Credit Traps and Credit Cycles[1]

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Credit Traps and Credit Cycles

Abstract

We develop a simple model of credit market imperfections, in which the agents have access to a variety of investment projects, which differ in productivity, in the investment size, and in the severity of the agency problems behind the borrowing constraints. A movement in borrower net worth can shift the composition of the credit between projects with different productivity levels. The model thus suggests how investment-specific technological change may occur endogenously through credit channels. Furthermore, such endogenous changes in investment technologies in turn affect borrower net worth. These interactions could lead to a variety of nonlinear phenomena, such as credit traps, credit collapse, leapfrogging, credit cycles, and growth miracles in the joint dynamics of the aggregate investment and borrower net worth.

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1. Introduction

The recent literature on macroeconomics of credit market imperfections, following the seminal work of Bernanke and Gertler (1989), emphasizes the credit multiplier (or financial accelerator) mechanism, which introduces persistence in the dynamics of the aggregate investment and borrower net worth. As the argument goes, a rise (a fall) in borrower net worth eases (aggravates) the borrowing constraint, thereby stimulating (discouraging) investment, which leads to further rise (fall) in borrower net worth. These studies typically consider the case where the investment projects facing the borrowing constraint are homogeneous. The alternatives available to the lenders are normally restricted to either consumption or the simple storage technology. Although such a framework is useful for understanding how the credit market imperfections affect the aggregate investment through the *volume* of the credit, it is ill-equipped to investigate how they affect the aggregate investment through the *composition* of the credit.

In this paper, we propose a simple macroeconomic model of credit market imperfections with heterogeneous investment projects in order to investigate the composition effects. Of course, the importance of the composition effects may depend on the applications. For example, it might be reasonable to ignore them on a first approximation, when applied to the high frequency dynamics to deal with the issues such as the short-run monetary policy analysis (see, e.g., Bernanke and Gertler 1995; Bernanke, Gertler, and Gilchrist 1999). It is only as a first approximation, however, because the existing studies in this area often assume exogenous productivity shocks to study the role of borrower net worth. Arguably, some of the productivity shocks may be caused by an endogenous shift in the composition of the credit across projects with different productivity levels. The composition effects would be of central importance in the low frequency dynamics. The development strategy is concerned about the composition of the credit at least as much as the volume of the credit, and many government and semi-government financial institutions, so-called "development banks", are set up precisely with the objective of redirecting the credit flow towards more "socially productive" and "growth oriented" investments.

In the model developed below, the homogenous agents have access to a variety of heterogeneous investment projects, which differ in productivity, in the investment size, and in the severity of the agency problems behind the borrowing constraints.[2] Furthermore, to highlight the composition effects, we deliberately set up the model in such a way that the investment dynamics would be identical to those in the standard neoclassical growth model, if the composition of the credit never changed. In this model, credit always goes to the projects that generate the highest rate of return to the lenders. However, due to the credit friction, these projects are not necessarily the most productive projects. Furthermore, which projects generate the highest rate of return depends on, among other things, borrower net worth. Along the equilibrium path, a movement in borrower net worth affects the composition of the credit, causing an endogenous switch between investment projects with different productivity levels. The model thus suggests how investment-specific technological change may occur endogenously through credit channels.[3] Furthermore, such endogenous changes in investment technologies in turn affect borrower net worth. These interactions lead to a variety of nonlinear phenomena, such as credit traps, credit collapse, leapfrogging, credit cycles and growth miracles, in the joint dynamics of the aggregate investment and borrower net worth.

The model's implications on the rate of return might also be of independent interest. A rise in borrower net worth not only eases the borrowing constraint, but it may also cause the composition of the credit to shift towards more productive projects. These effects can dominate the usual capital deepening effect. As a result, the rate of return may move pro-cyclically.[4]

The rest of the paper is organized as follows. Section 2 introduces the model and derives the system of equations that governs the equilibrium dynamics. Section 3 looks at two benchmark cases, in which the composition of the credit never changes along the equilibrium path, either due to the absence of the credit frictions or due to the homogeneity of projects. It is

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[2]We deliberately rule out the other sources of heterogeneity to keep the analysis simple. For example, it is assumed that all the projects produce the same capital stock (but in different quantity) and that the agents are homogeneous. It turns out that introducing the heterogeneity along these dimensions in a nontrivial way makes the analysis of the dynamics considerably more demanding. Nevertheless, we have made some progress for a few isolated cases. Matsuyama (2004a) considers the cases where some projects produce the consumption good, while others produce the capital good. The world economy model of Matsuyama (2004b) may be viewed as an example of the cases, where different agents run different projects that produce different capital goods (the agents and the capital goods differ in their locations.) We will offer more discussion on some differences between the present model and the model of Matsuyama (2004a) in Section 6.

[3]For investment-specific technological change, see Greenwood, Hercowitz, and Krusell (1997, 2000).

[4] This implication is absent in most existing macroeconomic models of credit market imperfections, as they typically assume the perfectly elastic supply of the aggregate saving, which pins down the rate of return.

shown that, in these cases, the aggregate investment dynamics are characterized by monotone convergence, as in the standard neoclassical growth model. These cases provide useful benchmarks against which to identify the composition effects in the presence of credit frictions. Sections 4 and 5 are the main parts of the paper. Section 4 looks at cases where there are tradeoffs between productivity and agency problems across different projects. These cases capture the situation where some advanced projects that use leading edge technologies are subject to bigger agency problems than some mundane projects that use well-established technologies. In the presence of such trade-offs, a rise in borrower net worth may cause the credit to switch towards more productive projects. This effect gives rise to the possibility of credit traps and credit collapses. Section 5 looks at cases where some projects that are less productive and subject to bigger agency problems have an advantage of having relatively small investment requirement, so that the agents need to borrow less for these projects. These cases capture the situation where the investments run by small family businesses compete with those in the corporate sector, or where traditional light industries, such as textile and furniture, compete with modern heavy industries, such as steel and petrochemical. In the presence of such trade-offs, a rise in borrower net worth may cause the credit to switch towards less productive projects. This effect gives rise to the possibility of leapfrogging, credit cycles and growth miracles. Section 6 concludes.

2. The Model.

The basic framework used is the Diamond overlapping generations model with two period lives. The economy produces a single final good, using the CRS technology, $Y_t = F(K_t,L_t)$, where K_t is physical capital, and L_t is labor. The final good produced in period t may be consumed in period t or may be allocated to investment projects. Let $y_t \equiv Y_t/L_t = F(K_t/L_t,1) \equiv f(k_t)$, where $k_t \equiv K_t/L_t$ and f(k) satisfies $f'(k) > 0 > f''(k)$. The markets are competitive, and the factor rewards for physical capital and for labor are equal to $\rho_t = f'(k_t)$ and $w_t = f(k_t)-k_tf'(k_t) \equiv W(k_t) > 0$, which are both paid in the final good. For simplicity, physical capital is assumed to depreciate fully in one period.

