ECON - C11
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Spring 2000

Homework 3 - Solutions

Exercise 3 page 99

(a) The equilibrium level of output is determined by solving the goods market equilibrium condition:

$$
\begin{aligned}
Z &= C + I + G = c_0 + c_1(Y - T) + I + G \\
Y &= Z
\end{aligned}
$$

Solving for Y:

$$Y = m_0 (c_0 - c_1 T + I + G)$$

where $m_0 = \frac{1}{1 - c_1}$ is the multiplier.

(b) Repeat the previous steps replacing I with $b_0 + b_1 Y - b_2 i$. The equilibrium level of output is:

$$Y = \frac{1}{1 - c_1 - b_1}(c_0 - c_1 T + b_0 - b_2 i + G) \qquad \text{(IS)}$$

A change in autonomous spending ΔAE generates a change in output equal to $\Delta Y = m_1 \Delta AE$, where $m_1 = \frac{1}{1 - c_1 - b_1}$ is the multiplier.

For a given level of the interest rate, the effect of a change in autonomous expenditure is bigger than the one in (a) since $m_1 > m_0$.

An initial increase in autonomous expenditure increases output by ΔAE. This produces an increase in both consumption and investment, which increases output further, and so on. The amount of the increase in actual output over time is ΔAE, $\Delta AE\,(c_1 + b_1)$, $\Delta AE\,(c_1 + b_1)^2$. The total change in output is $\sum_{i=0}^{\infty} \Delta AE\,(c_1 + b_1)^i = m_1 \Delta AE$. In part (a), the initial increase in output affects consumption only, implying that at each step the increase in output is smaller. Hence the overall increase in equilibrium output will be smaller.

(c) The money market equilibrium condition can be written as:

$$i = \frac{d_1}{d_2} Y - \frac{1}{d_2}\frac{M}{P} \qquad \text{(LM)}$$

We want to find the value of i and the value of Y such that the goods market and the money market are in equilibrium. We can do this by solving IS and LM for Y and i: To solve for Y; substitute LM into the IS curve derived in (b):

$$Y = m_2 \left(c_0 - c_1 T + b_0 + \frac{b_2}{d_2}\frac{M}{P} + G \right) \qquad (Y_{equil})$$

where $m_2 = \frac{1}{1 - c_1 + \frac{b_2 d_1}{d_2} - b_1}$ is the equilibrium multiplier.

(d) If $\frac{b_2 d_1}{d_2} - b_1 > 0$, you are dividing by a bigger number, and $m_2 < m_0$. Viceversa if $\frac{b_2 d_1}{d_2} - b_1 < 0$, $m_2 > m_0$.

In order to interpret the previous conditions, it is convenient to rewrite them as:

$$\frac{d_2}{d_1} < \frac{b_2}{b_1}, \quad m_0 > m_2$$
$$\frac{d_2}{d_1} > \frac{b_2}{b_1}, \quad m_0 < m_2$$

$\frac{d_2}{d_1}$ tells us how responsive is the real money demand to a change in the interest rate, as opposed to a change in income. $\frac{b_2}{b_1}$ represents the responsiveness of investment demand to the interest rate relative to output.

The effect of a change in autonomous expenditure is different because there are two forces at play in the IS-LM model which are absent in the Keynesian cross model in part (a).

First, firms decide how much to invest taking into account income and the interest rate. Second, there is a money market that needs to clear in order to achieve the equilibrium.

Consider an increase in AE. Assume $\frac{d_2}{d_1}$ is large. This means that the LM curve is flat[1]. Hence the shift of the IS curve translates into a relatively large increase in output with respect to the increase in the interest rate. If firms weigh output a lot in making their investment decisions, i.e. $\frac{b_2}{b_1}$ is small, the effect of ΔAE is magnified ($m_0 < m_2$).

If the LM curve is steep, i.e. $\frac{d_2}{d_1}$ is small, and firms care a lot about interest rates, i.e. $\frac{b_2}{b_1}$ is large, the increased autonomous expenditure will affect interest rates more, reducing investment a lot. In this case the effect is smaller with respect to the Keynesian cross model ($m_0 > m_2$).

[1] The slope of the LM curve is $\frac{d_1}{d_2}$.

Exercise 4 page 99

(a) A decrease in government spending shifts the IS curve to the left. The new equilibrium will have a lower level of output and a lower interest rate.

The reduction of the interest rate stimulates investment and the decrease in income tends to reduce it. Since the two effects work in opposite directions, the overall effect is ambiguous.

(b) See exercise 3.

(c) Substituting the equilibrium level of output computed above into the LM equation:

$$i = \frac{d_1}{d_2} m_2 \left(c_0 - c_1 T + b_0 + \frac{b_2}{d_2} \frac{M}{P} + G \right) - \frac{d_1}{d_2} \frac{M}{P} \qquad (i_{equil})$$

(d) Plugging the expressions for the equilibrium level of output and interest rate into the investment function:

$$I = b_0 + b_1 Y - b_2 i = b_0 + b_1 m_2 A - b_2 \frac{d_1}{d_2} m_2 A + b_2 \frac{d_1}{d_2} \frac{M}{P}$$

(e) The change in investment is simply:

$$\Delta I = b_1 \Delta Y - b_2 \Delta i$$

where ΔY and Δi can be computed from the expressions for the equilibrium level of output and interest rate derived above.

$$\Delta Y = m_2 \Delta G; \quad \Delta i = \frac{d_1}{d_2} m_2 \Delta G$$

The overall effect on investment is:

$$\Delta I = b_1 m_2 \Delta G - b_2 \frac{d_1}{d_2} m_2 \Delta G = \left(b_1 - b_2 \frac{d_1}{d_2} \right) m_2 \Delta G$$

$\Delta G < 0$, since we are considering a decrease in government spending, and $m_2 > 0$, since $b_1 + c_1 < 1$. So, in order to get an increase in investment ($\Delta I > 0$), the term in brackets has to be negative:

$$\left(b_1 - b_2 \frac{d_1}{d_2} \right) < 0$$

(f) The condition derived in (e) can be expressed as $\frac{b_2}{b_1} > \frac{d_2}{d_1}$. If $\frac{d_2}{d_1}$ is small, it means that real money demand responds a lot to changes in income and very little to variations in the interest rate. This implies a steep LM curve. When the IS curve shifts left for the decrease in public spending a relatively large reduction in the interest rate is required to restore the equilibrium in the money market. A large $\frac{b_2}{b_1}$ means that firms put a lot of weight on the interest rate in making their investment decisions. If this is the case, the interest rate effect prevails and investment increases.