

# Two-Period Version of Gertler-Karadi, Gertler-Kiyotaki Financial Friction Model

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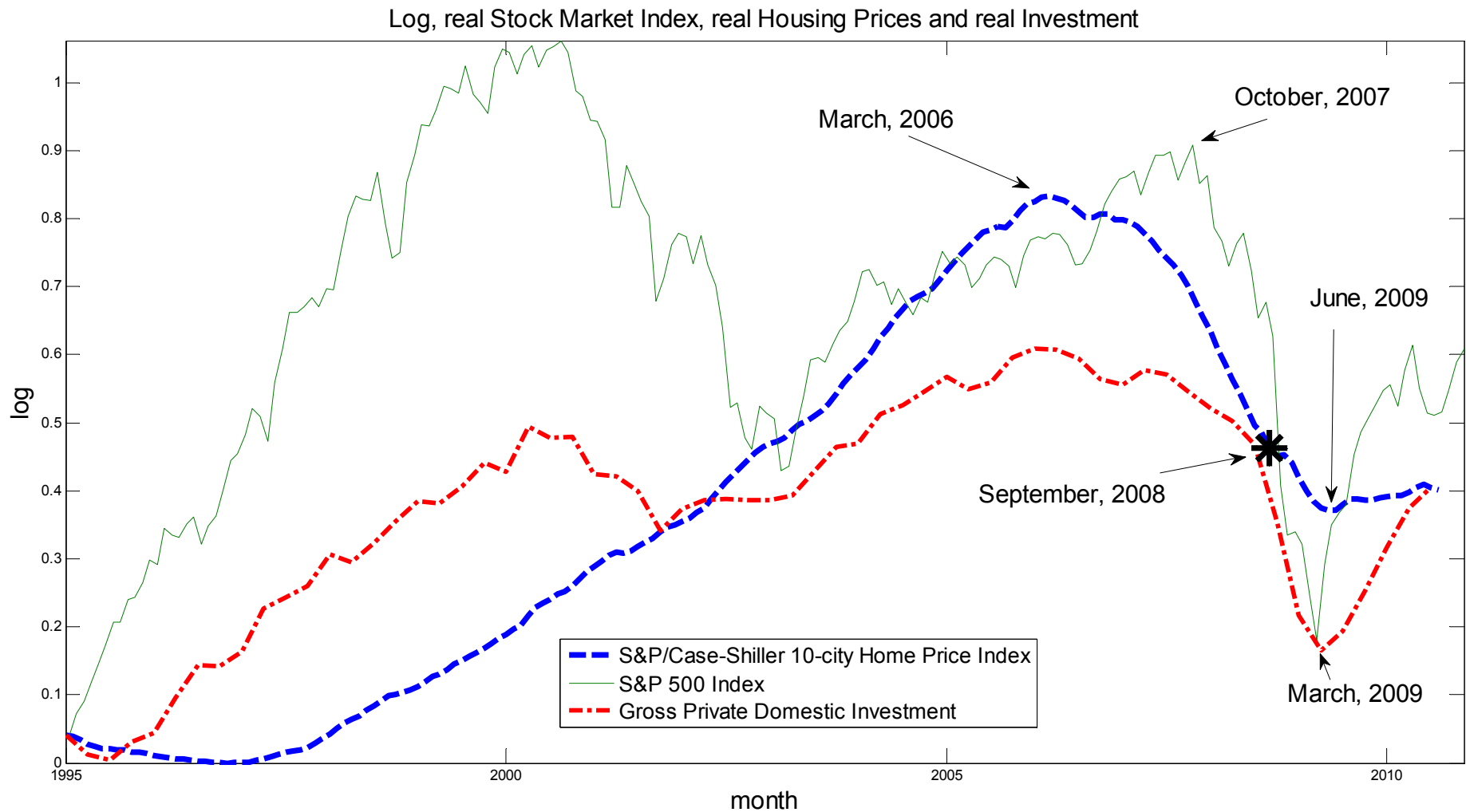
Summary of Christiano-Ikeda, 2012, 'Government Policy, Credit Markets and Economic Activity,' in Federal Reserve Bank of Atlanta conference volume,

*A Return to Jekyll Island: the Origins, History, and Future of the Federal Reserve*, Cambridge University Press.

# Motivation

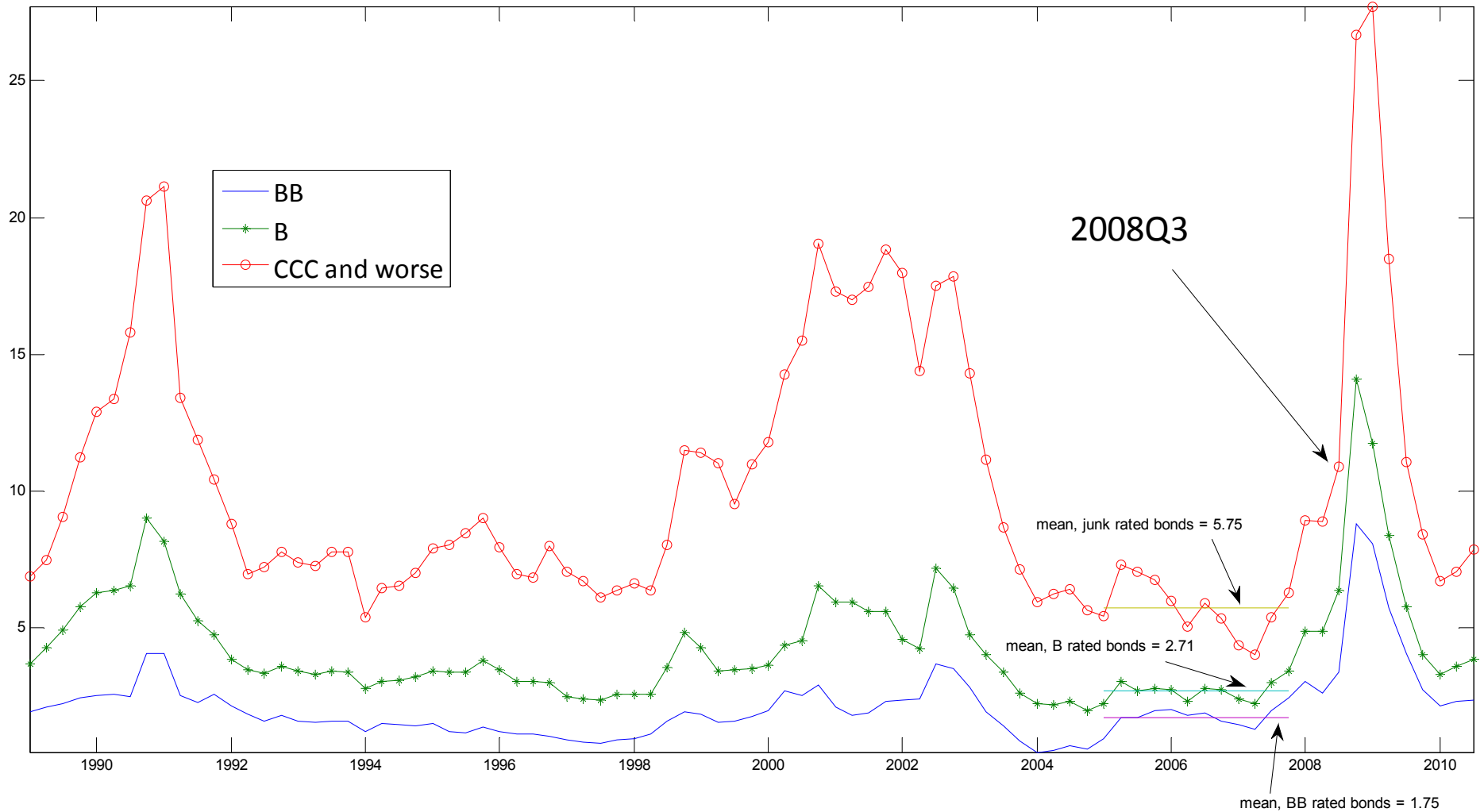
- Beginning in 2007 and then accelerating in 2008:
  - Asset values (particularly for banks) collapsed.
  - Intermediation slowed and investment/output fell.
  - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
  - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

