Specification, Estimation and Analysis of DSGE Models

Lawrence Christiano

Overview

- A consensus model has emerged as a device for forecasting, analysis, and as a platform for additional analysis of financial frictions and labor markets.

- Use the VAR evidence discussed by Eichenbaum to motivate the basic, platform DSGE model.

- Use of the model to analyze the economics of the zero bound
  - Why does a binding zero bound expose the economy to risk?
  - What can monetary and fiscal policy do to help?

- Practicum

  - Solve and estimate dsge models with Dynare.
  - Basic dynamic properties of dsge models.
  - Some implications for dsge models for monetary policy.
    - Taylor principle – foundation and challenges.

The Baseline Consensus DSGE Model and the Role of VAR Impulse Response Functions in Constructing It.

Lawrence Christiano

Overview

- A consensus has emerged about the rough outlines of a model for the analysis of monetary policy.
  - Consensus influenced heavily by estimated impulse response functions from Structural Vector Autoregression (SVARs)
- Eichenbaum described empirical SVAR results.
- Now, construct the consensus models based on SVAR results.
  - CEE (2005), SW (2007)
  - Christiano, Trabandt and Walentin (CTW handbook of monetary economics chapter, 2010)
- Also, describe additional developments consensus model
  - Labor market
  - Financial frictions: though work on this started long ago, it became urgent with the financial crisis of 2008.

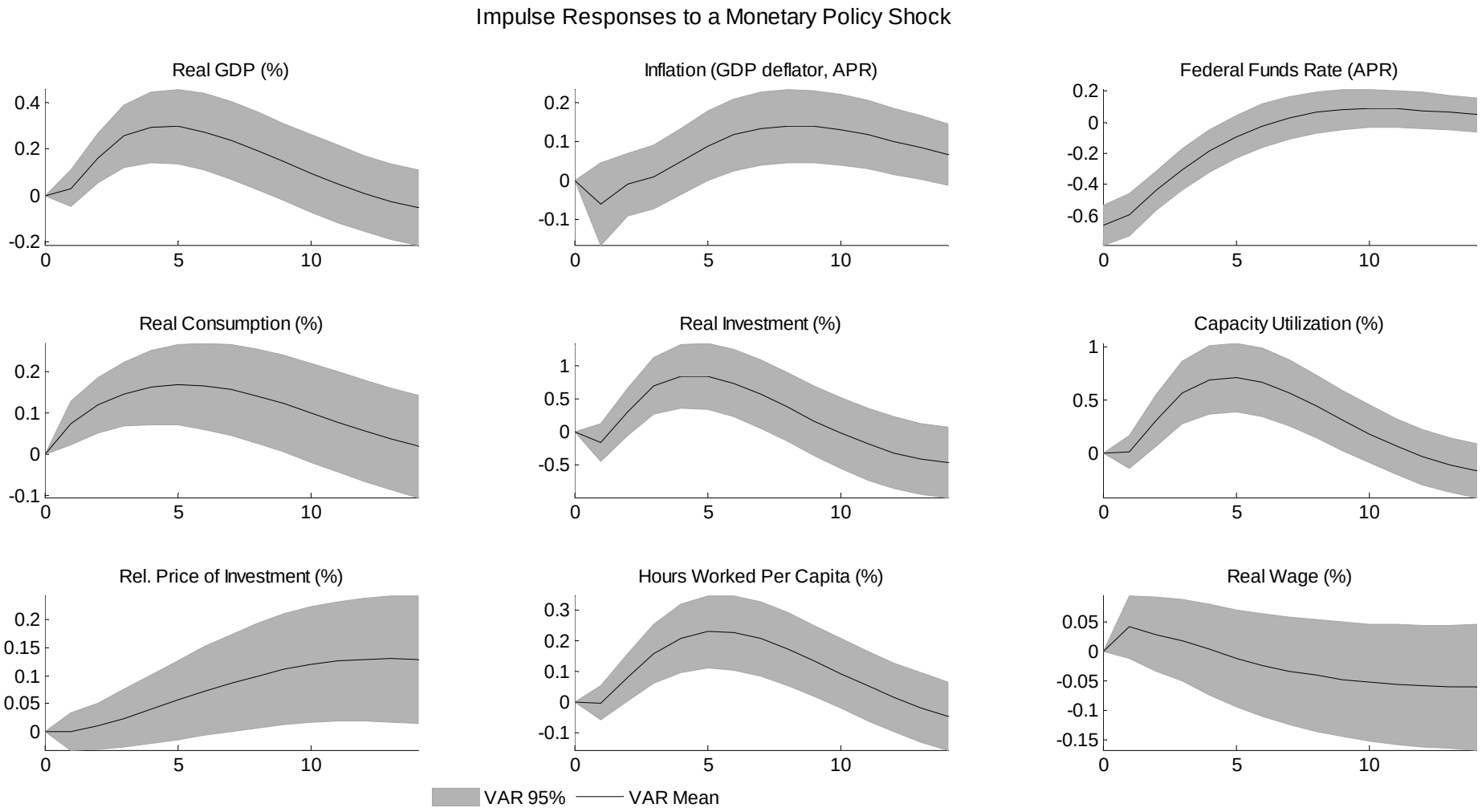
- Very brief review of Marty Eichenbaum’s discussion of SVARs.

