Notes on Financial Frictions Under Asymmetric Information and Costly State Verification

by

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Incorporating Financial Frictions into a Business Cycle Model

- General idea:
  - Standard model assumes borrowers and lenders are the same people..no conflict of interest
  - Financial friction models suppose borrowers and lenders are different people, with conflicting interests
  - Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Bernanke-Gertler.
- Integrating the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (2003,2012).

Simple Model

- There are entrepreneurs with all different levels of wealth, N.
  - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.

- For each value of N, there are many entrepreneurs.

- In what follows, we will consider the interaction between entrepreneurs with a specific amount of N with competitive banks.

- Later, will consider the whole population of entrepreneurs, with every possible level of N.

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1 + R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

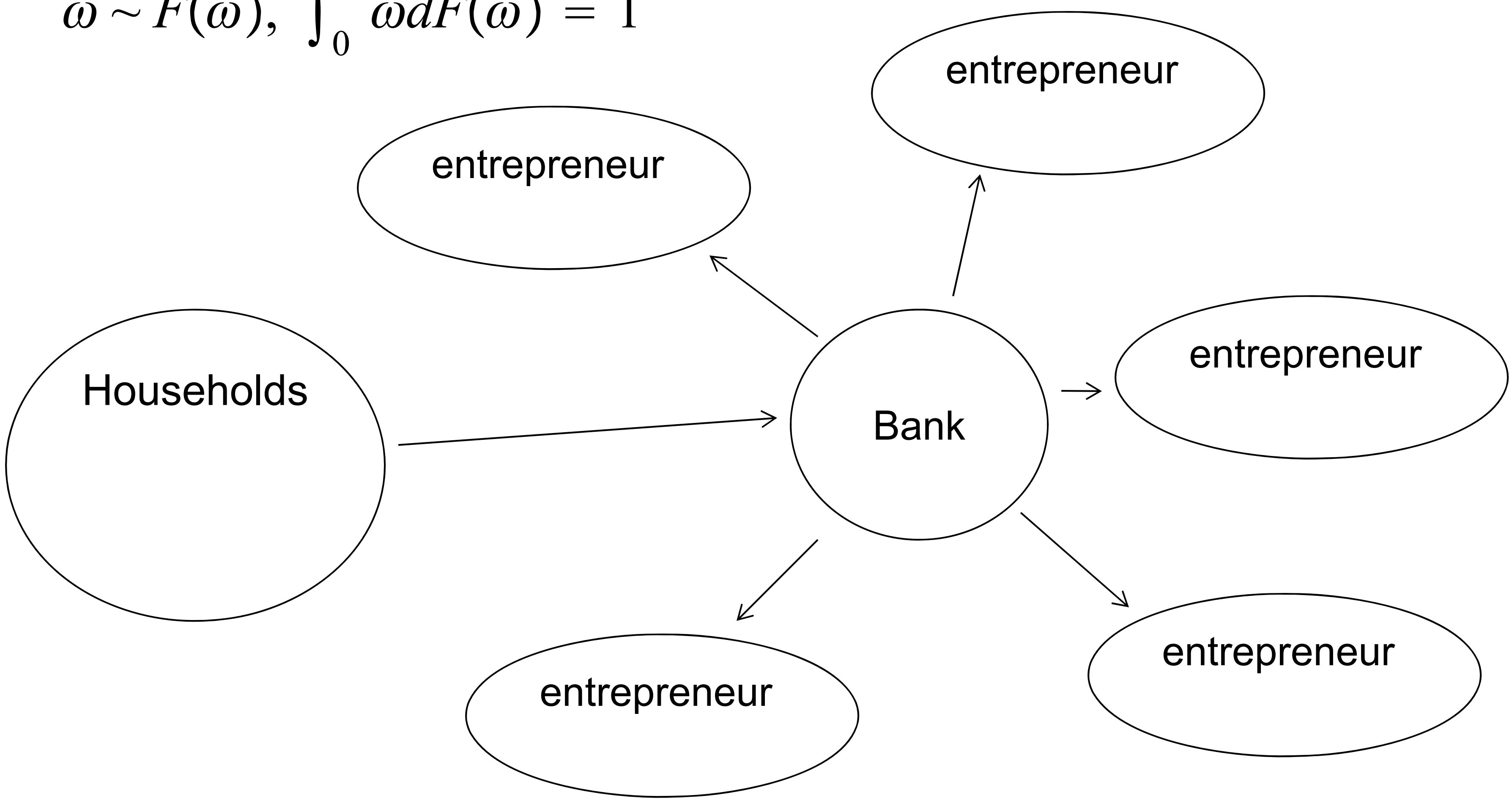
$$\int_0^\infty \omega dF(\omega) = 1$$

- The shock, ω, is privately observed by the entrepreneur.