# Collapse in Asset Values and Investment



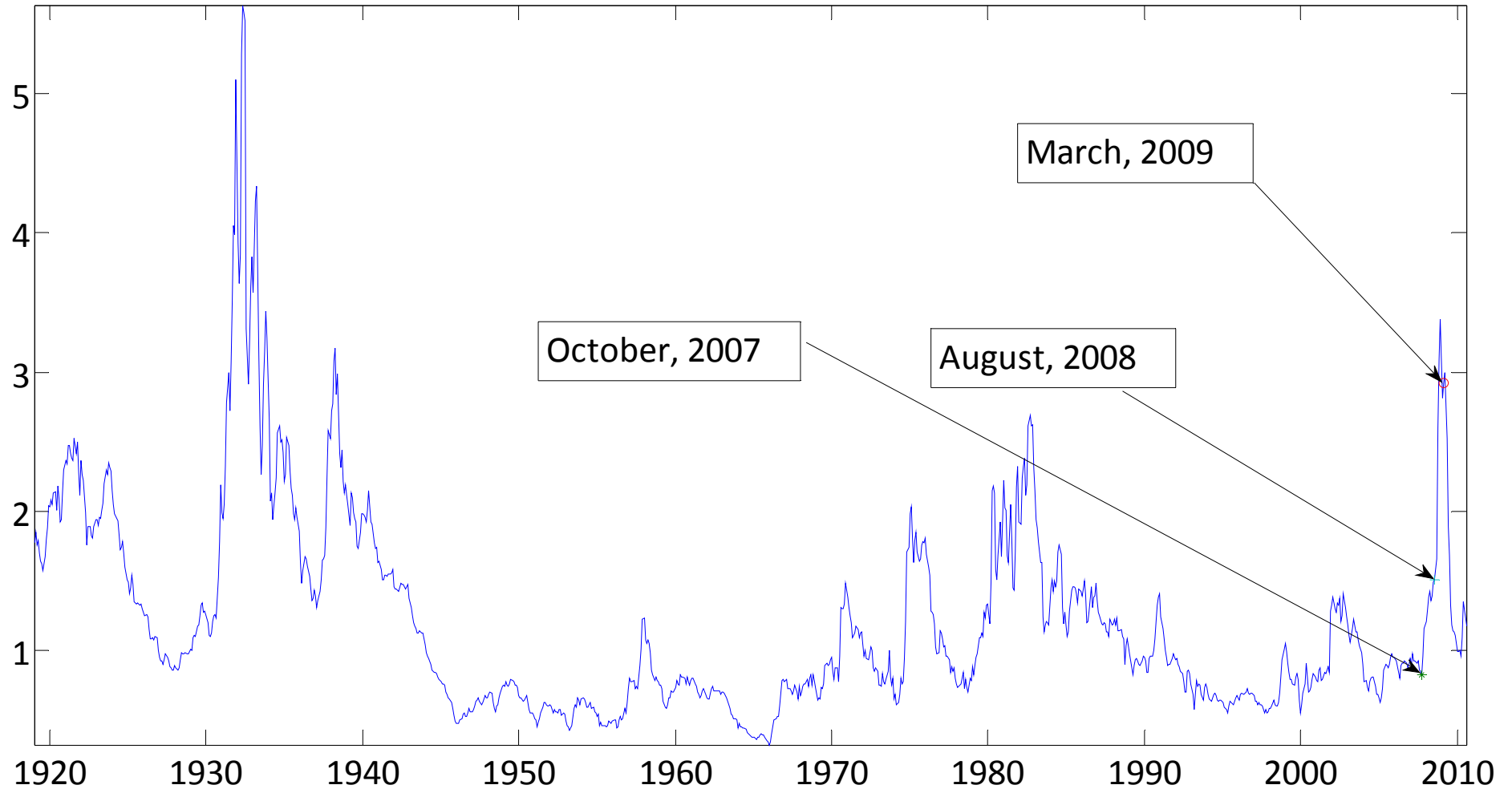
# Spreads for 'Risky' Firms Shot Up in Late 2008

Interest Rate Spread on Corporate Bonds of Various Ratings Over Rate on AAA Corporate Bonds

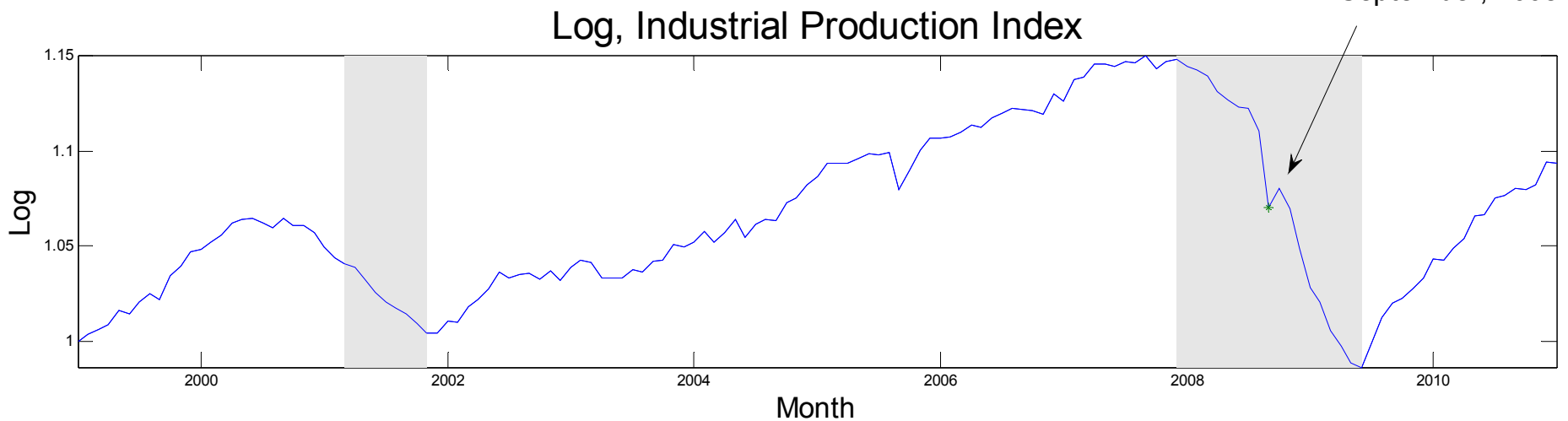
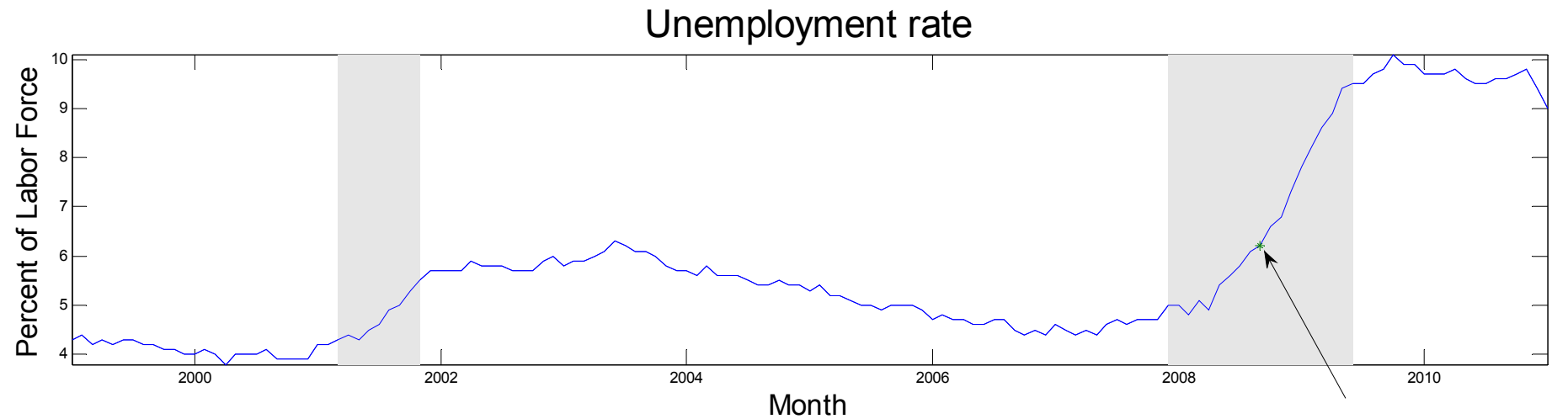


