The Effects of Balance Sheet Constraints on Non Financial Firms

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January 9, 2018

Background

- Several shortcomings of standard New Keynesian model.
  - It assumes that the interest rate satisfies an Euler equation with the consumption of a single, representative household.
  - Evidence against that Euler equation is strong (Hall (JPE1978), Hansen-Singleton (ECMA1982), Canzoneri-Cumby-Diba (JME2007)
- Here, discuss Buera-Moll (AEJ-Macro2015) model of heterogeneous households and firms.
  - Shows how a model with heterogeneous households breaks Euler equation.
  - Shows how deleveraging can lead to many of the things observed in the Financial Crisis and Great Recession.
    - fall in output, investment, consumption, TFP, real interest rate.
- 'Toy' model that can be solved analytically, great for intuition.
- Earlier, similar models: Kahn-Thomas (JPE2013), Liu-Wang-Zha (ECMA2013).

Outline

- Hand-to-mouth workers
- Entrepreneurs (where all the action is)
- Aggregates: Loan Market, GDP, TFP, Consumption, Capital, Consumption
- Equilibrium
  - Computation.
  - Parameter values.
  - The dynamic effects of deleveraging.

Hand-to-mouth Workers

- Hand-to-mouth workers maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^W\right)^{1-\sigma}}{1-\sigma} - \frac{1}{1+\chi} L_t^{1+\chi} \right]$$

subject to:

$$C_t^W \leq w_t L_t.$$

- Solution:

$$L_t^{\frac{\chi+\sigma}{1-\sigma}} = w_t, \tag{1}$$

and labor supply is upward-sloping for $0 < \sigma < 1$.

Entrepreneurs

- i^{th} entrepreneur would like to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right), \; u\left(c\right) = \log c.$$

- i^{th} entrepreneur can do one of two things in t:
  - use time t resources plus debt, $d_{i,t} \geq 0$, to invest in capital and run a production technology in period $t+1$.
    - will do this if i's technology is sufficiently productive.
  - use time t resources to make loans, $d_{i,t} < 0$, to financial markets.
    - will do this if i's technology is unproductive.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur can invest $x_{i,t}$ and increase its capital in $t+1$:

$$k_{i,t+1} = (1-\delta)\, k_{i,t} + x_{i,t},\ \delta \in (0,1)$$

- In $t+1$ entrepreneur can use $k_{i,t+1}$ to produce output:

$$y_{i,t+1} = (z_{i,t+1} k_{i,t+1})^{\alpha}\, l_{i,t+1}^{1-\alpha},\ \alpha \in (0,1)\,,$$

  where $l_{i,t+1}$ ~ amount of labor hired in $t+1$ for wage, w_{t+1}.

- Technology shock, $z_{i,t+1}$, observed at time t, and
  - independent and identically distributed:
    - across i for a given t,
    - across t for given i.
  - Density of z, $\psi(z)$; CDF of z, $\Psi(z)$.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur's time $t+1$ profits:

$$\max_{l_{i,t+1}} \left[\left(z_{i,t+1} k_{i,t+1} \right)^{\alpha} l_{i,t+1}^{1-\alpha} - w_{t+1} l_{i,t+1} \right] = \pi_{t+1} z_{i,t+1} k_{i,t+1}$$

$$\pi_{t+1} \equiv \alpha \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}.$$

- Rate of return on one unit of investment in t :

$$\pi_{t+1} z_{i,t+1} + 1 - \delta.$$

The Decision to Invest or Lend

- The i^{th} entrepreneur can make a one period loan at t, and earn $1 + r_{t+1}$ at $t + 1$.
- Let $\bar{z}_{t+1}$ denote value of $z_{i,t+1}$ such that return on investment same as return on making a loan:

$$\pi_{t+1}\bar{z}_{t+1} + 1 - \delta = 1 + r_{t+1}.$$

- If $z_{i,t+1} > \bar{z}_{t+1}$,
  - borrow as much as possible, subject to collateral constraint:

    $$d_{i,t+1} \leq \theta_t k_{i,t+1},\ \theta_t \in [0, 1],$$

    and invest as much as possible in capital.
  - In this case borrow:

    $$d_{i,t+1} = \theta_t k_{i,t+1}.$$

- If $z_{i,t+1} < \bar{z}_{t+1}$, then set $k_{i,t+1} = 0$ and make loans, $d_{i,t} < 0$.