In each period, a new generation of potential entrepreneurs, a unit measure of homogeneous agents, arrives with one unit of the endowment, called labor. They stay active for two periods. In the first period, they sell the endowment and earn $w_t = W(k_t)$. They consume only in the second period. Thus, they save all of the earning, w_t, and allocate it to maximize their second period consumption. They may become lenders or entrepreneurs. If they become lenders, they can earn the gross return equal to r_{t+1} per unit in the competitive credit market and consume $r_{t+1}w_t$ in the second period. Alternatively, they may become entrepreneurs by using their earning, w_t, to partially finance an investment project. They can choose from J-types of projects. All projects come in discrete, indivisible units and each entrepreneur can run only one project. A type-j ($j = 1,2,\ldots,J$) project transforms m_j units of the final good in period t into m_jR_j units of physical capital in period t+1. Because of the fixed investment size, m_j, an entrepreneur needs to borrow by m_j-w_t at the rate equal to r_{t+1}. (If $w_t > m_j$, they can entirely self-finance the project and lend $w_t - m_j$.)

Let X_{jt} denote the measure of type-j projects initiated in period t. Then, the aggregate investment, the amount of the final good allocated to all the projects, is $I_t = \sum_j(m_jX_{jt})$. Since the aggregate saving is $S_t = W(k_t)$, the credit market equilibrium requires that

$$(1) \qquad W(k_t) = \sum_j(m_jX_{jt}).$$

The capital stock adjusts according to

$$(2) \qquad k_{t+1} = \sum_j(m_jR_jX_{jt}).$$

Let us now turn to the investment decisions. To invest in a project, the entrepreneurs must be both willing and able to borrow. By becoming the lenders, they can consume $r_{t+1}w_t$. By running type-j projects, they can consume $m_jR_j\rho_{t+1} - r_{t+1}(m_j-w_t)$. Thus, the agents are *willing* to borrow and to run a type-j project if and only if $m_jR_j\rho_{t+1} - r_{t+1}(m_j-w_t) \geq r_{t+1}w_t$, which can be simplified to

(PC-j) $\qquad R_j f'(k_{t+1}) \geq r_{t+1}$,

where PC stands for the *profitability constraint*.

Even when (PC-j) holds, the agents may not be able to invest in type-j projects, due to the borrowing constraint. The borrowing limit exists because borrowers can pledge only up to a fraction of the project revenue for the repayment, $\lambda_j m_j R_j \rho_{t+1}$, where $0 \leq \lambda_j \leq 1$. Knowing this, the lender would lend only up to $\lambda_j m_j R_j \rho_{t+1}/r_{t+1}$. The agent can borrow to run a type-j project iff

(BC-j) $\qquad \lambda_j m_j R_j f'(k_{t+1}) \geq r_{t+1}(m_j - W(k_t))$,

where BC stands for the *borrowing constraint*.[5]

Suppose that $R_j f'(k_{t+1}) > r_{t+1}\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$, so that both (PC-j) and (BC-j) are satisfied with strict inequalities. Then, any agent would be able to borrow and run a type-j project and would be better off by doing so than by lending. This means that no agent would become a lender. Hence, in equilibrium, $R_j f'(k_{t+1}) \leq r_{t+1}\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$.[6] If this inequality holds strictly for some j, then at least one of (PC-j) and (BC-j) is violated, so that $X_{jt} = 0$. Since (1) requires that $X_{jt} > 0$ for some j, we have

(3) $$r_{t+1} = \max_{i=1,\ldots J}\left\{\frac{R_i f'(k_{t+1})}{\max\{1,[1-W(k_t)/m_i]/\lambda_i\}}\right\} \geq \frac{R_j f'(k_{t+1})}{\max\{1,[1-W(k_t)/m_j]/\lambda_j\}},$$

[5]We have used this specification of the credit market imperfections elsewhere, e.g., Matsuyama (2000, 2004a, b, 2005a, 2006). It is possible to give any number of agency stories to justify the assumption that borrowers can pledge only up to a fraction of the project revenue. The simplest story would be that they strategically default, whenever the repayment obligation exceeds the default cost, which is proportional to the project revenue. Alternatively, each project is specific to the borrower, and requires his services to produce R_j units of physical capital. Without his services, it produces only $\lambda_j R_j$ units. Then, the borrower, by threatening to withdraw his services, can renegotiate the repayment obligation down to $\lambda_j R_j \rho_{t+1}$. See Hart and Moore (1994) and Kiyotaki and Moore (1997). It is also possible to use the costly-state-verification approach used by Bernanke and Gertler (1989), or the moral hazard approach used by Tirole (2005). Nevertheless, the reader should interpret this formulation simply as a black box, a convenient way of introducing the credit market imperfection in a dynamic macroeconomic model, without worrying about the underlying causes of imperfections.

[6]It is implicitly assumed here that the agents cannot entirely self-finance the projects, so that some agents must become lenders in equilibrium. This condition is satisfied unless the production is too productive. That is to say, if we let $f(k) = Ag(k)$ with $g'(k) > 0 > g''(k)$, it suffices to assume that A is not too big. (Alternatively, we can make R_j proportionately smaller, or m_j proportionately larger, which is isomorphic to choosing a smaller A.)

where $X_{jt} > 0$ (j = 1, 2,…J) only if the inequality in (3) holds with the equality.

Eq. (3) plays a central role in the following analysis. Hence, it is worth thinking of the intuitive meaning behind it. The RHS of the inequality in (3) is the rate of return that the agents could offer both willingly and credibly to the lenders by running type-j projects. If this falls short of the equilibrium rate of return, type-j will not be run, because one of the two constraints is violated for j. In other words, the saving flows only to the projects for which the RHS of the inequality in (3) is the highest among all the projects. What matters in the following analysis is that the ranking of the projects, based on the RHS of the inequality in (3), determines the allocation of the credit, and that the ranking depends on the borrower net worth, $W(k_t)$. For example, if $W(k_t) < (1-\lambda_j)m_j$, the RHS of the inequality in (3) becomes $\{\lambda_jR_j/[1-W(k_t)/m_j]\}f'(k_{t+1})$, which depends on the pledgeable rate of return and the down payment ratio. In the limit, $W(k_t) \to 0$, this converges to $\lambda_jR_jf'(k_{t+1})$ for all j, which means that, with a sufficiently low net worth, the credit goes to the project with the highest λ_jR_j. On the other hand, if $W(k_t) > (1-\lambda_j)m_j$, the RHS of the inequality in (3) becomes $R_jf'(k_{t+1})$. Hence, for a sufficiently high $W(k_t)$, the credit goes to the project with the highest R_j. It is also noteworthy that the ranking of the projects, based on the RHS of the inequality in (3), is entirely independent of the allocation of the credit. This implies that all the credit generally goes to only one type of the projects and that, when the composition changes, it switches from one type from another completely. This "bang-bang" nature of compositional swifts, while not a realistic feature of the model, makes the analysis of the dynamics highly tractable, as will be seen below.