# Must Go Back to Great Depression to See Spreads as Large as the Recent Ones

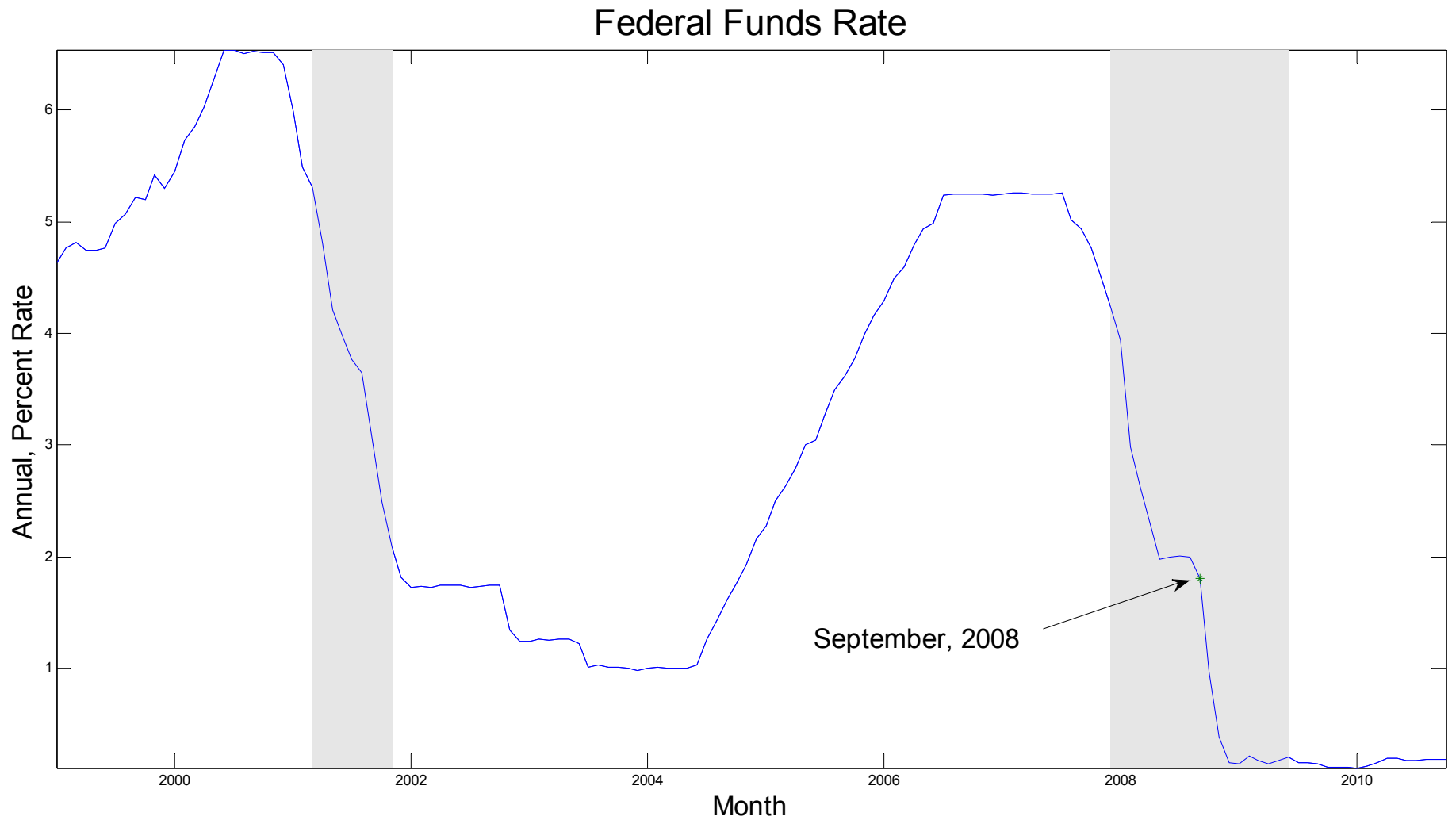
Spread, BAA versus AAA bonds



# Economic Activity Shows (anemic!) Signs of Recovery June, 2009



# Banks' Cost of Funds Low



# Characterization of Crisis to be Explored Here

- Bank Asset Values Fell.
- Banking System Became ‘Dysfunctional’
  - Interest rate spreads rose.
  - Intermediation and economy slowed.
- Monetary authority:
  - Transferred funds on various terms to private companies and to banks.
  - Sharply reduced cost of funds to banks.
- Economy in (tentative) recovery.
- Seek to construct models that links these observations together.



# Objective

- Keep analysis simple and on point by:
  - Two periods
  - Minimize complications from agent heterogeneity.
  - Leave out endogeneity of employment.
  - Leave out nominal variables: just look ‘behind the veil of monetary economics’
- Models:
  - Gertler-Kiyotaki/Gertler-Karadi
  - In two-period setting easy to study an interesting nonlinearity that is possible:
    - Participation constraint may be binding in a crisis and not binding in normal times.

# Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
  - Some members are ‘bankers’
  - Some members are ‘workers’
  - Perfect insurance inside households...everyone consumes same amount.
- Period 1
  - Workers endowed with  $y$  goods, household makes deposits,  $d$ , in a bank
  - Bankers endowed with  $N$  goods, take deposits and purchase securities,  $d$ , from a firm.
  - Firm issues securities,  $s$ , to produce  $sR^k$  in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits,  $\pi$ , from banker.
  - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d} [u(c) + \beta u(C)]$	

Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = R^d$	$c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$

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Household budget constraint when gov't buys private assets using tax receipts,  $T$ , and gov't gets the same rate of return,  $R^d$ , as households:

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

No change!  
(Ricardian-Wallace  
Irrelevance)

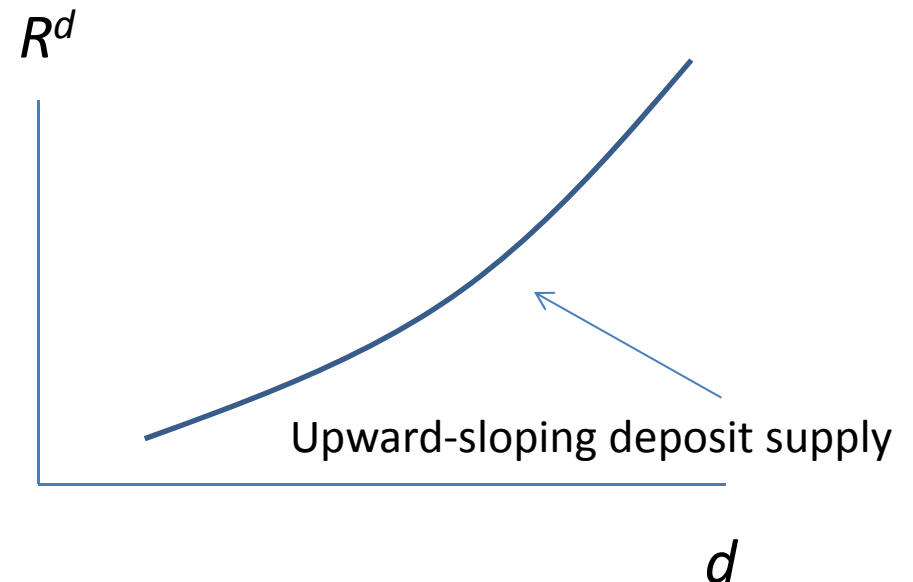
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# Household Supply of Deposits

- For given  $\pi$ ,  $d$  rises or falls with  $R^d$ , depending on parameter values.
- But, in equilibrium  $\pi = R^k(N+d) - R^d d$ .
- Substituting into the expression for  $c$  and solving for  $d$ :

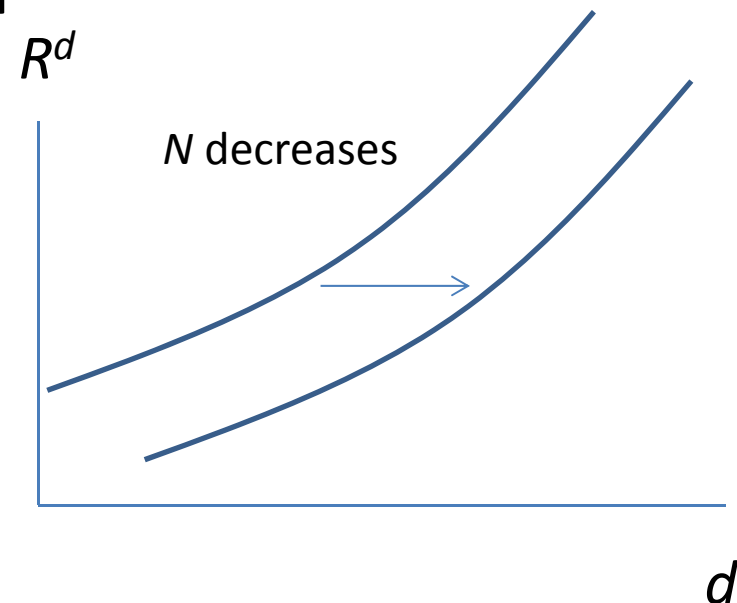
$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$



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# Properties of Equilibrium Household Supply of Deposits

- Deposits increasing in  $R^d$ .
- Shifts right with decrease in  $N$  because of wealth effect operating via bank profits,  $\pi$ .
  - rise in deposit supply smaller than decrease in  $N$ .

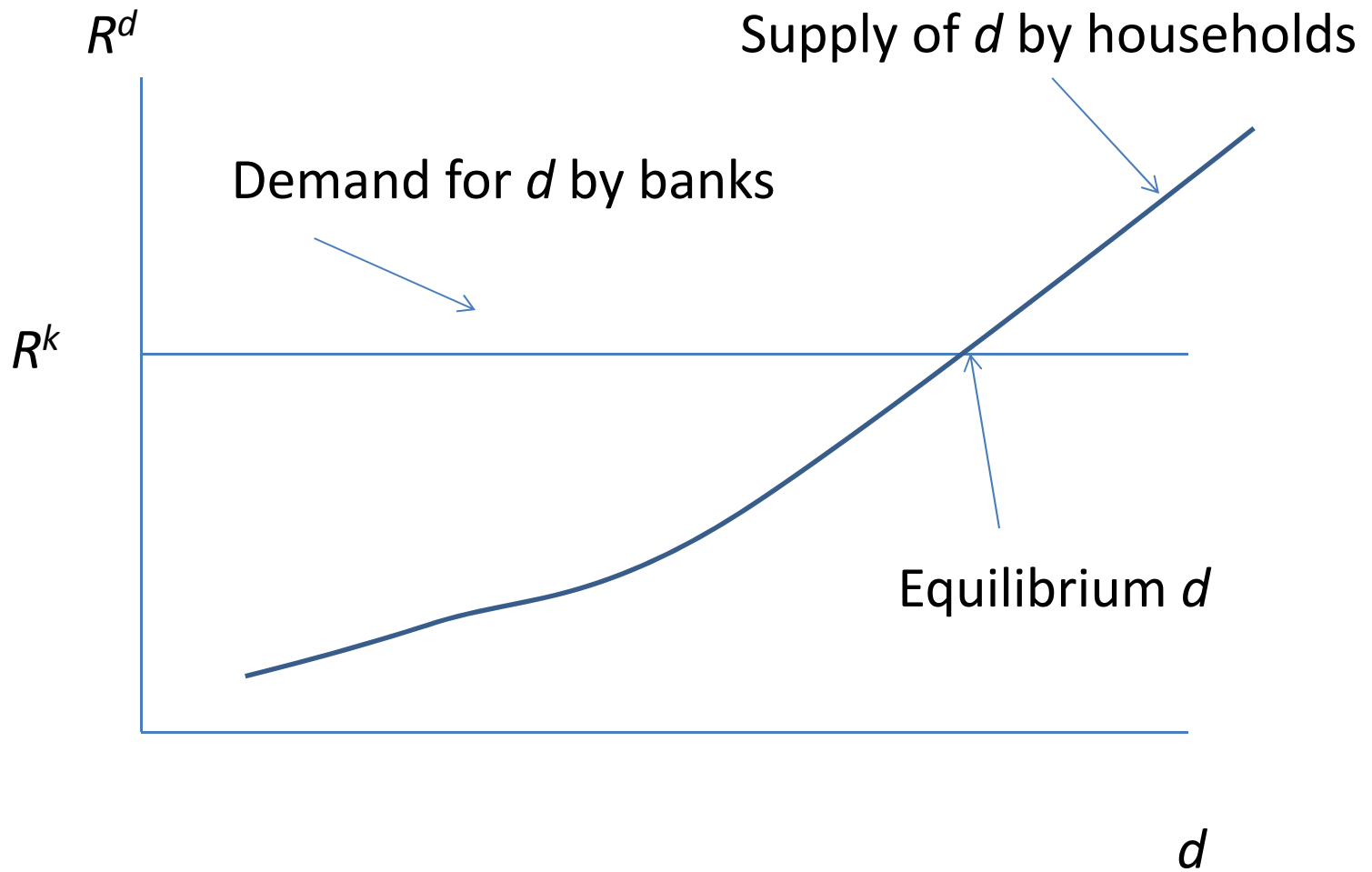
$$\frac{\partial d}{\partial N} = - \overbrace{\left[ \frac{R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} \right]}^{>0, <1}$$



