# Simple New Keynesian Model without Capital, II

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# Standard New Keynesian Model

- Taylor rule: designed so that in steady state, inflation is zero $(\bar{\pi} = 1)$

- Employment subsidy extinguishes monopoly power in steady state:
$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} = 1$$

# Equilibrium Conditions of NK Model with Taylor Rule

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu) Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[ (1-\theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$R_t / R = (R_{t-1}/R)^{\alpha} \exp\left[ (1-\alpha) \phi_{\pi} (\bar{\pi}_t - \bar{\pi}) + \phi_x x_t \right] \text{ (7)'}.$$

In steady state: $R = \frac{1}{\beta}, p^* = 1, F = K = \frac{1}{1-\beta\theta}, N = 1$

# Natural Rate of Interest

- Intertemporal Euler equation in Natural equilibrium:

$$\overbrace{a_t}^{\text{optimal consumption in } t} = -\left[r_t^* - rr\right] + E_t a_{t+1}$$

where a * indicates 'natural' equilibrium and $r_t^* = log\left(R_t^*\right)$.

- Back out the natural rate and ignoring constant

$$r_t^* = E_t \Delta a_{t+1}$$

- Law of motion for technology:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a.$$

# NK IS Curve

- Euler equation in two equilibria (ignoring variance adjustment):

$$\overbrace{c_t = -\left[r_t - E_t\pi_{t+1} - rr\right] + E_t c_{t+1}}^{\text{Taylor rule equilibrium, NK model}}$$

$$\overbrace{c_t^* = -\left[r_t^* - rr\right] + E_t c_{t+1}^*}^{\text{Natural equilibrium } (\theta = 0, \nu \text{ kills monopoly power})}$$

where lower case letters mean log, $\pi_t = log\left(\bar{\pi}_t\right)$and $^*$ means 'natural equilibrium'.

- Subtract:

$$\overbrace{x_t}^{\text{output gap}} = \left[r_t - E_t x_{t+1} - r_t^*\right]$$

# Output in the NK Equilibrium

- Aggregate output relation:

$$y_t = log\left(p_t^*\right) + n_t + a_1,\ log\left(p_t^*\right) = \begin{cases} = 0 & \text{if } P_{i,t} = P_{j,t} \text{ all i,j} \\ \leq 0 & \text{otherwise} \end{cases}.$$

- To first approximation (given that we set inflation to zero in steady state):

$$p_t^* = 1.$$

# Phillips Curve

- Equations pertaining to price setting:

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu)\, Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}\ , \quad \frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1}$$

- Log-linearly expand about zero-inflation steady state:

$$\hat{\bar{\pi}}_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}(1+\varphi)\, x_t + E_t \hat{\bar{\pi}}_{t+1}$$

  where $\hat{z}_t$ denotes $z_t - z)/z$

- See this for details.

# Collecting the Log-linearized Equations

$$
\begin{aligned}
x_t &= x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right] \\
r_t &= \phi_\pi \pi_t \\
\pi_t &= \beta \pi_{t+1} + \kappa x_t \\
r_t^* &= E_t \left(a_{t+1} - a_t\right),
\end{aligned}
$$

where

$$
\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi).
$$

# The Equations, in Matrix Form

- Representation of the shock:

$$
\begin{aligned}
s_t &= \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t \\
s_t &= P s_{t-1} + \epsilon_t
\end{aligned}
$$

- Matrix representation of system

$$
\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix}
+
\begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -1 & 1 \\ \phi_\pi & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{pmatrix}
$$

$$
+
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} s_{t+1}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} s_t
$$

- $E_t \left[ \alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0$

# Solving the Model

- Linearized equilibrium conditions:

$$E_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0$$
$$s_t = P s_{t-1} + \epsilon_t$$

- Data generated using this equation,

$$z_t = A z_{t-1} + B s_t$$

with $A$ and $B$ chosen as follows:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0, \quad (\beta_0 + \alpha_0 B) P + [\beta_1 + (\alpha_0 A + \alpha_1) B] = 0$$

- Standard strategy:
  - pick the unique $A$ with all eigenvalues less 1 in absolute value, that solves the first equation,
  - conditional on $A$, choose $B$ to solve the second equation.
  - Strategy breaks down if there is no such $A$ or there is more than one.

# Simulation

- Draw $\epsilon_1, \epsilon_2, ..., T$
- Solve

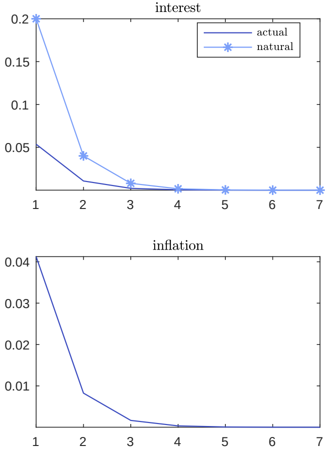
$$s_t = P s_{t-1} + \epsilon_t$$
$$z_t = A z_{t-1} + B s_t$$

,for $t = 1, ..., T$, and given $z_0$ and $s_0$.

- If $\epsilon_t = 0$ for $t > 1$ and $\epsilon_1 = 1$, this is an *impulse response function*.
- If $\epsilon_t$, $t = 1, 2, ..., T$ are *iid* and drawn from a random number generator, then it is a *stochastic simulation*.

