1. OBJECTIVE

Now, we’re going to look at economic activity over longer periods of time. Now fluctuations fade in importance and growth - the steady increase in output over time - dominates the picture.

Over long periods of time, movements in real GDP are dominated not by SR fluctuations, but by long-term trends.

2. GROWTH IN RICH COUNTRIES SINCE 1950

- **LARGE INCREASE IN THE STANDARD OF LIVING SINCE 1950**
  
  Growth since 1950 has increased real output per capita by a factor of 2.0 in the US, by a factor of 4.3 in Germany, and by a factor of 10.6 in Japan.

- **SLOWDOWN IN GROWTH SINCE THE MID-1970s**
  
  US (1950-73, 2.2% of growth - 1973-92, 1.2% of growth)  
  Japan (1950-73, 8.1% of growth - 1973-92, 3.0% of growth)  
  Germany (1950-73, 4.9% of growth - 1973-92, 1.9% of growth)

- **CONVERGENCE AMONG RICH COUNTRIES**
  
  Levels of output per capita across “rich” countries have converged over time. Put another way, those countries that were behind have grown faster, reducing the gap between them and the US.
But some other countries have not converged - Uruguay, Argentina and Venezuela, for instance. The most striking example is that of Argentina. In 1950 had a GDP per capita similar to that of France and in 1990 it was far below France (3 times).

Many Asian countries (besides Japan) are growing considerably: Singapore, Taiwan, South Korea, Hong-Kong (average annual growth rates of GDP per capita in excess of 6% over the last 30 years).

Convergence has not materialized for African countries, many of whom have exhibited negative growth rates in per capita terms since 1960.

**GROWTH HAS NOT ALWAYS OCCURRED**

- Till roughly year 1500 there was essentially no growth of output per capita in Europe.
- 1500-1700, growth of output per capita positive but small (around 0.1%).
- 1700-1820, around 0.2%.
- 1820-1950, in the period after the Industrial Revolution, around 1.5%.

In human history, growth of output per capita is a recent phenomenon.

**3. A FRAMEWORK TO ANALYZE GROWTH: THE SOLOW GROWTH MODEL**

We are going to use the Solow growth model to develop the analytics of LR growth.

The main questions we’ll try to answer with it are:

1. How do we explain growth?
2. What are the roles of capital accumulation and technological progress in growth?
The Aggregate Production Function (Determination of Output)

Neoclassical Production Function

\[ Y = F(K, N) \]

where,

- **Y**: Aggregate Output;
- **K**: Capital (Machines, plants, office buildings, housing in the economy);
- **N**: Labor (number of workers in the economy);
- **F**: Aggregate Production Function.

**F** depends on the *state of the technology*.

**Assumptions:**

- **Constant returns to scale (CRS)**
  
  Example: \( F(2K, 2N) = 2Y \)

- **Diminishing Marginal Products**
  
  \[ MPK = \frac{dF}{dK} \quad \text{and} \quad MPL = \frac{dF}{dN} \]

  MPK (MPL) decreases as K (N) increases.

**Output and Capital per worker**

With the above assumptions, we can write:

\[ \frac{d}{dK} F \]

Output per worker depends on the amount of capital per worker.

**The Sources of Growth:**

We can think of growth as coming from capital accumulation and/or from technological progress (changes in \( K/N \) and in \( F \)).
4. SAVING, CAPITAL ACCUMULATION, AND OUTPUT

We are now going to see the effects of the saving rate on capital and output per capita.

**Interactions between output and capital**

- **CAPITAL STOCK** → **OUTPUT/INCOME**
- **CHANGE IN THE CAPITAL SAVING/INVESTMENT STOCK**

The effects of capital on output

To simplify notation

\[
\frac{Y_t}{N} = F\left(\frac{K_t}{N}, I_t\right) \equiv f\left(\frac{K_t}{N}\right)
\]  

[1]

Assumption: \( N \) constant, \( f \) does not change over time (no technological progress).

The effects of output on capital accumulation

Private Saving proportional to income, so that \( S = sY \)

where \( s \) is the saving rate \((0 \leq s \leq 1)\).

Assume that in equilibrium \( I = S \) (budget deficit = 0, closed economy).

Then \( I = sY \)

**Capital accumulation** is given by:

\[
K_{t+1} = (1-\delta) K_t + I_t
\]

\( \delta = \) depreciation rate of capital.

The above equation can be transformed in the following one:

\[
\frac{K_{t+1}}{N} - \frac{K_t}{N} = s \frac{Y_t}{N} - \delta \frac{K_t}{N}
\]

[2]

[1] tells us that capital determines output.
[2] tells us that output in turn determines capital accumulation.
**Dynamics of output and capital**

[2] can be rewritten as:

\[
\frac{K}{N} \quad \quad \quad [3]
\]

Change in capital = investment - depreciation capital

The implications of this model for the behavior of the economy over time can best be described diagrammatically.

AB- output per worker for \( K_0/N \);
AC- investment per worker for \( K_0/N \);
AD- depreciation for \( K_0/N \);
CD = AC - AD = change in capital per worker.

**Characterizing the evolution of K/N over time:** At \( K_0/N \), investment exceeds depreciation => \( K/N \) increases till we reach \( K^*/N \) => LR eq. level.

\( K^*/N \) is called the *steady-state* of the economy and is defined by [3] when the left-hand side is equal to zero:

\[
sf(K^*/N) = \delta(K^*/N)
\]

From here, we can find

\[
Y^*/N = f(K^*/N)
\]
The Saving rate and output

How does the saving rate affect the level of output per worker?

1. The saving rate has no effect on the growth rate in the LR, which is equal to zero.
2. Nonetheless, the saving rate determines the level of output per worker in the LR.
3. An increase in the saving rate will lead to higher growth for some time but not forever.

The Saving rate and the golden rule

The golden rule level of capital is the level of capital at which consumption is maximized in the LR. There exists a corresponding saving rate consistent with the golden rule level of capital. There are two cases to consider.

- K/N below the golden rule level of K/N

Here, s must increase. As s increases, consumption will increase at the new steady state (i.e., consumption will increase in the LR). In the SR, however, since K and Y do not immediately change in response to the increase in s, an increase in the saving rate will cause a reduction in consumption.
• K/N above the golden rule level of K/N

Here, the saving rate must fall. As s falls, consumption will increase at the new steady state. In the SR, consumption will also increase since s is now lower.

Analytically:

Consumption per worker given by:

\[ \frac{C^*}{N} = \frac{Y^*}{N} - \frac{I^*}{N} = \frac{Y^*}{N} - \delta \frac{K^*}{N} = f \left( \frac{K^*}{N} \right) - \delta \frac{K^*}{N} \] [4]

Can compute value of K^*/N that maximizes [4].

That value is then given by the first derive of expression [4] with respect to K^*/N,

\[ f'(K^*/N) = \delta \]

This has a graphical interpretation.