Identifying Monetary Policy Shocks

- Rule that relates Fed's actions to state of the economy.

$$R_t = f(\Omega_t) + e_t^R$$

  - f is a linear function
  - Ω_t: set of variables that Fed looks at.
  - e_t^R: time t policy shock, orthogonal to Ω_t


Impulse Responses to a Monetary Policy Shock
Real GDP (%)
Inflation (GDP deflator, APR)
Federal Funds Rate (APR)
Real Consumption (%)
Real Investment (%)
Capacity Utilization (%)
Rel. Price of Investment (%)
Hours Worked Per Capita (%)
Real Wage (%)
VAR 95%
VAR Mean

Interesting Properties of Monetary Policy Shocks

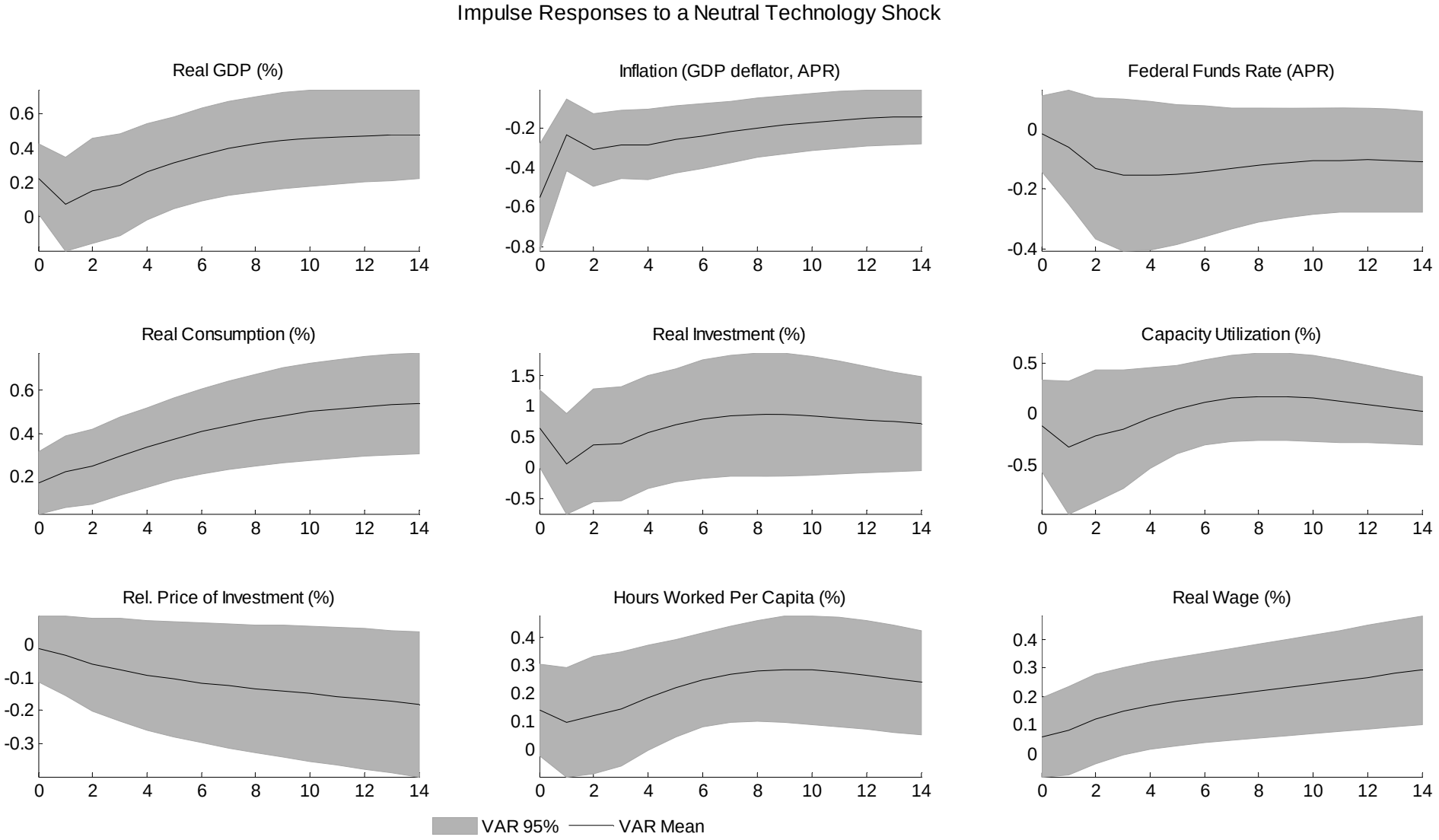
- Plenty of endogenous persistence:
  - money growth and interest rate over in 1 year, but other variables keep going….
- Inflation slow to get off the ground: peaks in roughly two years
  - It has been conjectured that explaining this is a major challenge for economics
  - Chari-Kehoe-McGrattan (*Econometrica*), Mankiw.
  - Kills models in which movements in *P* are key to monetary transmission mechanism (Lucas misperception model, pure sticky wage model)
  - Has been at the heart of the recent emphasis on sticky prices.
- Output, consumption, investment, hours worked and capacity utilization hump-shaped

