Strategic Complementarities: Some Macroeconomic Perspectives

By

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Some Static Games of Strategic Complementarities **(SC)** in Reduced Form

Example 1:

Continuum of Identical Agents choose $k \in R_+$, given the average choice of the others, k^e, to maximize

$U(k, k^e)$, $U_{11} < 0$, $U_{12} > 0$

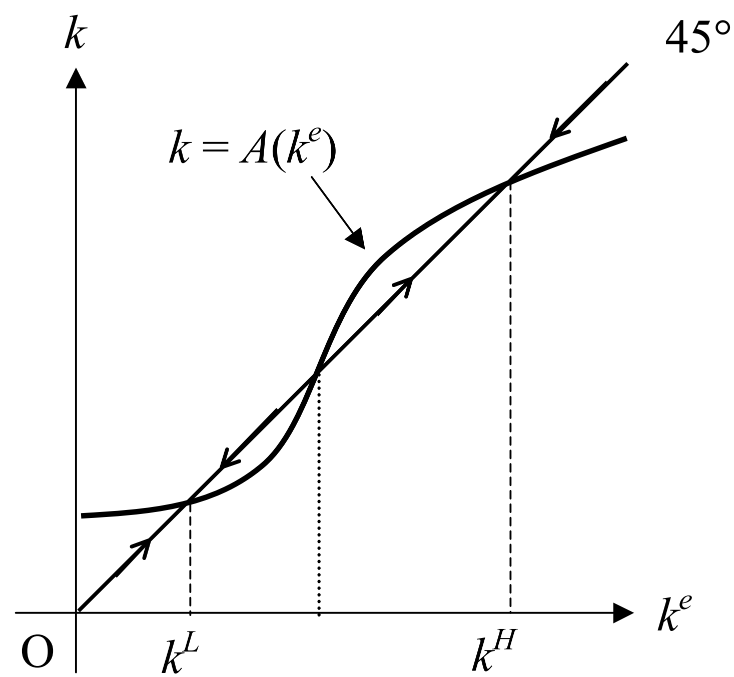
FOC: $U_1(k, k^e) = 0$

→ Best Response:

$k = A(k^e)$ increasing in k^e.

→ Nash Equilibrium

$k^* = A(k^*)$.



Example 2:

Continuum of agents, indexed by j, make a binary choice between {0, 1}.

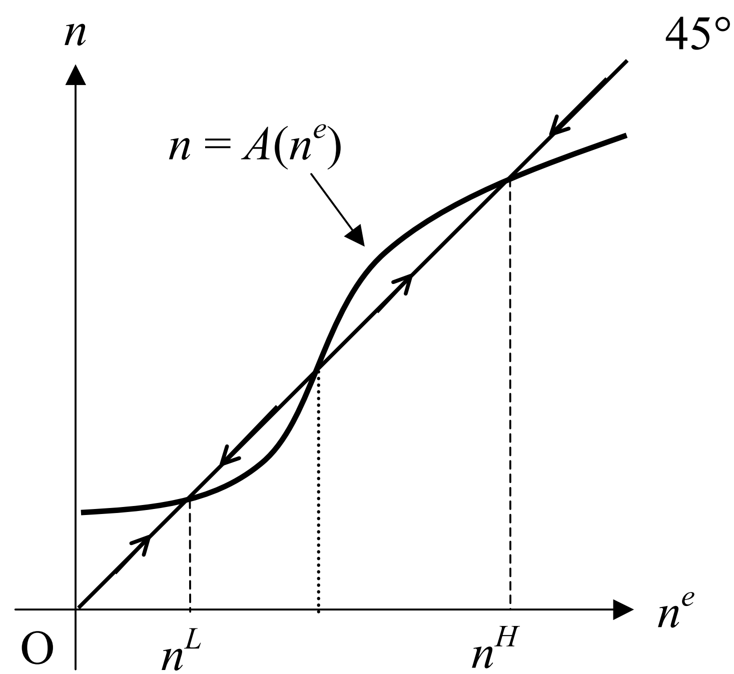
$U_j = \Pi(j, n^e)x_j$;

- x_j : probability with which 1 is chosen instead of 0.
- n^e : the measure of the agents choosing 1.
- $\Pi(j, n^e)$ is strictly decreasing in j, and strictly increasing in n^e.

Define $n = A(n^e)$ by $\Pi(n, n^e) = 0$.

→ Nash Equilibrium

$n^* = A(n^*)$.



Plan of the Talk

1. A VERY QUICK overview of macro applications of SC
2. Macroeconomic (a.k.a. **Dynamic, General Equilibrium**) Perspectives:

- Interactions between SC and GE Resource Constraints
  - Application: Regional and Cross-Country Inequality
  - Application: Business Cycles; Temporal Agglomeration
- Dynamics as a Selection Mechanism
- Multi-sector (high-dimensional) models

3. Macro (and Development) Policy Implications

- Do SC justify policy activism?
- Do SC-based multiple equilibriums imply Big Push?

A Quick Overview of Macro literature, classified by the sources of SC.

- *Intersectoral Investment Demand Spillovers;*

Rosenstein-Rodan, Murphy-Shleifer-Vishny (JPE 1989), Matsuyama (JEL 1995), etc.

- ***Market Size and Specialization;***

Adam Smith, Young (EJ 1928), Romer (AER, 1987), Matsuyama (JEL 1995), Rodriguez-Clare (JDE1996), Ciccone-Matsuyama (JDE 1996, Ecta 1999), etc.

- *Search and Trading Externalities*

Diamond (JPE 1982), Diamond-Fudenberg (JPE 1989), Kiyotaki-Wright (JPE 1989), Matsuyama-Kiyotaki-Matsui (REStud 1993) etc.

- *Development of Financial Markets and Economic Development*

Saint-Paul (EER 1992), Acemoglu-Zilibotti (JPE 1997), etc.

