A General Theory of Competitive Trade

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Very Preliminary; (Permanently) Work in Progress

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Why Study "International" Economics?

- It is misleading to analyze the World Economy as a mere collection of the national economies, as if each of these national economies were an isolated, autonomous closed economy (which is what most naïve cross-country comparative studies assume). There are enough flows of trade, capital, and technologies between national economies.
- However, the integration of these national economies is far from complete, so that it would also be misleading to treat the World Economy as if it were a single closed economy.
- To me, the ultimate goal of international economics is to understand the working of the World Economy, while explicitly recognizing that the World Economy consists of many semi-autonomous sub-systems (nations as well as regions) that are affecting each other.

Obviously, this is a very challenging problem, so we need to make some special assumptions as a first step….

"Standard" Assumptions in International Economics

- The world consists of "nations" or "countries" (a two-level hierarchy).

But, what is a "nation" or "country," anyway?

- Within each nation, the market is fully integrated, so that goods and factors move freely across different activities (sectors).
- Across nations, the market is only partially integrated, so that only a limited set of goods and factors can cross borders (possibly with some costs).

And sometimes, it is assumed that

- Consumers within each nation share the same taste
- Each nation has the government, who only cares about its "national" interest.

"Standard" Assumptions in Neoclassical Trade Models:

1. Perfect Competition; Convex and CRS Technologies; No Externalities
2. All goods that enter in the utility function are produced and tradeable at zero cost
3. All factors of production are in fixed supply and nontradeable.
4. Representative Consumer Within Each Country
5. Static Model (Long-Run View)

Many of these "classical" assumptions are not as restrictive as they seem, as more general cases may be accommodated by a clever reinterpretation of "goods" and "factors."

- DRS can be converted into CRS by defining some "hidden factors" in fixed supply.
- Trade in some factors can be viewed as trade in the good that use those factors only.
- Goods and factors can be distinguished by locations, time, and states of nature.

etc.

Some of these assumptions can also be dropped (often with significant complications in the notation.)

Most "crucial" assumptions are

- Perfect Competition
- No Externalities, No Market Failure
- Absence of Aggregate Increasing Returns

Part I through Part III deal with Neoclassical Competitive Models of Trade.
(Part IV deals with models of trade that drop these crucial assumptions.)

In Part I, we will look at **General Theory** of competitive models.

Some Cautions:

Do not expect to find many useful predictions here. To obtain such predictions, we need to make specific assumptions about the preferences, technologies, etc, which we will do in Part II and Part III.

Instead, what we will find here is a general framework, a kind of "template," where we can find "blanks," that we would have to fill in to get some useful predictions.

At the same time, we also need to understand what kind of restrictions that this template would impose, which is what we are trying to do in Part I.

But, first, let us have a quick review of what we teach to college students.

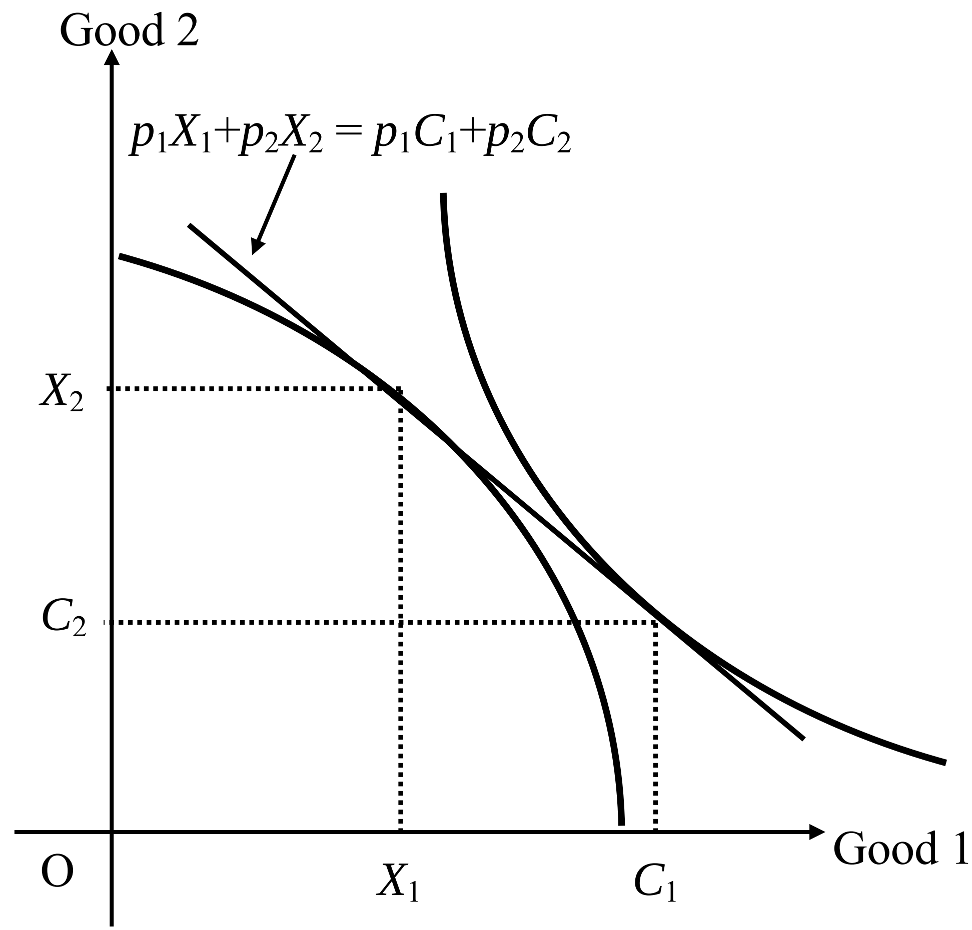
A Graphic Presentation of the Two-Sector Model

Three Key Elements:

Production Possibility Frontier:
Market allocates resources to maximize the value of production within the production possibility set.

Budget Constraint:
The Country faces the Budget Constraint, $p_1X_1 + p_2X_2 = p_1C_1 + p_2C_2$, which implies the Balanced Trade, $p_1(C_1 - X_1) = p_2(X_2 - C_2)$.

Indifference Curves
The representative agent chooses its consumption to maximize its utility subject to the Budget Constraint.

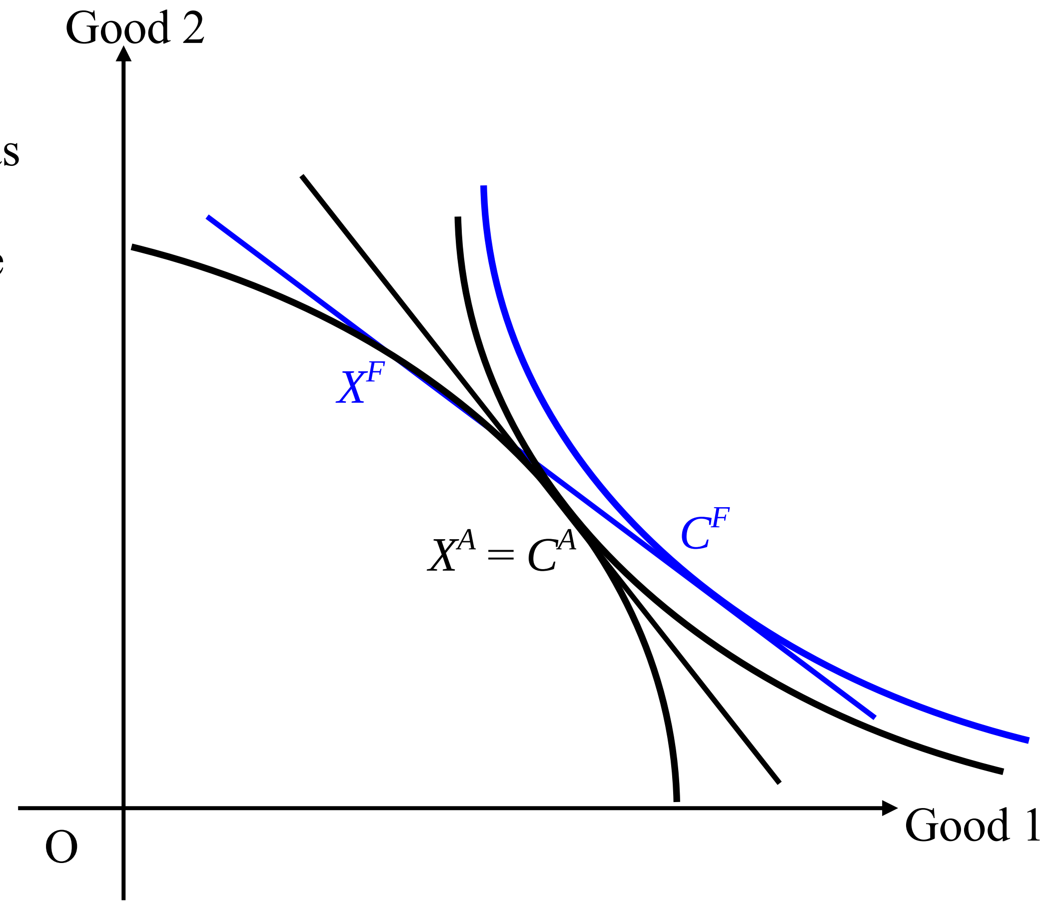


Three Key Results:

Gains from Trade:

The Country enjoys higher utility by moving *from Autarky to Free Trade* (as long as the relative prices under Free Trade do not coincide with the relative prices under Autarky).