- *F* is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega), \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.
  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

$$\overbrace{(1+R^k)\bar{\omega}}^{\text{gross rate of return experience by entrepreneur with 'luck', } \bar{\omega}} \times \overbrace{A}^{\text{total assets}}$$

$$= \overbrace{ZB}^{\text{interest and principle owed by the entrepreneur}}$$

$$(1+R^k)\bar{\omega}A = ZB \rightarrow$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{\overbrace{\frac{A}{N}}^{\text{leverage} = L} - 1}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

- Cutoff higher with:
  - higher leverage, *L*
  - higher $Z/(1+R^k)$

- Expected return to entrepreneur from operating risky technology, over return from depositing net worth in bank:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty} [(1+R^k)\omega A - ZB] dF(\omega)}{N(1+R)}$$

For lower values of ω , entrepreneur receives nothing 'limited liability'.

gain from depositing funds in bank ('opportunity cost of funds')

- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L}$$

Gets smaller with *L*

Larger with *L*

- Rewriting entrepreneur's rate of return:

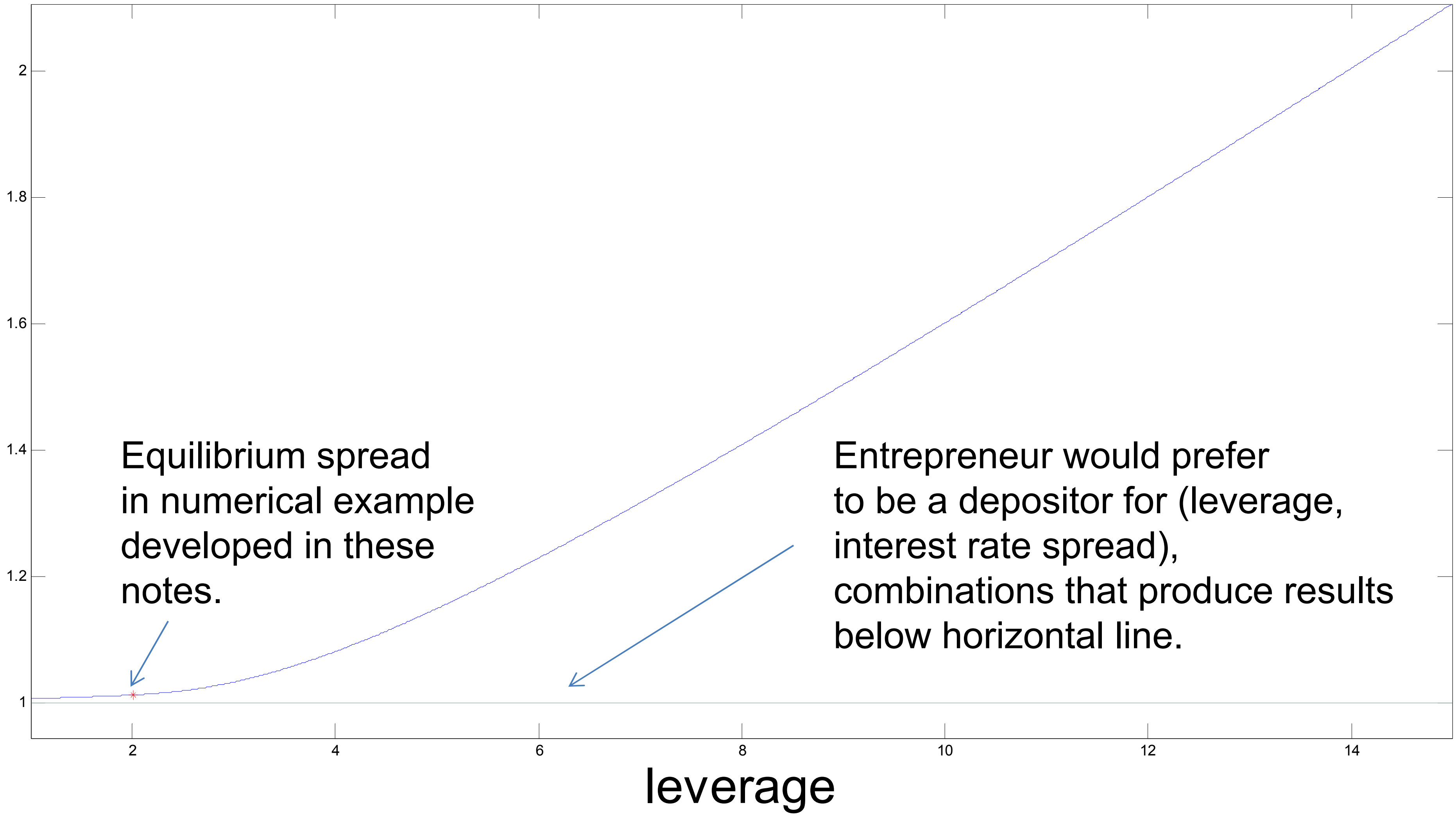
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