Entrepreneur's Problem

- At t, maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{i,t+j}\right)$$

subject to:

  - given $k_{i,t}$ and $d_{i,t}$
  - borrowing constraint
  - budget constraint:

$$c_{i,t} + \overbrace{k_{i,t+1} - (1-\delta)\, k_{i,t}}^{\text{investment, } x_{it}} \leq \overbrace{\pi_t z_{i,t} k_{i,t}}^{y_{i,t} - w_t l_{i,t}, \text{ if entrepreneur invested in } t-1}$$

$$+ \overbrace{d_{i,t+1} - (1+r_t) d_{i,t}}^{\text{increase in debt, net of financial obligations}}$$

- Alternative representation of budget constraint:

$$c_{i,t} + \overbrace{a_{i,t+1}}^{\equiv k_{i,t+1} - d_{i,t+1}, \text{ 'net worth'}} \leq \overbrace{\left[\pi_t z_{i,t} + 1 - \delta\right] k_{i,t} - (1+r_t) d_{i,t}}^{\equiv m_{i,t}, \text{ 'cash on hand'}}$$

Entrepreneur's Problem

- At t, maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{i,t+j}\right),$$

$u(c) = log(c)$, subject to:
  - given $k_{i,t}$ and $d_{i,t}$
  - borrowing constraint
  - budget constraint:

$$c_{i,t} + a_{i,t+1} \leq m_{i,t}$$

  where,

$$a_{i,t+1} = k_{i,t+1} - d_{i,t+1}, \quad m_{i,t} = \left[\pi_t z_{i,t} + 1 - \delta\right] k_{i,t} - (1 + r_t) d_{i,t}$$

- Optimal choice of next period's net worth:

$$a_{i,t+1} = \beta m_{i,t}, \quad c_{i,t} = (1 - \beta)\, m_{i,t}.$$

Entrepreneur's Problem

- For $z_{i,t+1} \geq \bar{z}_{t+1}$, max debt and capital:

$$d_{i,t+1} = \theta_t k_{i,t+1} = \theta_t \left(d_{i,t+1} + a_{i,t+1} \right)$$
$$\rightarrow d_{i,t+1} = \frac{\theta_t}{1-\theta_t} a_{i,t+1}, \quad k_{i,t+1} = \frac{1}{1-\theta_t} a_{i,t+1}$$

  - Example:
    - if $\theta_t = \frac{2}{3}$, then leverage $= 1/\left(1-\theta_t\right) = 3$.
    - if net worth, $a_{i,t+1} = 100$, then $k_{i,t+1} = 300$ and $d_{i,t+1} = 200$.

- For $z_{i,t+1} < \bar{z}_{t+1}$, $k_{i,t+1} = 0$ and $d_{i,t+1} < 0$ (i.e., lend)
  - upper bound on lending: $d_{i,t+1} = -m_{i,t}$, all cash on hand.
  - won't go to upper bound with log utility.

Aggregates: Demand for Loans

- The total amount of cash on hand for all entrepreneurs, M_t, is

$$M_t = \int_i m_{i,t} di.$$

- Total demand for loans:
  - Since the $z_{i,t+1}$'s are distributed randomly to entrepreneurs, the cash in hand of the $\left[1 - \Psi\left(\bar{z}_{t+1}\right)\right]$ investing entrepreneurs is:

$$\left[1 - \Psi\left(\bar{z}_{t+1}\right)\right] M_t.$$

  - Each of these entrepreneurs borrows $d_{i,t+1} = \theta_t / \left(1 - \theta_t\right) \beta m_{it}$, so total borrowing by investing entrepreneurs is

$$\beta \frac{\theta_t}{1 - \theta_t} \left[1 - \Psi\left(\bar{z}_{t+1}\right)\right] M_t.$$

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Aggregates: Supply of Loans and Loan Market Clearing