Note: It does *not* say that, when the country already trades (restrictively) with the rest of the world (ROW) would benefit from *freer* trade.

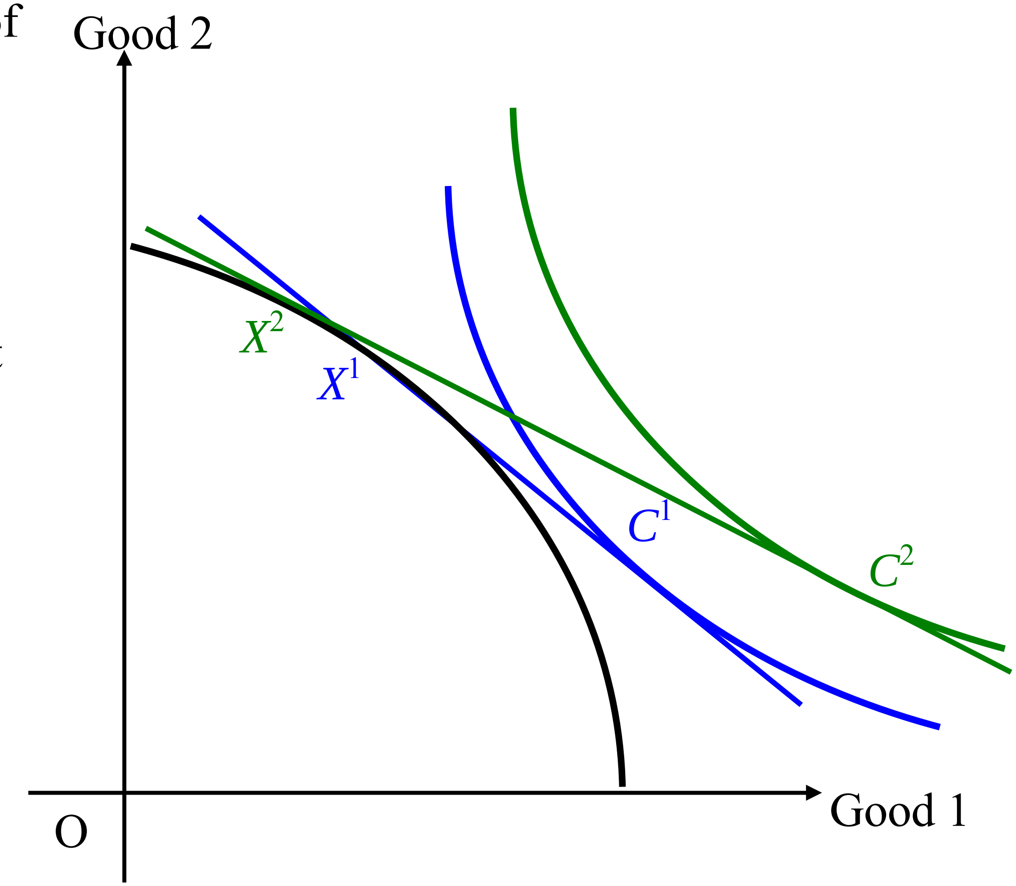


Terms of Trade Effect:

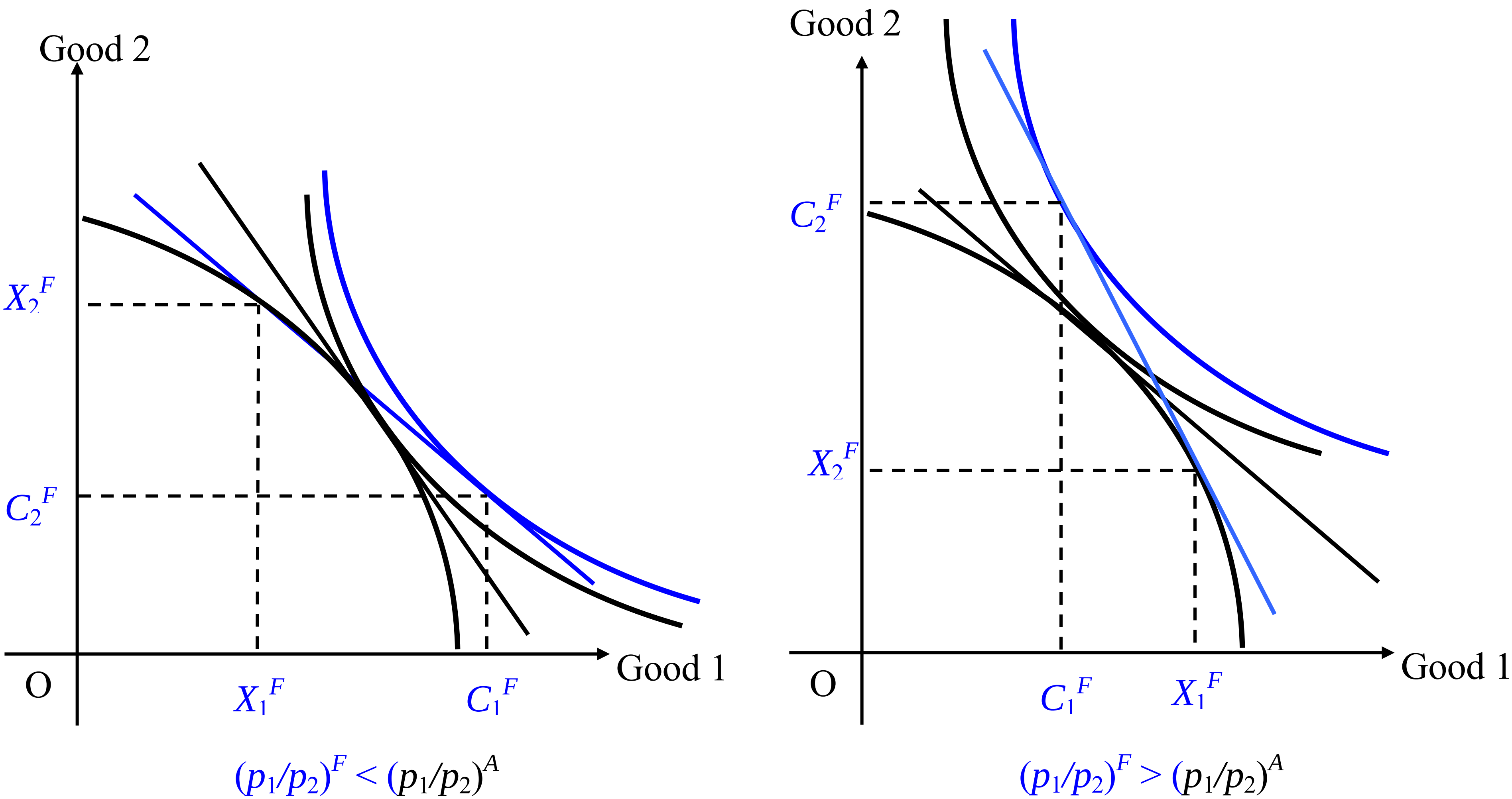
The Country's welfare improves (deteriorates) when the relative price of its Export goods goes up (down).

