Professor Christiano
Economics 311, Winter 2004

# Solutions to First Midterm

1. Multiple Choice.

   (a) v.
   (b) ii.
   (c) v.
   (d) iv.
   (e) ii.
   (f) ii.
   (g) iv.

2.

   (a) The goods market is in equilibrium when total demand equals total production, i.e. $Y = Z$. In the standard IS-LM model, $Z = C^d + I^d + G^d = c_0 + c_1 (Y - T) + \bar{I} - b \times i + \overline{G}$. So using the equilibrium condition, the equilibrium output as a function of the exogenous parameters and of the rate of interest, $i$ is:

   $$Y = \frac{1}{1 - c_1} \left( c_0 - c_1 \bar{T} + \bar{I} + \bar{G} - b \times i \right).$$

   (b) For a given interest rate, the total desired spending curve is:

   $$Z(Y) = c_0 + c_1 (Y - T) + \bar{I} - b \times i + \overline{G},$$

   so that the vertical intercept is $c_0 - c_1 T + \bar{I} - b \times i + \overline{G}$, while the slope is $c_1$. (In the figure, 'goods market equilibrium' is indicated by $Y^* (i)$, to remind ourselves it is determined for a given interest rate.)

   (c) The financial market is in equilibrium when money demand equals money supply, i.e.: $\frac{M^d}{P} = \frac{M^s}{P}$. Solving the equilibrium condition:

   $$\frac{M^s}{P} = Y + \bar{L} - L_1 \times i$$

   for $i$, $i = \frac{1}{L_1} \left( Y + \bar{L} - \frac{M^s}{P} \right)$, where all variables on the right hand side are parameters but for the level of output $Y$. (In the figure, the 'money market equilibrium' interest rate is indicated by $i^* (Y)$, to remind ourselves that it is determined for a given level of income).

   (d) The demand for money, expressed in terms of the interest rate is: $i = \frac{1}{L_1} \left( Y + \bar{L} - \frac{M^d}{P} \right)$, so that the vertical intercept is: $\frac{1}{L_1} \left( Y + \bar{L} \right)$ while the slope is $-\frac{1}{L_1}$.

(e)