For any initial value, $k_0 > 0$, the sequence of k_t that solves (1), (2), and (3) is the equilibrium trajectory of the economy.[7]

Remark 1: The careful reader must have undoubtedly noticed that we deliberately avoid the use of the terminologies such as "debt capacity," "interest rate," and "loan market," and

[7] Strictly speaking, eqs. (1)-(3) do not fully describe the equilibrium. It is also necessary to add the condition stating that, when (3) holds with equality for two or more types of projects, entrepreneurs would choose the one that would give them the highest second period consumption. However, this situation occurs only for a finite number of k_t, which means that the equilibrium trajectory would not encounter such a situation for almost all initial values of k_0. Hence, we omit the discussion of this condition for the ease of exposition.

instead use "borrowing limit," "rate of return," and "credit market." This is because the present paper is concerned with dynamic general equilibrium implications of credit market imperfections, arising from the difficulty of external finance in general. Note that the borrowing constraint arises due to the inability of the borrowers to pledge the project revenue fully, not due to any restriction on the menus of the financial claims that they can issue. The main issues addressed here are general enough that they are independent of the financial structure. Indeed, the model is too abstract to make a meaningful distinction between the equity, the debt, the bonds, or any other forms of financial claims, which we view as an advantage of the model.[8]

3. Two Benchmarks: Monotone Convergence

In this section, we present special cases, where the composition of the credit never changes along the equilibrium path, either due to the absence of the credit frictions or due to the homogeneity of projects. These cases provide useful benchmarks against which we identify the composition effects in later sections.

Benchmark I: The Case of Full Pledgeablity

The first is the case where the revenues from the most productive projects are fully pledgeable. Then, obviously, all the credit goes to the most productive projects. Let $R = \max\{R_1, R_2, \ldots, R_J\}$ denote the productivity of the most productive ones, and let m denote its investment size.[9] Then, eqs. (1)-(3) become

(4) $X_t = W(k_t)/m < 1,$

(5) $k_{t+1} = RW(k_t),$

(6) $r_{t+1} = Rf'(k_{t+1}) = Rf'(RW(k_t)).$

[8] See Tirole (2005, pp.119), who also argues for the benefits of separating the general issues of credit market imperfections and the questions of the financial structure.

[9] For the expositional ease, we assume that one type of the projects strictly dominates all the others in productivity.

Note that the equilibrium trajectory of k_t is determined entirely by eq. (5), which is depicted in Figure 1 under the following assumption:

(A) $W(k)/k$ is strictly decreasing in k, with $\lim_{k\to+0} W(k)/k = \infty$ and $\lim_{k\to+\infty} W(k)/k = 0$,

which holds for many standard production functions, including a Cobb-Douglas, $f(k) = Ak^{\alpha}$ with $0 < \alpha < 1$. Under this assumption, the economy converges monotonically towards its unique steady state, k^*, given by $k^* \equiv RW(k^*)$, as seen in Figure 1. We maintain this assumption for the rest of the paper.

In this case, the model is essentially of the textbook Solow model variety.[10] Since the credit always goes to the most productive project, the composition of the credit never changes, and the dynamics is driven entirely by the aggregate saving, which is inelastic here. Eq. (6) shows that the equilibrium rate of return declines as k_t, and $W(k_t)$, increases. Without credit market imperfections, the rate of return is always equal to the marginal productivity of the project, which declines with capital deepening, just as in the neoclassical growth model.

Note also that the dynamics is independent of the investment size, m. The indivisibility plays no role here, because there is a continuum of homogeneous agents, all of whom can initiate the identical (indivisible) investment project, so that the aggregate investment can change through the extensive margin, as the measure of the projects initiated (and the measure of the agents who become entrepreneurs) adjusts endogenously to equalize the investment and the saving in the aggregate. In other words, this is the environment in which "convexification by aggregation" applies. In spite of the nonconvexity of each investment project, the aggregate investment technology is linear, just as in the standard neoclassical growth model.[11] Of course, the nonconvexity of each project implies that only a fraction of the agents, $X_t = W(k_t)/m < 1$,

[10]It is also isomorphic to the standard Diamond overlapping generations model in which the agents consume only in the second period.

[11]In the neoclassical growth model, the productivity of this linear technology, R, is commonly normalized to be one, which can be done without any loss of generality because of the homogeneity of the investment technologies. The important exception is the literature on investment-specific technological changes, such as Greenwood, Hercowitz, and Krusell (1997, 2000), which allow R to change stochastically. Likewise, we do not use the normalization here, because we consider the cases where the investment projects differ in productivity.

become entrepreneurs, while the others will become lenders.[12] In this case, however, the agents are indifferent, as (PC) holds with equality for the most productive project.

Remark 2: We assume that all agents are homogeneous, because it helps to keep the analysis simple and to highlight the role of changing composition of the credit across heterogeneous projects in the aggregate dynamics.[13] We assume the nonconvexity of each project in order to ensure that some agents become entrepreneurs and others become lenders, so that some credit transactions take place between homogeneous agents. We assume that a continuum of the agents has access to the identical (nonconvex) projects, so that the nonconvexity at the micro level will not carry over to the macro level. In this sense, the nonconvexity here differs fundamentally from the nonconvexity in Kiyotaki (1988), Murphy, Shleifer, Vishny (1989), and other monopolistic competition models surveyed in Matsuyama (1995). These models also have a continuum of agents, each of whom has access to a nonconvex technology. However, these technologies produce imperfectly substitutable goods. Hence, the logic of "convexification by aggregation" does not apply and the nonconvexity at the micro level generally carries over to the macro level in monopolistic competition models.

Benchmark II: The Case of Homogeneous Projects

The second is the case, where there is only one type of projects, i.e., $J = 1$. Then, obviously, all the savings have to go to finance these projects, even if the project revenue is not fully pledgeable. By dropping the subscript $j = 1$ to simplify the notation, we obtain from eqs. (1)-(3)

(4) $$X_t = W(k_t)/m < 1,$$

(5) $$k_{t+1} = RW(k_t),$$

[12]The same procedure described in footnote 4 ensures that $W(k_t)/m < 1$.

