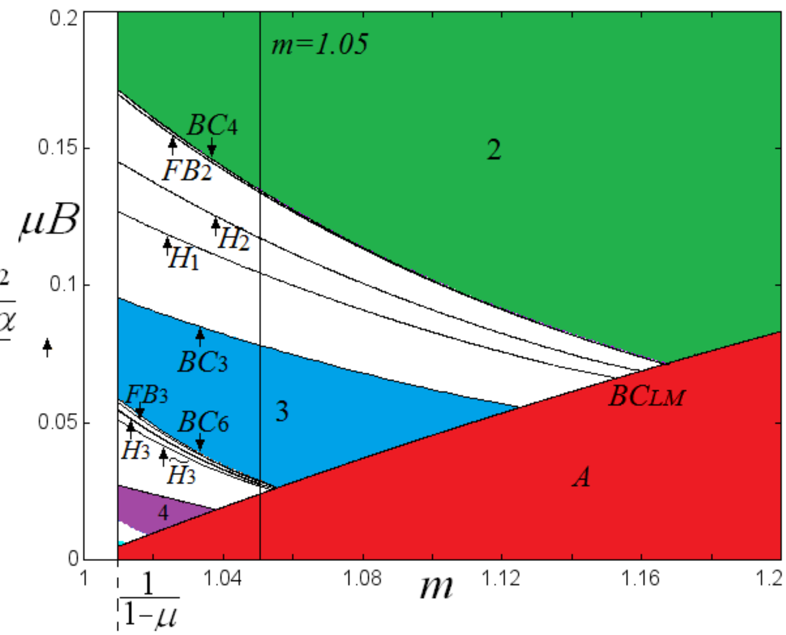
White: chaotic attractor with multiple intervals (the second subscript indicates the number of intervals)

With $b < -1$, the Red region for the skew tent map (the region of stable ss) would map into Gray in this figure, which is outside of our parameter range.

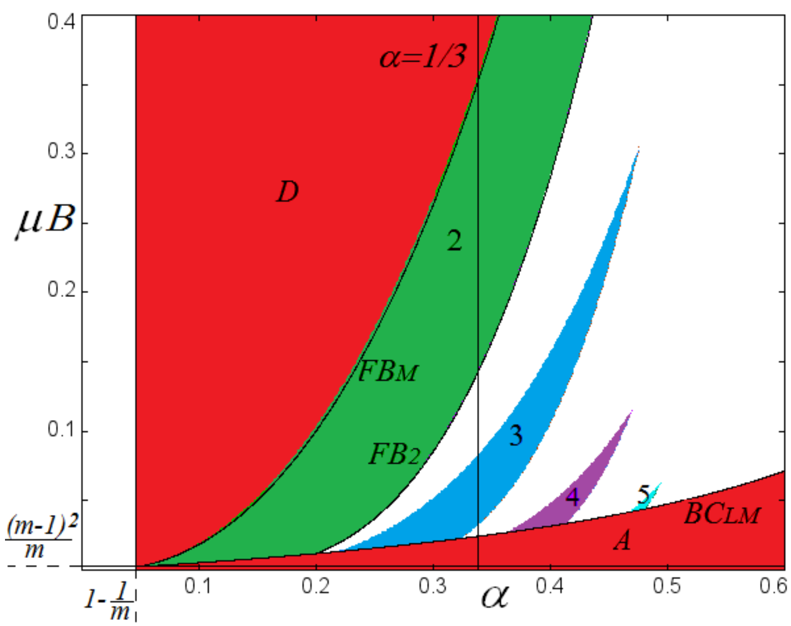
Two Bifurcation Diagrams: Inside Region E-I