- Total supply of loans:
  - Since the $z_{i,t+1}$'s are distributed randomly to entrepreneurs, the cash in hand of the $\Psi\left(\bar{z}_{t+1}\right)$ non-investing entrepreneurs is:
$$\Psi\left(\bar{z}_{t+1}\right) M_t.$$
  - Each of these entrepreneurs lends $-d_{i,t+1} = \beta m_{it}$, so total borrowing by investing entrepreneurs is
$$\beta \Psi\left(\bar{z}_{t+1}\right) M_t.$$
- Loan market clearing implies:
$$\beta \frac{\theta_t}{1-\theta_t}\left[1-\Psi\left(\bar{z}_{t+1}\right)\right] M_t = \beta \Psi\left(\bar{z}_{t+1}\right) M_t,$$
or,
$$\Psi\left(\bar{z}_{t+1}\right) = \theta_t. \tag{2}$$

Aggregates: Gross Domestic Product

- The i^{th} firm's production function is:

$$y_{it} = (z_{i,t}k_{i,t})^{\alpha}\, l_{i,t}^{1-\alpha} = \overbrace{\left(\frac{z_{i,t}k_{i,t}}{l_{i,t}}\right)^{\alpha}}^{\text{same for each } i\text{, because all face same } w_t} l_{i,t}.$$

- Ratios equal ratio of sums:

$$\frac{z_{i,t}k_{i,t}}{l_{i,t}} = \frac{\int_i z_{i,t}k_{i,t}di}{\int_i l_{i,t}di} = \frac{\int_i z_{i,t}k_{i,t}di}{L_t}.$$

- GDP

$$Y_t = \int_i y_{i,t}di = \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^{\alpha} \int_i l_{i,t}di$$

$$= \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^{\alpha} L_t$$

Aggregates: GDP, TFP and wage

- With some algebra, can establish:

$$Y_t = \left(\overbrace{\int_i z_{i,t} k_{i,t} di}^{=E[z|z>\bar{z}_t]\times K_t} \right)^{\alpha} L_t^{1-\alpha} = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{3}$$

$$Z_t \equiv \left(E\left[z | z > \bar{z}_t \right] \right)^{\alpha}. \tag{4}$$

- Simple intuition:
  - Aggregate output, Y_t, a function of aggregate capital and labor, and (endogenous) TFP, Z_t.
  - Z_t average TFP of firms in operation.

- Aggregate wage:

$$w_t = (1-\alpha) \frac{Y_t}{L_t}. \tag{5}$$

Aggregates: Consumption

- Integrating over entrepreneurs' budget constraints:

$$\int_i \left[c_{i,t} + k_{i,t+1} - d_{i,t+1}\right] di$$
$$= \int_i \left[y_{i,t} - w_t l_{i,t} + (1-\delta)\, k_{i,t} - (1+r_t) d_{i,t}\right] di$$

- Using loan market clearing, $\int_i d_{i,t} di = 0$:

$$C_t^E + K_{t+1} - (1-\delta)\, K_t = Y_t - \overbrace{(1-\alpha)\, Y_t}^{\overbrace{= w_t \int_i l_{i,t} di}^{=C_t^W}},$$

where

$$C_t^E = \int_i c_{i,t} di$$

- Then,

$$C_t^E + K_{t+1} - (1-\delta)\, K_t = \alpha Y_t. \tag{6}$$

Aggregates: Capital Accumulation

- Entrepreneur decision rule:

$$\begin{aligned} a_{i,t+1} &\equiv k_{i,t+1} - d_{i,t+1} \\ &= \beta \left[y_{i,t} - w_t l_{i,t} + (1-\delta)\, k_{i,t} - (1+r_t) d_{i,t} \right] \end{aligned}$$

- Integrating over all entrepreneurs (using $\int_i d_{i,t} di = 0$):

$$K_{t+1} = \beta \left[\alpha Y_t + (1-\delta)\, K_t \right] \tag{7}$$

- Note: K_{t+1} is not a direct function of θ_t.
  - If θ_t falls, then borrowing drops by investing entrepreneurs, driving down r_{t+1}.
  - Lower r_{t+1} encourages unproductive entrepreneurs who previously were lending, to switch to borrowing and buying more capital.
  - The positive and negative effects on capital purchases cancel, which is why K_{t+1} is not a function of θ_t.