Identification of Technology Shocks

- Two technology shocks:
  - One perturbs price of investment goods
  - One perturbs total factor productivity

- Identification assumptions:
  - They are the only two shocks that affect labor productivity in the long run

  - Only the shock to investment good prices have an impact on investment good prices in the long run.


Impulse Responses to a Neutral Technology Shock
Real GDP (%)
Inflation (GDP deflator, APR)
Federal Funds Rate (APR)
Real Consumption (%)
Real Investment (%)
Capacity Utilization (%)
Rel. Price of Investment (%)
Hours Worked Per Capita (%)
Real Wage (%)
VAR 95%
VAR Mean

Observations on Neutral Shock

- Generally, results are ‘noisy’, as one expects.
  - Interest, money growth, velocity responses not pinned down.

- Interestingly, inflation response is immediate and *precisely* estimated.
  - Does this raise a question about the conventional interpretation of the response of inflation to a monetary shock?

Importance of Three Shocks

- According to VAR analysis, they account for a large part of economic fluctuations.

Variance Decomposition (ACEL)

Variable	BP(8,32)
Output	86 [18]
Money Growth	23 [11]
Inflation	33 [17]
Fed Funds	52 [16]
Capacity Util.	51 [16]
Avg. Hours	76 [17]
Real Wage	44 [16]
Consumption	89 [21]
Investment	69 [16]
Velocity	29 [16]
Price of investment goods	11 [16]

Next

- Use Impulse Responses to Estimate a DSGE Model
  - Motivate the Basic Model Features.
  - Model Estimation.
- Determine if there is a conflict regarding price behavior between micro and macro data.
  - Macro Evidence:
    - Inflation responds slowly to monetary shock
    - Single equation estimates of slope of Phillips curve produce small slope coefficients.
  - Micro Evidence:
    - Bils-Klenow, Nakamura-Steinsson report evidence on frequency of price change at micro level: 5-11 months.
- Finding: no micro macro puzzle.