- *Credit Traps*

Bernanke-Gertler (AER 1989), Banerjee-Newman (JPE 1993), Matsuyama (Ecta 2004), etc.

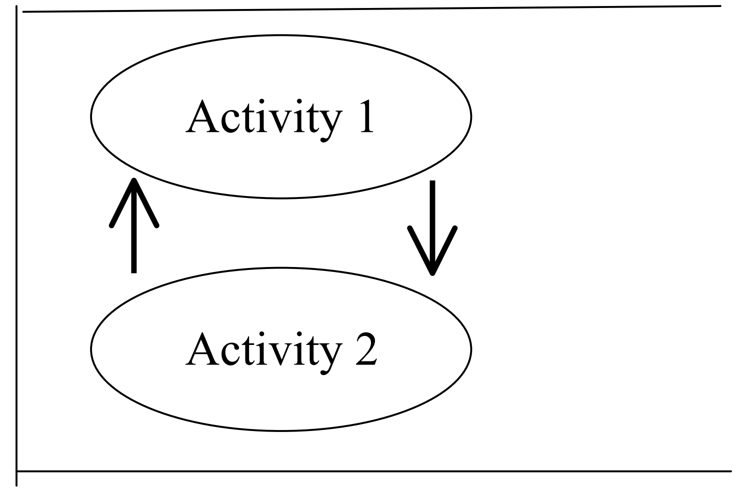
And many, many more

Prominent Features of Macro Complementarity Models (in contract to IO)

Macro	IO
(Mostly) General Equilibrium	(Mostly) Partial Equilibrium
(Frequently) Dynamic	(Frequently) Static
(Mostly) Anonymous Games with infinitesimal players	(Mostly) Games with a few large players
(Occasionally) Multi-sector	(Rarely) Multi-sector

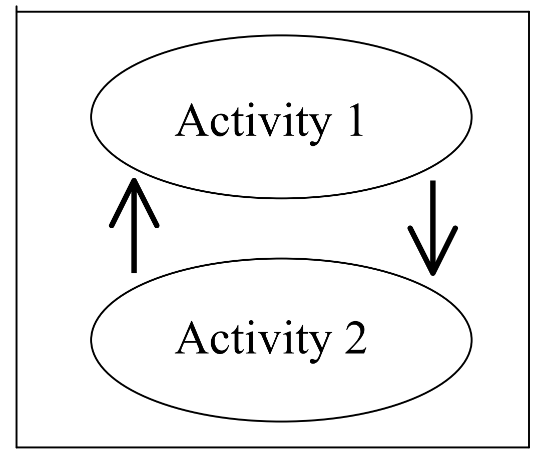
Interactions between SC and GE Resource Constraints

In Partial Equilibrium,
the economy is an open system.



Mutually complementary activities can
expand by drawing more resources from
the rest of the economy.

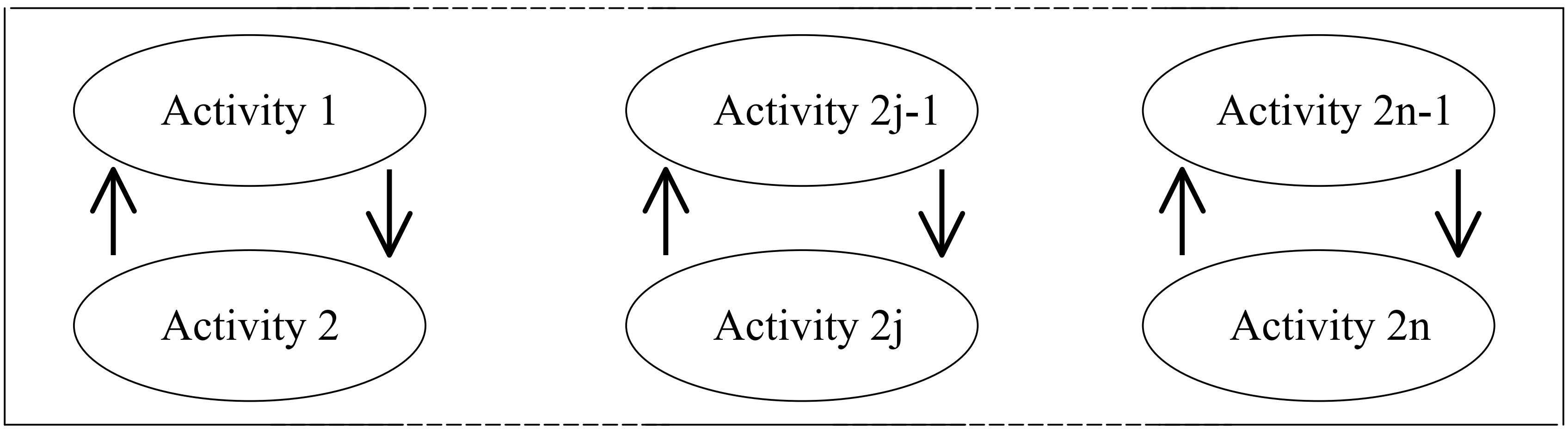
In GE, the economy is a closed system.
(Box indicates the resource base of the economy.)



With GE resource constraint, and different
activities competing for the use of the resource,
an expansion in one activity has to come at the
expense of other activities.

For multiple equilibriums, positive feedback
generated by SC must be strong enough to overcome

What if the scope of SC is different from the scope of GE resource constraint?



Local Positive Feedback and **Global** Resource Constraint

A group of mutually complementary activities can expand (only at the expense of other groups).

Multiple equilibriums even if positive feedback is relatively weak.
All stable equilibria imply variations across groups.

I will illustrate this point with a concrete example.

Market Size and Specialization

Adam Smith: "Division of labor is limited by the extent of the market."
Allyn Young; "The extent of the market is also limited by the division of labor."