# Efficient Benchmark

Problem of the Bank	
period 1	period 2
take deposits, $d$	pay $dR^d$ to households
buy securities, $s = N + d$	receive $sR^k$ from firms
problem: $\max_d [sR^k - R^d d]$	

# Bank demand for $d$



# Equilibrium in Absence of Frictions

Interior Equilibrium:  $R^d, \pi, d, c, C$

(i)  $c, d, C > 0$

(ii) household problem is solved

(iii) bank problem is solved

(iv) goods and financial markets clear

- Properties:

- Household faces true social rate of return on saving:

$$R^k = R^d$$

- Equilibrium is ‘first best’, i.e., solves

$$\max_{c, C, k} u(c) + \beta u(C)$$

$$c + k \leq y + N, \quad C \leq kR^k$$

# Friction

- bank combines deposits,  $d$ , with net worth,  $N$ , to purchase  $N+d$  securities from firms.
- bank has two options:
  - ('no-default') wait until next period when  $(N+d)R^k$  arrives and pay off depositors,  $R^d d$ , for profit:

$$(N+d)R^k - R^d d$$

- ('default') take  $\theta(N+d)$  securities, refuse to pay depositors and wait until next period when securities pay off:
$$\theta(N+d)R^k$$
- Bank must announce what value of  $d$  it will choose at the beginning of a period.

# Incentive Constraint

- Recall, banks maximize profits
- Choose 'no default' iff

$$\overbrace{(N + d)R^k - R^d d}^{\text{no default}} \geq \overbrace{\theta(N + d)R^k}^{\text{default}}$$

- Next: derive banking system's demand for deposits in presence of financial frictions.

# Result for a no-default equilibrium:

- Consider an individual bank that contemplates defaulting.

- It sets a  $d$  that implies default,

$$R^k(N + d) - R^d d < \theta R^k(d + N) \quad , \text{ or}$$

what the household gets in the other banks

$$\overbrace{R^d}$$

>

what the household gets in the defaulting bank

$$\frac{\overbrace{(1 - \theta)R^k(d + N)}}{d}$$

- A deviating bank will in fact receive no deposits.
- **An optimizing bank would never default**

# Problem of the bank in no-default, interior equilibrium

- Maximize, by choice of  $d$ ,

$$R^k(N + d) - R^d d$$

If interest rate is REALLY low, then bank has no incentive to default because it makes lots of profits not defaulting

subject to:

$$R^k(N + d) - R^d d - R^k \theta(N + d) \geq 0,$$

or,

$$(1 - \theta)R^k N - [R^d - (1 - \theta)R^k]d \geq 0.$$

- Note that  $0 < d < \infty$  requires

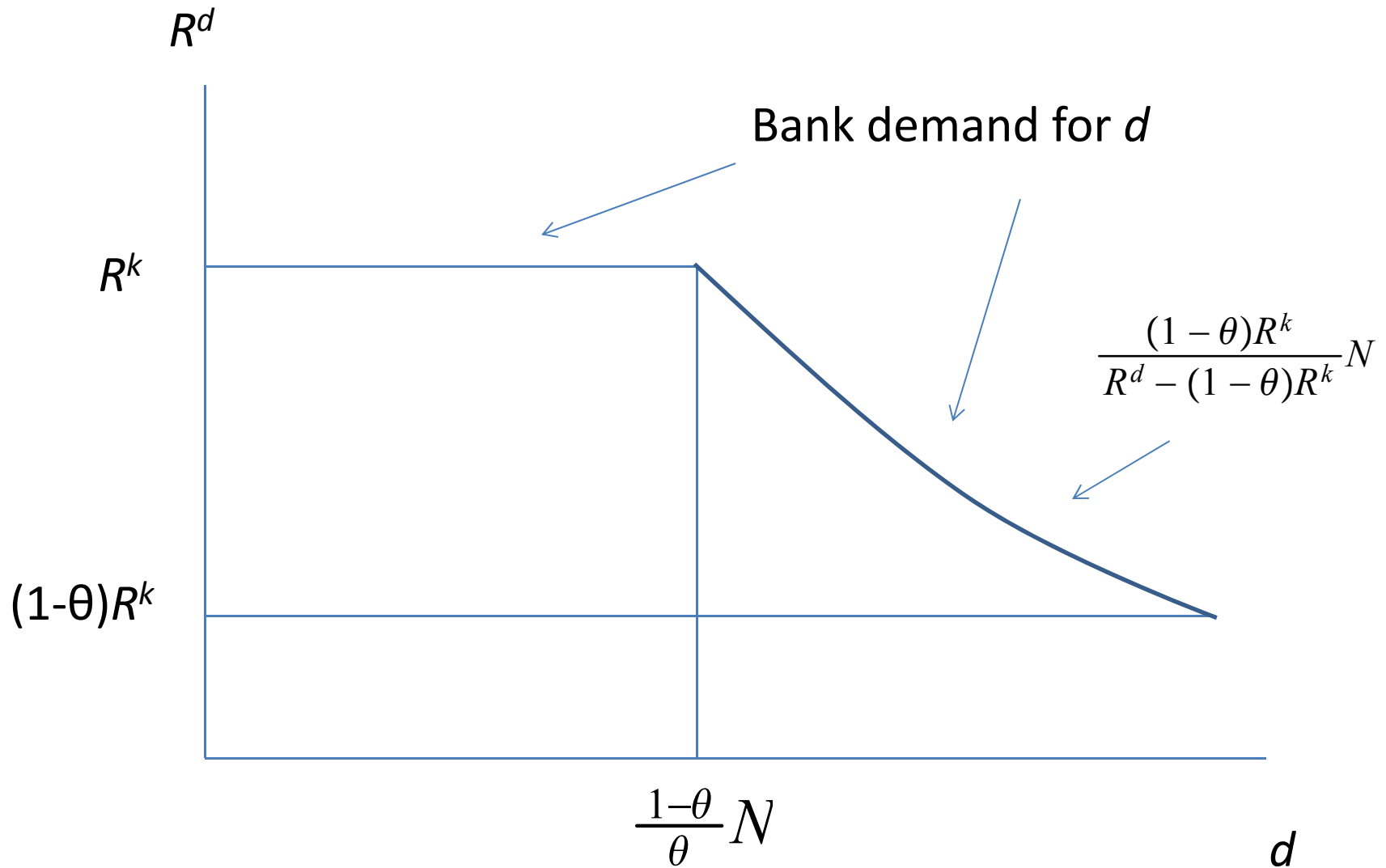
$(1 - \theta)R^k$	$\underbrace{<}_{\text{if not, then } d=\infty}$	$R^d$	$\underbrace{\leq}_{\text{if not, then } d=0}$	$R^k.$
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# Problem of the bank in no-default, interior equilibrium, cnt'd