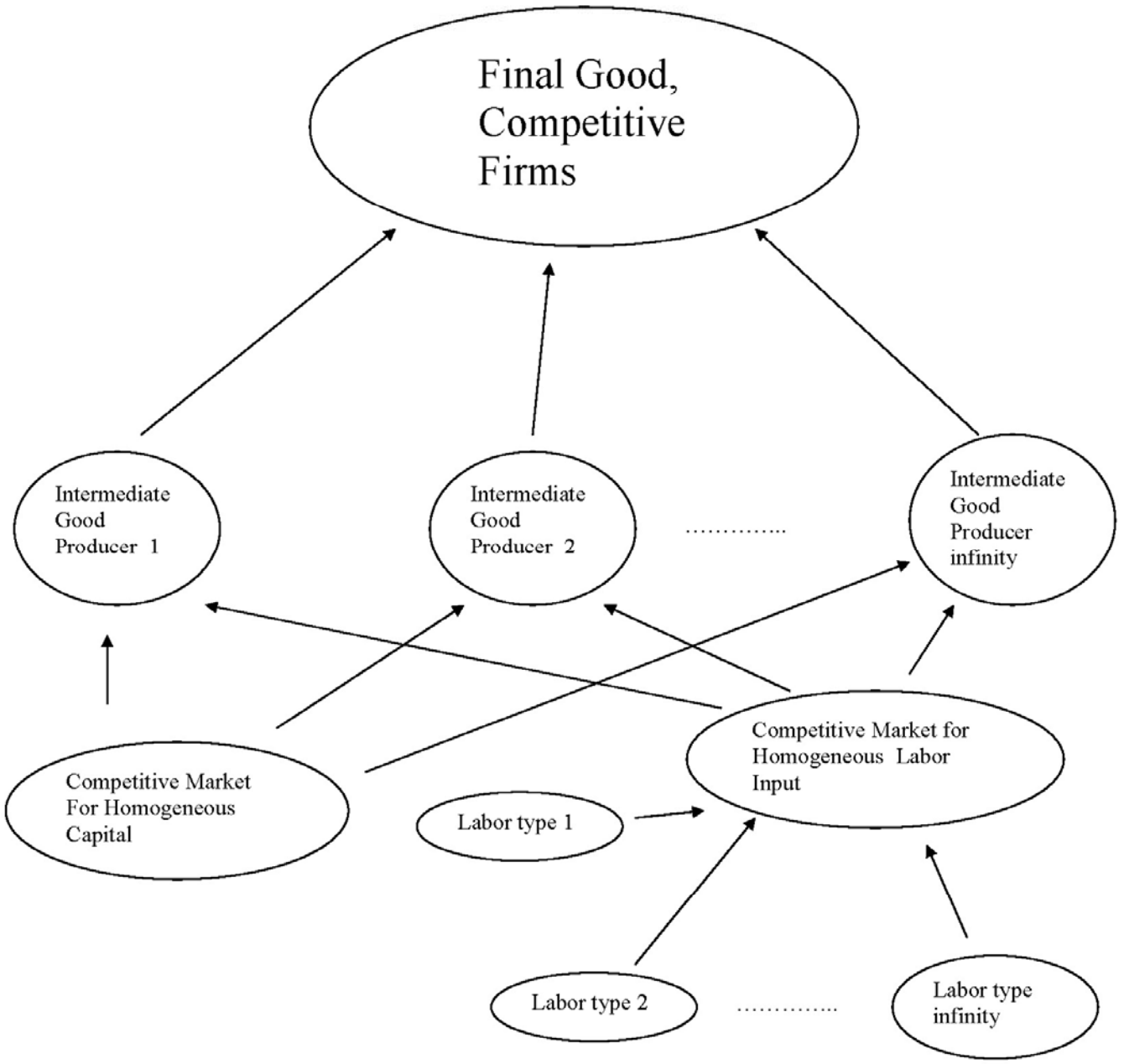
Description of Model

- Timing Assumptions
- Firms
- Households
- Monetary Authority

Timing

- Technology Shocks Realized.
- Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
- Monetary Policy Shock Realized.
- Production, Employment, Purchases Occur, and Markets Clear.
- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firm Sector