Free Entry Game Based on Monopolistic Competition Model a la Dixit-Stiglitz

- **Competitive final good industry**; Y = C = F(X, H)

  H; direct labor input, the numeraire

  $$X = \left\{ \int_0^n [x(z)]^{1-1/\sigma} dz \right\}^{\sigma/(\sigma-1)} \ (\sigma > 1); \quad P = \left\{ \int_0^n [p(z)]^{1-\sigma} dz \right\}^{1/1-\sigma}$$

  n: active number of input producers (product variety, specialization)

- **MC Input Producers**; to produce x(z) requires F + (1–1/σ)x(z) units of labor

- **Labor Market**: $L = H + nF + (1-1/\sigma) \int_0^n [x(z)] dz$

In symmetric allocation, $x(z) = x$, and $p(z) = p$,

$X/(nx) = n^{1/(\sigma-1)}$, increasing in n.

$P/p = n^{1/(1-\sigma)}$, decreasing in n.

→ **Productivity gains from division of labor (specialization), potential for SC.**

$L = H + n[F + (1-1/\sigma)x]$

→ **GE Resource Constraint**

For a given n, one can calculate the profit of MC firms, $\pi(n)$.

$\pi(n) > 0 \rightarrow$ entry $\quad \pi(n) < 0 \rightarrow$ exit.

In equilibrium, $\pi(n) = 0$.

Are there multiple equilibriums? **Depends!!**

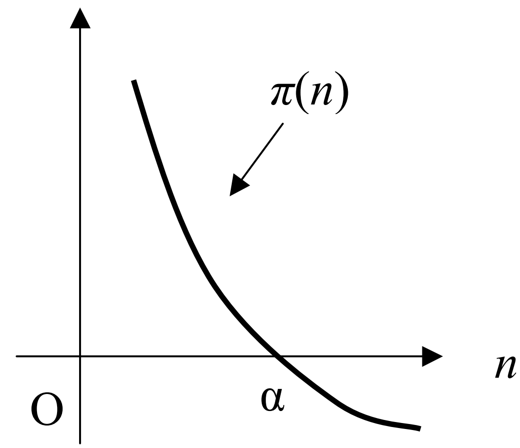
whether $\pi(n)$ increasing or decreasing in n

or

whether entry is Strategically Complementary (SC) or Substitutes

Case I: $F(X, H) = AX^{\alpha}H^{1-\alpha}$.

$\pi(n) = (\alpha L/n - \sigma F)/(\sigma - \alpha) \rightarrow n = \alpha L/\sigma F$.



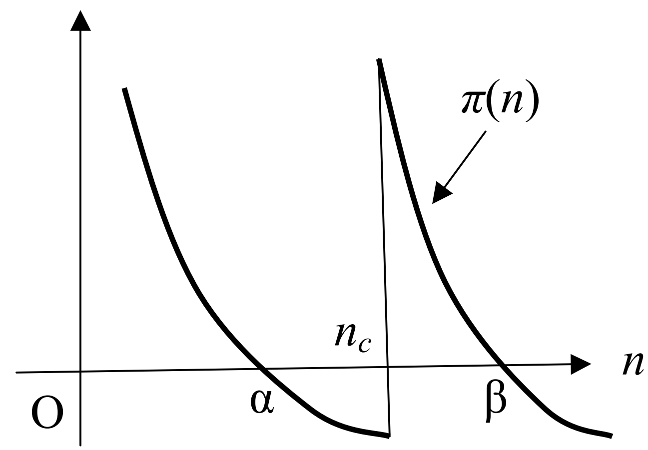
No multiple equilibrium.

GE resource constraint is stronger than SC in the entry decision.

Case II: *Any* convex combination of $AX^{\alpha}H^{1-\alpha}$ and $BX^{\beta}H^{1-\beta}$ $(0 \le \alpha < \beta \le 1)$.

$$F(X, H) = \text{Max } \{A(X_\alpha)^{\alpha}(H_\alpha)^{1-\alpha} + B(X_\beta)^{\beta}(H_\beta)^{1-\beta}\}$$
$$\text{s.t } X_\alpha + X_\beta \le X,\ H_\alpha + H_\beta \le H,\ X_\alpha \ge 0,\ X_\beta \ge 0,\ H_\alpha \ge 0,\ H_\beta \ge 0.$$

$$\pi(n) = \begin{cases} (\alpha L/n - \sigma F)/(\sigma - \alpha) & \text{if } n < n_c \\ (\beta L/n - \sigma F)/(\sigma - \beta) & \text{if } n > n_c \end{cases}$$



multiple equilibria iff $\alpha L/\sigma F < n_c < \beta L/\sigma F$.

Entry decisions are SC

Limited Market Size ↔ Limited Specialization

A *Naive* Approach to explain Cross-Country Differences;

Some countries are in α-equilibrium, with low output, low income, low TFP,
while other countries are in β-equilibrium, with high output, low income, high TFP.

This argument implicitly assumes that each country is a closed system, independent, autarky.

Why can all countries be in β-equilibrium?
More sophisticated approach can answer this criticism.