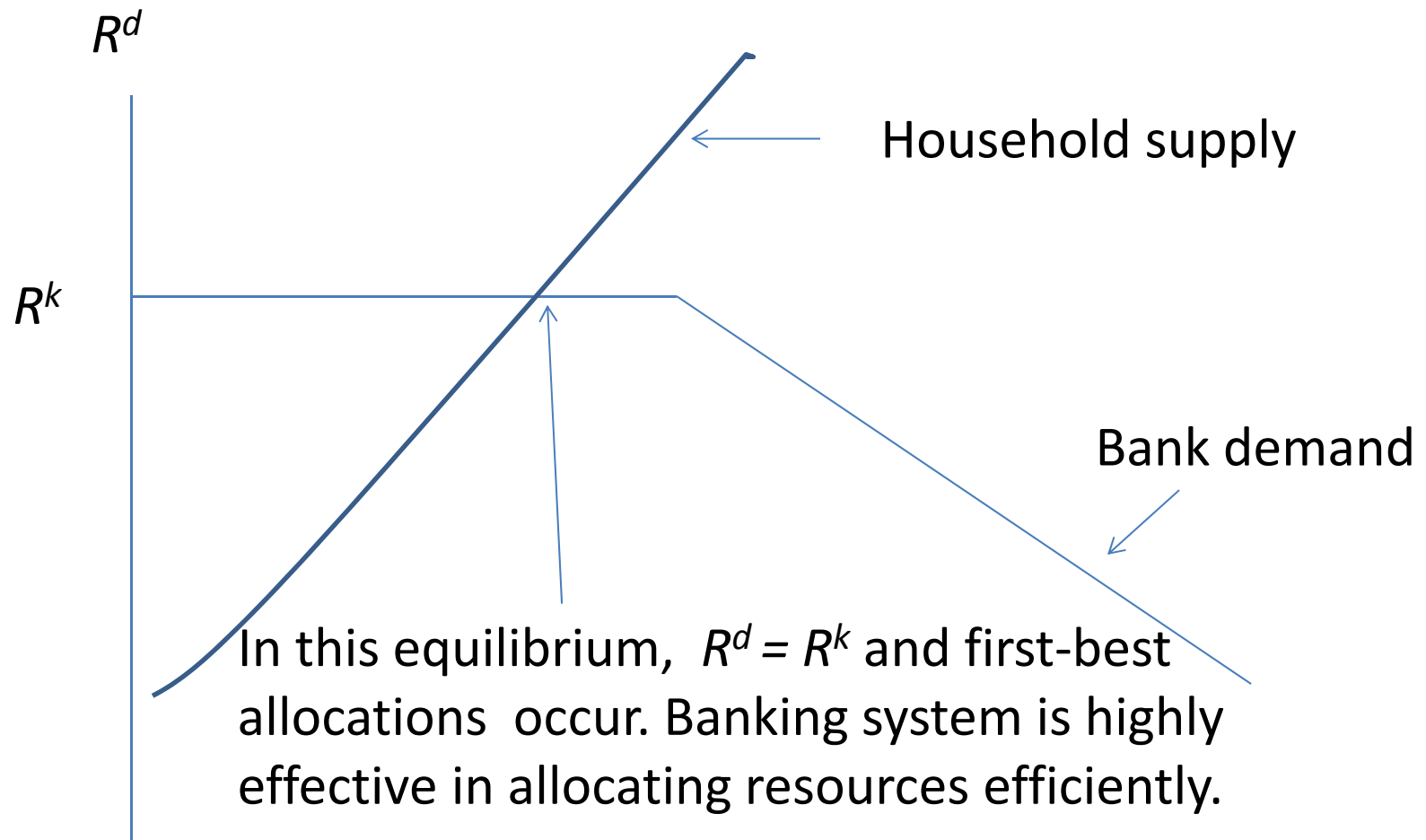
- For  $R^d = R^k$ 
  - a bank makes no profits on  $d$  so – absent default considerations - it is indifferent over all values of  $0 \leq d$
  - Taking into account default, a bank is indifferent over  $0 \leq d \leq N(1-\theta)/\theta$
- For  $(1-\theta)R^k < R^d < R^k$ 
  - Bank wants  $d$  as large as possible, subject to incentive constraint.
  - So,  $d = R^k N(1-\theta) / (R^d - (1-\theta)R^k)$



# Bank demand for $d$



# Interior, no default equilibrium

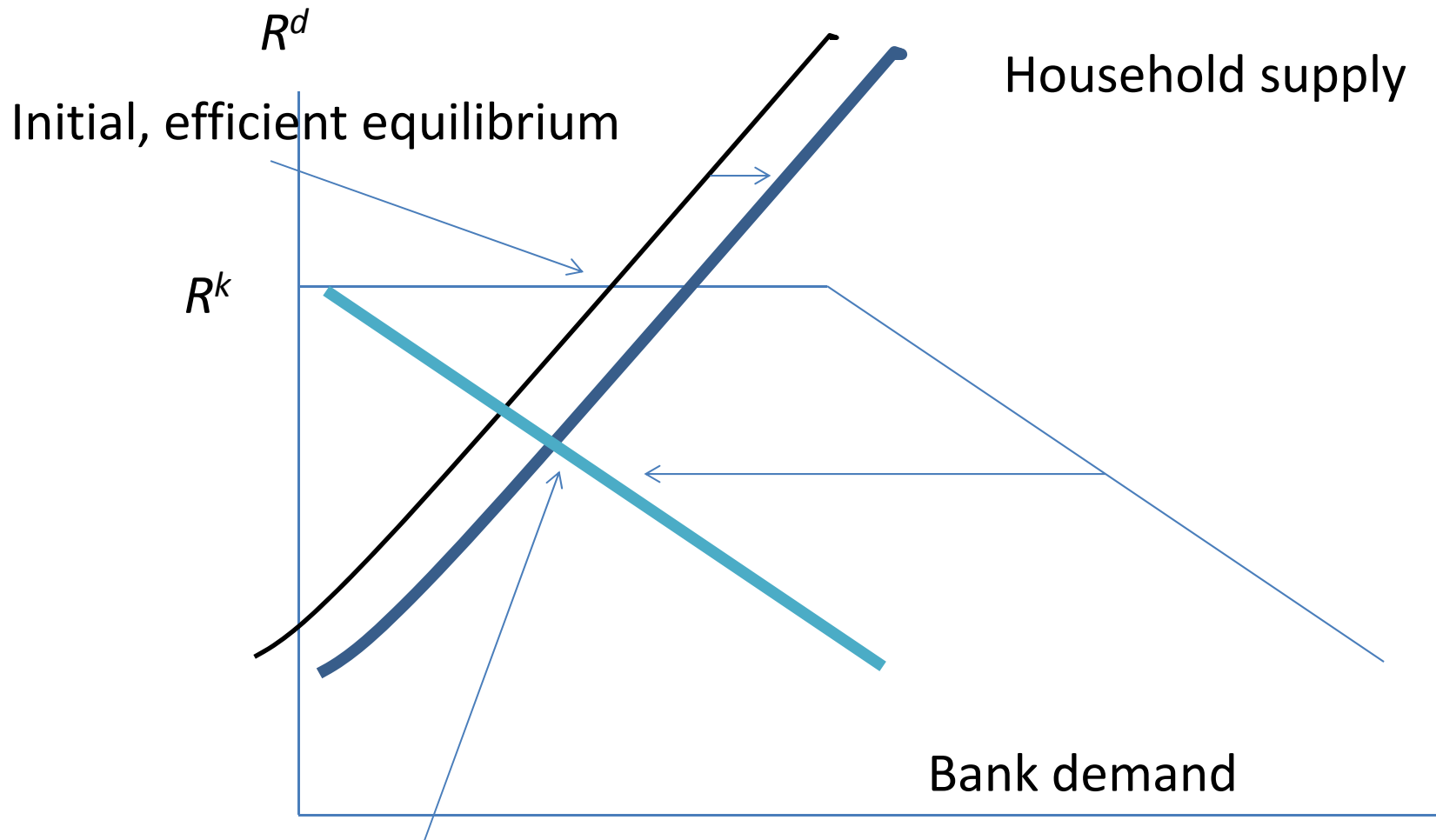


$d$

# Collapse in Bank Net Worth

- Suppose that the economy is represented by a sequence of repeated versions of the above model.
- In the periods before the 2007-2008 crisis, net worth was high and the equilibrium was like it is on the previous slide: efficient, with zero interest rate spreads.
  - In practice, spreads are always positive, but that reflects various banking costs that are left out of this model.
- With the crisis,  $N$  dropped a lot, shifting demand to the right and supply to the left.