a) $\alpha = 1/3$

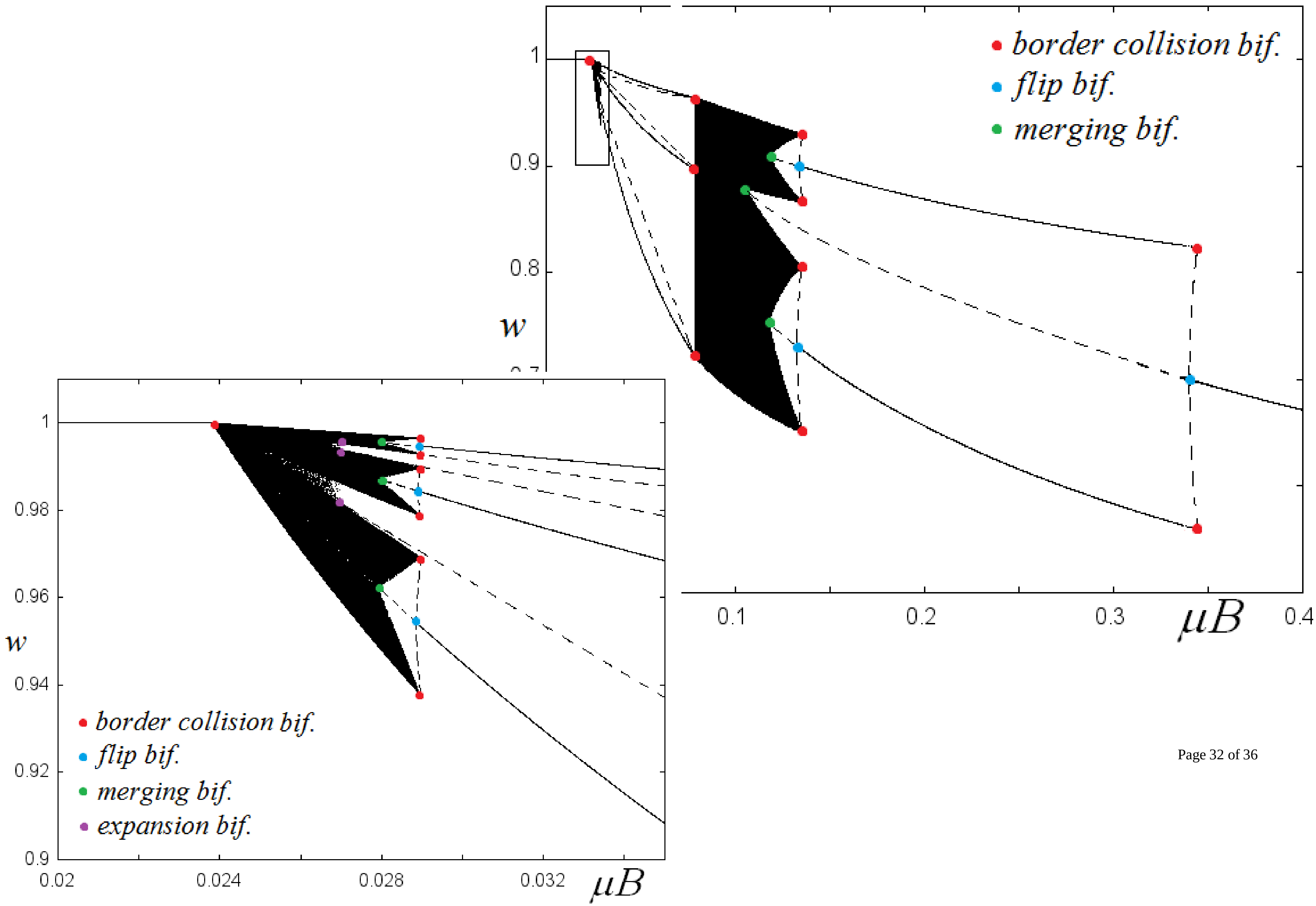




b) $m = 1.05$



Effects of μB: A Typical Bifurcation Scenario ($\alpha = 1/3,\ m = 1.05$)