[13]In some models of the aggregate investment dynamics, the heterogeneity of agents plays critical roles in generating the credit frictions; see, e.g., Azariadis and Smith (1998) and Caselli and Gennaioli (2005).

as in the previous case. However, the rate of return is now given by

$$(6') \qquad r_{t+1} = \frac{Rf'(RW(k_t))}{\max\left\{1, [1 - W(k_t)/m]/\lambda\right\}}.$$

Again, the model predicts the monotone convergence to the unique steady state under (A). Eq. (5) is independent of m for the reason already discussed above. The nonconvexity of each project implies that, in equilibrium, only the fraction of the agents, $X_t = W(k_t)/m < 1$ becomes entrepreneurs, while the rest becomes the lenders. In spite of the nonconvexity of each project, the aggregate investment technology is linear through the adjustment of X_t, or through "convexification by aggregation."

What is also noteworthy is that eq. (5) is independent of λ, as well. This is because, with all the projects being the same, the fact that an entrepreneur cannot fully pledge their project revenue does not affect the allocation of credit, and hence the dynamics is driven entirely by the aggregate saving, which is inelastic in this model. A change in λ would be entirely offset by a change in the equilibrium rate of return, which adjusts to equate the saving and investment.

Unlike in the previous case, however, the implication on the rate of return is different from the standard neoclassical model. For $W(k_t) < (1-\lambda)m$, eq. (6′) becomes $r_{t+1} = \lambda Rf'(RW(k_t))/(1-W(k_t)/m) < Rf'(RW(k_t))$. That is, (BC) is binding, while (PC) holds with strict inequality. In this case, the rate of return for the lender falls short of the marginal productivity of the project, and hence the agents strictly prefer borrowing to become entrepreneurs to lending. Pinned down by (BC), r_{t+1} cannot adjust to make them indifferent. This means that the equilibrium allocation involves credit rationing, i.e., the credit is allocated randomly to the fraction, X_t, of the agents, while the rest of the agents is denied the credit. The latter have no choice but to become the lenders; they would not be able to entice the potential lenders by promising a higher return, because that would violate (BC).[14] Because of the binding (BC), the

[14] An alternative to the credit rationing, suggested by one of the referees, is that would-be entrepreneurs "outbid" each other by offering random financing contrasts, in which they would give w_t in exchange of a probability of being funded. Then, the equilibrium probability would be equal to X_t. In either way, the allocation mechanism has to be random in order to allocate the credit among the homogeneous agents.

equilibrium rate of return may be procyclical. Furthermore, this can occur in the neighborhood of the steady state. For example, let $f(k) = Ak^{\alpha}$ with $\lambda < 1/(2-\alpha) < 1$. Then, eq. (6′) implies that r_{t+1} is increasing in $W(k_t)$ over $[(1-\alpha)/(2-\alpha)m, (1-\lambda)m]$. This interval includes the steady state, $W(k^*)$, if $[(1-\alpha)/(2-\alpha)]^{(1-\alpha)} < (1-\alpha)[AR^{\alpha}/m^{(1-\alpha)}] < (1-\lambda)^{(1-\alpha)}$.

Remark 3: Although the random allocation of credit is important to understand the working of this model, one should not make too much out of it, because it is an artifact of the assumption that the agents are homogeneous, which we made for the expositional and pedagogical reasons. The homogeneity of the agents means that, whenever some agents face the binding borrowing constraint, all the agents must face the binding borrowing constraint, so that coin tosses or some random devices must be evoked to determine the allocation of the credit. It is possible to extend the model to eliminate the random allocation without changing the essential feature of the model. For example, suppose that the labor endowment of the agents is given by $1 + \varepsilon z$, where ε is a small positive number and z is distributed with the mean equal to zero, no mass point, and a finite support. Then, the allocation of the credit in period t is determined by a critical value, z_t, i.e., the agents, whose endowments are greater than or equal to $1 + \varepsilon z_t$, become entrepreneurs and those whose endowments are less than $1 + \varepsilon z_t$ become the lenders. Our model can be viewed as the limit case, where ε goes to zero.

Remark 4: Two of the results above, (i) the dynamics converge to the unique positive steady state and (ii) the dynamics of k_t is independent of λ, when $J = 1$, are not robust features of the model. The first result is ensured by (A). Without this assumption, the dynamics may have multiple steady states, or it may have no steady state for a sufficiently large R, or its unique steady state may be zero for a sufficiently small R. The second result depends on the assumption that the aggregate saving is inelastic.[15] The point is not to show that these results are inherent features of the dynamics in the absence of the composition effect, because they are not. The

[15]For example, suppose that, in their first periods, the agents can store the final good at the gross rate of return, ρ. When this storage technology is used (i.e., $k_{t+1} < RW(k_t)$), the credit supply becomes perfectly elastic at $r_{t+1} = \rho$. For $W(k_t) < (1-\lambda)m$, the dynamics is given by $f'(k_{t+1}) = (\rho/R\lambda)(1 - W(k_t)/m)$, which shows the credit multiplier (financial accelerator) effect. Indeed, this case is effectively a reproduction of the Bernanke-Gertler (1989) model in its essentials.

point is to offer benchmarks, against which we can identify the role of the changing compositions of the credit across heterogeneous projects in the dynamics of the aggregate investment and borrower net worth.

4. Credit Traps and Credit Collapses

First, let us consider the case, where $R_1 < R_2 < \ldots < R_J$ and $\lambda_1 R_1 > \lambda_2 R_2 > \ldots > \lambda_J R_J$. In words, there are trade-offs between productivity and pledgeability. Higher-indexed projects are more productive, hence appealing to the borrowers (and the next generations of the agents), while lower-indexed projects offer more pledgeable revenues per unit of investment, which make them potentially better alternatives for the lenders. Such trade-offs between productivity and pledgeability can be important when some advanced projects that use leading edge technologies may be subject to bigger agency problems than some mundane projects that use well-established technologies.