Inequality in the Global Economy; Matsuyama (JEL 1995)

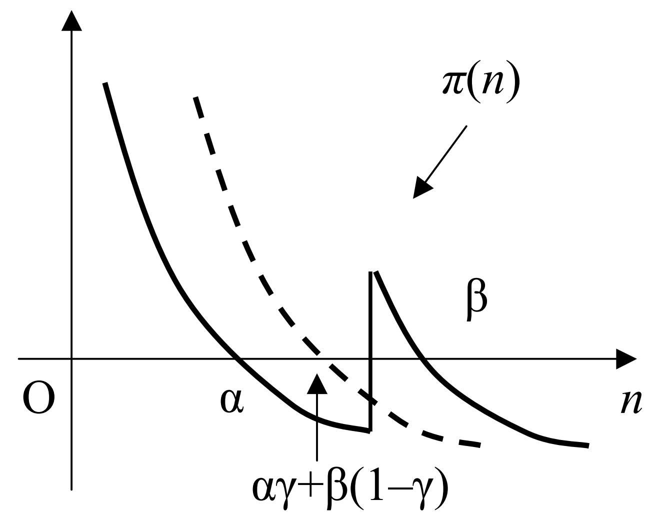
Two consumption goods, **α-good**, and **β-good**, with the preferences, $(C_\alpha)^\gamma(C_\beta)^{1-\gamma}$.
α-good is produced by $AX^\alpha H^{1-\alpha}$ and β-good is produced by $BX^\beta H^{1-\beta}$.

Autarky: like Case I (a single good produced with Cobb-Douglas with $\alpha\gamma+\beta(1-\gamma)$.

Small Open Economy:
(α-good, and **β-good** are traded at an exogenously given relative price; the inputs are nontraded.)

just like Case II,

The economy specializes, but we can't say which!



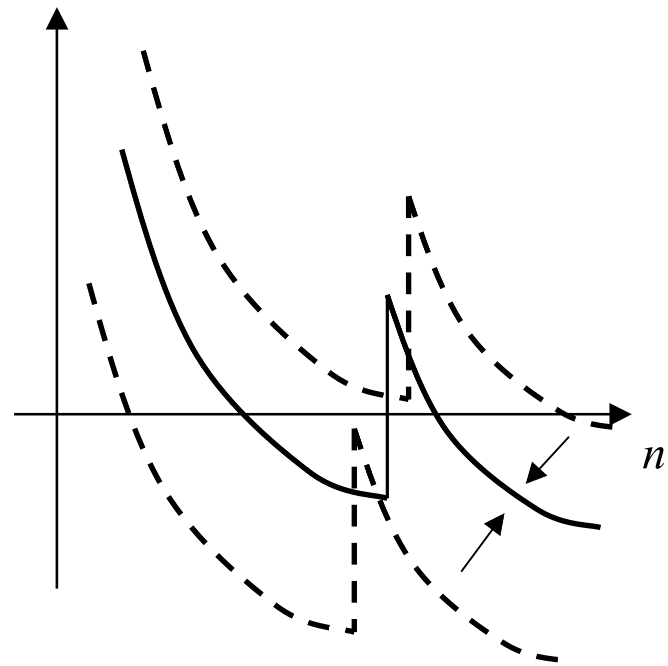
World Economy: (the relative price of **α-good** and **β-good** is now *endogenous*)

It is impossible that all economies produce the same good.
Some economies *must* produce α-good, while others produce β-good.

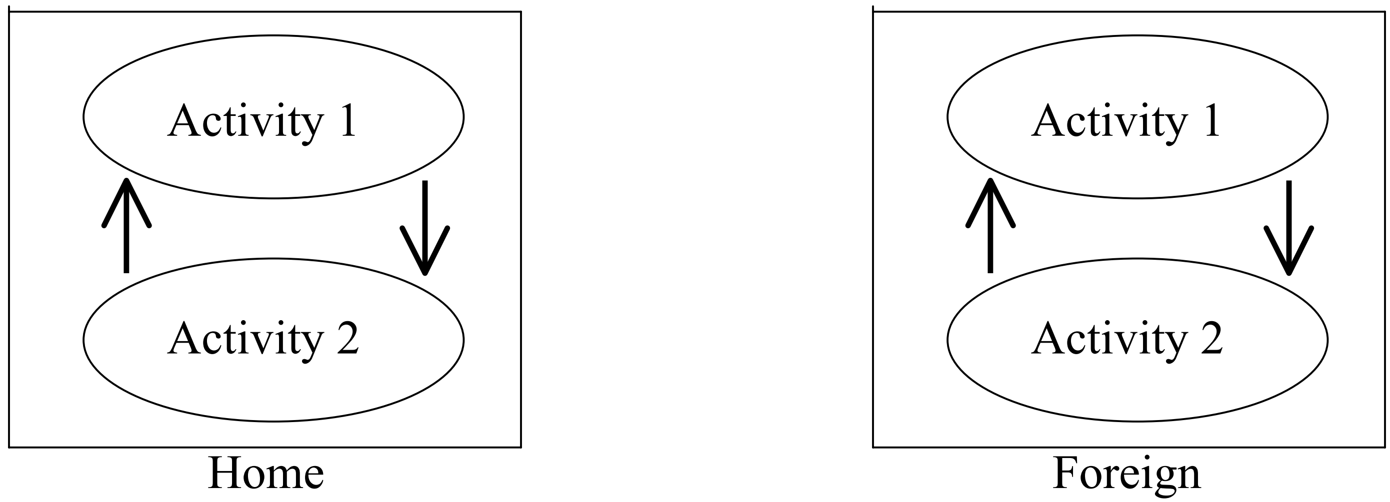
Why?

If very few countries produce β-good, the relative price of β-good goes up, eliminating α-equilibrium.

