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C = 10 + 0.8(Y - T),
I = 10,
G = 10,
T = 10
Q = 0.3Y
X = 0.3Y

Real exchange rate = 1

(a)
Equilibrium given Y*:
Aggregate demand = Z = C + I + G + X - Q = 10 + 0.8(Y - 10) + 20 + 0.3Y* - 0.3Y = 22 + 0.3Y* + 0.5Y

Equil.: Y = Z. So, Y = 2*(22 + 0.3 Y*) = 44 + 0.6Y*

Multiplier = 2.

In the closed economy (without imports nor exports), the multiplier would be 5.

In an open economy the multiplier is lower because there is a leakage for every increase in output (ie: imports).

(b)
Suppose that the foreign economy has the same equations as the domestic economy. Then the 2 equations which determine Y and Y* are:
Y = 44 + 0.6 Y* and Y* = 44 + 0.6 Y

Plugging one into the other: Y = 44 + 0.6 (44 + 0.6 Y) = 44 + 26.4 + 0.36 Y.

So, Y = Y* = (1/0.64) * 70.4 = 110

In order to determine the multiplier, define A = I + G + c_0 - c_1 T = 22.

We have, Z = A + 0.5Y + 0.3Y*, so in equilibrium, Y = 2A + 0.6 Y*.

Again, computing the world equilibrium, we have that:
Y = 2A + 0.6 (2A* + 0.6 Y) = 2A + 1.2A* + 0.36Y, so that Y = 1/0.64 (2 A + 1.2A*) = 3.125A + 1.875A*

The multiplier is 3.125, which is larger than the multiplier we obtained above (2). This is because we are endogenizing Y*. An increase in domestic A, by increasing Y and hence domestic imports, increases foreign output Y*. This provides an extra push to domestic output Y through a higher demand for its exports.

(c)
Suppose both countries have a target of \(Y=Y^*=125\).
Let’s denote domestic and foreign government expenditure as \(G^D\) and \(G^F\), respectively.
Suppose that \(G^F = 10\). What should \(G^D\) be in order to set \(Y = Y^* = 10\).

We know that:
\[Y = 2A + 2G^D + 0.6 \ Y^* \text{ and } Y^* = 2A + 2G^F + 0.6 \ Y,\text{ where now } A=12 \text{ excludes government expenditures.}\]

Substituting:
\[Y = 2A + 2G^D + 0.6 \ (2A + 2G^F) + 0.6 \ Y.\]

Solving for \(Y\) and \(Y^*\):
\[Y = \frac{1}{0.64} \ (3.2A + 2G^D + 1.2G^F) \text{ and } Y^* = \frac{1}{0.64} \ (3.2A + 1.2G^D + 2G^F)\]

Let’s fix \(G^F=10\). In order to set \(Y=125\), we need to solve for \(G^D\) in the following equation:
\[125 = \frac{1}{0.64} \ (3.2*12 + 2G^D + 12)\]
So \(G^D = \frac{0.5*(125*0.64-3.2*12-12)}{3.2} = 14.8\).

With \(G^D=14.8\), \(Y^*=119\).

Let’s now compute the budget deficit and net exports:

**Domestic country:**
- Budget deficit = \(G - T = 14.8 - 10 = 4.8\)
- Net exports = \(X - Q = 0.3 \times (Y^* - Y) = -0.3 \times 6 = -1.8\)

**Foreign country:**
- Budget deficit = \(G - T = 10 - 10 = 0\)
- Net exports = \(X - Q = 0.3 \times (Y - Y^*) = 0.3 \times 6 = 1.8\)

(d) Now let’s suppose that both countries coordinate an equal \(G\) to set \(Y=Y^*=125\).
\(G\) solves the following equation:
\[125 = \frac{1}{0.64} \ (3.2*12 + 2G + 1.2G).\]
Solving for \(G\):
\[G = \frac{(125*0.64 - 3.2*12)}{3.2} = 13\]

(e) In this open economy scenario there is a clear externality: If a country increases \(G\), it will not only increase its own output, but also the foreign country’s output. Assuming that fiscal deficits are expensive to sustain, then it will be in the interest of countries to free ride on the expansionary fiscal policy of the neighbor in order to increase its own output. Hence, fiscal coordination will be difficult to achieve.
real exchange = \( E = S \times P^* / P \), where \( S \) is the nominal exchange rate (local currency per foreign currency), \( P^* \) is the foreign price level, and \( P \) is the domestic price level.

Proposition 7 and 8 in appendix 2 imply that if \( z = w / y \), then \( g_z = g_w + g_x - g_y \), where \( g \) denotes a growth rate.

Then, from the definition of the real exchange rate, we will have that

\[ g_e = g_s + g_{P^*} - g_P \],

which is exactly equal to what we are asked to prove.