Corollary:

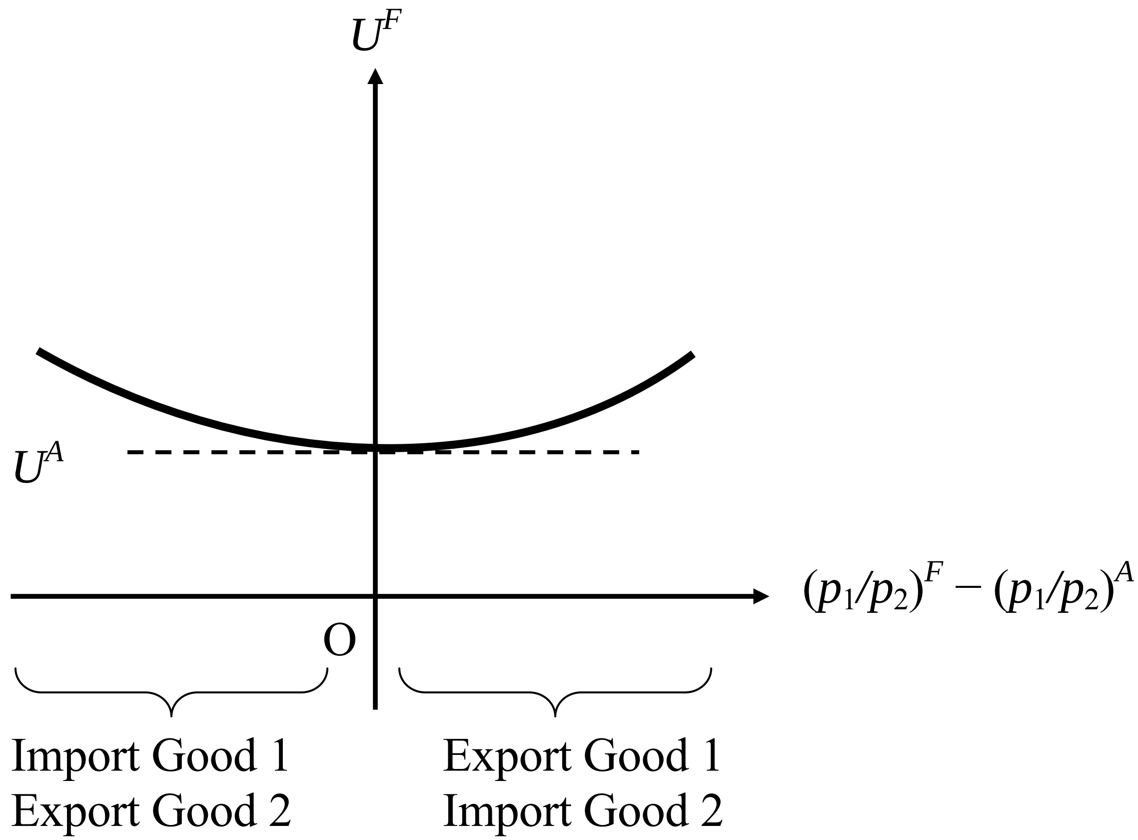
The Country's gains from trade are greater when it trades at the prices that are more different from the autarky prices.



Law of Comparative Advantage: The Country exports (imports) the good whose relative price is lower (higher) in autarky than under free trade.



The Three Key Results Summarized in One Picture



Can these results be extended to more general settings?

General Theory

M (Nontradeable) Factors of Production:

Endowments = Supply:	M-Dim. *Column* Vectors;	$V = (V_1, V_2, \ldots, V_M)^T$
Factor Prices:	M-Dim. *Row* Vectors;	$w = (w_1, w_2, \ldots, w_M)$

N (Tradeable) Commodities Produced

Outputs:	N-Dim. *Column* Vectors;	$X = (x_1, x_2, \ldots, x_N)^T$
Output Prices:	N-Dim. *Row* Vectors;	$p= (p_1, p_2, \ldots , p_N)$

GDP and Total Expenditure: Inner Products

$$Y = pX = p_1x_1 + \ldots + p_Nx_N = wV = w_1V_1 + \ldots + w_MV_M = E$$

Production Possibility Set: $(x, V) \in \Omega$; Ω is a M+N dimensional *convex cone*.

All the information about Nation's Technology can be summarized by

Output and Revenue (GDP) Functions:

$x(p, V) \equiv \text{Argmax}_x \{px \mid (x, V) \in \Omega\}$; Note that x(p,V) may be a set.

$R(p, v) \equiv px(p,V) = \text{Max}_x \{px \mid (x, V) \in \Omega\}$.

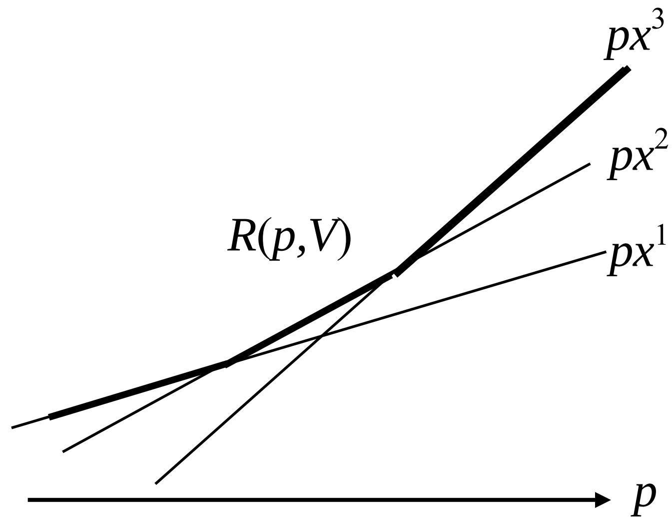
Key Properties:

(R1) : $R(p, V) = px(p,V) \geq px$ for any $(x, V) \in \Omega$.
By definition.

(R2): $(p^1-p^2)(x^1-x^2) \geq 0$ for $x^1 \in x(p^1, V)$ and $x^2 \in x(p^2, V)$.
By definition, $p^1x^1 \geq p^1x^2$ & $p^2x^2 \geq p^2x^1 \rightarrow p^1(x^1-x^2) \geq 0$ & $p^2(x^2-x^1) \geq 0 \rightarrow (p^1-p^2)(x^1-x^2) \geq 0$.
Thus, on average, the outputs are increasing in prices.

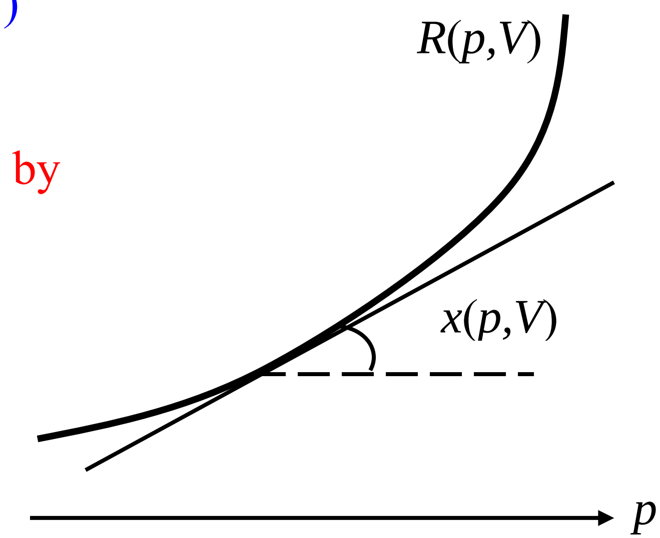
(R3) : R(p,V) is linear homogeneous and convex in p.
(Linear homogeneity): By definition.
(Convexity): let $p^\lambda = \lambda p^1 + (1-\lambda)p^2$. Then, $R(p^\lambda, V) = p^\lambda x(p^\lambda,V) = \lambda p^1 x(p^\lambda,V) + (1-\lambda)p^2 x(p^\lambda,V) \leq \lambda p^1 x^1 + (1-\lambda)p^2 x^2 = \lambda R(p^1,V) + (1-\lambda)R(p^2,V)$.

Intuitively, R(p,V) is convex in p since it is the upper envelope of linear functions, px.



(R4) : If R is differentiable in p, $x(p,V) = R_p(p,V)$.
Differentiating $R(\lambda p, V) = \lambda R(p,V)$ by λ yields $R(p,V) = pR_p(p,V)$.

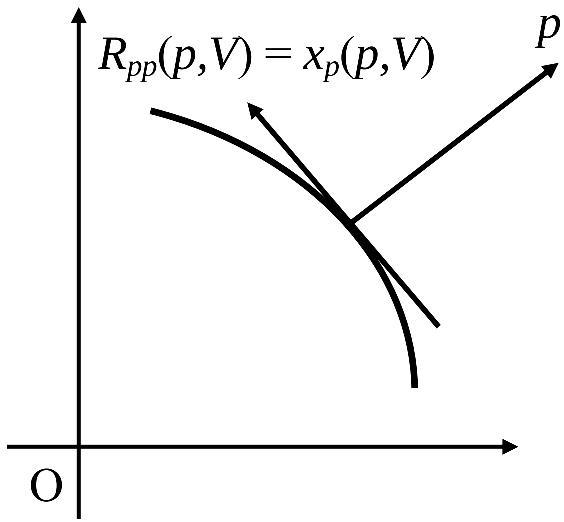
With slight abuse of notation, we often denote x(p,V) by $R_p(p,V)$.