Aggregates: Consumption Euler Equation

- Interestingly, aggregate entrepreneurial consumption satisfies Euler equation:

$$\begin{aligned}
\frac{C_{t+1}^E}{C_t^E} &= \frac{(1-\beta)\left[\alpha Y_{t+1} + (1-\delta) K_{t+1}\right]}{(1-\beta)\left[\alpha Y_t + (1-\delta) K_t\right]} \\
&= \beta \frac{\alpha Y_{t+1} + (1-\delta) K_{t+1}}{K_{t+1}} \\
&= \beta \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right]
\end{aligned}$$

- But,
  - does not hold for aggregate consumption, $C_t = C_t^W + C_t^E$.
  - does not hold relative to the interest rate.

Equilibrium

- Seven variables:

$$L_t, w_t, C_t^E, Y_t, K_{t+1}, \bar{z}_t, Z_t.$$

- Seven equations: (1), (2), (3), (4),(5), (6), (7).
- Exogenous variables:

$$K_1, \theta_0, \theta_1, \theta_2, ..., \theta_T$$

Equilibrium Computation

- Responses to exogenous variables:
  - For $t = 1, 2, ..., T$, $\bar{z}_t = \Psi^{-1}(\theta_{t-1})$ using (2); $Z_t \equiv (E[z|z > \bar{z}_t])^{\alpha}$ using (4),
  - Using (1) and (5) for L_t and w_t; (3) for Y_t; (7) for K_{t+1}; and (6) for C_t^E:

$$
\begin{aligned}
L_t &= \left[(1-\alpha) Z_t K_t^{\alpha}\right]^{\frac{1-\sigma}{\chi+\sigma+(1-\sigma)\alpha}} \\
w_t &= L_t^{\frac{\chi+\sigma}{1-\sigma}} \\
Y_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} \\
K_{t+1} &= \beta\left[\alpha Y_t + (1-\delta) K_t\right] \\
C_t^E &= (1-\beta)\left[\alpha Y_t + (1-\delta) K_t\right]
\end{aligned}
$$

sequentially, for $t = 1, 2, 3, ..., T$.

Equilibrium Computation

- Other variables: interest rate and profits for $t = 1, 2, ..., T$:

$$\pi_t = \alpha \left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}}$$

$$1 + r_t = \pi_t \bar{z}_t + 1 - \delta$$

- Pareto distribution:

$$\psi(z) = \eta z^{-(\eta+1)},\ \eta = 2.1739,\ 1 \leq z$$

$$\Psi(\bar{z}) = 1 - \bar{z}^{-\eta},\ Ez = \frac{\eta}{\eta - 1} = 1.85.$$

Parameter Values and Steady State

- Other parameters:

$$\alpha = 0.36, \delta = 0.10, \beta = 0.97, \chi = 1, \sigma = 0.9.$$

- Steady state, with $\theta = \frac{2}{3}$:

$$\begin{aligned} Y &= 3.45, K = 9.50, L = 1.04, C = 2.50, \\ w &= 2.12, \bar{z} = 1.66, Z = 1.50, \\ Z^{\frac{1}{\alpha}} &= E\left[z|z > \bar{z}\right] = 3.07, 1 + r = 0.97, \\ C^E/C &= 0.12, C^W/C = 0.88, \end{aligned}$$

$()$after rounding.