Final Good, Competitive Firms
Intermediate Good Producer 1
Intermediate Good Producer 2
.....
Intermediate Good Producer infinity
Competitive Market For Homogeneous Capital
Competitive Market for Homogeneous Labor Input
Labor type 1
Labor type 2
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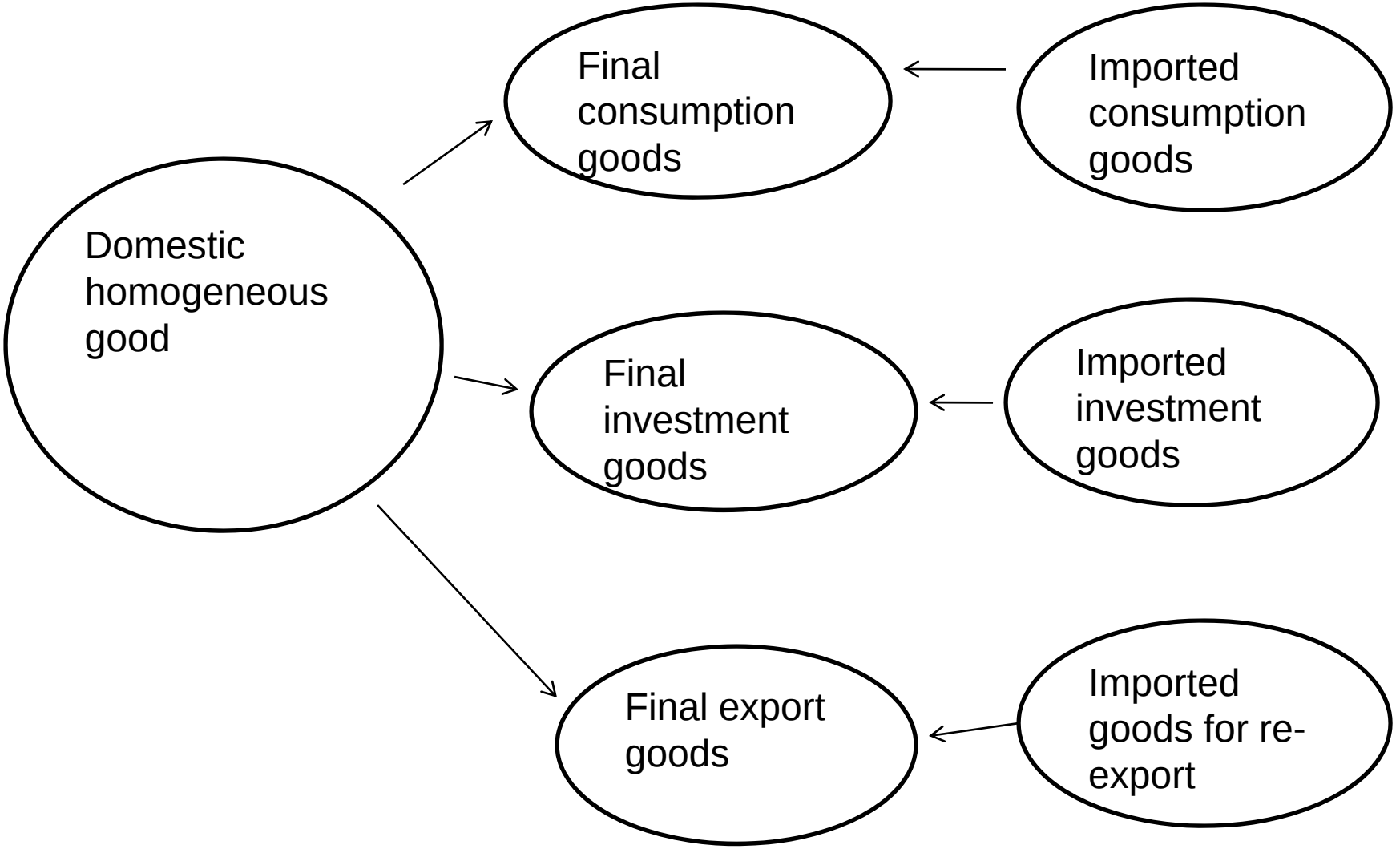
Extension to small open economy (Christiano, Trabandt, Walentin (2009))