(R5) : If R is twice differentiable in p, $pR_{pp}(p,V) = px_p(p,V) = 0$.

$R_p(p,V)$ is homogeneous of degree zero, $R_p(\lambda p, V) = R_p(p,V)$. Differentiating it by λ yields $pR_{pp}(p,V) = 0$.

$R_{pp}(p,V) = x_p(p,V)$ is an NxN positive semi-definite matrix, with the rank at most equal to N−1.



Later, we will discuss the properties of R(p,V) as a function of V, as well as the conditions ensuring the differentiability of R.

Representative Consumer (endowed with V)

Max u(c) subject to $pc \leq Y = wV = pX$,

where $u: R_+^N \rightarrow R$ is increasing, strictly concave and non-satiated.

Hicksian (Compensated) Demand and Expenditure Function:

$c(p, U) \equiv \text{Argmin}_c \{pc \mid u(c) \geq U\}$;

$E(p, U) \equiv pc(p, U) = \text{Min}_c \{pc \mid u(c) \geq U\}$,

E(p, U) is strictly increasing in U, from which we may define.

Indirect Utility Function: $U = U(p,y) \leftrightarrow y = E(p, U)$.

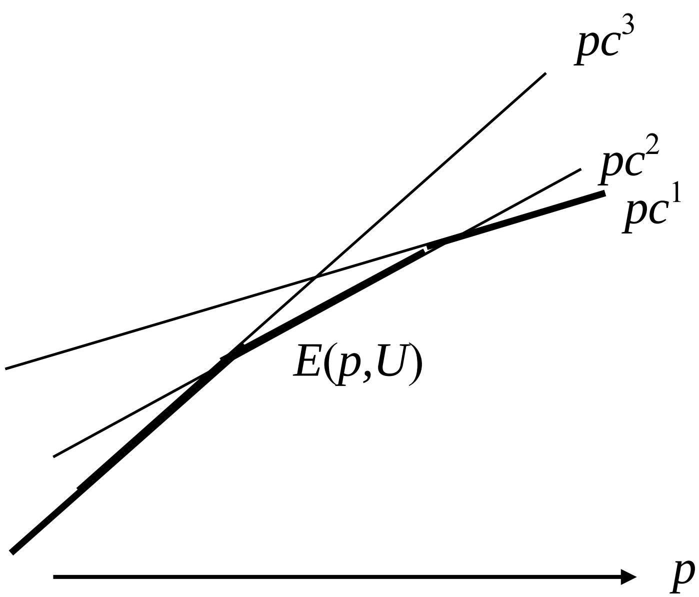
Budget Constraint: $E(p, U) = wV = pX$

Key Properties:

(E1) : $E(p, U) = pc(p, U) \leq pc$ for any $u(c) \geq U$.
By definition.

(E2): $(p^1-p^2)(c^1-c^2) \leq 0$ for $c^1 = c(p^1, U)$ and $c^2 = c(p^2, U)$.

By definition, $p^1c^1 \leq p^1c^2$ & $p^2c^2 \leq p^2c^1 \rightarrow$
$p^1(c^1-c^2) \leq 0$ & $p^2(c^2-c^1) \leq 0 \rightarrow (p^1-p^2)(c^1-c^2) \leq 0$.
Thus, on average, demand are decreasing in prices

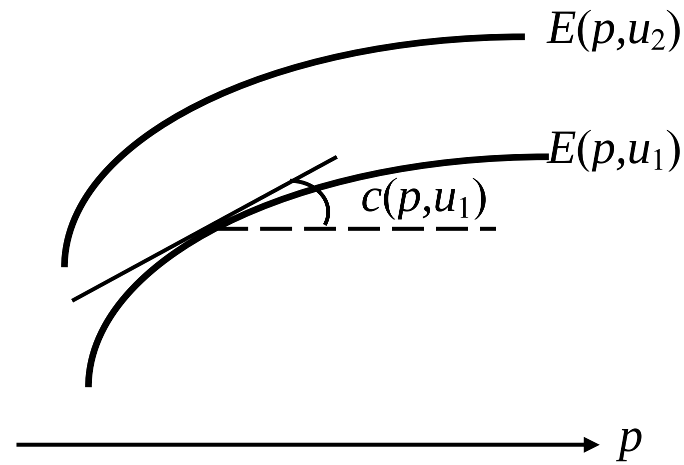


(E3) : $E(p,U)$ is linear homogeneous and concave in p.
(Linear homogeneity): By definition. (Convexity): Let $p^\lambda = \lambda p^1 + (1-\lambda)p^2$. Then, $E(p^\lambda, U) = p^\lambda c(p^\lambda,U) = \lambda p^1 c(p^\lambda,U) + (1-\lambda)p^2 x(p^\lambda,U) \geq \lambda p^1 c^1 + (1-\lambda)p^2 c^2 = \lambda E(p^1,U) + (1-\lambda)E(p^2,U)$.

Intuitively, $E(p,U)$ is concave in p since it is the lower envelope of linear functions, pc.

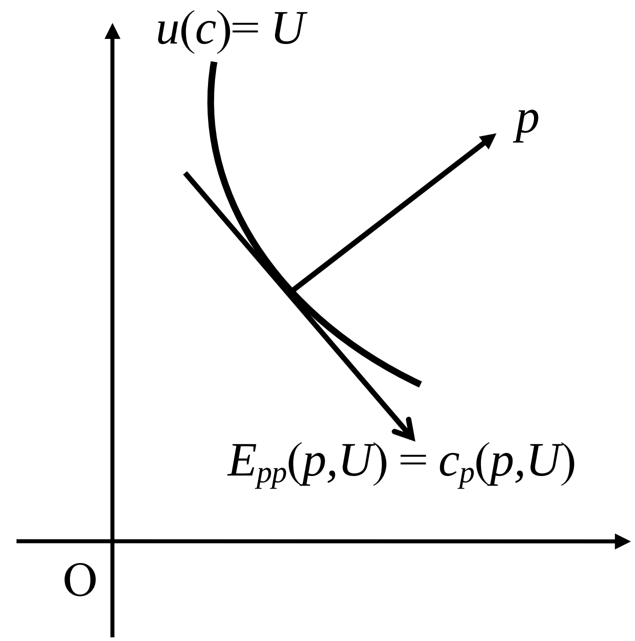
(E4) : If E is differentiable in p, $c(p,U) = E_p(p,V)$.
Differentiating $E(\lambda p, U) = \lambda E(p,U)$ by λ yields $E(p,V) = pE_p(p,V)$.

With slight abuse of notation, we often denote $c(p,U)$ by $E_p(p,U)$.



(E5) : If E is twice differentiable in p, $pE_{pp}(p,U) = pc_p(p,U) = 0$.
$E_p(p,U)$ is homogeneous of degree zero, $E_p(\lambda p, U) = E_p(p,U)$. Differentiating it by λ yields $pE_{pp}(p,U) = 0$.

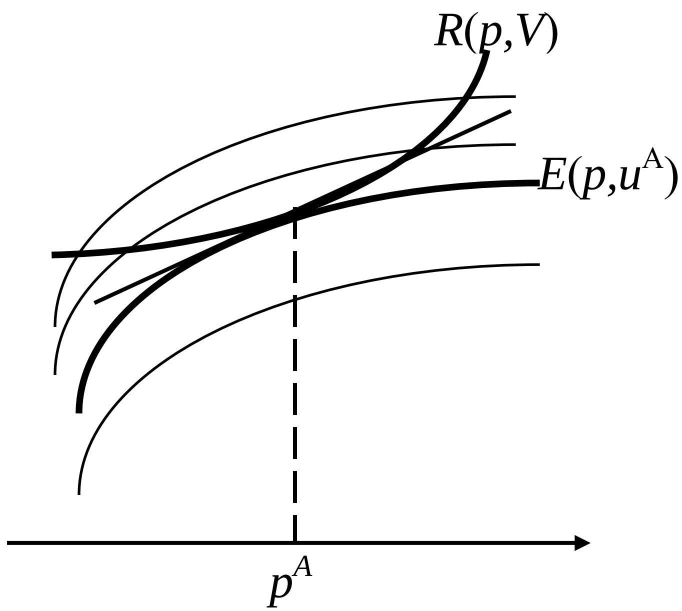
$E_{pp}(p,U) = c_p(p,U)$ is an NxN negative semi-definite matrix, with the rank at most equal to N−1.