For much of the discussion in this section, we focus on the case where J = 2, because it is straightforward (but cumbersome) to extend the analysis for the cases where J > 2. Figures 2a and 2b show the graphs of

$$\frac{R_j}{\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}} \qquad (j = 1, 2)$$

as functions of $W(k_t)$. These graphs, when multiplied by $f'(k_{t+1})$, show the RHS of the inequality in (3), i.e., the rate of return that each project type can offer willingly and credibly to the lender. As shown, each graph is increasing in $W(k_t)$ for $W(k_t) < (1-\lambda_j)m_j$, i.e., when (BC-j) is the relevant constraint. The reason is that an increase in $W(k_t)$ eases the borrowing constraint, as the entrepreneurs need to borrow less. This makes it possible for them to promise a higher rate of return to the lenders. The graphs are flat for $W(k_t) > (1-\lambda_j)m_j$, i.e., when (PC-j) is the relevant constraint. With $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$, the two graphs intersect once at k_c. At this intersection, (BC-2) is always binding. For type-1 projects, (PC-1) is binding, if $m_2/m_1 > (1-\lambda_1)/(1-\lambda_2 R_2/R_1)$, as shown in Figure 2a; (BC-1) is binding if $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2R_2/R_1) < 1$, as shown in Figure 2b. In either case, for $k_t < k_c$, type-1 projects can offer a higher rate of return to the lender than type-2 projects, and hence, all the saving flows into type-1 projects; $X_{1t} = W(k_t)/m_1$ and $X_{2t} = 0$. Therefore, from (2), $k_{t+1} = R_1W(k_t)$. Likewise, for $k_t > k_c$, $k_{t+1} = R_2W(k_t)$. To summarize,

$$(7) \qquad k_{t+1} = \begin{cases} R_1W(k_t) & \text{if} \quad k_t < k_c, \\ R_2W(k_t) & \text{if} \quad k_t > k_c. \end{cases}$$

The intuition behind eq. (7) should be clear. When the entrepreneurs have low net worth, they have to rely heavily on borrowing. Thus the saving flows into type-1 projects, which generate the higher rate of pledgeable return. When the net worth improves, the borrowers need to borrow less, which enables the entrepreneurs to offer the higher return to the lender with type-2 projects, despite they generate the lower pledgeable return per unit of investment.

Since $R_1 < R_2$, the map defined in eq. (7) jumps up as k_t passes k_c, which means that there are three generic cases, depending on whether $k_c < k^*$ (Figure 3a), or $k^* < k_c < k^{**}$ (Figure 3b), or $k^{**} < k_c$ (Figure 3c), where k^* and k^{**} ($> k^*$) are defined by $k^* \equiv R_1W(k^*)$ and $k^{**} \equiv R_2W(k^{**})$, respectively. One can easily verify that all three cases are feasible.[16]

In Figure 3b, both k^* and k^{**} are stable steady states. The lower steady state, k^*, may be interpreted as a *credit trap*. In this steady state, the borrower net worth is low, so that the saving flows into the projects that generate the higher pledgeable return per unit of investment, although they produce less physical capital. The resulting lower supply of physical capital leads to a lower price of the endowment held by the next generation of the agents, hence, a low borrower net worth. Which steady state the economy will converge to depends entirely on the initial condition. If the economy starts below k_c, it converges monotonically to k^*. If the economy starts above k_c, it converges monotonically to k^{**}. Thus, k_c may be viewed as the critical threshold level for economic development.

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[16]To see this, note that k^* and k^{**} are independent of the parameters, λ_1, λ_2, m_1, and m_2, and that k_c can take any positive value by changing these parameters without violating the assumption, $R_2 > R_1 > \lambda_1R_1 > \lambda_2R_2$.

Even when a credit trap does not exist, a low net worth can contribute to a slow growth of the economy, as illustrated by Figure 3a. In this case, if the economy starts well below k_c, the saving will fail to flow into more productive projects for long time, thereby slowing down an expansion of the economy. In Figure 3c, the higher steady state fails to exist, and the saving will eventually stop flowing into more productive type-2 projects, even if the economy starts with a high value of k_0. This case may be called a *credit collapse*.

What is the implication on the rate of return? Instead of going through the taxonomical analysis, let us focus on the case, where there are two stable steady states, $k^* < k_c < k^{**}$, characterized by $k^* = R_1W(k^*)$ and $k^{**} = R_2W(k^{**})$, and where (BC-1) is binding at k* and (PC-2) is binding at k**. Then, from (3), the rates of return in these steady states are given by

$$r^* = \frac{\lambda_1 R_1 f'(k^*)}{1 - W(k^*)/m_1}; \qquad\qquad r^{**} = R_2 f'(k^{**}),$$

respectively. Note that three distinctive factors affect the relative rates of return in the two steady states. First, the credit friction keeps the rate of return strictly below the marginal productivity of the project at k*, but not at k**: $\lambda_1/(1-W(k^*)/m_1) < 1$. Second, the credit friction prevents the credit from flowing into the more productive project at k*, but not at k**: $R_1 < R_2$. These two factors work in the direction of the lower rate of return at k*. Offsetting these factors is the third factor, the standard neoclassical capital deepening effect due to the diminishing return: $f'(k^*) > f'(k^{**})$. For the Cobb-Douglas case, $f(k) = Ak^{\alpha}$, simple algebra can show that the second and third factors exactly offset each other: $R_1f'(k^*) = R_2f'(k^{**})$, from which the overall effect is

$$\frac{r^{**}}{r^*} = \frac{1 - W(k^*)/m_1}{\lambda_1} > 1.$$

The model thus suggests that the rate of return can be higher in a more developed or booming economy than in a less developed or stagnating economy.

It should be obvious to the reader how the above analysis can be extended to the case with J > 2. With J types of the projects, there can be as many as J stable steady states and J −1 credit traps. Furthermore, it is also possible that credit traps and credit collapses may exist at any level of k_t. While this may seem trivial, it helps to clarify some widespread misunderstandings on the implications of models with multiple stable steady states.[17] For example, it is often argued that models with stable multiple steady states offer an explanation for variations of economic performances across the countries. When a graph similar to Figure 3b is used to make this point, the lower (higher) steady state is interpreted as representing the location of poorer (richer) countries. One should not conclude from this, however, that the argument suggests that the poor "developing" countries are in the trap, while the rich "developed" countries are out of the trap. It is also false to say that the argument suggests that the distribution must be bimodal. The logic of the argument does not require that there are only two stable steady states. Models with multiple stable steady states mean that there are many states towards which a country may gravitate. If the countries are scattered across an arbitrary number of stable steady states, there is no reason to believe that the argument suggests a bimodal distribution. Furthermore, it may well be the case that no country has succeeded in reaching the highest stable steady state. If so, all the countries are in the traps, and in this sense, they are all "developing."[18]

Before moving to the next section, let us briefly consider the implications of an increase in pledgeability. In the above analysis, one reason why the saving may fail to flow into the more productive projects is that the borrowers cannot fully pledge their project revenues. So, one might think that a better corporate governance or contractual enforcement technology, which helps to improve pledgeability would always cause the saving to flow into the more productive investment projects. That is certainly the case, if the improvement means a higher λ_J, i.e., a higher pledgeability of the most productive projects. How about a higher pledgeability of the other projects? To answer this question, let us go back to the case where J = 2. In particular, look at the case illustrated in Figure 2b. Note that a higher λ_1 leads to a higher k_c. Since k* and k** are independent of λ_1, this means that the dynamics could change from Figure 3a to Figure

[17] For broad methodological issues on poverty traps, see Azariadis and Stachurski (2005) and Matsuyama (2005b).
[18] Indeed, one could allow for J = ∞, and an infinite number of stable steady states, in which case it is impossible for any country to reach the highest stable steady state, because there is no highest stable steady state.