Robust Chaos: Merging and Expansion Bifurcations

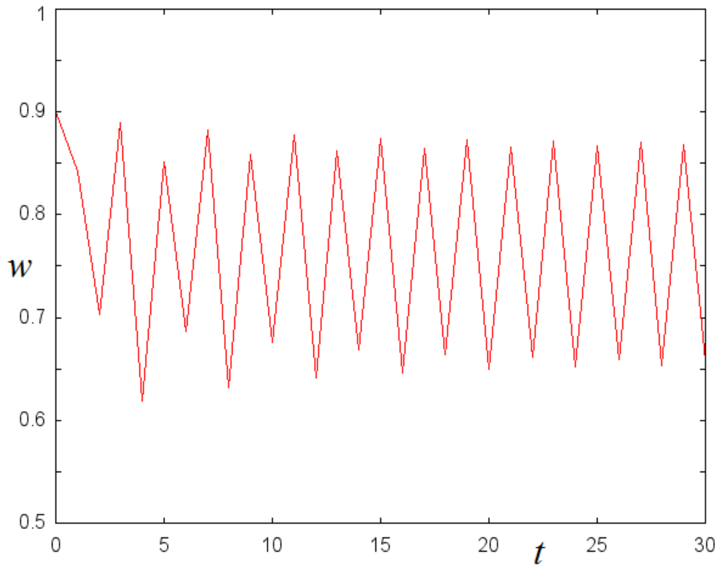
A Robust Chaotic Attractor existing in an open region (without periodic windows) with subregions related to different numbers of pieces of the attractor. Number of pieces changes due to merging or expansion bifurcations.

- **Merging Bifurcation**: Transition from $2n$- to n-cyclic chaotic attractor via pairwise merging due to the homoclinic bif. of a repelling cycle with negative eigenvalue, located at the immediate basin boundary.

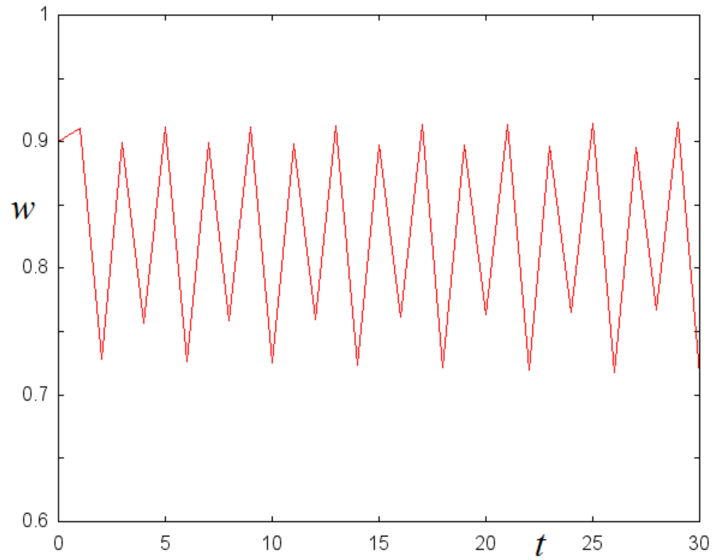
- **Expansion Bifurcation:** The attractor discontinuously increases in size due to the homoclinic bif. of a repelling cycle with positive eigenvalue (suf. cond.).