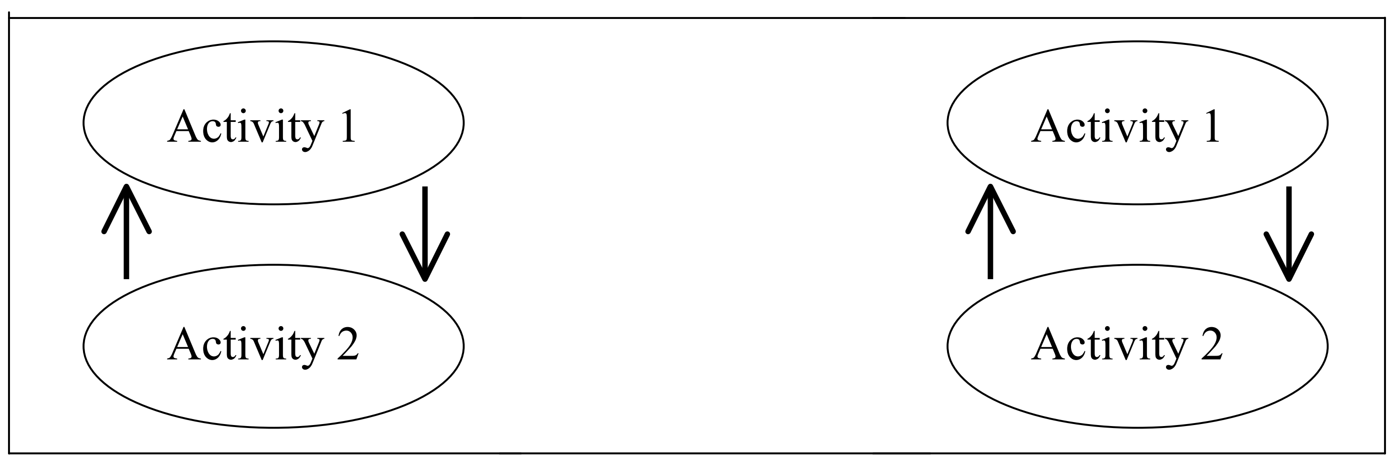
If very few countries produce α-good, the relative price of α-good goes up, eliminating β-equilibrium.



In the Absence of Globalization



In the Presence of Globalization



Local Positive Feedback & *Global* Resource Constraints

- Entry is SC only within the same country
- Resource Constraint applied globally.

→ Intraregional Complementarities & Interregional Substitutions

→ Self-Organized (a.k.a. Endogenous) Inequality and Patterns of specialization

World as "System;" regional economies its "Components"

See Fujita, Krugman, and Venables (1999).

Applications to Business Cycles; Temporal Agglomeration

A *Naive* Approach: Replication of a static GE model of SC.

Economy plays a High equil. in some periods and a Low equil. in others, High is interpreted as a boom, and Low as a recession.

Fluctuations are realized by some coordination devices (sunspots).

The argument implicitly assumes that there is little interconnection across periods.

Why does the economy have to jump between different equilibriums?
Why can the economy always stay in a High equilibrium, generating a permanent boom?
Why can the economy always stay in a Low equilibrium, generating a permanent recession?

Need for a more sophisticated approach, which is immune to these criticisms.

Shleifer (JPE 1986), Aghion and Howitt (Ecta 1992), Gale (REStud 1996), Matsuyama (Ecta 1999), Francois-Lloyd-Ellis (AER 2003) etc.

Players choose the timing of investment/innovation

Intratemporal Complementarities and Intertemporal Substitution
→ Temporal Agglomeration and Variations

With a high intertemporal substation, only small complementarities are necessary to generate cycles.

Analogy with

Intraregional Complementarities and Interregional Substitution
→ Spatial Agglomeration and Variations

See Matsuyama (AER 2002, New Palgrave 2005); Symmetry-Breaking

What would happen in the case of Intertemporal Complementarity?

Dynamics as a Selection Mechanism

In the presence of ***Intertemporal Strategic Complementarity***,

Can ***History*** (past action) dictate the current and future actions?

Or

Can the current actions be affected by the ***Expectations*** (of future actions)?

A Small Open Economy with Two Sectors: CRS Agriculture and IRS Industry

Static Model:

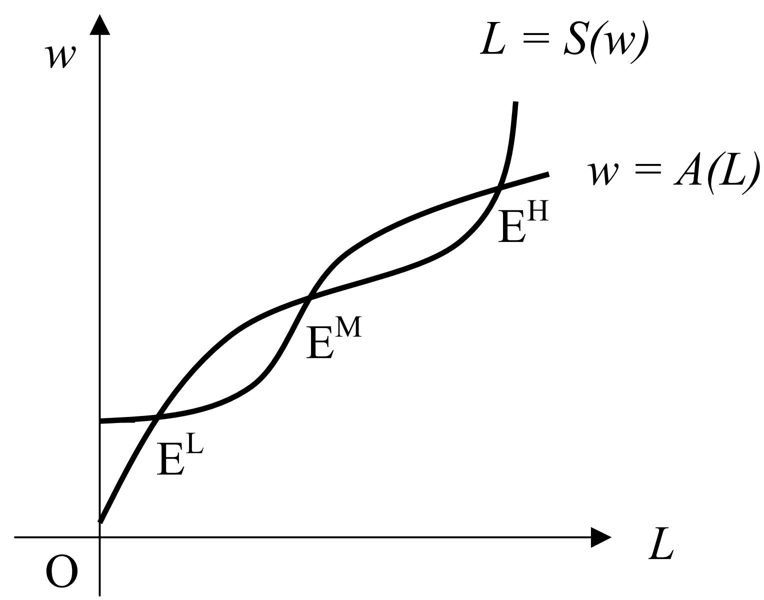
Agriculture; Labor productivity is one
Industry; $y = A(L)\ell$,; y and ℓ are the output and the labor input of each firm
A(L); labor productivity, increasing in L, the total labor input in Industry.

Labor Supply in Industry: increasing in the relative wage, L = S(w)

(1) $w = A(L)$

(2) $L = S(w)$

Can Dynamics help to select?



Version I; Learning-By-Doing

(3) $w_t = A(Q_t)$, where $Q_t \equiv \delta \int_{-\infty}^{t} L_s \exp[\delta(s-t)]ds$ is the cumulative employment.