3b, in which case the credit trap is created as a result of "an improvement" in the credit market. Or the dynamics could change from Figure 3b to Figure 3c, in which case the credit collapse occurs as a result of "an improvement in the credit market. This suggests the following possibility. If an attempt to improve corporate governance is effective only for the well-established industries, whose nature of the agency problems are relatively understood (type-1 projects), it would end up preventing the saving from flowing into new, but more productive technologies, run by small venture capital, whose nature of the agency problems are poorly understood (type-2 projects).[19] More generally, a higher pledgeability of the projects, except those most productive, could end up causing credit traps and credit collapses.

5. Leapfrogging, Credit Cycles, and Growth Miracles

In the previous section, we considered cases where there are trade-offs between productivity and pledgeability, so the interests of the borrowers and the lenders are diametrically opposed when it comes to the choice of the project to be funded. This does not mean that the heterogeneity of the projects and the composition effects play no role when there is no such conflict of interest. To see this, consider the case where $J = 2$ with $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$, and $m_1/m_2 < (1-\lambda_2 R_2/R_1)/(1-\lambda_1) < 1$. Thus, type-1 projects produce less physical capital *and* generate less pledgeable rate of return than type-2 projects. However, the investment size is much smaller for type-1 projects, so the agents need to borrow much less to invest into these projects, which may give type-1 projects advantage over type-2 projects. For example, type-1 projects could represent family operated farms or other small businesses, while type-2 projects represent the investments in the corporate sector. Or, type-1 projects represent traditional light industries, such as textile and furniture, that require a relatively small initial expenditure, while type-2 projects represent modern heavy industries, such as steel, industrial equipments, petrochemical, and pharmaceutical industries that require a relatively large initial expenditure.

Figure 4 shows the two graphs, $R_j/\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$ $(j = 1, 2)$, as functions of $W(k_t)$ for this case. This time, the two graphs intersect twice, at k_c and k_{cc}. For an intermediate range, $k_c < k_t < k_{cc}$, type-1 projects offer a higher return to the lenders than type-2 projects, and

[19] Recall that Figure 2b is applied when $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2 R_2/R_1) < 1$.

hence all the saving flows into type-1 projects and $k_{t+1} = R_1W(k_t)$. Otherwise, $k_{t+1} = R_2W(k_t)$. To summarize,

$$(8) \qquad k_{t+1} = \begin{cases} R_2W(k_t) & \text{if} \quad k_t < k_c \text{ or } k_t > k_{cc} \\ R_1W(k_t) & \text{if} \quad k_c < k_t < k_{cc}. \end{cases}$$

Since $R_1 < R_2$, the map defined in eq. (8) jumps down as k_t passes k_c and jumps up as k_t passes k_{cc}. The intuition should be clear. When the net worth is very low, the entrepreneurs must rely almost entirely on external finance, so that the saving flows into type-2 projects that generate more pledgeable return per unit of investment. As the net worth rises, the entrepreneurs can offer more attractive rate of return with type-1 projects than with type-2 projects, because they need to borrow little for type-1 projects. Hence, a rise in the net worth leads to a shift of the credit toward less productive projects. If the net worth rises even further, then the borrowing need becomes small enough for type-2 projects that the credit shifts back to more productive type-2 projects.

Figures 5a through 5c depict some possibilities generated by eq. (8). In Figure 5a, where $k_c < k^* < k_{cc} < k^{**}$, there are two stable steady states, k^* and k^{**}, again defined by $k^* \equiv R_1W(k^*)$ and $k^{**} \equiv R_2W(k^{**})$. If $k_c < k_0 < k_{cc}$, the economy converges monotonically to k^*. If $k_0 > k_{cc}$, the economy converges monotonically to k^{**}. Hence, as long as we focus our attention to the range above k_c, the dynamics look similar to Figure 3b. However, it can be more complicated if the economy starts below k_c. After the initial phase of growth, if the economy falls into the intermediate interval, (k_c, k_{cc}), then it will converge to k^*. However, if $R_2W(k_c) > k_{cc}$, the economy could bypass this stage and converge to k^{**}, as indicated by the arrows in Figure 5a. In this case, the long run performance of the economy could sensitively depend on the initial condition.[20] Furthermore, this case suggests the possibility of *leapfrogging*. That is, an economy that starts at a lower level may take over another economy that starts at a higher level. For example, imagine that only type-1 projects, textile and others emerged at the time of the first

[20] Mathematically, for any $\varepsilon > 0$, there exist open intervals, I^* and $I^{**} \subset (0, \varepsilon)$, such that, as $t \rightarrow \infty$, $k_t \rightarrow k^*$ for $k_0 \in I^*$ and $k_t \rightarrow k^{**}$ for $k_0 \in I^{**}$.

industrial revolution, are available initially, and some countries, say Britain, have succeeded in reaching the steady state, k*. Then, the second industrial revolution arrives and type-2 projects, some new technologies like chemical and steel industries, are born. Britain, located in k*, is unable to switch to the new technologies, while some, but not all, latecomers, say Germany, come from behind and take over the technology leadership by successfully adopting the new technologies.[21]

In Figure 5b, where $k^* < k_c < k^{**} < k_{cc}$, the equilibrium path fluctuates forever for all k_0.[22] Along these *credit cycles*, an improvement in the current net worth causes a shift in the credit towards the less productive projects that help less to create the future net worth. The resulting decline in the net worth causes the credit to shift back towards the projects that help more to build the net worth in the following period. In Figure 5c, where $k^* < k_c < k_{cc} < k^{**}$, these endogenous fluctuations co-exist with the steady state, k**. If $R_2W(k_c) < k_{cc}$, the economy fluctuates indefinitely for $k_0 < k_{cc}$, while it converges to k** for $k_0 > k_{cc}$. Thus, this is the case where the credit trap takes the form of credit cycles around k_c, instead of the lower steady state, k*. The situation is far more complicated if $R_2W(k_c) > k_{cc}$. This case may be viewed as a hybrid of Figure 5a and Figure 5b. Starting from $k_0 < k_{cc}$, the economy may fluctuate forever around k_c, or, depending on the value of k_0, it may escape and succeed in reaching k**, possibly after long periods of fluctuating around k_c. Thus, this case suggests the possibility of *growth miracles*, where some countries succeed in escaping the trap, and which countries succeed and which countries fail may depend on subtle differences in the initial conditions.

