Credit Traps and Credit Cycles

By

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Topics:

Aggregate Investment Dynamics under Credit Market Imperfections

Standard View: Bernanke-Gertler and many others

*Credit Multiplier (Financial Accelerator) & Persistence*

Higher Borrower Net Worth $\rightarrow$ Less Credit Frictions  
$\rightarrow$ Higher Investment $\rightarrow$ Higher Borrower Net Worth

Focus: Impacts on BNW on the *Volume* of the Credit
**Question:** What would be the *Composition* Effects?

**What We Do:** a Simple BG-type Model with Heterogeneous Investment Projects that differ in

- Productivity
- Severity of Agency Problems (Pledgeability)
- Investment Requirement (Initial Setup Cost)

to study how a BNW change shifts the *Composition* of the Credit
Results:

- Endogenous *Investment Specific Technical Change* via Credit Channels

- A Wide Range of *Nonlinear* Behaviors (not necessarily persistence)
  - Credit Traps, Credit Collapse
    (if a higher BNW shifts the credit towards the more productive)
  - Leapfrogging, Credit Cycles, Growth Miracles
    (if a higher BNW shifts the credit towards the less productive)

- *Pro-cyclical Rates of Return*
2. The Model: A Variation of the Diamond OG model

**Final Good:** \( Y_t = F(K_t, L_t) \), with physical capital, \( K_t \) and labor, \( L_t \).

\[
y_t = \frac{Y_t}{L_t} = F(K_t/L_t, 1) = f(k_t), \text{ where } k_t = K_t/L_t; \quad f'(k) > 0 > f''(k).
\]

**Competitive Factor Markets:**

\[
\rho_t = f'(k_t); \quad w_t = f(k_t) - k_t f'(k_t) = W(k_t) > 0.
\]

**Agents:** A unit measure of homogeneous agents.

In the 1\(^{\text{st}}\) period, they supply one unit of labor, earn and save \( W(k_t) \).
In the 2\(^{\text{nd}}\) period, they consume.

They maximize the 2\(^{\text{nd}}\) period consumption.
**Investment Technologies:** Agents can choose one (and only one) from $J$ indivisible projects ($j = 1,2, \ldots J$).

<table>
<thead>
<tr>
<th>Period $t$</th>
<th>Period $t+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project-$j$:</td>
<td>$m_j$ units of final good $\rightarrow m_j R_j$ units of capital</td>
</tr>
</tbody>
</table>

$m_j$: the (fixed) set-up cost, $R_j$: the project productivity

**To Invest or Not to Invest?**

By starting a project-$j$, $C_t = m_j R_j \rho_{t+1} - r_{t+1}(m_j - w_t)$, ($j = 1,2,\ldots,J$)

By lending, $C_t = r_{t+1} w_t$

**Profitability Constraint:** $R_j f'(k_{t+1}) \geq r_{t+1}$, (PC-$j$)

**Note:** In the perfect credit market, (PC) would hold with equality for the most productive (and with strict inequality for the others) in equilibrium.
Credit Market Imperfections:

Borrowing Constraint: \[ \lambda_j m_j R_j f'(k_{t+1}) \geq r_{t+1}(m_j - W(k_t)), \quad \text{(BC-}j) \]

\( \lambda_j \): the pledgeable fraction of the project revenue

Both (PC-\(j\)) and (BC-\(j\)) must be satisfied for the credit to flow into type-\(j\) projects.
What is the rate of return the lenders can expect from a type–j project?

From (PC-j) and (BC-j),

\[
\frac{r_{t+1}}{f'(k_{t+1})} = \frac{R_j}{\max\{1, \left\lbrack 1 - \frac{W(k_t)}{m_j} \right\rbrack / \lambda_j\}}
\]
Equilibrium Conditions

(1) \( W(k_t) = \sum_j (m_j X_{jt}) \).

(2) \( k_{t+1} = \sum_j (m_j R_j X_{jt}) \).

(3) \( \frac{r_{t+1}}{f'(k_{t+1})} \geq \frac{R_j}{\max\{1, [1 - W(k_t) / m_j] / \lambda_j}\} \quad (j = 1, 2, \ldots J) \)

where \( X_{jt} \) is the measure of type-\( j \) projects initiated in period \( t \), and \( X_{jt} > 0 \) \( (j = 1, 2, \ldots J) \) implies the equality in (3).

**Note:** The projects are ranked according to the RHS of (3), which does not depend on the allocation of the credit. \( \Rightarrow \) Generally, \( X_{jt} = 1 \) or 0.