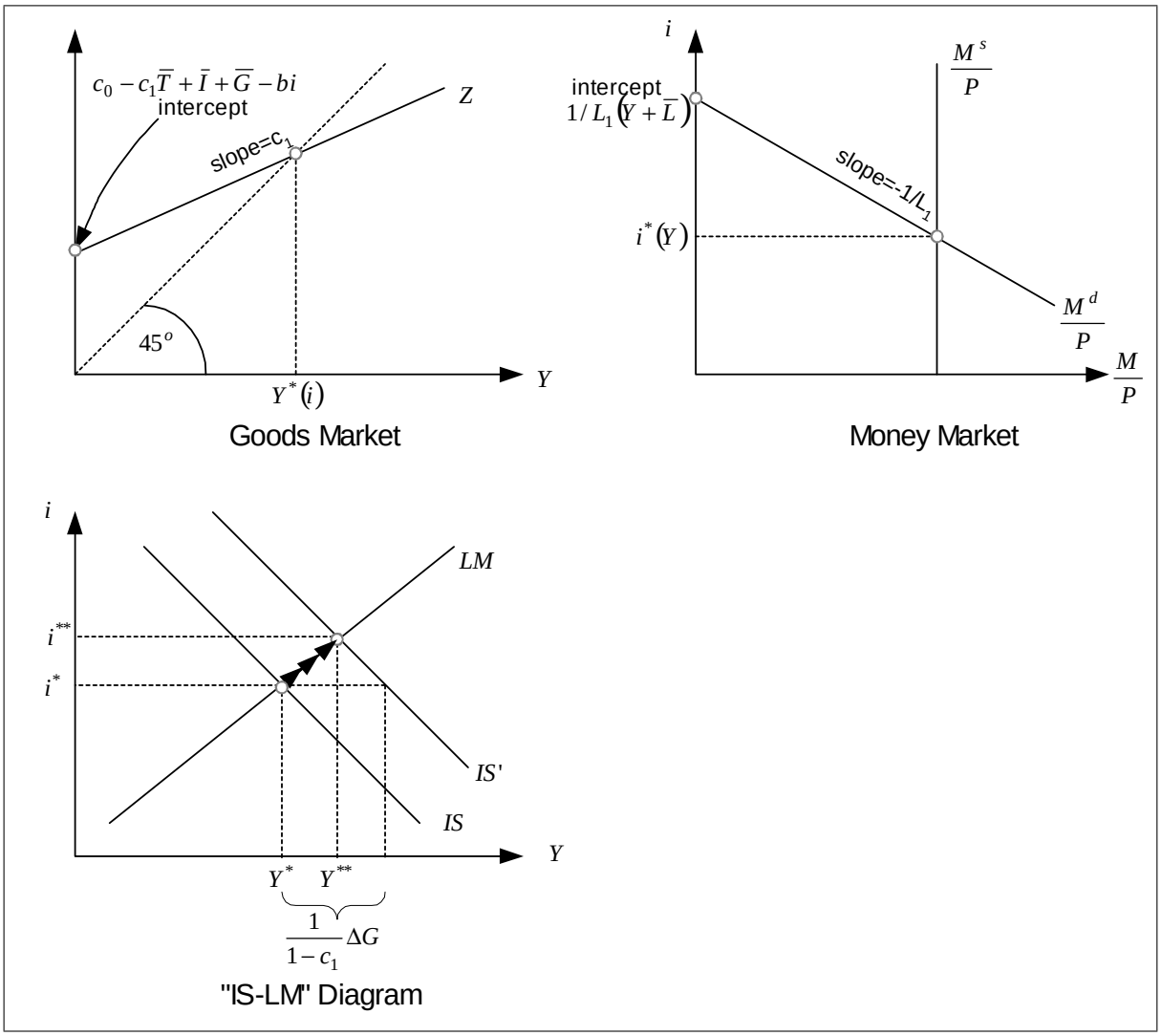
i. The IS curve represents combinations $(Y, i)$ such that the goods market is *in equilibrium*. The IS curve has a negative slope because, as the interest rate increases, desired investment decreases and so does goods market equilibrium output (with the latter change being larger in absolute value than the former, because of the multiplier effect).
The LM curve represents combinations $(Y, i)$ such that financial markets are *in equilibrium*. The LM curve has a positive slope because as income increases, money demand increases and bond demand decreases for a given interest rate. But since money supply and bond supply are fixed, equilibrium in financial markets requires an increase in the interest rate to increase bond demand and reduce money demand back to their initial equilibrium levels.

ii. Above (below) the IS there is excess supply (demand) in the goods market.
For a given $Y$, the interest rate is higher (lower) than required for $Y = Z$. Because desired investment is decreasing in the interest rate, desired investment is low (high), and so $Y > Z$ ($Y < Z$). The stock of unintended inventory investment is positive (negative).

iii. Above (below) the LM there is excess supply (demand) in the money market.
For a given $Y$, the interest rate is higher (lower) than required for $\frac{M^s}{P} = \frac{M^d}{P}$. Because money demand is decreasing in the interest rate (bonds are more attractive), real money demand is smaller (greater) than money supply, $\frac{M^s}{P} < \frac{M^d}{P}$.

iv. If $\bar{G}$ increases by $\Delta G > 0$, the $IS$ curve shifts rightward. The magnitude of the rightward shift is given by the *change in equilibrium $Y$ in the goods market* **for a given interest rate.** In the standard IS/LM model, this change is always bigger than the equilibrium change in $Y$, since as $Y$ increases, the equilibrium interest rate must also increase to guarantee market clearing in financial markets, inducing a decrease in investment demand. However, when calculating the rightward shift of the IS-curve, we are not requiring financial markets to be in equilibrium: The interest rate is taken as given. From the previous answers: $Y = \frac{1}{1-c_1}\left(c_0 - c_1\bar{T} + \bar{I} + \bar{G} - b \times i\right)$, so that the rightward shift is: $\frac{\Delta G}{1-c_1}$ for any value of the interest rate (parallel shift).