Again, the above analysis can be extended to the case with $J > 2$. In particular, it is possible that the map jumps down and up for many times, creating fluctuations around different levels of k_t. Therefore, one should *not* conclude by looking at Figure 5b or Figure 5c that only the poor countries are subject to credit cycles.[23]

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[21] The story here is only meant to be suggestive, and we do not intend to rule out many other hypotheses that have been proposed as explanations for the stagnation of the Victorian Britain relatively to the Imperial Germany in the late nineteenth and early twentieth centuries. Some of these hypotheses focus on the credit market (e.g., Kennedy 1987). Also, there are many theories of leapfrogging in the national technological leadership. However, to the best of our knowledge, no theory of leapfrogging based on the credit friction exists in the literature.

[22] Although these figures depict period-2 cycles, the fluctuations can take a more complicated form. Providing a full characterization of the dynamics could easily double the length of this paper, without adding much economic insight.

[23] Empirically, it may be the case that poor countries are more volatile. However, this is not a robust implication of the model presented here.

6. Concluding Remarks

The recent macroeconomic literature on credit market imperfections emphasizes the importance of borrower net worth in the aggregate investment dynamics. The existing models are, however, designed to investigate the role of credit market imperfections through its effects on the volume of credit, but not through its effects on the composition of credit. In this paper, we proposed a model of credit market imperfections with heterogeneous investment projects, and studied how a movement in borrower net worth causes the composition of the credit to switch between investment projects between different productivity levels, which in turn affect borrower net worth. The model is simple enough to be tractable and yet rich enough to capture many implications of the composition effects in the joint dynamics of the aggregate investment and borrower net worth.

Keep in mind that this paper offers only a glimpse of what might happen in the investment dynamics in the presence of credit market frictions, when we allow for the composition of the credit to change. The model presented here does not take into account all the potential sources of the heterogeneity across the investment projects. They are assumed to be different only in productivity, pledgeability, and the investment size. Among other things, it is assumed that all the investment projects produce the same capital good, and that the agents are homogeneous. These restrictions are responsible for certain unrealistic features of the equilibrium. For example, the model has the property that, in any period, all the credit goes to only one type of the projects. When a change in borrower net worth causes the credit to switch from one type to another, the switch occurs quite abruptly. Although it helps to makes it tractable, this is neither a realistic nor robust feature of the model. And this abrupt switch causes the discontinuity of the dynamical systems studied here. One could remove these features of the models by relaxing the above restrictions.

Such an attempt has been made in Matsuyama (2004a), which assumes that some projects produce the consumption good, while others produce the capital good. Introducing this additional element of heterogeneity makes the dynamical system continuous and prevents any abrupt change in the composition of the credit along the equilibrium path. It also enables us to address certain issues that cannot be addressed in the present model. For example, all the projects produce the same capital good in the present model, which means that the interest of the agents as the borrower/entrepreneur is completely aligned with the interest of the next generation of the agents. In the model of Matsuyama (2004a), on the other hand, the borrower/entrepreneur may invest in the projects that produce the consumption good, although such projects do not improve the net worth of the next generation. This feature of the model makes it easier to generate endogenous credit cycles under less stringent conditions. Furthermore, this mechanism can easily be combined with the credit multiplier mechanism of Bernanke and Gertler (1989) to generate asymmetric fluctuations, where the economy experiences a long and slow process of recovery from a recession, followed by a rapid expansion, and possibly after a period of high volatility, plunges into a recession. Such an asymmetry would be harder to generate in the present model.[24]

While these additional features might make the model of Matsuyama (2004a) more appealing in some respects, it also makes it technically demanding, as the analysis requires the use of fairly sophisticated techniques from the nonlinear dynamical system theory, which are not among the standard tools in economics. One advantage of the model presented above is that it is simple enough that it could be analyzed by relatively simple graphic techniques that are familiar to many economists. The message here is that, *even* in such a simple model, an endogenous shift in the composition of the credit can generate investment-specific technological change and lead to a wide range of phenomena, such as traps, collapses, leapfrogging, cycles and miracles, in the joint dynamics of the aggregate investment and borrower net worth. What has been uncovered in this paper is only the tip of the iceberg.

[24] The two models also differ in the welfare implications of endogenous cycles. In the present model, the credit goes to the more productive projects in booms, and shifts towards the less productive projects in recessions. In the model of Matsuyama (2004a), booms occur due to *over*-investment into the capital good project, and recessions occur when such inefficiency is corrected as the credit shifts towards the more productive consumption good project.

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Figure 1: Two Benchmarks (Fully Pledgeability, Homogeneous Projects): Monotone Convergence

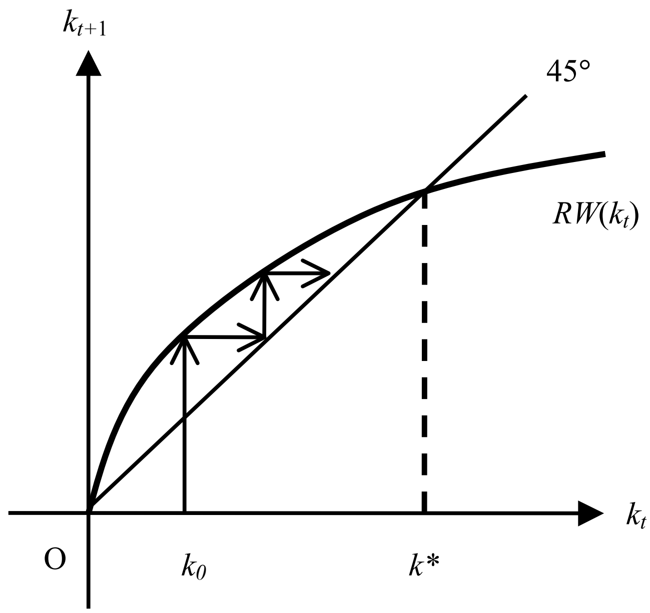


Figure 2a: $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$; $m_2/m_1 > (1-\lambda_1)/(1-\lambda_2 R_2/R_1)$

