1. Show that an increase in $a$ has no impact on the natural rate of unemployment.

The natural rate of unemployment—the unemployment rate in the medium run—is obtained from the intersection of the $PS$ curve with the (medium run) $WS$ curve. Equating the real wage from the two curves:

$$\frac{1}{1 + \mu} = F(u^n, z),$$

which (implicitly) defines the natural level of unemployment as a function of $z$ and $\mu$. As $a$ does not appear in the relation, a change in $a$ has no effect on the natural level of unemployment.

2. Show what happens to the AS curve after a rise in $a$.

First note that from the definition of unemployment: $u = 1 - \frac{N}{L}$. Substituting total employment from the aggregate production function: $Y = aN$, we can rewrite the unemployment rate as:

$$u = 1 - \frac{Y}{aL}.$$  

Then the $AS$ curve is:

$$(AS) \ P = P^e(1 + \mu)F \left( 1 - \frac{Y}{aL}, z \right).$$

If $a$ increases, for a given $Y$, the price level is lower; in other words, the $AS$ curve shifts down/right.
3. Indicate the short run equilibrium after a rise in $a$, and the medium run equilibrium, in the AD-AS diagram, and in the IS-LM diagram.

The initial equilibrium is 1, the short run equilibrium is 2 and the medium run equilibrium is 3:

![AD-AS Diagram](image)

4. What happens to unemployment in the short run equilibrium as a result of an increase in $a$.

From part 1., in the medium run the unemployment rate equals the initial (medium run) rate. In the short run $Y < Y'_n$, hence the unemployment rate increases in the short run.
5. What happens to I, i, P after the rise in a.

- $P, i$: decrease both in the short and in the medium run.
- $I$: increases both in the short and in the medium run

**Question #2.**

Let the $PS$ equation be: $\frac{W}{P} = \frac{1}{1+1}$, and the $WS$ equation: $\frac{W}{P} = 1 - (u_s + \frac{1}{2}u_L)$, where $u_l = \beta u$ and $u_s = (1 - \beta)u$, are respectively the ratio (to $L$) of the short-term and long-term unemployed, and $u = u_s + u_b$.

1. Which type of unemployment has the greater impact on the real wage? Explain.

   Short term unemployment. The long-term unemployed may not be searching as much and/or may not be as employable as the short-term.

2. Derive the natural rate of unemployment.

   Equating the real wage from $WS$ and $PS$, and using $u_s = \beta u$ and $u_l = (1 - \beta)u$, the natural rate of unemployment is:

   $$ u = \frac{1}{1.1(1 - 0.5*\beta)} $$

3. What is the unemployment rate for $\beta = 0, 0.4, 0.8$? Explain.

   If $\beta = 0, 0.4, 0.8$ the natural rate is: $u = 9\%, 11.4\%, 15.2\%$; an increase in the fraction of short-term unemployed increases the unemployment rate. To understand this, first note that from the wage setting equation, an increase in the portion of the long term unemployed has the effect of increasing the real wage. This is because short term unemployed have a greater (negative) impact on the real wage. However from the price setting equation, the real wage is fixed. Hence, the only way in which the the wage setting equation can be satisfied is for $u$–the overall unemployment rate– to increase.