Simple Example to Illustrate Bayesian Inference

Lawrence J. Christiano

Model and Priors

- We have one model, and its parameters are θ .
 - $\theta = \theta^{RBC}$, prices flexible, so model corresponds to RBC model (with money).
 - $\dot{\theta} = \theta^{NK}$, same model but with sticky prices.

• Priors
$$p(\theta = \theta^{NK}) = p^{NK}$$
, $p(\theta = \theta^{RBC}) = p^{RBC}$.

- RBC model, $\theta = \theta^{RBC}$, prediction:
 - Inflation equals money growth minus GDP growth \simeq money growth.
- NK model, $\theta = \theta^{NK}$, prediction:
 - Inflation determined by marginal cost, which is a function of output gap.

Data

- Right after 2008 crisis in US, money growth extremely high and US in recession.
 - Prediction with $\theta = \theta^{RBC}$:
 - high inflation, $\pi = \pi^h$, with probability 0.8 $p\left(\pi = \pi^h | \theta = \theta^{RBC}\right) = 0.8.$

– Prediction with $\theta = \theta^{NK}$:

•
$$\pi = \pi^h$$
 probability 0.1 - $p\left(\pi = \pi^h | \theta = \theta^{NK}\right) = 0.1.$

• Data: $\pi = \pi^l$, low inflation.

Inference Question

- In light of the evidence, how should you feel about the plausibility of $\theta = \theta^{RBC}$ versus $\theta = \theta^{NK}$?
 - The RBC predicted high inflation that did not happen.
 - The NK predicted low inflation.
- Common sense suggests that your beliefs will shift towards NK in light of the evidence.
 - Your posterior distribution over θ should be heavily tilted towards $\theta=\theta^{NK}$ by the evidence
- Bayesian theory is fancy common sense.

What Odds Should We Assign Model Parameters In Light of the Evidence?

• Posterior density of parameters in light of data:

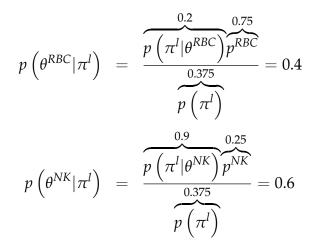
$$p\left(\theta^{RBC}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC}}{p\left(\pi^{l}\right)}$$
$$p\left(\theta^{NK}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{NK}\right)p^{NK}}{p\left(\pi^{l}\right)}$$

• Marginal likelihood of the high inflation:

$$p\left(\pi^{l}\right) = p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC} + p\left(\pi^{l}|\theta^{NK}\right)p^{NK}$$

Plugging in the Numbers

- Suppose you are a big fan of $RBC : p^{RBC} = 3/4, p^{NK} = 1/4.$
- Posterior density of parameters in light of data:



Conclusion

Even if your prior experience and training had made you a devoted RBC fan before, the recent US data would have moved your posteriors in the direction of NK.