v. Disequilibrium dynamics involves two things: which market variable adjusts and how fast. Output shifts slowly in the goods market and the interest rate shifts extremely quickly in the financial market. As a result of these assumptions – along the adjustment path – $(i, Y)$ is always on the $LM$-curve, i.e. financial markets are always in equilibrium.
Because of the expansionary fiscal shock, the $IS$ shifts rightward by $\frac{\Delta G}{1-c_1}$ to $IS'$, intersecting the $LM$ at $(Y^{**}, i^{**})$ where $Y^* < Y^{**} < Y^* + \frac{\Delta G}{1-c_1}$ and $i^{**} > i^*$. Because of the slow adjustment in the goods market, the economy does not jump directly to $(Y^{**}, i^{**})$.
Immediately after the shock, $Z = Y^* + \Delta G > Y^*$ : There is excess demand in the goods market and hence a decrease in inventories. Firms adjust their production upwards accordingly. As they do so, income increases, and this in turn increases money demand, requiring interest rate to increase at the same time in order to maintain equilibrium in financial markets This process of output and interest rate adjustment continues until output and interest rates reach $(Y^{**}, i^{**})$. (Along the adjustment path, the rise in the interest rate reduces desired investment demand, but there remains a (decreasing) excess aggregate desired demand as we are always at $(Y, i)$ pairs below the new IS-curve.).




3.

(a) The equilibrium condition in the Keynesian Cross model is

$$Y^* \stackrel{!}{=} Z \equiv c_0 + c_1\left(Y^* - \overline{T}\right) + \overline{I} + \overline{G}$$

Solving this equation for equilibrium $Y^*$ yields

$$Y^* = \tfrac{1}{1-c_1}\left[c_0 - c_1\overline{T} + \overline{I} + \overline{G}\right]$$

The multiplier on investment is

$$\frac{\Delta Y^*}{\Delta \overline{I}} = \frac{\frac{1}{1-c_1}\left[c_0 - c_1\overline{T} + \overline{I} + \Delta\overline{I} + \overline{G}\right] - \frac{1}{1-c_1}\left[c_0 - c_1\overline{T} + \overline{I} + \overline{G}\right]}{\Delta\overline{I}} = \frac{1}{1-c_1}.$$

(b) The equilibrium condition in the Keynesian Cross model with $t_1 > 0$ is

$$Y^*_{t_1>0} \stackrel{!}{=} Z = c_0 + c_1\left(Y^*_{t_1>0} - \overline{T} - t_1 Y^*_{t_1>0}\right) + \overline{I} + \overline{G}$$

Solving this equation for equilibrium $Y$ yields

$$Y^*_{t_1>0} = \tfrac{1}{1-c_1(1-t_1)}\left[c_0 - c_1\overline{T} + \overline{I} + \overline{G}\right]$$

The multiplier on investment is now

$$\frac{\Delta Y^*_{t_1>0}}{\Delta \overline{I}} = \frac{1}{1-c_1(1-t_1)}.$$

Since $0 < t_1 < 1$, $\frac{1}{1-c_1(1-t_1)} < \frac{1}{1-c_1}$, i.e. the investment multiplier is smaller if $t_1 > 0$ (and it is decreasing in $t_1$).
*Intuition:* Taxes reduce consumption and hence demand. If taxes increase with income ($t_1 > 0$), they will also decrease the effect of a change in income on consumption and hence demand. This will dampen the multiplier effect of a change in autonomous expenditure on equilibrium output.

