Simple Example to Illustrate Bayesian Inference

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Model and Priors

- We have one model, and its parameters are θ.
  - $\theta = \theta^{RBC}$, prices flexible, so model corresponds to RBC model (with money).
  - $\theta = \theta^{NK}$, same model but with sticky prices.

- Priors $p(\theta = \theta^{NK}) = p^{NK}$, $p\left(\theta = \theta^{RBC}\right) = p^{RBC}$.

- RBC model, $\theta = \theta^{RBC}$, prediction:
  - Inflation equals money growth minus GDP growth $\simeq$ money growth.

- NK model, $\theta = \theta^{NK}$, prediction:
  - Inflation determined by marginal cost, which is a function of output gap.

Data

- Right after 2008 crisis in US, money growth extremely high and US in recession.
  - Prediction with $\theta = \theta^{RBC}$:
    - high inflation, $\pi = \pi^h$, with probability 0.8 - $p\left(\pi = \pi^h | \theta = \theta^{RBC}\right) = 0.8.$
  - Prediction with $\theta = \theta^{NK}$:
    - $\pi = \pi^h$ probability 0.1 - $p\left(\pi = \pi^h | \theta = \theta^{NK}\right) = 0.1.$
- Data: $\pi = \pi^l$, low inflation.

Inference Question

- In light of the evidence, how should you feel about the plausibility of $\theta = \theta^{RBC}$ versus $\theta = \theta^{NK}$?
  - The RBC predicted high inflation that did not happen.
  - The NK predicted low inflation.

- Common sense suggests that your beliefs will shift towards NK in light of the evidence.
  - Your posterior distribution over θ should be heavily tilted towards $\theta = \theta^{NK}$ by the evidence

- Bayesian theory is fancy common sense.

What Odds Should We Assign Model Parameters In Light of the Evidence?

- Posterior density of parameters in light of data:

$$p\left(\theta^{RBC}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC}}{p\left(\pi^{l}\right)}$$

$$p\left(\theta^{NK}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{NK}\right)p^{NK}}{p\left(\pi^{l}\right)}$$

- Marginal likelihood of the high inflation:

$$p\left(\pi^{l}\right) = p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC} + p\left(\pi^{l}|\theta^{NK}\right)p^{NK}.$$

Plugging in the Numbers

- Suppose you are a big fan of RBC : $p^{RBC} = 3/4$, $p^{NK} = 1/4$.
- Posterior density of parameters in light of data:

$$p\left(\theta^{RBC}|\pi^{l}\right) = \frac{\overbrace{p\left(\pi^{l}|\theta^{RBC}\right)}^{0.2}\overbrace{p^{RBC}}^{0.75}}{\underbrace{p\left(\pi^{l}\right)}_{0.375}} = 0.4$$

$$p\left(\theta^{NK}|\pi^{l}\right) = \frac{\overbrace{p\left(\pi^{l}|\theta^{NK}\right)}^{0.9}\overbrace{p^{NK}}^{0.25}}{\underbrace{p\left(\pi^{l}\right)}_{0.375}} = 0.6$$

Conclusion

Even if your prior experience and training had made you a devoted RBC fan before, the recent US data would have moved your posteriors in the direction of NK.