# Impulse Response Function

**Figure:** Dynamic Response to a Technology Shock




Note: $\beta = 0.97, \phi_\pi = 1.5, \rho = 0.2, \varphi = 1, \theta = 0.75, \kappa = 0.1817.$

# Interpretation

- What happened in the figure?
  - The positive technology shock creates a surge in spending because of consumption smoothing. The natural rate has to rise, to prevent excessive consumption (an 'aggregate demand problem'). In the efficient allocations, log, natural consumption is equal to $a_t$ and natural employment is constant.
  - The Taylor rule response is too weak, so the surge in aggregate demand is not pulled back and the economy over reacts to the shock. The excessive aggregate demand causes the economy to expand too much, which raises wages and costs and inflation.
  - The weak response under the Taylor rule is not a function of the value of $\rho$, whether the time series representation is difference or trend stationary (see).
- How to get a better response?
  - Put the natural rate of interest in the Taylor rule:

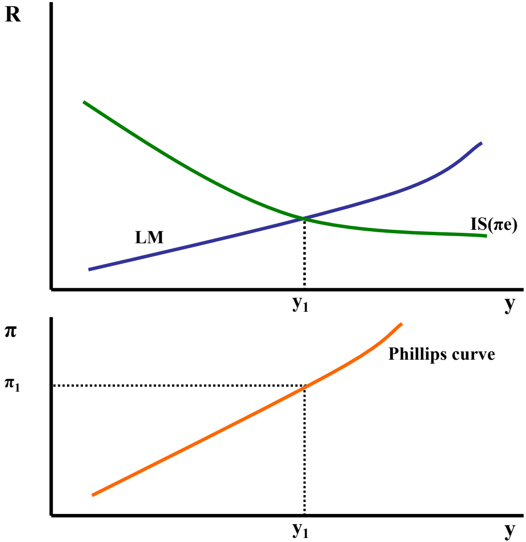
$$r_t = r_t^* + \phi_\pi \pi_t$$

  - Either measure it exactly, or put in a proxy.

# How to Proxy the Natural Rate?

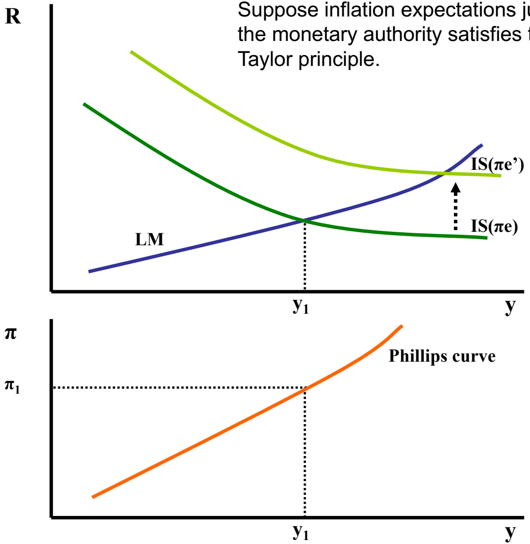
- How to proxy the natural rate?
  - It is consumption growth that appears in the natural rate, so something that signals good future consumption prospects would be a good proxy.
    - Credit growth?
    - Stock market growth?
- DSGE model simulations can be helpful for thinking about a good proxy.
  - For an example, see Christiano-Ilut-Motto-Rostagno (Jackson Hole, 2011).
- Why keep inflation in the Taylor rule?
  - To have a locally unique equilibrium, must have $\phi_\pi > 1$, i.e., *Taylor Principle*.

# Uniqueness Under Taylor Principle



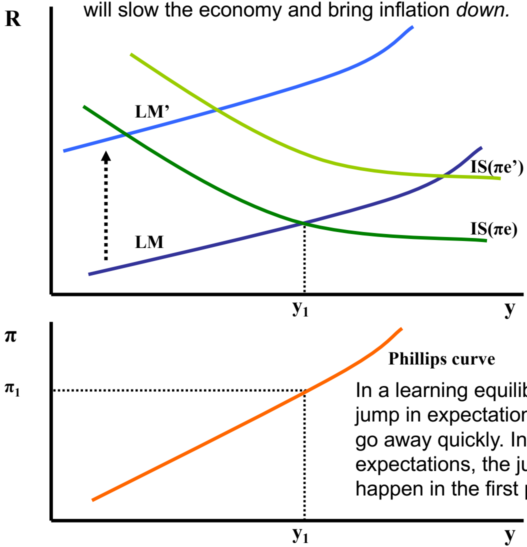
# Uniqueness Under Taylor Principle

Suppose inflation expectations jump and the monetary authority satisfies the Taylor principle.



# Uniqueness Under Taylor Principle

The sharp rise in interest rate
will slow the economy and bring inflation *down.*




In a learning equilibrium the jump in expectations for no reason go away quickly. In rational expectations, the jump would not happen in the first place.

# Conclusion

- With sticky prices, the real allocations of the economy are not determined independently of monetary policy.
- So, performance of the economy will depend in part monetary policy design.
  - Put the natural rate (or some good proxy) in the Taylor rule, and policy works well.
  - Violate the Taylor principle and you could have trouble (‘sunspots’).
  - Other potential dysfunctions associated with poorly designed monetary policy are described in Christiano-Trabandt-Walentin (Handbook of Monetary Economics 2011).
- Without the proper design of monetary policy, aggregate demand can go wrong.
  - In the example described above, the response of aggregate demand to a shock is excessive.
  - Other examples can be found in which the response of aggregate demand is inadequate, and even the wrong sign.