For \( k_0 > 0 \), (1)-(3) determine the equilibrium trajectory.
3. Two Benchmarks: Monotone Convergence

Benchmark 1: *Full Pledgeability* (\(\lambda = 1\))

All the credit goes to the most productive.

\[
0 < X_t = \frac{W(k_t)}{m} < 1,
\]

\[
k_{t+1} = RW(k_t),
\]

\[
r_{t+1} = R'f(RW(k_t))
\]

*Monotone Convergence* under

- \(W(k)/k\) is strictly decreasing in \(k\),
- \(\lim_{k \to +0} W(k)/k = \infty; \lim_{k \to +\infty} W(k)/k = 0.\)
Benchmark 2: *Homogenous Projects* (J = 1)

\[(4) \quad 0 < X_t = W(k_t)/m < 1,\]

\[(5) \quad k_{t+1} = RW(k_t),\]

\[(6') \quad r_{t+1} = \frac{Rf'(RW(k_t))}{\max\{1,\frac{[1 - W(k_t)/m]}{\lambda}\}}\]

For \(W(k_t) < (1 - \lambda)m,\)

\[r_{t+1} = \frac{\lambda Rf'(RW(k_t))}{1 - W(k_t)/m} < Rf'(RW(k_t)). \quad \text{BCbinding; PCnonbinding.}\]

For \(W(k_t) > (1 - \lambda)m,\)

\[r_{t+1} = Rf'(RW(k_t)) < \frac{\lambda Rf'(RW(k_t))}{1 - W(k_t)/m}. \quad \text{BCnonbinding; PCbinding.}\]
4. Credit Traps and Credit Collapses ($R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$)

Productivity vs. Pledgeability:
Project 2 is more productive, but less pledgeable than Project 1.

e.g. Some advanced projects that use leading edge technologies may be subject to bigger agency problems than some mundane projects that use well-established technologies.

![Figure 2a](image1)

![Figure 2b](image2)
(7) \[ k_{t+1} = \begin{cases} R_1 W(k_t) & \text{if } k_t < k_c, \\ R_2 W(k_t) & \text{if } k_t > k_c. \end{cases} \]
Rate of Return Movement:

With a higher $k$,
- Borrowers can pledge more with higher BNW (procyclical)
- Credit Composition may shift towards more productive projects (procyclical)
- Neoclassical capital deeping effect (counter-cyclical)

For Figure 3b, the last two exactly offset each other for the Cobb-Douglas case. The overall effect is procyclical.

The rate of return may be higher in the developed or in a booming economy than the undeveloped or in a stagnating economy.

Effect of a Higher Pledgeability: A higher $\lambda_1$ can make things worse (e.g., may create a credit trap or cause a credit collapse).
5. Leapfrogging, Credit Cycles and Growth Miracles:
\[ R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1, \] and
\[ \frac{m_2}{m_1} > \frac{(1-\lambda_1)}{(1-\lambda_2 R_2/R_1)}. \]

- Project 1 is less productive and less pledgeable than Project 2.
- Project 1 requires the smaller set-up cost than Project 2.

e.g. Project 1: family operated firms or other small businesses,
    Project 2: the investments in the corporate sector.
e.g. Project 1: traditional light industries (textile and furniture)
    Project 2: modern heavy industries (steel, industrial equipments, petrochemical, and pharmaceutical)

Figure 4
\begin{align*}
\text{(8) } k_{t+1} &= \begin{cases} 
R_2 W(k_t) & \text{if } k_t < k_c \text{ or } k_t > k_{cc} \\
R_1 W(k_t) & \text{if } k_c < k_t < k_{cc}.
\end{cases}
\end{align*}

\begin{figure}
\centering
\begin{subfigure}{0.3\textwidth}
\begin{tikzpicture}
\draw[->] (0,0) -- (6,0) node[below] {$k_t$};
\draw[->] (0,0) -- (0,6) node[left] {$k_{t+1}$};
\draw[dashed] (0,0) -- (5,5);
\draw[thick] (0,0) -- (0,2) -- (2,4) -- (4,2) -- (5,0);
\draw[thick] (0,2) -- (2,2) -- (2,4) -- (4,4) -- (5,0);
\end{tikzpicture}
\caption{Leapfrogging}
\end{subfigure}
\hspace{1cm}
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\end{tikzpicture}
\caption{Credit Cycles}
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\end{tikzpicture}
\caption{Cycles as a Trap}
\end{subfigure}
\caption{Figure 5a – 5c}
\end{figure}
6. Concluding Remarks:

One Special (Peculiar) Feature of the Model

• All the Credit goes to one type of the project.
• A change in BNW causes a Bang-Bang shift in the composition.

How to remove this feature without losing too much tractability?

• Allow projects to differ in goods that they produce
• Allow heterogeneous agents