Autarky Equilibrium: p^A, $x^A = c^A$, and **U^A** satisfying

- Budget Constraint: $E(p^A, U^A) = R(p^A,V)$

- Market Clearing: $M(p^A, U^A) \equiv E_p(p^A, U^A) - R_p(p^A,V) = 0_N$



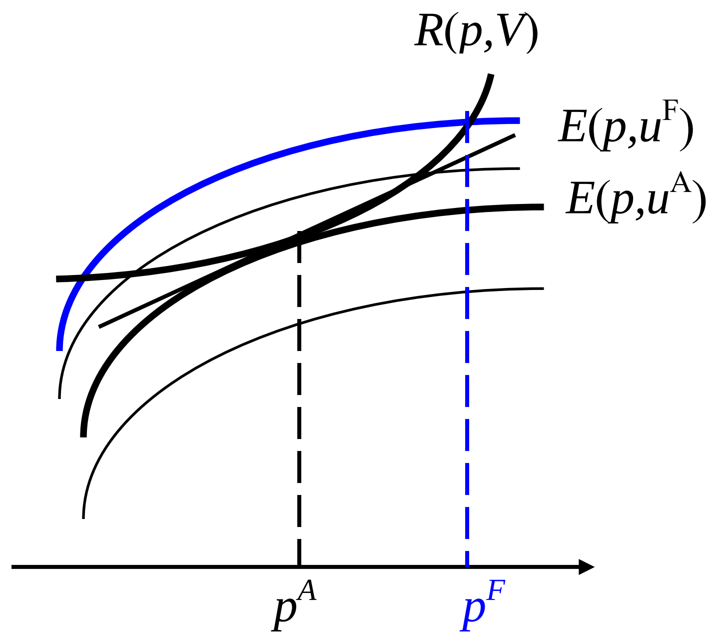
There are N + 1 equations. The Walras' Law, $E(p^A, U^A) = p^A E_p(p^A, U^A) = p^A R_p(p^A,V) = R(p^A,V)$, implies N independent conditions, which determines N endogenous variables, i.e., U^A and (N−1) relative commodity prices, p^A.

Alternatively, using the indirect utility function, $U^A = U(p^A, R(p^A,V))$, the market clearing condition can be rewritten as

$$m(p^A) \equiv M(p^A, U(p^A, R(p^A,V))) = E_p(p^A, U(p^A, R(p^A,V))) - R_p(p^A,V) = 0_N,$$

where m(p) is the *net import vector.*

Trade Equilibrium: If the economy trades with the ROW at the price, p^F,



- Budget Constraint: $E(p^F, U^F) = R(p^F,V)$
- Net Import Vector: $M(p^F, U^F) \equiv E_p(p^F, U^F) - R_p(p^F,V)$.

From the Budget Constraint (or Walras' Law),

Balanced Trade: $p^F M(p^F, U^F) = 0$.

Using the indirect utility function, $U^F = U(p^F, R(p^F,V))$, the net import vector can be written simply,

$$m(p^F) \equiv M(p^F, U^F) = E_p(p^F, U(p^F, R(p^F,V))) - R_p(p^F,V)$$

and it satisfies $p^F m(p^F) = 0$.

Two Country World Economy: Home and Foreign, each characterized by its own revenue and expenditure functions;

	Home	Foreign
Revenue Function	R(p,V)	R*(p,V*)
Expenditure Function	E(p, U)	E*(p, U*)

Or

	Home	Foreign
Indirect Utility Function	U(p,y) defined by y = E(p,U(p,y))	U*(p, y) defined by y = E*(p,U*(p,y))

Or

	Home	Foreign
Net Import Vector	m(p) = $E_p(p,U(p,R(p,V))) - R_p(p,V)$	m*(p) = $E^*_p(p,U^*(p,R^*(p,V^*))) - R^*_p(p,V^*)$

No Trade Equilibrium: described by two sets of Autarky Equilibriums

Home: $E(p^A, U^A) = R(p^A,V)$ & $E_p(p^A,U^A) = R_p(p^A,V)$. $\rightarrow m(p^A) = 0$.

Foreign: $E^*(p^{*A}, U^{*A}) = R^*(p^{*A},V^*)$ & $E^*_p(p^{*A},U^{*A}) = R^*_p(p^A,V^*)$ $\rightarrow m^*(p^{*A})=0$.

Free Trade Equilibrium (in the absence of trade costs),

Budget Constraints at H & F: $E(p^F, U^F) = R(p^F,V)$ & $E^*(p^F, U^{*F}) = R^*(p^F,V^*)$

World Market Clearing: $E_p(p^F,U^F) + E^*_p(p^F,U^{*F}) = R_p(p^F,V) + R^*_p(p^F,V^*)$

- Free Trade and Zero Trade Cost are implicit because the same price vector, p^F, prevails in both countries. No government intervention is also implicit as the budget constraint in each country has no terms representing the government net surplus.
- N+2 Equations; Due to the Walras's Law, N+1 independent conditions for N+1 endogenous variables; i.e., U^F, U^{*F} and $(N-1)$ relative commodity prices, p^F.

Or, simply,

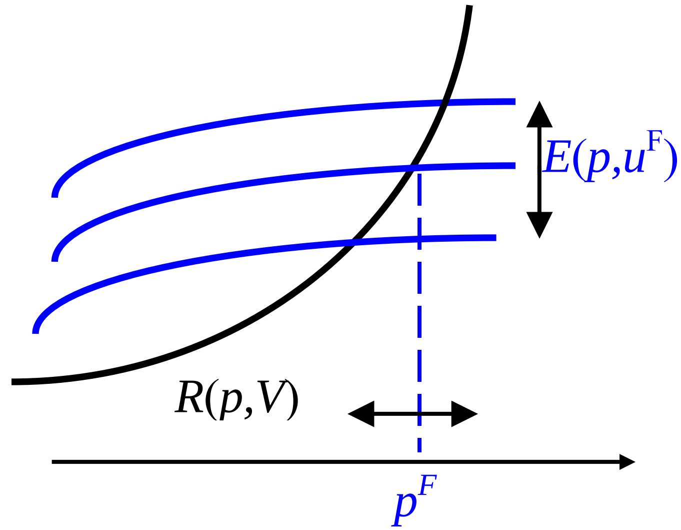
$$m(p^F) + m^*(p^F) = 0 \qquad \text{with} \qquad p^F m(p^F) = -p^F m^*(p^F) = 0.$$

Welfare Effects of a Change in p^F,

(caused by some exogenous change in Foreign)

Differentiate the Budget Constraint, $E(p^F, U^F) = R(p^F, V)$.

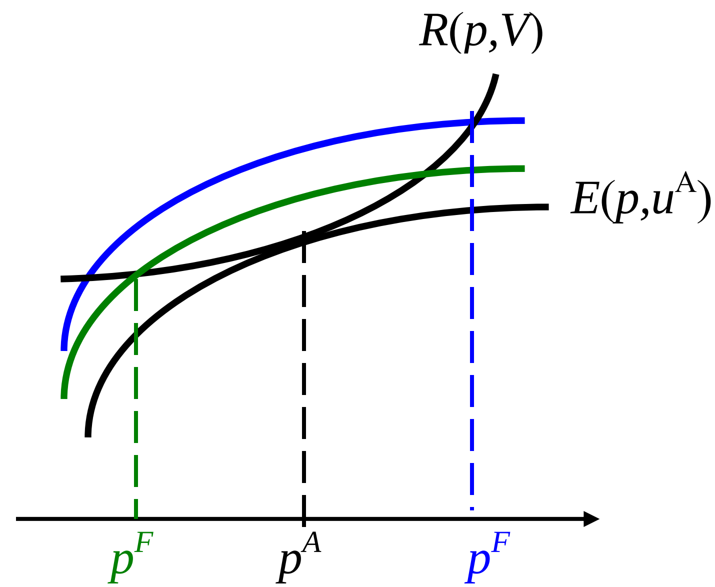
$$E_U dU^F = dp^F R_p - dp^F E_p = -dp^F m(p^F)$$



Index Theory for the Terms of Trade:

See Hamada and Iwata (1984?), Feenstra (2004, App. A)

Gains from (Free) Trade:



Proof that Free Trade is better than Autarky:

$$E(p^F, U^A) \le p^F c^A = p^F x^A$$
$$\le p^F x^F = R(p^F,V) = E(p^F,U^F)$$

$$\rightarrow U^A \le U^F.$$

Notes:

- Gains from trade are of the 2^{nd} order in $p^F - p^A$, if R & E are both differentiable at $p = p^A$.