Expected entrepreneurial return, over opportunity cost, N(1+R)



Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

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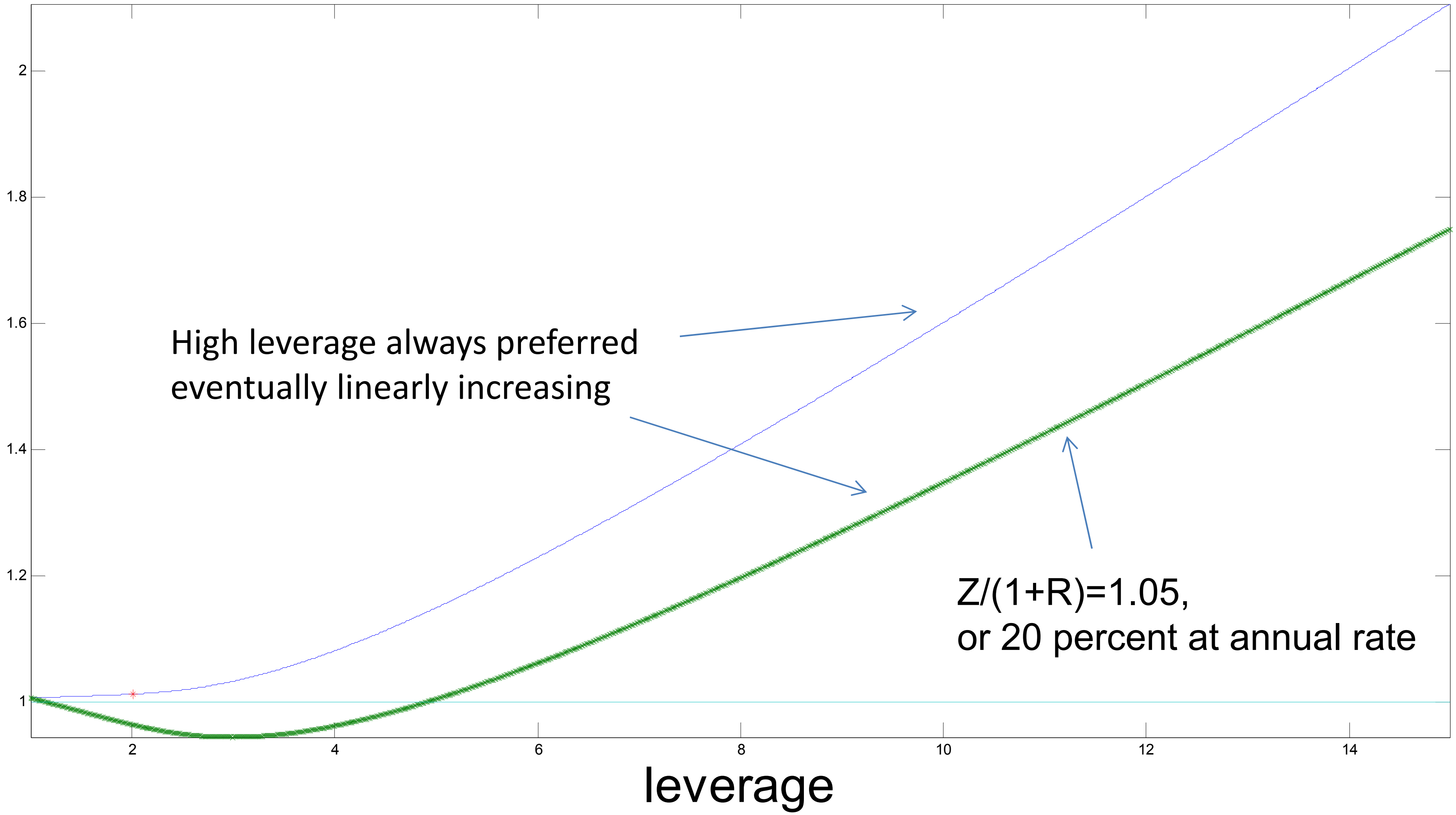


Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

- In equilibrium, bank can't lend an infinite amount.

- This is why a loan contract must specify *both* an interest rate, Z, and a loan amount, B.

Simplified Representation of Entrepreneur Utility

- Utility:

$$\int_{\bar{\omega}}^{\infty}[\omega-\bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}L$$

$$=[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L$$

Share of gross entrepreneurial earnings kept by entrepreneur

- Where

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}(1-F(\bar{\omega}))+G(\bar{\omega})$$

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}}\omega dF(\omega)$$

- Easy to show:

$$0 \leq \Gamma(\bar{\omega}) \leq 1$$

$$\Gamma'(\bar{\omega}) = 1-F(\bar{\omega}) > 0,\ \Gamma''(\bar{\omega}) < 0$$

$$\lim_{\bar{\omega}\to 0}\Gamma(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}\Gamma(\bar{\omega}) = 0$$

$$\lim_{\bar{\omega}\to 0}G(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}G(\bar{\omega}) = 1$$

Banks

- Source of funds from households, at fixed rate, *R*

- Bank borrows *B* units of currency, lends proceeds to entrepreneurs.

- Provides entrepreneurs with standard debt contract, *(Z,B)*

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1+R^k)\omega A$$

- Bank zero profit condition

$$\overbrace{[1-F(\bar{\omega})]}^{\text{fraction of entrepreneurs with } \omega>\bar{\omega}} \qquad \overbrace{ZB}^{\text{quantity paid by each entrepreneur with } \omega>\bar{\omega}}$$

$$+ \quad \overbrace{(1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}^{\text{quantity recovered by bank from each bankrupt entrepreneur}}$$

$$= \quad \overbrace{(1+R)B}^{\text{amount owed to households by bank}}$$

Banks, cont'd

- Zero profit condition:

$$[1-F(\bar{\omega})]ZB+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\frac{[1-F(\bar{\omega})]ZB+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)(1+R^k)A}{B} = (1+R)$$

The risk free interest rate here is equated to the 'average return on entrepreneurial projects'.
This is a source of inefficiency in the model. A benevolent planner would prefer that the market price savers correspond to the *marginal* return on projects (Christiano-Ikeda).

Banks, cont'd

- Simplifying zero profit condition:

$$[1-F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$[1-F(\bar{\omega})]\bar{\omega}(1+R^k)A + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