# Effect of Substantial Drop in Bank Net Worth

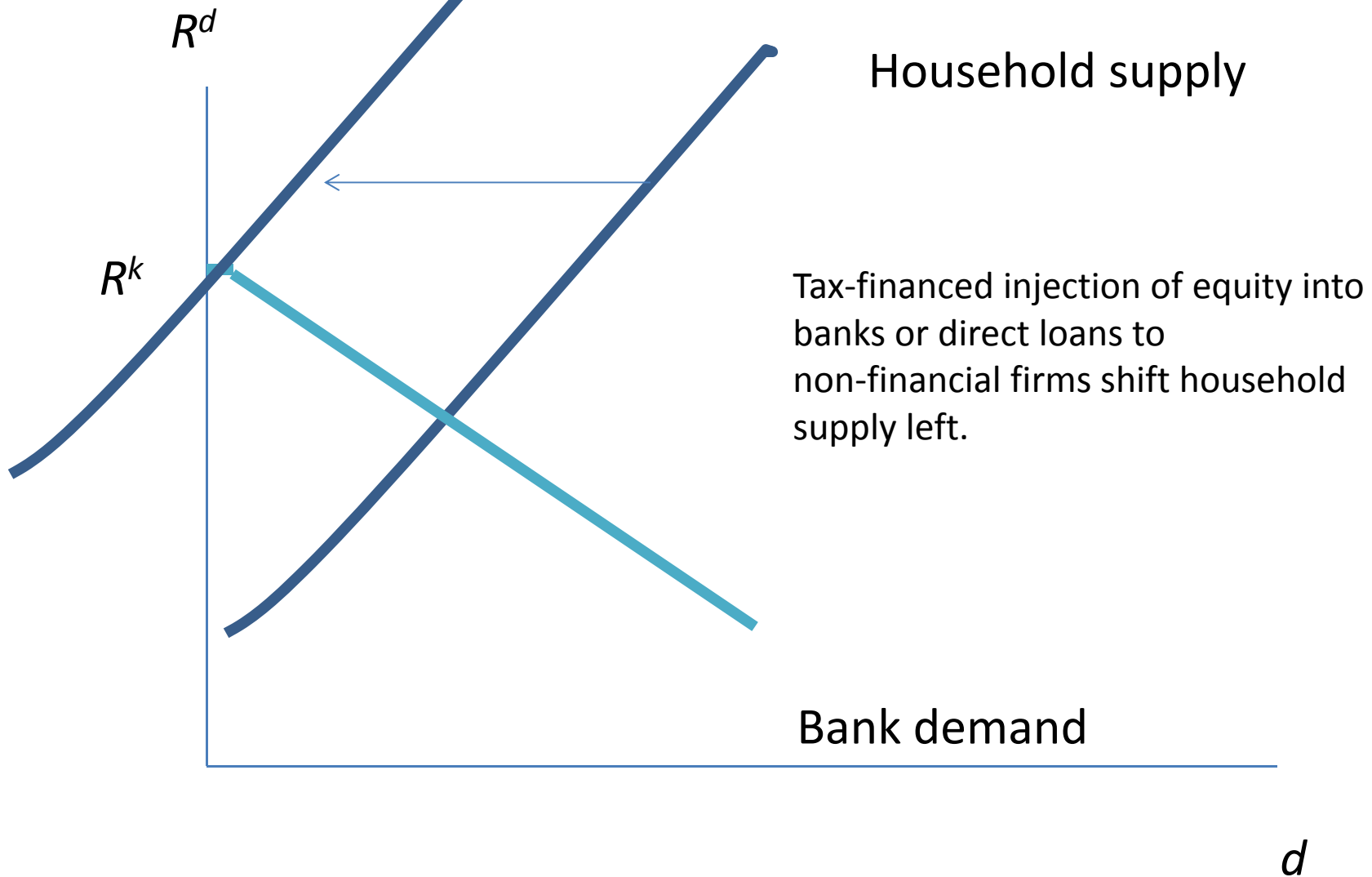


Equilibrium after  $N$  drops is inefficient because  $R^d < R^k$ .  $d$

# Government Intervention

- Equity injection.
  - Government raises  $T$  in period 1, provides proceeds to banks and demands  $R^k T$  in return at start of period 2.
  - Rebates earnings to households in 2.
- Has no impact on demand for deposits by banks (no impact on default incentive or profits).
- Reduces supply of deposits by households.
  - $d+T$  rises when  $T$  rises (even though  $d$  falls) because  $R^d$  rises.
- Direct, tax-financed government loans to firms work in the same way.
- An interest rate subsidy to banks will shift their demand for deposits to the right....no impact on supply curve when subsidy financed by period 2 lump sum tax on households.

# Equity Injection and Drop in $N$



# Recap

- Basic idea:
  - Bankers can run away with a fraction of bank assets.
  - If banker net worth is high relative to deposits, friction not a factor and banking system efficient.
  - If banker net worth falls below a certain cutoff, then banker must restrict the deposits.
    - Bankers fear (correctly) that otherwise depositors would lose confidence and take their business to another bank.
  - Reduction in banker demand for deposits:
    - makes deposit interest rates fall and so spreads rise.
    - Reduced intermediation means investment drops, output drops.
  - Equity injections by the government can revive the banking system.

# Is the Model Narrative Consistent with the Evidence?

- Model says that reduced intermediation of funds through the financial system reflected reduced demand for credit by financial institutions.
- Prediction: interest rate to financial institutions fall.