Exercise: Derive a 2^{nd} order approximation of $U^F = U(p^F, R(p^F, V))$ around $p^F = p^A$.

- Two similar countries gain little from trading with each other.
- Two identical countries cannot gain from trading with each other. Without any economies of scale, gains from trade in the classical trade model arise only from taking advantage of the differences across countries.

So far, we have compared Free Trade with Autarky. What about Gains from Freer Trade, or Gains from Partially Free Trade?

Consider a Restrictive Trade Regime, B, with $q^B = \tau^B + p^B$ be the Internal (Domestic) Price; τ^B, the Trade Tax, and p^B, the External Price under Regime B.

When is Free Trade better than a restrictive trade, B?

$E(p^F, U^B) \leq p^F c^B = p^F x^B + p^F m^B \leq p^F x^F + p^F m^B = E(p^F, U^F) + p^F m^B$.

$\rightarrow U^B \leq U^F$ if $p^F m^B = (p^F - p^B) m^B \leq 0$.

Sufficient Condition: A shift to Free Trade does not worsen the terms-of-trade.
Collorary: Free Trade is the best for the *small* open economy, which cannot affect the terms-of-trade.

Technical Notes:

- Proof makes use of $p^F x^F = E(p^F, U^F)$, which implies that there is no government net surplus under Regime F (i.e., the government imposes no trade taxes.)
- Proof makes use of $p^B m^B = 0$. Thus, the Balanced Trade continues to hold even with the Trade Taxes. Implicitly, this means that the Home government redistributes its tax revenues to the Home agent (or finance its deficit by imposing lump-sum taxes on the Home agents.)

When is a restrictive trade, B, better than Autarky?

$E(q^B, U^A) \leq q^B c^A = q^B x^A \leq q^B x^B = q^B c^B - q^B m^B = E(q^B, U^B) - q^B m^B.$

$\rightarrow U^A \leq U^B$ if $q^B m^B = (q^B - p^B) m^B = \tau^B m^B \geq 0.$

Sufficient Condition: With Self-financing trade taxes (i.e., the revenue generating), some trade is better than Autarky. (Trading with trade subsidies may be harmful than autarky, as it effectively gives the gift to Foreign country even if the government does not distribute its trade tax revenue to Foreign country.)

This line of argument cannot use to compare two restrictive trade regimes. So, we need to try something else.

Optimal Trade Policy: The Case of a Small Country Case, with p* given.

We already know that Free Trade is the Best Policy. But, to confirm it, consider the following maximization,

$$Max\mathcal{L}(q,U,\lambda) = U + \lambda p^* \{R_p(q,V) - E_p(q,U)\},$$

where q is the internal price and λ is the Lagrange multiplier. The F.O.C. are given by

$$\lambda p^* E_{pU}(q,U) = 1$$

&

$$p^* R_{pp}(q,V) = p^* E_{pp}(q,U).$$

Since $pR_{pp}(p,V) = pE_{pp}(p,U) = 0$, $q = kp^*$ is an optimum.

Furthermore,
if $R_{pp} - E_{pp}$ has the rank equal to $N - 1$, $q = kp^*$ is the unique optimum.

Optimal Trade Policy: The Case of a Large Country, which takes $m^*(p^*)$ as exogenously given. (We ignore the possibility of retaliation from the ROW.)

$$Max\,\mathcal{L}(q, p^*, U, \lambda) = U + \lambda\{R_p(q,V) - E_p(q,U) - m^*(p^*)\},$$

where λ is now a N-dimensional row vector. The F.O.C. are given by

$$\lambda E_{pU}(q,U) = 1; \quad \lambda R_{pp}(q,V) = \lambda E_{pp}(q,U); \;\&\; \lambda m^*_p(p^*) = 0.$$

If $R_{pp} - E_{pp}$ has the rank equal to $N - 1$, $\lambda = kq$. Hence, $qm^*_p(p^*) = 0$. Differentiating $p^*m^*(p^*) = 0$ yields $m^*(p^*) + p^*m^*_p(p^*) = 0$, so that $(q - p^*)m^*_p(p^*) = m^*(p^*)$, or

$$\tau m^*_p(p^*) = m^*(p^*).$$

Optimal export tax rates are inversely related to the elasticity of the Foreign import curve. Intuition: A large country has monopoly power in the world market, and hence may gain from manipulating its terms of trade. Yet, being competitive, its private export sector cannot do so. But, the government can do so by restricting its export by imposing the export tax. (Note: in general equilibrium, there is an equivalence between the export tax and the import tax.)

Let us now drop the assumption of the Representative Consumer. In particular, we are interested in possible conflicts of interest that arise from the fact that different agents have different factor endowments. To this end, we need to review:

Properties of $R(p, V) \equiv px(p,V) = \mathrm{Max}_x \{px \mid (x, V) \in \Omega\}$ as a function of V.

Key Properties:

(R6) : R(p,V) is linear homogeneous and concave in V. Linear homogeneity comes from the fact that Ω is a cone (i.e. CRS). For the convexity, for $x^1 \in x(p, V^1)$ and $x^2 \in x(p, V^2)$, let $x^\lambda = \lambda x^1 + (1-\lambda)x^2$ and $V^\lambda = \lambda V^1 + (1-\lambda)V^2$. Then, $(x^\lambda, V^\lambda) \in Y$. Thus, $R(p, V^\lambda) \geq px^\lambda = p\{\lambda x^1 + (1-\lambda)x^2\} = \lambda px^1 + (1-\lambda)px^2 = \lambda R(p,V^1) + (1-\lambda)R(p,V^2)$.

(R7) : If R is differentiable in V, $R(p,V) = R_V(p,V)V = w(p,V)V$. Differentiating $R(p,\lambda V) = \lambda R(p,V)$ by λ yields $R(p,V) = R_V(p,V)V$. $R_V(p,V) = w(p,V)$ is a row-vector.

(R8) : If R is twice differentiable in V, $R_{VV}(p,V)V = w_V(p,V)V = 0$.

$R_V(p,V)$ is homogeneous of degree zero, $R_V(p, \lambda V) = R_V(p,V)$. Differentiating it by λ yields $R_{VV}(p,V)V = 0$. $R_{VV}(p,V) = w_V(p,V)$ is a negative semi-definite MxM matrix, with the rank at most equal to M−1.

(R9): **[Reciprocity]** If R is twice differentiable in p and V, $x_V(p,V) = R_{pV}(p, V) = [R_{Vp}(p,V)]^T = [w_p(p,V)]^T$. $x_V(p,V) = R_{pV}(p, V)$ is a MxN matrix; $R_{Vp}(p,V) = w_p(p,V)$ is a NxM matrix.

Non-Representative Consumer Cases: Households are indexed by $h \in H$, each endowed by factor V^h; $V = \Sigma_h V^h$

Household Budget Constraints: $E^h(p, U^h) = R_V(p,V)V^h$

Excess Demand (Net Import): $m(p) \equiv \Sigma_h E^h{}_p(p, U^h) - R_p(p,V)$

Aggregation: If all the households have *identical homothetic preferences*, $E^h(p, U^h) = e(p)U^h$. By defining $U \equiv \Sigma_h U^h$,

Aggregate Budget Constraint: $e(p)U = R_V(p,V)V = R(p,V)$

Excess Demand (Net Import): $m(p) = e_p(p)U - R_p(p,V)$.

For the purpose of positive analysis, we can treat as if there were a representative household. We may also argue that the aggregate gains from trade, if the national welfare is measured by $U \equiv \Sigma_h U^h$. But, different households are affected differently, because they have different endowments. Some households may be worse off without government interventions. Can the government ensure that all households gain from trade?

Autarky Equilibrium with Many Households

Household Budget Constraints: $E^h(p^A, U^{hA}) = R_V(p^A,V)V^h$

Zero Excess Demand: $\Sigma_h E^h{}_p(p^A, U^{hA}) - R_p(p^A,V) = 0.$

Pareto-Superior Trade:

Lump-sum transfer can make trade Pareto-superior to Autarky.

Let $\tau^h \equiv E^h(p^T, U^{hA}) - R_V(p^T,V)V^h$, the lump-sum transfer necessary to keep h indifferent with the price change. Since $\tau^h \leq p^T E^h{}_p(p^A, U^{hA}) - R_V(p^T,V)V^h$,

$$\Sigma_h \tau^h \leq p^T \Sigma_h E^h{}_p(p^A, U^{hA}) - R_V(p^T,V)(\Sigma_h V^h) = p^T R_p(p^A,V) - R(p^T,V) \leq 0.$$

This system of lump-sum transfer is highly informationally intensive. In the presence of private information on preferences (or factor endowments), it will be impossible to implement.