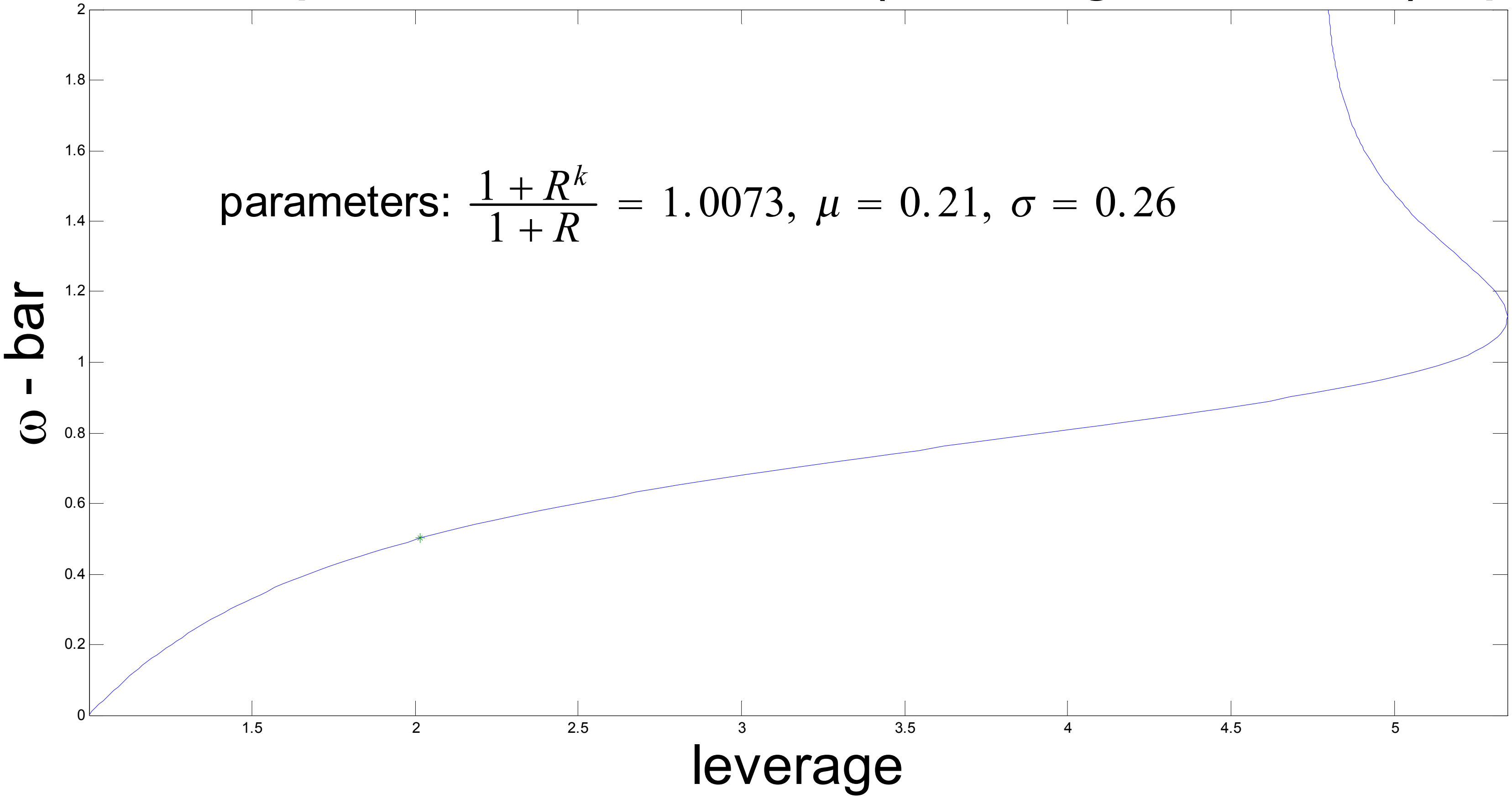
share of gross return, $(1+R^k)A$, (net of monitoring costs) given to bank

$$\overbrace{\left([1-F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)\right)}\quad (1+R^k)A = (1+R)B$$

$$[1-F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega) = \frac{1+R}{1+R^k}\frac{B/N}{A/N}$$
$$= \frac{1+R}{1+R^k}\frac{L-1}{L}$$

Expressed naturally in terms of $(\bar{\omega}, L)$

Bank zero profit condition, in (leverage, ω - bar) space



Our value of $\frac{1+R^k}{1+R}$, 290 basis points at an annual rate, is a little higher than the 200 basis point value adopted in BGG (1999, p. 1368); the value of μ is higher than the one adopted by BGG, but within the range, 0.20-0.36 defended by Carlstrom and Fuerst (AER, 1997) as empirically relevant; the value of $Var(\log\omega)$ is nearly the same as the 0.28 value assumed by BGG (1999,p.1368).