Some Trajectories

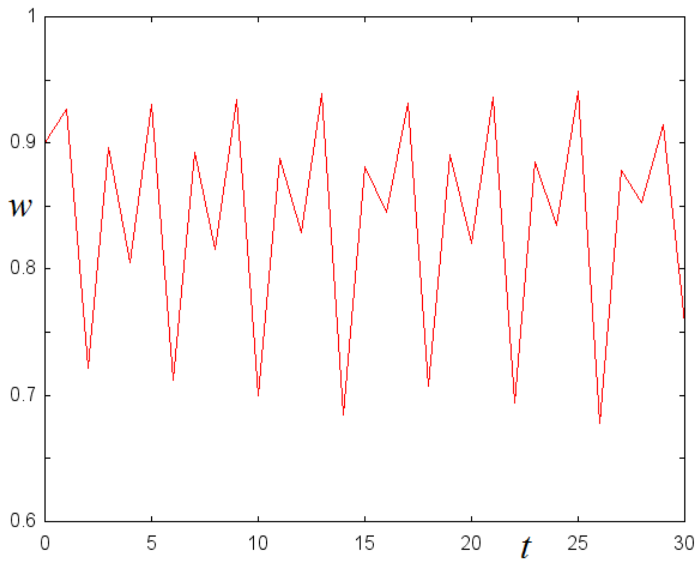
μB=0.2 (2-cycle), with w_0 = 0.9



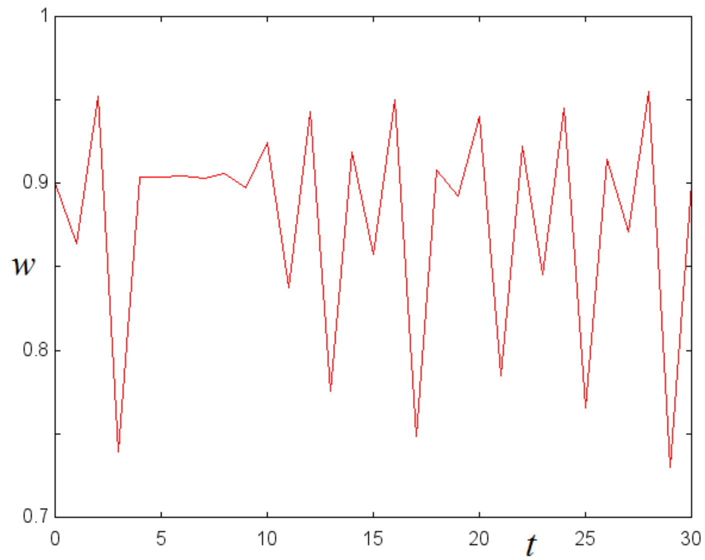
.μB = 0.125 ($G_{2,4}$), with w_0 = 0.9.



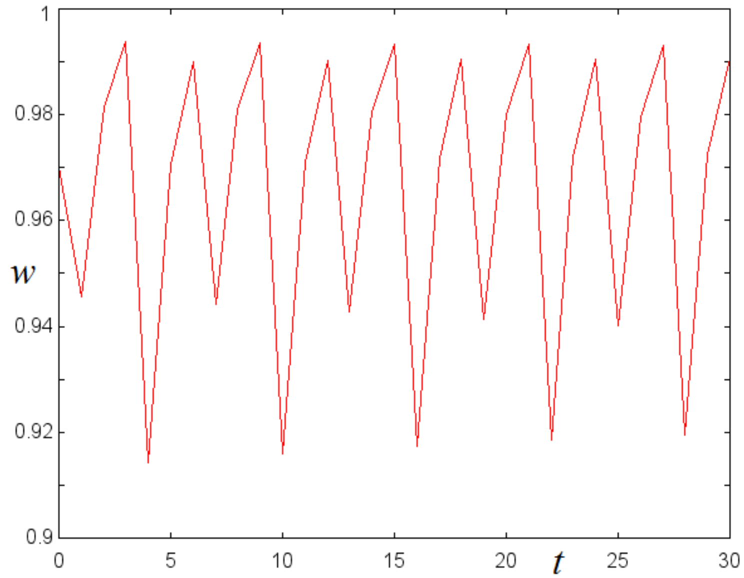
μB = 0.1125 ($G_{2,2}$), with w_0 = 0.9.



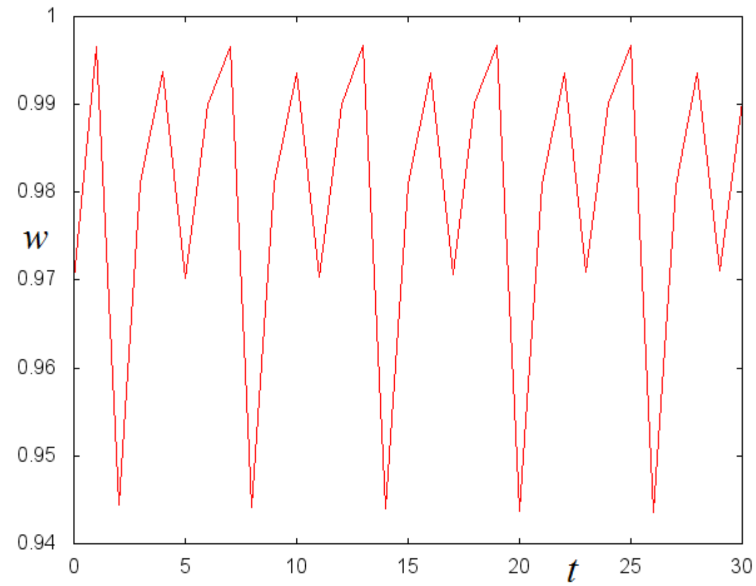
μB = 0.085 (G_1), with w_0 = 0.9.