By differentiating Q_t,

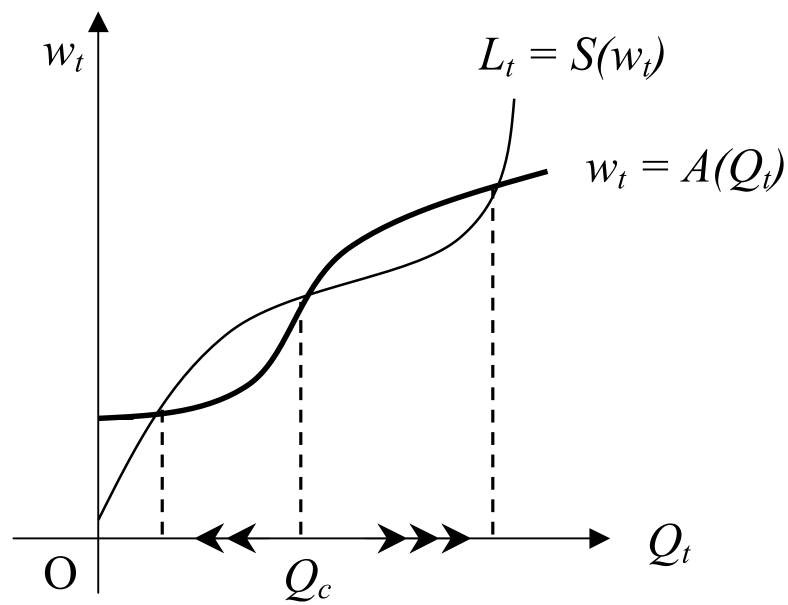
(4) $\dot{Q}_t = \delta\{L_t - Q_t\} = \delta\{S(w_t) - Q_t\} = \delta\{S(A(Q_t)) - Q_t\}$.

(5) $L_t = S(w_t)$.

Q_0 is given.

History dictates the outcome.

E^L becomes a Poverty Trap.



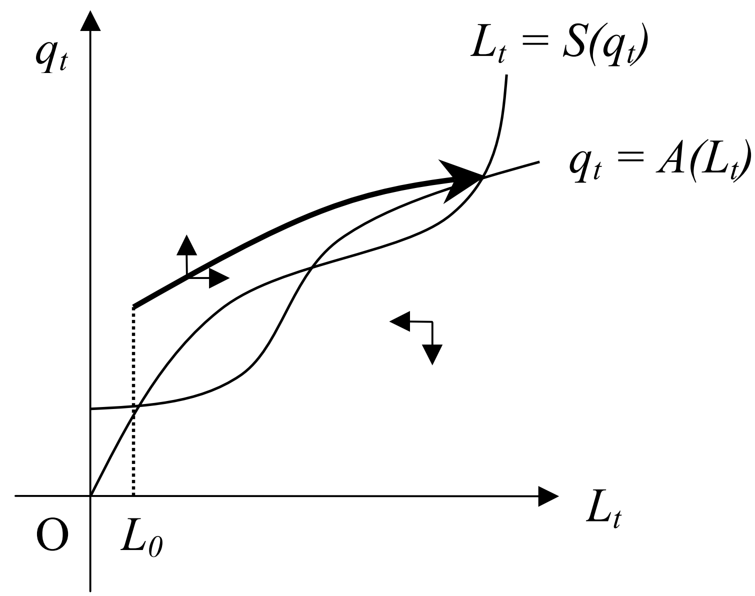
Version II; Irreversible Sectoral Choices (Matsuyama QJE 1991)

(7) $w_t = A(L_t)$.

The workers die at the rate equal to λ, replaced by the new workers of the same size. The new workers must make irreversible sectoral choices in a forward looking way.

(8) $\dot{L}_t = \lambda[S(q_t) - L_t]$, $\quad q_t \equiv (r+\lambda)\int_t^{\infty} w_s \exp[(r+\lambda)(t-s)]ds$

$\lambda S(q_t)$: gross inflow
λL_t: gross outflow
q_t; expected discounted future wage
r: pure discount rate.



(9) $\dot{q}_t = (r+\lambda)[q_t - A(L_t)]$.

Self-Fulfilling Expectations!!

See also Matsui-Matsuyama (JET 1995), Hofbaufer-Sorger (JET 1999), Burdzy, Frankel, Pauzer (Ecta 2001), Frankel-Pauzer (QJE 2000), Oyama (JET 2002)

Some Multi-sector (High-dimensional) Issues:

Imagine a multi-sector economy with intra-sectoral SC and inter-sectoral resource constraint.

Learning-By-Doing Model with Many Industries (j =1, 2,…,J)

$L^j_t = A^j(Q^j_t)Y^j_t$,

L^j : Employment in j.

$A^j(Q^j)$: Unit labor requirement as a decreasing function of Q^j, cumulative experience in industry j, following $\dot{Q}^j_t = \delta\left(L^j_t - Q^j_t\right)$.

The state space; J-dimensional; $\mathbf{Q_t} = [Q^j_t]$.

Case I:

Suppose that all the industries produce the perfect substitutes.

$Y_t = \sum_j Y^j_t$; $L = \sum_j A^j(Q^j_t) Y^j_t$

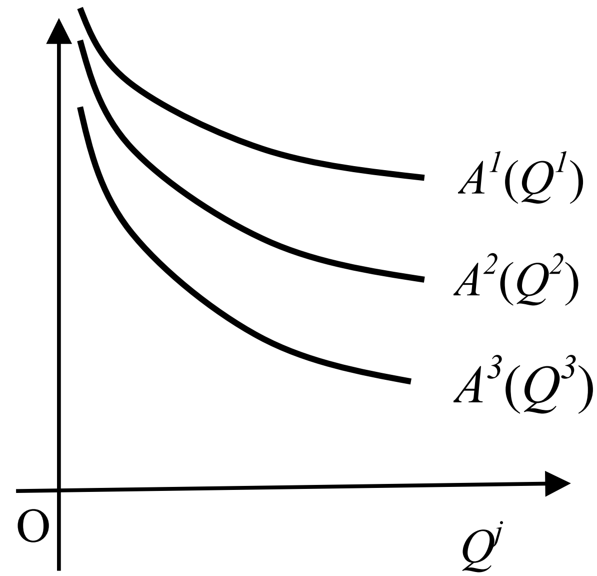
$L^j_t = A^j(Q^j_t) Y^j_t > 0$ only if $A^j(Q^j_t) = \min_k \{A^k(Q^k_t)\}$.