(c) The equilibrium condition in the Keynesian Cross model with $q > 0$ (and $t_1 = 0$) is

$$Y^*_{q>0} \stackrel{!}{=} Z = c_0 + c_1\left(Y^*_{q>0} - \overline{T}\right) + \overline{I} + q \cdot Y + \overline{G}$$

Solving this equation for equilibrium $Y$ yields

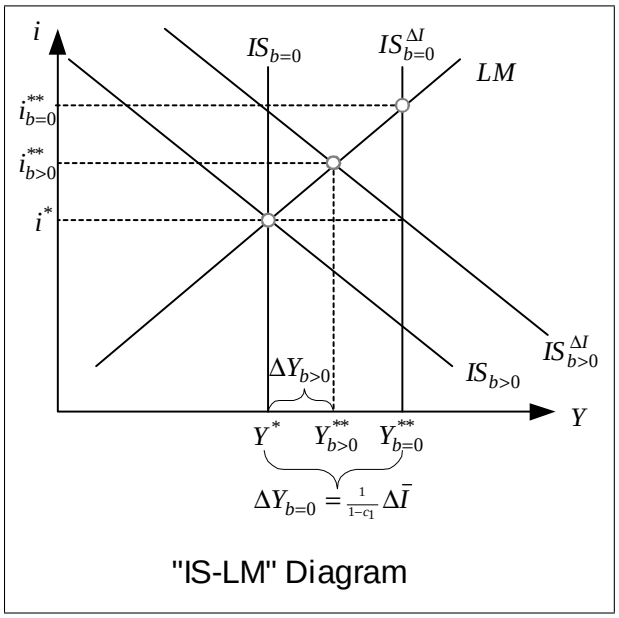
$$Y^*_{q>0} = \frac{1}{1-c_1-q}\left[c_0 - c_1\overline{T} + \overline{I} + \overline{G}\right]$$

The multiplier on investment is now

$$\frac{\Delta Y^*_{q>0}}{\Delta \overline{I}} = \frac{1}{1-c_1-q}.$$

Since $q > 0$, $\frac{1}{1-c_1-q} > \frac{1}{1-c_1}$ (as long as $q < 1 - c_1$), i.e. the investment multiplier is larger for $q > 0$ (and increasing in $q$).
*Intuition:* If investment increases with income ($q > 0$), the multiplier process is amplified. The multiplier effect on equilibrium income of an increase in autonomous expenditures will work through not only consumption but also investment. Hence the investment multiplier $\frac{\Delta Y}{\Delta \overline{I}}$ is larger for $q > 0$ (Accelerator Effect).

(d) The magnitude of the horizontal shift of the IS curve is always equal to the product of the corresponding 'Keynesian Cross-Multiplier' and the change in autonomous expenditure. This would be the effect for a given interest rate. *Equilibrium changes in output* will however involve changes in the interest rate as financial markets must also be in equilibrium.
If $b = 0$,investment and hence aggregate demand does **not** depend on the interest rate. Equilibrium output is solely determined in the goods market: The IS curve is vertical. Even though the equilibrium interest rate rises in response to $\Delta\overline{I} > 0$ to restore equilibrium in financial markets at a higher $Y$, the investment multiplier $\frac{\Delta Y^*}{\Delta \overline{I}}$ is identical to the one of the Keynesian Cross model.
If investment depends negatively on the interest rate if $b > 0$, the equilibrium response of output to $\Delta\overline{I}$ will be *smaller* than in the case $b = 0$,because higher output requires a higher interest rate for the financial markets to clear.
(In the figure, initial equilibrium is drawn at the same $(i, Y)$ pair to simplify the exposition)




4.

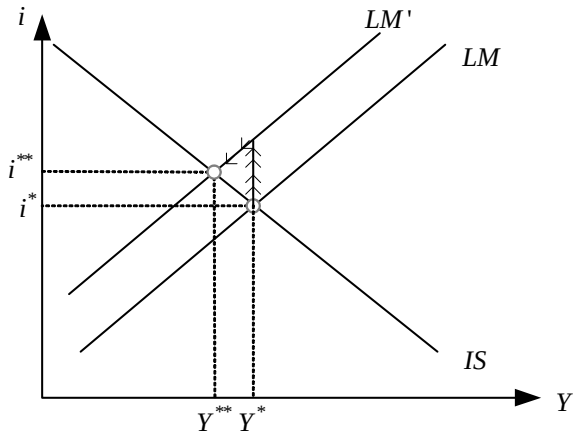
(a) In deciding whether to invest in a given project, firms have to decide on what rate of return the project will generate. This in turn depends on how active the economy is. If firms expect the level of activity in the economy to be high, then they will expect a high rate of return–in other words, they form their expectation of how high economic activity will be by looking at current output. So, the reason $q > 0$ is that in this case, a higher Y causes firms to revise upward their assessment of the rate of return on all projects.