Dixit and Norman (1980) argued that lump-sum transfers are not necessary.

Self-financing system of distortionary taxes can make trade Pareto-superior to Autarky.

Choose the taxes so that the households face the same prices. That is, $p^A - p^T$ on commodities and $R_V(p^T,V) - w^A$ on factors. This ensures that each household chooses the same consumption bundle, and achieves the same utility level. The government revenue is

$$
\begin{aligned}
&(p^A - p^T)\, \Sigma_h\, E^h(p^A, U^{hA}) + (R_V(p^T,V) - w^A)\, \Sigma_h\, V^h \\
&= \Sigma_h\, (p^A E^h(p^A, U^{hA}) - w^A V^h) + R_V(p^T,V)V - p^T\, \Sigma_h\, E^h(p^A, U^{hA}) \\
&= R(p^T,V) - p^T R(p^A, V) \geq 0.
\end{aligned}
$$

For some caveats and extensions of these results, see Dixit and Norman (1980, pp….), Kemp-Wan (1986), Dixit (1986) and Feenstra (2004, pp. 184-186).

Akerlof et. al. (1991) proposed such system of distortionary taxes/subsides to insulate the adverse effects of the German reunification on the Eastern Germany.

Patterns of Trade (The Law of Comparative Advantage):

Since $U^A \leq U^F$,

$$\begin{aligned} E(p^A, U^F) &\leq p^A c^F = p^A x^F + p^A m^F \\ &\leq p^A x^A + p^A m^F = E(p^A, U^A) + p^A m^F \\ &\leq E(p^A, U^F) + p^A m^F, \end{aligned}$$

Hence,

$$p^A m^F = (p^A - p^F) m^F \geq 0.$$

In the two-country case, since $m^F + m^{*F} = 0$,

$$(p^A - p^{*A}) m^F \geq 0 \quad \& \quad (p^{*A} - p^A) m^{*F} \geq 0.$$

- Deardorff (1980) extended this formula for some cases with trade impediments.
- Bernhofen and Brown (2004) tested this and similar inequality restrictions for the mid-19th century Japan.

The above inequality implies,

For N = 2, $m_1 > 0 > m_2$ iff $\frac{p_1^A}{p_2^A} > \frac{p_1^{*A}}{p_2^{*A}} \leftrightarrow \frac{p_1^A}{p_1^{*A}} > \frac{p_2^A}{p_2^{*A}}$,

Thus, differences in the autarky prices (the patterns of comparative advantage) predict the patterns of trade.

The problems are:

- The autarky prices are not generally observable. (see, however, Bernhofen and Brown)
- It is not easy to generalize to the case of N > 2. For example, *the chain of comparative advantage,*

$$\frac{p_1^A}{p_1^{*A}} > \frac{p_2^A}{p_2^{*A}} > \ldots > \frac{p_N^A}{p_N^{*A}},$$

does *not* necessarily imply that Home imports the lower-indexed goods and the Foreign imports the higher-indexed goods. To obtain such a generalization, we need further restrictions. See Ricardian Trade Theory.

Some Comparative Statics

Recall that Free Trade Equilibrium of Two Country World Economy (with the representative agents in each country) may be described by, in the absence of trade costs,

Budget Constraint at Home: $E(p, U) = R(p,V)$

Budget Constraint at Foreign: $E^*(p, U^*) = R^*(p,V^*)$

World Market Clearing: $E_p(p,U) + E^*_p(p,U^*) = R_p(p,V) + R^*_p(p,V^*)$

(I dropped superscript F to keep the notation simple.)

By adding some shift parameters, we may use this system of equations to conduct some comparative statics, to study the effects of

- Transfer Payments across Countries
- Factor Accumulation & Technical Change
- Trade Taxes,

etc.

There is a HUGE literature that conducts such exercises.

As pointed out by Dixit and Norman (1980; Chapter 5), however, we should NOT expect to find many robust predictions. Often, specific results may be obtained only in the Two-goods case (N = 2). These results cannot be easily extended to general cases (without making strong assumptions about the preferences and technologies).

Here,

We will discuss heuristically some notable results, often called "Paradoxes" since they are counter-intuitive to those who are used to think within the closed economy setting.

This is *not* because these results are robust, *but* because I find these results

- illustrate well possible complications that arise from the fact that the countries trade with each other (the Terms of Trade Effect).

- useful as intuition-building devices.

Factor Accumulation and Technical Progress in One Country

Autarky and Small Open Economy Cases:

Under the assumption of the efficient resource allocation, these changes that cause an outward shift of the Production Possibility Frontier clearly improves the national welfare if the country is in Autarky or a small open economy (so that the ToT is fixed).

In either case, the ROW is unaffected by the changes.

A Large Open Economy Case:

We need to take into account the secondary effect of the ToT change in addition to the primary effect of the production capacity gains.

If Home's ToT improves,
    Home gains more. Foreign suffers from a welfare loss, as its ToT deteriorates.
If Home's ToT deteriorates,
    Foreign gains as some of the Home capacity gains spillover.

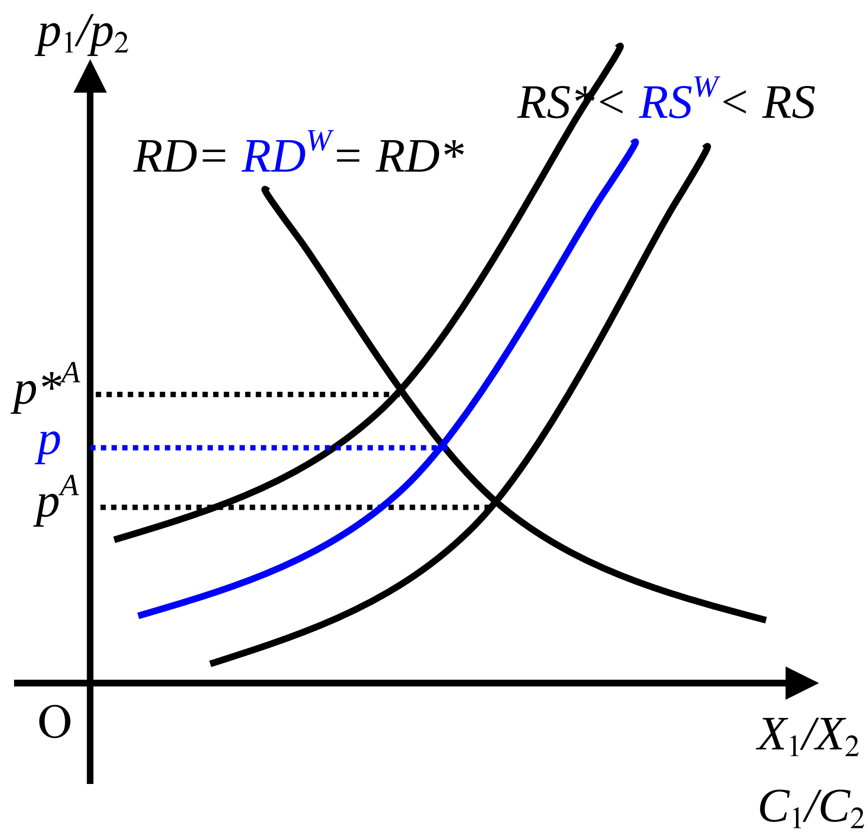
How is the ToT affected by the change?

In the 2-Country, 2-Goods Case, suppose that

- Two Countries Share the Same Homothetic Preferences
- Home's Relative Supply (RS) Curve is located to the right of Foreign's at any relative price.

