# Simple Example to Illustrate Bayesian Inference

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### **Model and Priors**

- We have one model, and its parameters are  $\theta$ .
  - $\theta = \theta^{RBC}$ , prices flexible, so model corresponds to RBC model (with money).
  - $\dot{\theta} = \theta^{NK}$ , same model but with sticky prices.

• Priors 
$$p(\theta = \theta^{NK}) = p^{NK}$$
,  $p(\theta = \theta^{RBC}) = p^{RBC}$ .

- RBC model,  $\theta = \theta^{RBC}$ , prediction:
  - Inflation equals money growth minus GDP growth  $\simeq$  money growth.
- NK model,  $\theta = \theta^{NK}$ , prediction:
  - Inflation determined by marginal cost, which is a function of output gap.

#### Data

- Right after 2008 crisis in US, money growth extremely high and US in recession.
  - Prediction with  $\theta = \theta^{RBC}$  :
    - high inflation,  $\pi = \pi^h$ , with probability 0.8  $p\left(\pi = \pi^h | \theta = \theta^{RBC}\right) = 0.8.$

– Prediction with  $\theta = \theta^{NK}$  :

• 
$$\pi = \pi^h$$
 probability 0.1 -  $p\left(\pi = \pi^h | \theta = \theta^{NK}\right) = 0.1.$ 

• Data:  $\pi = \pi^l$ , low inflation.

### **Inference Question**

- In light of the evidence, how should you feel about the plausibility of  $\theta = \theta^{RBC}$  versus  $\theta = \theta^{NK}$ ?
  - The RBC predicted high inflation that did not happen.
  - The NK predicted low inflation.
- Common sense suggests that your beliefs will shift towards NK in light of the evidence.
  - Your posterior distribution over  $\theta$  should be heavily tilted towards  $\theta=\theta^{NK}$  by the evidence
- Bayesian theory is fancy common sense.

## What Odds Should We Assign Model Parameters In Light of the Evidence?

• Posterior density of parameters in light of data:

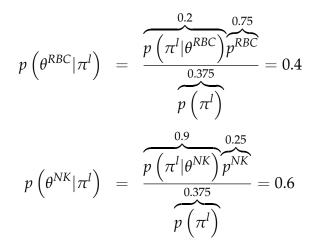
$$p\left(\theta^{RBC}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC}}{p\left(\pi^{l}\right)}$$
$$p\left(\theta^{NK}|\pi^{l}\right) = \frac{p\left(\pi^{l}|\theta^{NK}\right)p^{NK}}{p\left(\pi^{l}\right)}$$

• Marginal likelihood of the high inflation:

$$p\left(\pi^{l}\right) = p\left(\pi^{l}|\theta^{RBC}\right)p^{RBC} + p\left(\pi^{l}|\theta^{NK}\right)p^{NK}$$

### Plugging in the Numbers

- Suppose you are a big fan of  $RBC : p^{RBC} = 3/4, p^{NK} = 1/4.$
- Posterior density of parameters in light of data:



### Conclusion

Even if your prior experience and training had made you a devoted RBC fan before, the recent US data would have moved your posteriors in the direction of NK.