$w_t = 1/\min_k \{A^k(Q^k_t)\}$;

Ex: $A^j(x) = \lambda^{j-1} A(x)$; A is decreasing, and $\lambda < 1$.

There are J-stable steady states.

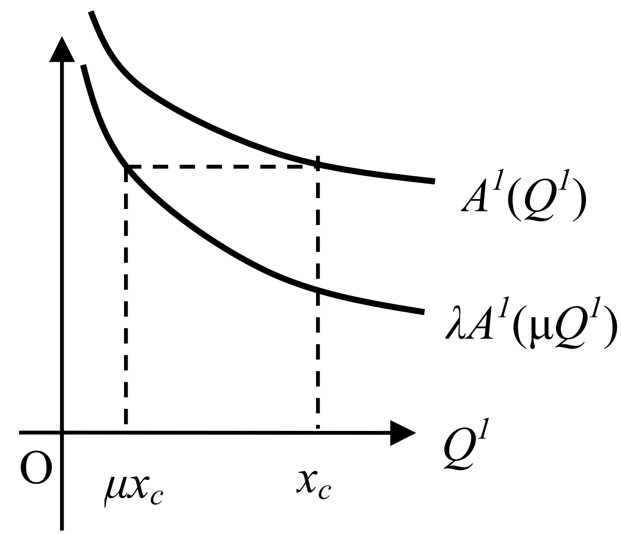
$Q = (0, 0, \ldots, L, 0, \ldots, 0)$.



What happens if there are some interindustry complementarities as well?

Case II: *Inter-Industry Spillovers;* Stokey (JPE 1988); Lucas (Ecta 1993)

Two Industries; $L^1_t = A(Q^1_t)Y^1_t$;
$L^2_t = \lambda A(\mu Q^1_t + Q^2_t)Y^2_t$, with $\mu < 1$.



A is strictly increasing
$\lambda A(\mu x)/A(x)$ is decreasing in x
$\lambda A(\mu x_c)/A(x_c) = 1$.
(e.g., $A(x) = 1+\alpha/x$ and $\mu < \lambda < 1$.)

If the economy starts with $0 < Q^1_t < x_c$ and $Q^2_t = 0$.

For $L < x_c$, trapped in Industry 1.
For $L > x_c$, successful transition from Industry 1 to 2.

Infinite Industries: $L = \sum A^j_t Y^j_t = \sum \lambda^j A(\mu Q^{j-1}_t + Q^j_t)Y^j_t$, with $\mu < \lambda < 1$.

For $L > x_c$, transition from 1 to 2, then from 2 to 3, then from 3 to 4, ….

The structure of stable steady states may be simplified by adding complementarities across industries.

Case III: *Demand Complementarities;* Matsuyama (JPE 2002)

J-consumption goods, ordered by the priority (or hierarchical needs)

The consumers buy one unit of Good 1 first, then one unit of Good 2, and then one unit of Good 3, and so on, as long as they can afford.

F() is the distribution of the purchasing power across households.

Rich consumers buy a more variety of goods.

$$D^j(Q_t) = N\left[1 - F\left(\sum_{k=1}^{j} A^k(Q_t^k)\right)\right]$$

$$\dot{Q}_t^j = \delta\left(D_t^j(Q_t) - Q_t^j\right) = \delta\left\{N\left[1 - F\left(\sum_{k=1}^{j} A^k(Q_t^k)\right)\right] - Q_t^j\right\}$$

The dynamical system is cooperative (in the sense of Hirsch).
The set of steady states is a lattice; it depends sensitively on F.

Some Macro (Development) Policy Issues

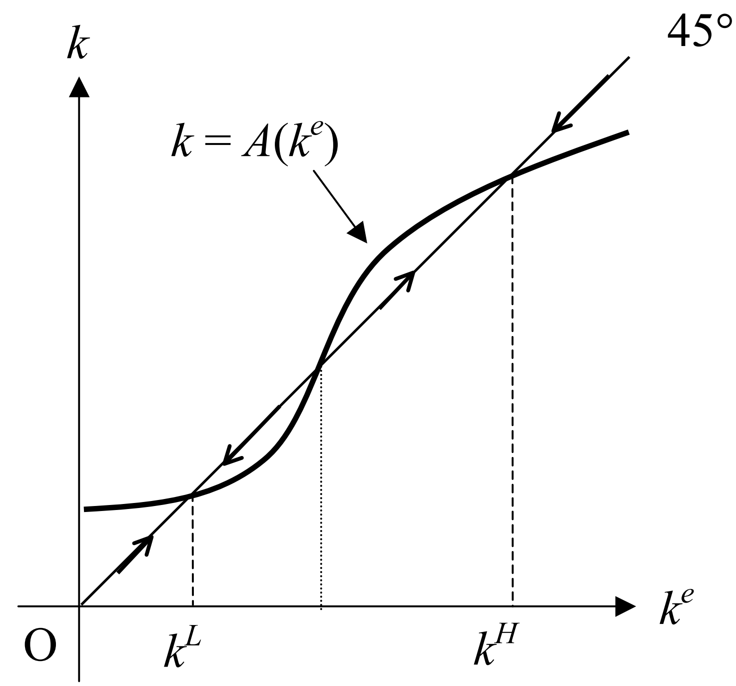
Do Macroeconomic Complementarity Games justify policy activism?