(b) Each firm has a bunch of potential projects laying around, each with an expected rate of return. To invest in a project, they have to convince someone to give them the cash. That person has other stuff to do with the cash (this is measured by $i$), and so the firm manager won't even make an attempt at a project if its rate of return is less than $i$. For any firm, the investment demand curve is a downward sloped step function. As you aggregate over the whole economy the steps become finer until it's just a smooth downward sloped curve. So aggregate investment negatively depends on the interest rate, i.e. $b > 0$.

5.

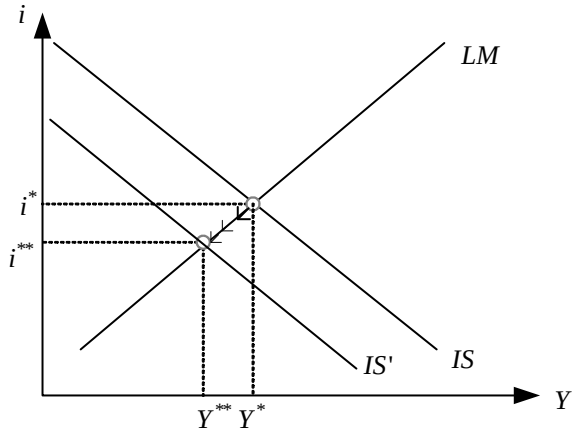
(a) A decrease in the money supply (or a positive shock to the liquidity demand parameter $\overline{L}$) would shift the LM curve upward. The new equilibrium in the goods and financial market involves a higher interest rate ($i^{**} > i^*$) and lower output ($Y^{**} < Y^*$). The direction of the adjustment

reflects the assumptions on the disequilibrium dynamics.




"IS-LM" Diagram

(b) A decrease in any component of autonomous expenditure would shift the IS curve leftward and the new equilibrium would involve both a lower interest rate ($i^{**} < i^*$) and lower output ($Y^{**} < Y^*$). The direction of the adjustment reflects the assumptions on the disequilibrium dynamics.




"IS-LM" Diagram

6. In the IS-LM model (a model of the short run), equilibrium output is determined by the demand side of the economy (how much households, businesses and governments want to buy). It is assumed that firms do whatever it takes to produce the output that demand requires. Now suppose that a positive technology shocks increases productivity in the production sector. The technology shock does not affect total demand (it does not appear anywhere in our desired consumption, investment or government consumption equations). As productivity increases and aggregate demand is unaffected, the number of workers shrinks, reducing employment (holding hours per-worker fixed).

7. Desired saving is $S_t = Y_t - C_t - T_t = -c0 + (1 - c1)(Y_t - T)$. As the loss of consumer confidence is unexpected by firms, $Y_1 = Y_0$, so that,as a consequence of the fall in $c_0$: $\Delta S_1 = S_1 - S_0 = -\Delta c0 = \$10$. As the economy is in equilibrium in period 0, total demand, which falls by 10\$ relative to $Z_0$, is $Z_1 = Z_0 - 10\$ = Y_0 - 10\$ = Y_1 - 10\$$. So unintended inventory investment in period one is $I_{u1} = Y_1 - Z_1 = 10\$$. As the economy is in equilibrium in the period 0 $\Delta I_{u1} = I_{u1} - I_{u0} = I_{u1} - 0 = 10\$$. As desired investment is unchanged, the change in actual investment is $\Delta I_a = 10\$$.

8. The equilibrium interest rate is found by equating money demand and money supply: $M^s = 100\$ = M^d = \$100 \times (1.05 - i)$, so that $i = .05$ or $i = 5\%$. Assuming that there are no assets other than money and bonds, wealth is invested in either of the two that is: $W = M^d + B^d$. From this, and the fact that the equilibrium level of nominal money is 100\$, the equilibrium demand for bonds is 900\$.