Simplest New Keynesian Model without Capital

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Objective

- Describe the model sufficiently, so that ‘homework #9’ can be done.
- Define the model, display its linearized equilibrium conditions.
- Define a model ‘solution’.

Clarida-Gali-Gertler Model

- Households maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}, \ \varepsilon_t^{\tau} \sim iid,$$

- Subject to:

$$P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + T_t$$

- Intratemporal first order condition:

$$C_t \exp(\tau_t) N_t^{\varphi} = \frac{W_t}{P_t}$$

Household Intertemporal FONC

- Condition:

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{1+\pi_{t+1}}$$

  – or

$$\begin{aligned} 1 &= \beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{1+\pi_{t+1}} \\ &= \beta E_t \exp[\log(R_t) - \log(1+\pi_{t+1}) - \Delta c_{t+1}] \\ &\simeq \beta \exp[\log(R_t) - E_t\pi_{t+1} - E_t\Delta c_{t+1}],\ c_t \equiv \log(C_t) \end{aligned}$$

  – take log of both sides:

$$0 = \log(\beta) + r_t - E_t\pi_{t+1} - E_t\Delta c_{t+1},\ r_t = \log(R_t)$$

  – or

$$c_t = -\log(\beta) - [r_t - E_t\pi_{t+1}] + c_{t+1}$$

Firms

- Competitive final good firms:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} di, \; \varepsilon > 1,$$

  – First order condition:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Intermediate good producer (monopolist in output, competitive in labor market):

$$Y_{i,t} = \exp(a_t) N_{i,t}, \; \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,$$

  – Calvo price frictions

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ P_{i,t} & \text{with probability } \theta \end{cases},$$

Marginal Cost

$$\text{real marginal cost} = s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{(1-\nu)W_t/P_t}{\exp(a_t)}$$

$$\overbrace{=}^{\text{household efficiency condition}} \frac{\overbrace{(1-\nu)}^{=\frac{1}{\lambda_f}} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)}$$

- in steady state marginal cost and product of labor equal ('first-best'):

$$s = \frac{1}{\lambda_f} = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)} \rightarrow \frac{C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)} = 1$$

Optimal Monetary Policy

- Properties of (Ramsey-) optimal monetary policy in CGG model when effects of monopoly power are extinguished with an employment subsidy to monopolists:
  - Inflation is zero for all *t* and for all realizations of shocks.
  - Allocations coincide with allocations in first-best ('natural') equilibrium.
  - Proof: see, among other places, http://faculty.wcas.northwestern.edu/~lchrist/course/optimalpolicyhandout.pdf

First Best Allocations

- Maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \; \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}, \; \varepsilon_t^{\tau} \sim iid,$$

- Subject to $C_t = \exp(a_t) N_t$
- Intratemporal first order condition:

$$\frac{\text{marginal utility of leisure}}{\text{marginal utility of consumption}} = C_t \exp(\tau_t) N_t^{\varphi} = \exp(a_t) = \text{marginal product of labor}$$

  – natural employment and consumption:

$$\log(N_t^*) = -\frac{\tau_t}{1+\varphi}, \; \log(C_t^*) = a_t - \frac{\tau_t}{1+\varphi}$$

Natural Rate of Interest

- Given natural consumption, intertemporal Euler equation defines natural rate of interest

$$1 = \beta E_t \frac{u^*_{c,t+1}}{u^*_{c,t}} \frac{R^*_t}{1+\pi^*_{t+1}}$$

- Applying the same log as before:

= Zero

$$c^*_t = -\log(\beta) - [r^*_t - E_t \pi^*_{t+1}] + c^*_{t+1}$$

$$c^*_t = \log(C^*_t),\ r^*_t = \log R_t,\ \pi^*_t = 0$$

- The natural rate:

$$\boxed{r^*_t = -\log(\beta) + E_t[c^*_{t+1} - c^*_t]}$$

Key Features of First-Best

- Employment does not respond to technology
  - Improvement in technology raises marginal product of labor and marginal cost of labor by same amount.
- First best consumption not a function of intertemporal considerations
  - Discount rate irrelevant.
  - Anticipated future values of shocks irrelevant.
- Natural rate of interest steers consumption and employment towards their natural levels.

Back to Actual Economy

- Output gap, x_t

$$x_t = c_t - c_t^*$$

- Intertemporal conditions in natural and actual equilibrium:

$$c_t = -\log(\beta) - [r_t - E_t\pi_{t+1}] + E_t c_{t+1}$$
$$c_t^* = -\log(\beta) - r_t^* + E_t c_{t+1}^*$$

- Subtract, to obtain familiar IS equation:

$$\boxed{x_t = E_t x_{t+1} - [r_t - E_t\pi_{t+1} - r_t^*]}$$

Actual Economy

- Marginal cost:

$$s_t = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)}$$

$$\underbrace{\simeq}^{C_t = \exp(a_t) N_t \text{ holds only as a first order approximation}} \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) \left[\frac{C_t}{\exp(a_t)} \right]^{\varphi}}{\exp(a_t)}$$

$$= \frac{1}{\lambda_f} C_t^{1+\varphi} \left[\frac{\exp\left(\frac{\tau_t}{1+\varphi} \right)}{\exp(a_t)} \right]^{(1+\varphi)} = \frac{1}{\lambda_f} \left(\frac{C_t}{C_t^*} \right)^{1+\varphi}$$

- Then,

$$\hat{s}_t \underbrace{\equiv}^{\text{a hat indicates log-deviation from steady state}} \log(s_t \lambda_f) = (1+\varphi)(c_t - c_t^*)$$

$$= (1+\varphi) x_t$$

Actual Economy

- Phillips curve summarizes price setting by intermediate good firms:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\hat{s}_t$$

- or, substituting from previous slide

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}(1+\varphi)x_t$$

Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \text{ (Calvo pricing equation)}$$

$$x_t = -[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} \text{ (intertemporal equation)}$$

$$r_t = \alpha r_{t-1} + (1-\alpha)[\phi_\pi \pi_t + \phi_x x_t] + u_t \text{ (policy rule)}$$

$$r_t^* = E_t(y_{t+1}^* - y_t^*) = E_t\left(\Delta a_{t+1} - \frac{1}{1+\varphi}\Delta \tau_{t+1}\right) \text{ (natural rate)}$$

$$y_t^* = a_t - \frac{1}{1+\varphi}\tau_t \text{ (natural output)}, \qquad x_t = y_t - y_t^* \text{ (output gap)}$$

also, time series representations for shocks listed above

Solving the Model

- Express the equations in matrix form:

$$z_t = \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r_t^* \end{pmatrix}, \; s_t = \begin{pmatrix} \tau_t \\ a_t \\ u_t \end{pmatrix}, \; \varepsilon_t = \begin{pmatrix} \varepsilon_t^\tau \\ \varepsilon_t^a \\ \varepsilon_t^u \end{pmatrix}$$

$$(*)\; \alpha_0 E_t z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 E_t s_{t+1} + \beta_1 s_t = 0$$

$$s_t = P s_{t-1} + \varepsilon_t$$

- Solution: *A* and *B* matrices such that (*) is satisfied and

$$z_t = A z_{t-1} + B s_t$$