— 1-Month AA Financial Commercial Paper Rate



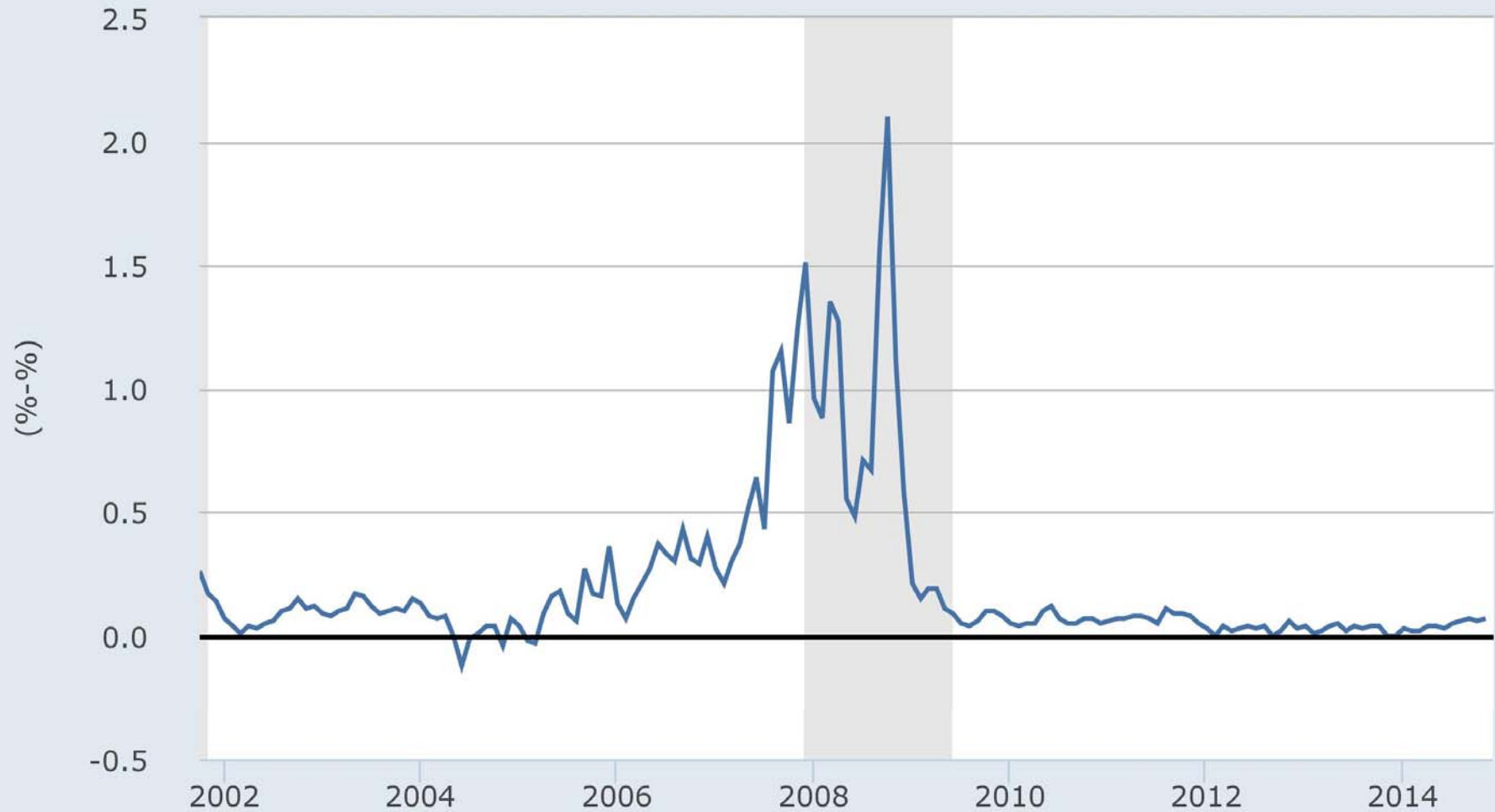
Source: Board of Governors of the Federal Reserve System (US)

Shaded areas indicate US recessions - 2014 [research.stlouisfed.org](http://research.stlouisfed.org)

- Model prediction for decline in cost of funds to financial institutions seems verified.
- But, other 'risk free' interest rates fell even more.
  - Interest rates on US government debt fell more than interest rate on financial firm commercial paper.



— 1-Month AA Financial Commercial Paper Rate-3-Month Treasury Bill: Secondary Market Rate



Shaded areas indicate US recessions - 2014 research.stlouisfed.org

# Assessment

- Fact that interest rates on US government debt went down more than cost of funds to financial institutions suggests that a complete picture of financial crisis may require two additional features:
  - Risky Banks:
    - Banks in the model are risk free. Default only occurs out of equilibrium.
    - Increased actual riskiness of banks is perhaps also an important part of the picture.
  - Liquidity:
    - Low interest rates on US government debt consistent with idea that high demand for liquidity played an important role in the crisis.

# Macro Prudential Policy

- In recent years there has been increased concern that banks may have a tendency to take on too much debt.
- Has accelerated thinking about debt restrictions on banks.
- There are several models of financial frictions in banks, but they do not necessarily provide a foundation for thinking about debt restrictions on banks.
  - A CSV model of banks implies they issue too *little* debt. (See Christiano-Ikeda).
  - The ‘running away’ model of banks does *not* rationalize debt restrictions. (See next).

# Optimal Debt Restriction in Two-Period Running Away Banking Model

- Debt restriction on banks:

$$d \leq \bar{d}$$

- What is the socially optimal level of  $\bar{d}$  ?
- To answer this, must take into account structure of private economy
  - The way households choose debt in competitive markets
  - The fact that banks will not choose a debt level that violates incentive constraints.

# Social Welfare Function

$$u(c) + \beta u(C)$$

$$= u\left(\underbrace{y-d}_{c}\right) + \beta u\left(\underbrace{\overbrace{R^d d}^{\text{=earnings on deposits}} + \overbrace{+R^k(N+d)-R^d d}^{\text{=bank profits}}}_{C}\right)$$

$$= u(y - d) + \beta u(R^k(N + d)).$$

# Household Saving

- Optimization:

$$u'(y - d) = R^d u'(C)$$

plus budget constraint and definition of profits  
(see above) implies:

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - R^k \frac{n}{y}}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$

or

$$R^d = \frac{1}{\beta} \left( \frac{d+n}{y-d} R^k \right)^\gamma \equiv f(d)$$



# Implementability Constraint

- Let  $d^*$  denote the value of deposits that a benevolent planner wishes the banks would choose.
- Planner must take into account:
  - banks will not choose a level of  $d$  which implies a violation of the incentive constraint.
  - market arrangement in which households make their deposit supply decision.
  - these considerations restrict  $d$  as follows:

$$(1 - \theta)(N + d)R^k - f(d)d \geq 0$$

# Planning Problem

- $d^*$  is solution to the following problem:

$$\max_d u(y - d) + \beta u(R^k(N + d)) + \mu[(1 - \theta)(N + d)R^k - f(d)d]$$

- Fonc

$=u'(y-d)/R^d$  by households

$$-u'(y - d) + \overbrace{\beta u'(C)} \times R^k + \mu[(1 - \theta)R^k - f'(d)d - f(d)] = 0$$

$$\mu \geq 0, [(1 - \theta)(N + d)R^k - f(d)d] \geq 0, \mu[(1 - \theta)(N + d)R^k - f(d)d] = 0 \quad .$$

# Planning Problem

- $d^*$  is solution to the following problem:

$$\max_d u(y - d) + \beta u(R^k(N + d)) + \mu[(1 - \theta)(N + d)R^k - f(d)d]$$

- Fonc

$$u'(y - d) \left[ \frac{R^k}{f(d)} - 1 \right] + \mu[(1 - \theta)R^k - f'(d)d - f(d)] = 0$$

Complementary Slackness

$$\mu \geq 0, [(1 - \theta)(N + d)R^k - f(d)d] \geq 0, \mu[(1 - \theta)(N + d)R^k - f(d)d] = 0$$

# Planning Problem

- First order conditions:

$$u'(y - d) \left[ \frac{R^k}{f(d)} - 1 \right] + \mu [(1 - \theta)R^k - f'(d)d - f(d)] = 0$$

Complementary Slackness

$$\mu \geq 0, [(1 - \theta)(N + d)R^k - f(d)d] \geq 0, \mu [(1 - \theta)(N + d)R^k - f(d)d] = 0$$

- Solving the problem:

- Try  $\mu = 0$  and solve ('savings supply crosses horizontal line at  $R^k$ )  $R^k = f(d)$

- Check incentive constraint. If satisfied,  $R^k = f(d^*)$

- Otherwise, conclude  $\mu > 0$  and

$$(1 - \theta)(N + d^*)R^k - f(d^*)d^* = 0$$

- ('Savings supply crosses incentive constraint').

# No Borrowing Restrictions Desired

- Deposits selected by government coincide with equilibrium deposits when there is no borrowing restriction.
- So, according to the model, restriction on bank borrowing not necessary.
- Model is not a good laboratory for thinking about leverage restrictions on banks, if you're firmly convinced that leverage restrictions are required.