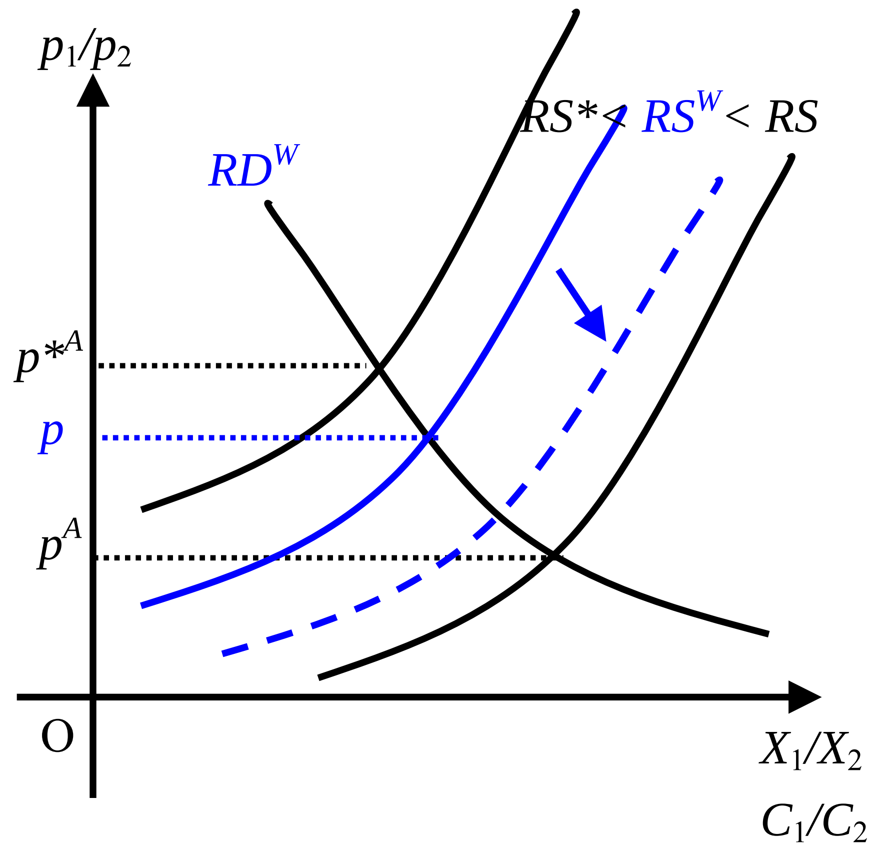
→

- Home exports Good 1.
- The World RS Curve is the weighted average of the Home and Foreign RS Curves.
- The World Relative Demand (RD) is the same with the Home and Foreign RD Curves.
- Home's growth will not affect the RD Curves.



Home's ToT deteriorates and Foreign gains if the World RS shifts to the right, which occurs when

- Home experiences *Uniform* Growth (the change that would not affect its RS at a given relative price)
- Home experiences *Export-Biased* growth (the change that would increase the RS of its export good at a given relative price)



On the other hand,

Home's ToT improves and Foreign loses if the World RS shifts to the left, which occurs if Home experiences sufficiently *Import-Biased* Growth.

Question: Is "Export versus Import-Biased" a useful dichotomy when $N > 2$?

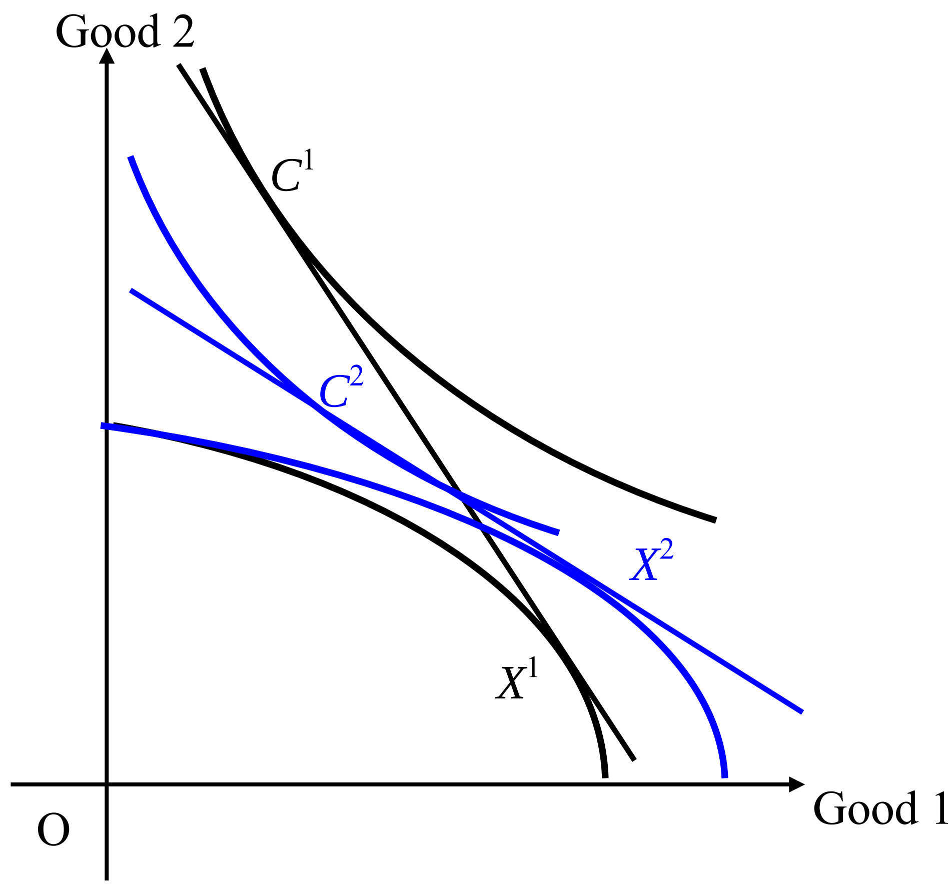
Immiserizing Growth; Bhagwati (1958)

Can Home's capacity gains that cause an outward shift of its PPF be harmful to Home? *Can* the primary gain of the production capacity increase be more than offset by the secondary effect of the ToT deterioration?

The answer is **Yes.**

A Graphic Illustration →

See Dixit and Norman (1980; pp133) for the analytics in the two-country, two-sector (2x2) model.



Some Remarks:

- Immiserizing Growth is more likely when
    1) Growth is concentrated in the country's export sector
    2) The country's share in the world output of its export good is larger
    3) The elasticity of demand for the export good is lower.

- When Home suffers from Immiserizing Growth, it is because its ToT moves in favor of the ROW. Thus, the ROW gains. This means that the entire world cannot suffer from the capacity gains. So, this is the question of how the gains are distributed across countries.

- When the condition for Immiserizing Growth is met, the country can gain by deliberately destroying its own capacity to produce the export good (at the expense of ROW).

- Immiserizing Growth occurs partly because the country's export sector, being competitive, cannot take advantage of the country's monopoly power in the world market.

- Indeed, if the country adopts the Optimal Trade Policy, which automatically adjusts when the capacity changes, the country never experiences Immiserizing Growth.

- Immiserizing Growth cannot occur in a small open economy (unless there are some distortions; see Johnson (1960)).

For an empirical study, see Sawada (2003) who claims to have identified many cases of immiserizing growth.

Terms of Trade Change as a Built-In Insurance:

Consider a large developing country, an exporter of primary commodities whose production is subject to random fluctuations. Then, The ToT improves in bad times, and deteriorates in good times, offering a built-in insurance.

On the other hand, a small exporter of primary commodities can be devastated when it experiences a harvest failure.

Transfer Payments

Imagine a gift or an aid from Home to Foreign, by *T* .

Budget Constraints at H&F: $R(p,V) - E(p,U) = T = E^*(p, U^*) - R^*(p,V^*)$

World Market Clearing: $E_p(p,U) + E^*_p(p,U^*) = R_p(p,V) + R^*_p(p,V^*)$

Such a transfer payment, by shifting the purchasing power, could change the ToT.

- If Home's ToT worsens, the ToT change amplifies the effect of the transfer.
  This could happen, for example, when Foreign (the recipient) has lower marginal propensity to consume Home's export good than Home (the donor).
- If Home's ToT improves, the primary effect of the transfer is offset by the secondary effect of the ToT change.
  This could happen, for example, when Foreign (the recipient) has higher marginal propensity to consume Home's export good than Home (the donor).

Transfer Paradox: *Can* the Donor's ToT improve (and the Recipient's ToT deteriorate) so much that the Donor gains from making the transfer and the Recipient loses from receiving it?

The answer is *Yes*, but only under more stringent conditions than Immiserizing Growth. Indeed, it can be ruled out in the Two-Goods, Two-Country Case (unless there are some other distortions).

Why is the Transfer Paradox more unlikely than Immiserizing Growth?

- Recall that, when the condition of Immiserizing Growth is met, the Country can gain by throwing away some of its export good.
- This is because, by doing so, it causes a shortage of its export good, thereby improving its ToT.
- This would work because the Country throws away its export good.
- It would not work if the Country gives it away to the ROW.

But, what if the Country give it to a subset of countries, not to the ROW?

Bilateral Transfers in a Multi-Country World; Bhagwati-Brecher-Hatta (1983)

(Unfinished)

Regional Trade Agreements (Unfinished)

Viner; Trade Creation versus Trade Diversion

Kemp-Wan

Feenstra (2004, pp.192…),

Panagariya (2000)