Expressing Zero Profit Condition In Terms of New Notation

$$\overbrace{(1-F(\bar{\omega}))\bar{\omega}+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)}^{\text{share of entrepreneurial profits (net of monitoring costs) given to bank}} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$\Gamma(\bar{\omega})-\mu G(\bar{\omega}) = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

Equilibrium Contract

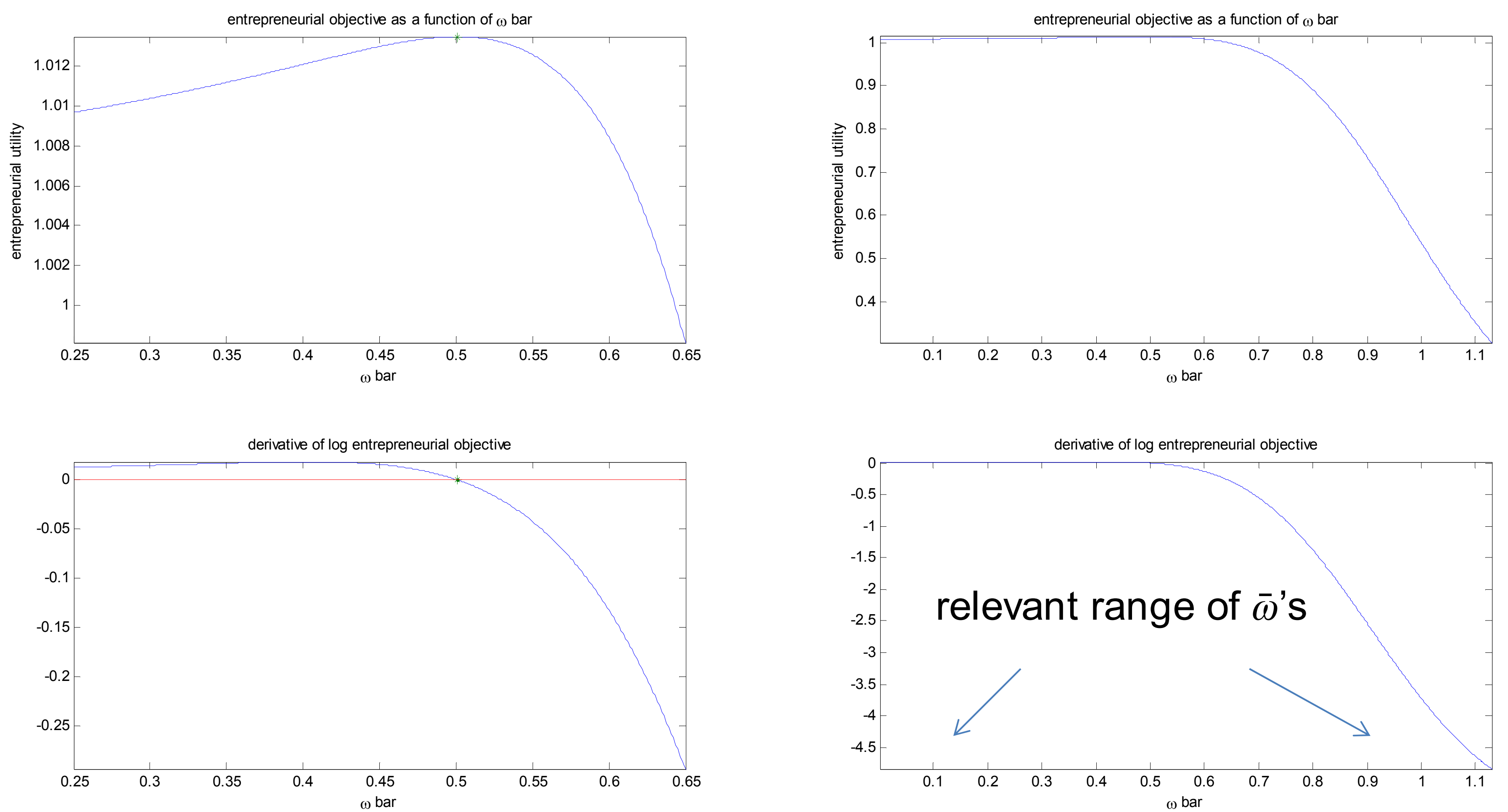
- Entrepreneur selects the contract is optimal, given the available menu of contracts.

- The solution to the entrepreneur problem is the $\bar{\omega}$ that maximizes, over the relevant domain (i.e., $\bar{\omega} \in [0, 1.13]$ in the example):

$$\log\left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

$$= \log \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1+R^k}{1+R} \overbrace{- \log\left(1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Entrepreneur Objective



Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\overbrace{\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R}\left[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})\right]}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R}\bar{\omega}\frac{L}{L-1}$$

Result

- Leverage, *L*, and entrepreneurial rate of interest, *Z*, **not a function of net worth**, *N*.

