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311, Spring 2000

Suggested Answers to Second Midterm

1. That the price level and output are both falling suggests that the shock is to the AD curve. That the interest rate is falling too, suggests that the shock is to the IS curve, and not the LM curve. These four shocks could account for a left shift in the IS curve: a fall in the autonomous component of consumption, of investment, or government spending, or a rise in taxes. Additional data on these variables would be needed to determine which was the culprit.

2. With free and flexible international capital markets, it must be that rates of return in every country is the same, for otherwise people would only hold the return-yielding assets of the country with the highest return. Rates of return must be the same, when denominated in the same currency. This implies that the domestic rate of return, i, must be equal to the foreign rate of return, i^*, plus the anticipated rate of depreciation of the domestic currency. The latter is what you get from exchanging the domestic currency for a foreign currency and reversing the operation later on, when the foreign asset pays off. Under a fixed exchange rate, the anticipated rate of depreciation is, of course, zero. So, the implication is that the domestic rate of interest must equal the foreign rate of interest. To implement this, the monetary authority must adjust the money supply so that the domestic nominal rate of interest is i^* always. This translates into the need to move M around so that, given the level of Y, the LM curve intersects the IS curve at i^*.

   (a) With P fixed, you can use the open economy $IS-LM$ model to do the analysis. A fall in Y^* produces a drop in exports, X, which has the effect of shifting the IS curve to the left. To keep the level of output unchanged, the central bank needs to shift the LM curve to the right. The left shift in the IS curve and the right-shift in the LM curve both have the effect of driving down i. The low rate of interest, given the unchanged value of E^e (the shocks we're talking about are temporary, so the whole economy returns to where it was in the medium run), means that the exchange rate depreciates (i.e., E rises). There is no effect on consumption, because there is no effect on Y or T. There is no effect on G because we've assumed that's exogenously set to $\bar{G}$. What happens, then, is that investment is increased by the full amount of the drop in X. All this is accomplished with a temporary rise in M, one that is reversed once Y^* returns to its original position.

   (b) To keep the exchange rate unchanged, the central bank has to keep the interest rate, i, equal to i^*, the foreign interest rate. This means that, as the IS curve shifts left, the central bank must *decrease* the

money supply. This leads to a large fall in output, certainly by comparison with what we get in part (a). The fall would only occur in the short run, since in the medium run the shock reverses itself, and everything goes back to where it was before. The other things that happen in the short run are: a fall in consumption, since Y falls; a rise in unemployment; a fall in the money supply; and no change in investment. The fixed exchange rate policy, in effect, forces the central bank to do the opposite of what it would like to do, if it only had domestic output and employment objectives to think about. It would certainly feel a lot of pressure to abandon the fixed exchange rate policy and let the currency depreciate.

(c) The fall in Y^* may lead everyone to believe, for the reasons sketched in (a) and (b), that the central bank will cut its interest rate and accept a depreciation of the currency. If they expect, say, a 10% depreciation next week, then markets will require that i jump by 10% per week if they are to be happy keeping their funds in the domestic economy. That translates into 520% at an annual rate! So, defending the exchange rate now will require not just keeping the interest rate constant (which is bad enough!) but raising it by an enormous amount! This high interest rate will reduce investment (because of the high value of i); reduce consumption (because of the low Y); and reduce output.

(d) It is possible that the central bank does not have the stomach to raise the interest rate by 520% to defend against an attack, even though it might well have been able to maintain the exchange rate if only Y^* had fallen and no attack had occurred.

(a) You get $i = sy$ from the national income identity, $c + i = y$. But, $c = (1 - s)y$, so that $(1 - s)y + i = y$, or, $i = sy$. You get the per-capita capital evolution equation by dividing by L. The production function is $y = k^\alpha$.

(b) Long-run equilibrium occurs where investment, i, equals the depreciation on capital, δk, so that $\Delta k = 0$. Graphically, this is where the investment function, expressed as a function of k, intersects the depreciation function, δk. If k is below this, then investment exceeds saving and investment and k has a tendency to rise. Similarly, if k is above this point, then it falls.

(c) The wage rate on labor corresponds to the marginal product of capital, MPL. Profit-maximizing firms hire labor up to the point where MPL equals the real wage rate, w, where MPL is the derivative of the production function with respect to labor. Similarly, they hire capital up to the point where MPK equals the real rental rate on capital, r, where MPK is the derivative of the production function with respect to capital. These results can easily be established graphically, by showing that profits, $y - w\bar{L} - r\bar{K}$, is highest when

$MPL = w$ and $MPK = r$, where $y = \bar{K}^{\alpha}\bar{L}^{1-\alpha}$ and $\bar{K}$, $\bar{L}$ denote the amount of capital and labor hired by an individual firm. The formula for MPK and MPL is:

$$MPK = \alpha\left(\frac{\bar{L}}{\bar{K}}\right)^{1-\alpha},\ MPL = (1-\alpha)\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}.$$

With all firms choosing labor and capital in this way, it must be that L/K, the economy's aggregate labor over capital ratio, is equal to $\bar{L}/\bar{K}$. Taking this into account, these equations can be used to solve for r and w as a function of K and L :

$$r = \alpha\left(\frac{L}{K}\right)^{1-\alpha},\ w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

(d) The fact that K drops by a smaller percent than L means that there is a rise in k. This is like capital destruction in a war, except the economic effect is the other way around. With every firm in the economy doing this, Initially, the marginal product of labor rises. This can be written as follows:

$$MPL = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

3. In the country with no total factor productivity growth, the increase in Y must be occurring because the economy is converging to its long-run equilibrium k. In this case, growth will eventually stop. This sort of situation could occur if growth is happening because the government has increased the saving rate or if something has caused the capital labor ratio to drop. In the country with total factor productivity growth, the increase in Y is probably a reflection of the rise in A. This bodes well for the future because experience suggests that this can go on for ever, unlike, for example, increases in s.