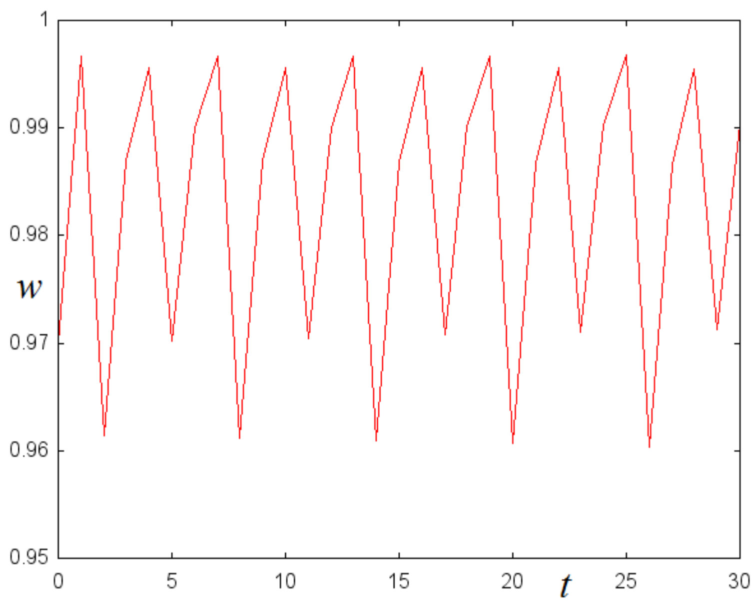
$\mu B = 0.032$ (3cycle), with $w_0 = 0.97$.



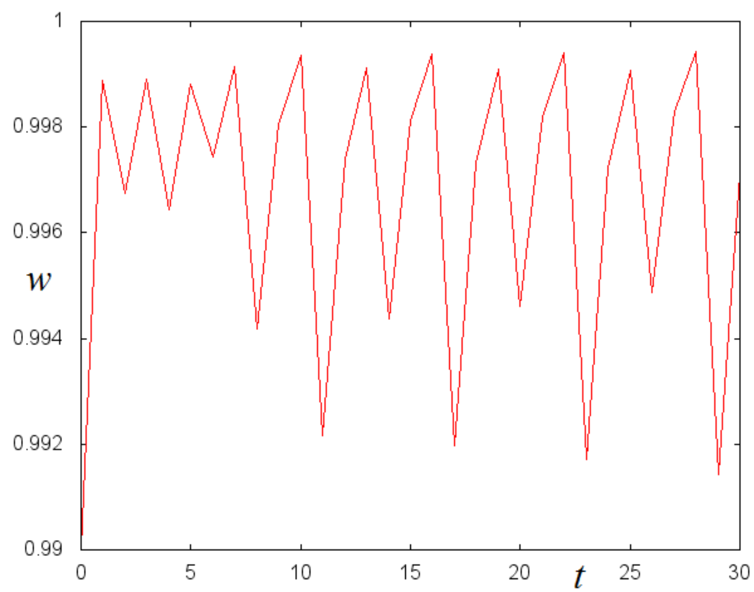
$\mu B = 0.0285$ ($G_{3,6}$), with $w_0 = 0.97$.



$\mu B = 0.0275$ ($G_{3,3}$), with $w_0 = 0.97$.



$\mu B = 0.0245$ (G_1), with $w_0 = 0.99$.



5. Concluding Remarks

Revisiting the model of endogenous credit cycles with Good & Bad projects (Matsuyama TE 2013 Sections 2-4)

- A much simpler presentation of the key mechanisms
- Detailed analysis of the nature of fluctuations under Cobb-Douglas
  - Subcritical flip bifurcation of the Steady State (SS)
    - ***Corridor Stability:*** SS stable against small shocks, unstable against large shocks

Message: Caution for studying nonlinear dynamic models by linearizing around SS!

    - ***Catastrophic and Irreversible*** transitions

Message: Even a small temporary shock could have a large permanent effect on volatility

  - An ***Immediate*** transition to ***Asymmetric Stable Cycles***

Message: Slow recovery followed by a quick recession

  - An ***Immediate*** transition to ***Robust*** Chaotic ***Attractors***

Message: These features, unique to "regime-switching" models, make chaos more relevant than those generated by smooth maps.

Since many nonlinear economic models are of "Regime-Switching" types, the techniques used here should have wide applications.