- Quantity of loans proportional to net worth:

$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

$$B = (L-1)N$$

- To compute *L, Z/(1+R),* must make assumptions about *F* and parameters.

$$\frac{1+R^k}{1+R},\ \mu,\ F$$

Numerical Example

- Parameters:

Percent of average product of entrepreneurial Projects, absorbed by monitoring costs: 0.06%

$$\frac{1+R^k}{1+R} = 1.0073,\ \sigma = 0.26,\ \mu = 0.21$$

- (Micro) equilibrium quantities:

$$\overbrace{\bar{\omega} = 0.50}^{\text{cutoff } \omega},\ \overbrace{\Gamma(\bar{\omega}) = 0.5008}^{\text{fraction of gross entrepreneurial earnings going to lender}},\ \overbrace{F(\bar{\omega}) = 0.0056}^{\text{bankruptcy rate: 0.56\%}},\ \overbrace{G(\bar{\omega}) = 0.0026}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 2.02}^{\text{leverage}},\ \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0015}^{0.62\text{ (APR)}},\ \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0135}^{\text{avg earnings of entrepreneur, divided by opportunity cost}} > 1$$

- Note: on average, entrepreneur better off leveraging net worth and investing in project, rather than depositing net worth in bank.

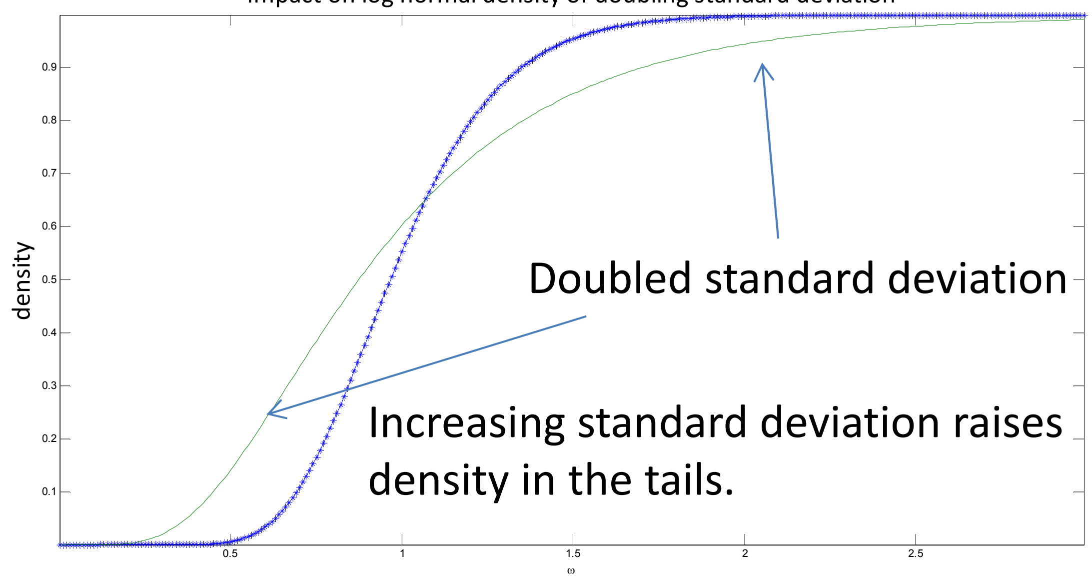
Effect of Increase in Risk, σ

- Keep

$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.

Impact on log normal density of doubling standard deviation



Jump in Risk

- σ replaced by $\sigma \times 3$

$$\overbrace{\bar{\omega} = 0.12}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.12}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0108}^{\text{bankruptcy rate: 1.08\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0011}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 1.1418}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0041}^{1.66 \text{ (APR)}}, \quad \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0080}^{\text{avg earnings of entrepreneur, per unit of net worth}} > 1$$

- Comparison with benchmark:

$$\overbrace{\bar{\omega} = 0.50}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.5008}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0056}^{\text{bankruptcy rate: 0.56\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0026}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 2.02}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0015}^{0.62 \text{ (APR)}}, \quad \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0135}^{\text{avg earnings of entrepreneur, divided by opportunity cost}} > 1$$