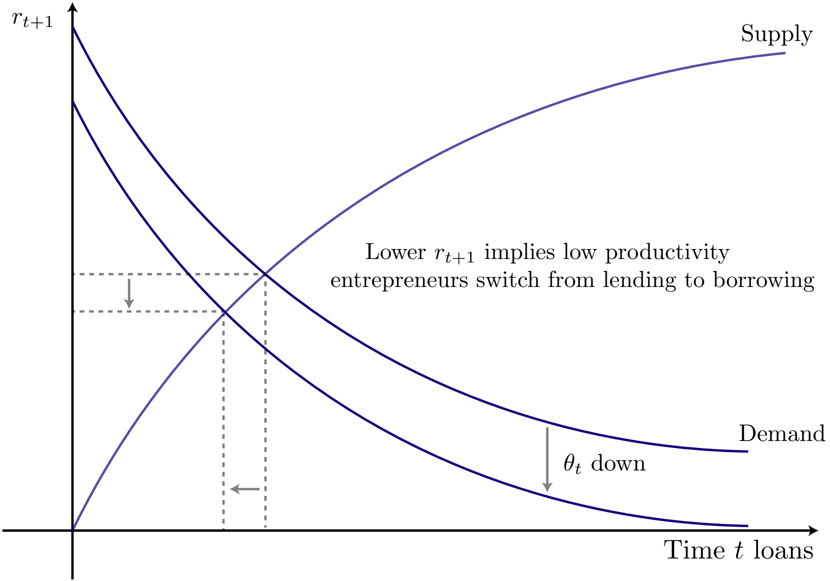
Tighter Lending Standards: θ_t down

- 'MIT shock'
  - economy in a steady state, $t = -\infty, ..., 1, 2$, and expected to remain there.
  - In $t = 3$, θ_3 drops unexpectedly from 0.67 to 0.60, and gradually returns to its steady state level:
    - $\theta_3 = \theta \times 0.9$, $\theta_t = (1 - \rho)\,\theta + \rho\theta_{t-1}$, for $t = 4, 5, ...$
    - $\rho = 0.8$.

Immediate Impact of Negative θ_t Shock

- Period $t = 3$ impact of shock:
  - Deleveraging associated with drop in θ_3 reduces demand for debt by each investing entrepreneur, driving down period $t = 3$ interest rate, r_4.
  - Marginally productive firms which previously were lending, switch to borrowing and making low-return investments with the drop in r_4.
    - No impact on total investment in period $t = 3$, as the cut-back by high productivity entrepreneurs is replaced by expanded investment by lower productivity entrepreneurs.
    - No impact on consumption, wages, etc., in period $t = 3$.

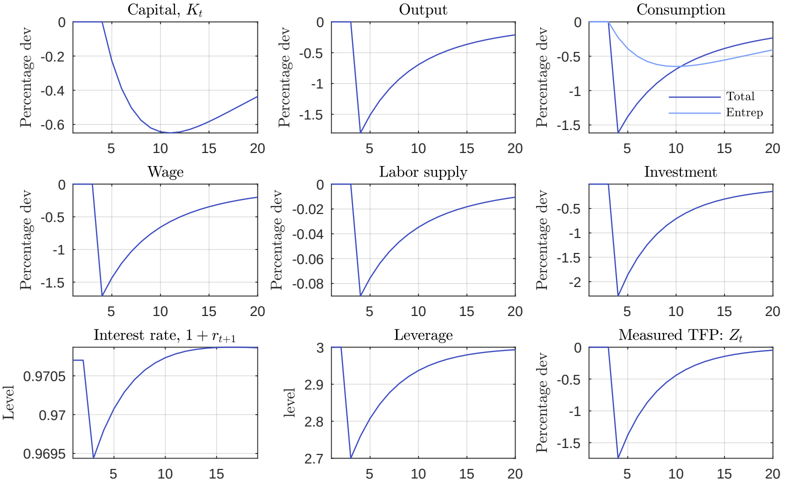
Immediate Impact of Negative θ_t Shock



Dynamic Effects of Drop in θ_t

- The cut in leverage by highly productive, but collateral-poor, firms is the trigger for the over 1.8 percent drop in TFP in period $t = 4$.
  - Until the drop in capital is more substantial, by say period $t = 20$, the drop in TFP is the main factor driving GDP down.
- Total consumption drops substantially, driven by the drop in income of hand-to-mouth workers, who consume $2/3$ of GDP.
  - Entrepreneurial consumption, directly related to GDP, also drops.
- Investment drops by over 2 percent.
- Employment drops by (a modest) 0.1 percent.

Response to Collateral Constraint Shock



Conclusion

- Buera-Moll model gives a flavor of the sort of analysis one can do with heterogeneous agent models with balance sheet constraints.
  - Illustrates the value of simple models for gaining intuition.
- Model provides an 'endogenous theory of TFP'.
  - Stems from poor allocation of resources due to frictions in financial market.
  - See also Song-Storesletten-Zilibotti (AER2011).
- Deleveraging shock gets a surprising number of things right, but
  - how important was deleveraging per se, for the crisis?
  - what is the 'deleveraging shock a stand-in for?'