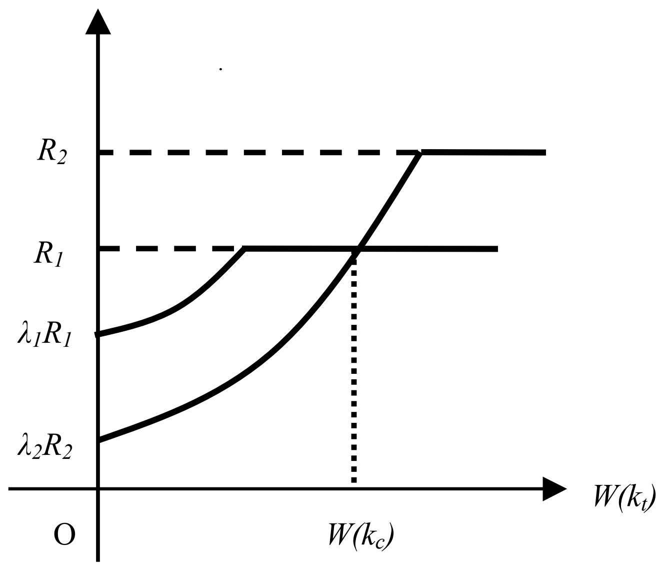
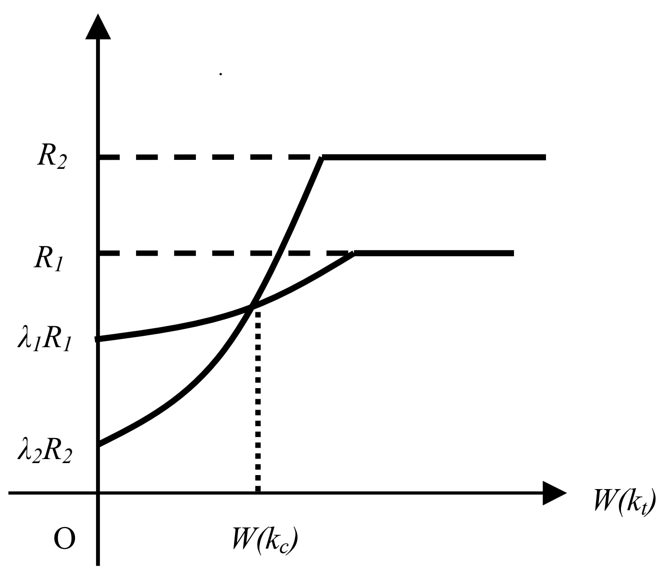


Figure 2b: $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$; $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2 R_2/R_1) < 1$



Figures 3: Credit Traps and Credit Collapse

Figure 3a: $k_c < k^* < k^{**}$

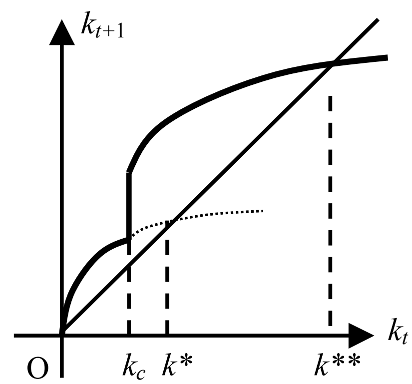


Figure 3b: $k^* < k_c < k^{**}$

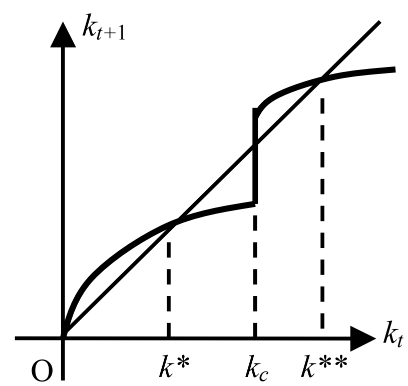


Figure 3c: $k^* < k^{**} < k_c$

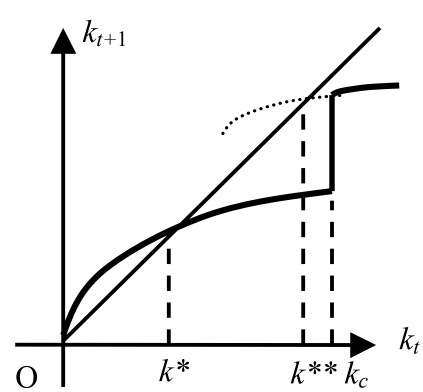
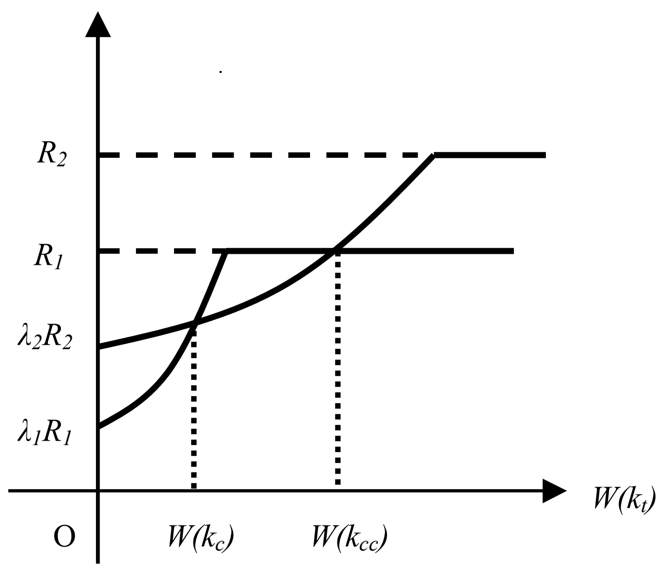


Figure 4: $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$; $m_1/m_2 < (1-\lambda_2 R_2/R_1)/(1-\lambda_1) < 1$



Figures 5: Leapfrogging, Credit Cycles, and Growth Miracles

Figure 5a: $k_c < k^* < k_{cc} < k^{**}$

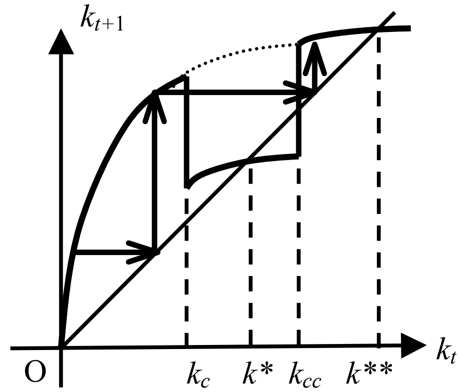


Figure 5b: $k^* < k_c < k^{**} < k_{cc}$

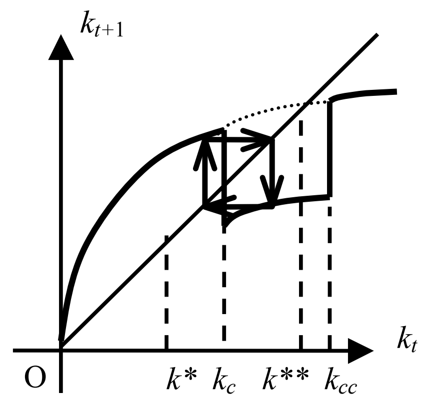


Figure 5c: $k^* < k_c < k_{cc} < k^{**}$