Cooper-John (1988) Argument:

$U_j(k_j; k^e) = A(k^e)k_j - (k_j)^2/2.$

→ For $k^* = A(k^*)$,
$U_j(k^*; k^*) = (k^*)^2/2$, increasing in k^*.

Pareto-rankable multiple equilibria!



Suppose instead

$$\underline{U}_j(k_j;\, k^e) = A(k^e)k_j - (k_j)^2/2 + B_j(k^e).$$

The same best response; the same set of equilibria.

→ For k* = A(k*),

$$U_j\,(k^*;\, k^*) = (k^*)^2/2 + B_j(k^*),$$

which can be any function of k*.

Battle of the Sexes, another SC game with the conflict of interest

Pareto-rankability is an implication of the homogeneity assumption, not of SC.
Welfare → 1st derivative, SC → 2nd derivative.

Do SC-Based Multiple Equilibria Imply Big Push?

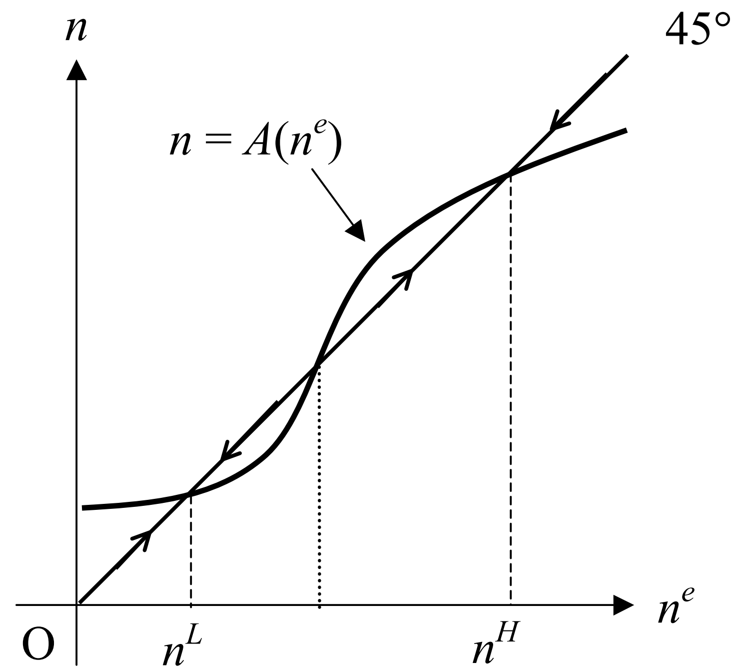
Rosenstein-Rodan (1943)

Theory of Big Push Industrialization

vs.

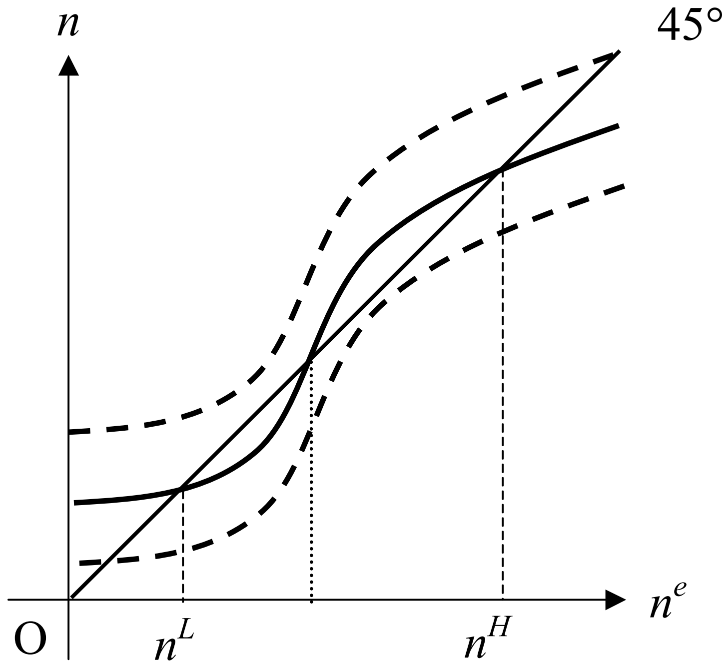
Hirschman (1958);

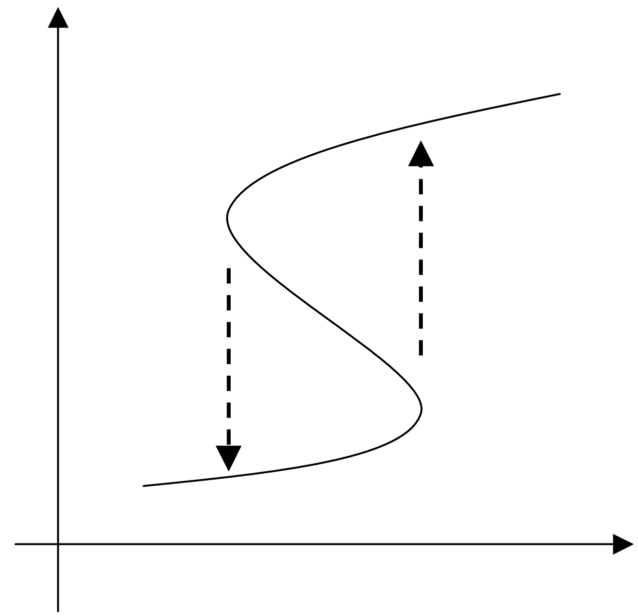
Theory of Linkages and Selective Intervention



Chain Reactions and Domino Effects

Nonlinearity and Sensitive Dependence





Complex Coordination Problems; see Matsuyama (1996)

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