Firms

- Final good firms
  - Technology:

$$Y_t = \left[\int_0^1 y_{it}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty$$

  - Objective:

$$\max_{Y_t, \{y_{it}, 0 \leq i \leq 1\}} P_t Y_t - \int_0^1 P_{it} y_{it} di$$

  - Foncs and prices:

$$\left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{y_{it}}{Y_t}, \; P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}$$

Firms, cont'd

- Intermediate good firms
  - Each y_{it} produced by a monopolist with demand curve:

$$y_{it} = \left(\frac{P_t}{P_{it}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t$$

  - Technology:

$$y_{it} = K_{it}^{\alpha}(z_t L_{it})^{1-\alpha},\ 0 < \alpha < 1$$

  - Random walk technology shock

$$\Delta \log z_t = \mu_z + \varepsilon_t^z,\ E(\varepsilon_t^z)^2 = \sigma_z^2$$

    - consistent with identifying assumption on technology.
    - consistent with time series properties of Fernald's direct measure of TFP (see CTW handbook chapter).

Firms, cnt'd

Nominal wage

Real rental rate of capital services

- Intermediate good firm marginal cost

$$MC\$ = [\psi + (1-\psi)R_t]\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_t r_t^k}{\alpha}\right)^{\alpha}\frac{1}{z_t^{1-\alpha}}$$

Fraction of wage and capital rental bill that must be borrowed in advance at gross nominal rate of interest, *R*

ψ < 1 creates 'working capital channel' for the interest rate, *R,* on the supply side of the economy.

Helps keep prices from rising after monetary injection (actually, may even help explain the 'price puzzle').

Firms, cnt'd

- Intermediate good firm marginal cost

$$MC\$ = [\psi + (1-\psi)R_t]\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_t r_t^k}{\alpha}\right)^{\alpha}\frac{1}{z_t^{1-\alpha}}$$

- Marginal cost divided by final good price:

$$s_t \equiv \frac{MC\$}{P_t} = [\psi + (1-\psi)R_t]\left(\frac{W_t/P_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\frac{1}{z_t^{1-\alpha}}$$

Calvo price frictions in intermediate good firms

- With probability, $1 - \xi_p$, firms may optimize price:

$$P_{it} = \tilde{P}_t$$

- With probability, ξ_p ,

Steady state inflation

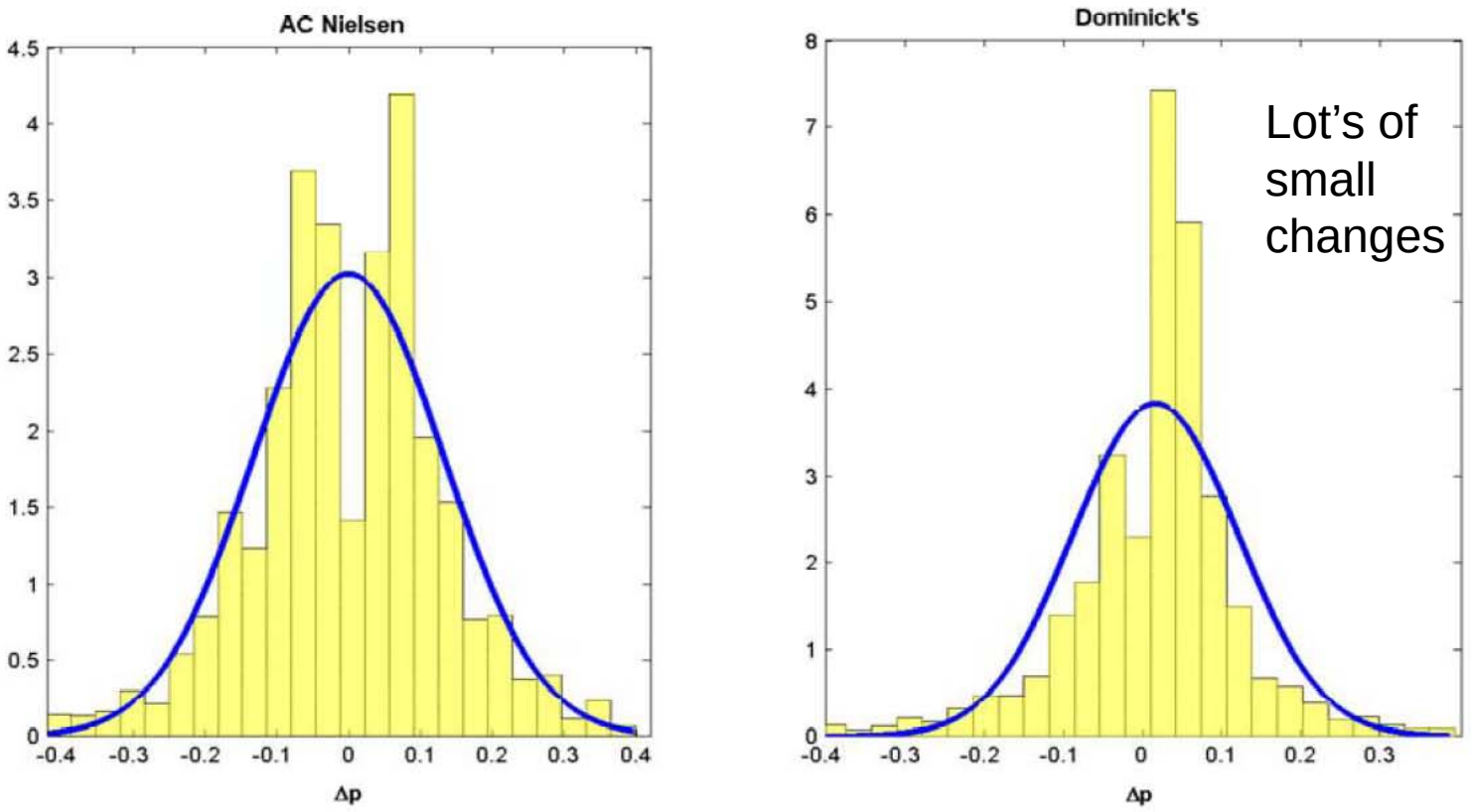
$$P_{i,t} = \pi P_{i,t-1}$$

- Alternative is that with probability ξ_p ,

$$P_{it} = P_{i,t-1}$$

Evidence from Midrigan, ‘Menu Costs, Multi-Product Firms, and Aggregate Fluctuations’

Figure 1: Distribution of price changes conditional on adjustment



Note: superimposed is the pdf of a Gaussian distribution with equal mean and variance

Histograms of $\log(P_t/P_{t-1})$, conditional on price adjustment, for two data sets pooled across all goods/stores/months in sample.

- Linearized equilibrium condition on inflation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} E_t \hat{s}_t$$

Households: Sequence of Events

- Technology shock realized.
- Decisions: Consumption, Capital accumulation, Capital Utilization.
- Wage rate set.
- Monetary policy shock realized.

Households

- Representative household solves:

number of workers of type *j* (parameter, ϕ, not Frisch elasticity)

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - bC_{t-1}) - A \int_0^1 \frac{h_{t,j}^{1+\phi}}{1+\phi} dj \right]$$

capital purchased from other households

$$P_t\left(C_t + \frac{1}{\Psi_t} I_t\right) + B_{t+1} + P_t P_{k',t} \Delta_t \leq \int_0^1 W_{t,j} h_{t,j} dj + X_t^k \bar{K}_t + R_{t-1} B_t$$

investment technology shock

real price of capital

physical capital

$$X_t^k = u_t P_t r_t^k - \frac{P_t}{\Psi_t} a(u_t).$$

capital utilization

Household and Labor Market Erceg-Henderson-Levin Model

- Each type of labor, *j*, in the household joins a union of all *j*-type labor from all other households.

- The union for *j*-type labor behaves as a monopolist on behalf of its members, setting the wage $W_{j,t}$ subject to a demand curve for *j*-type labor.

- With probability $1 - \xi_w$ the union may reoptimize the wage and with probability ξ_w it may not:

$$W_{j,t} = \pi_{t-1} \mu_{z^+} W_{j,t-1}$$

Labor market, cnt'd

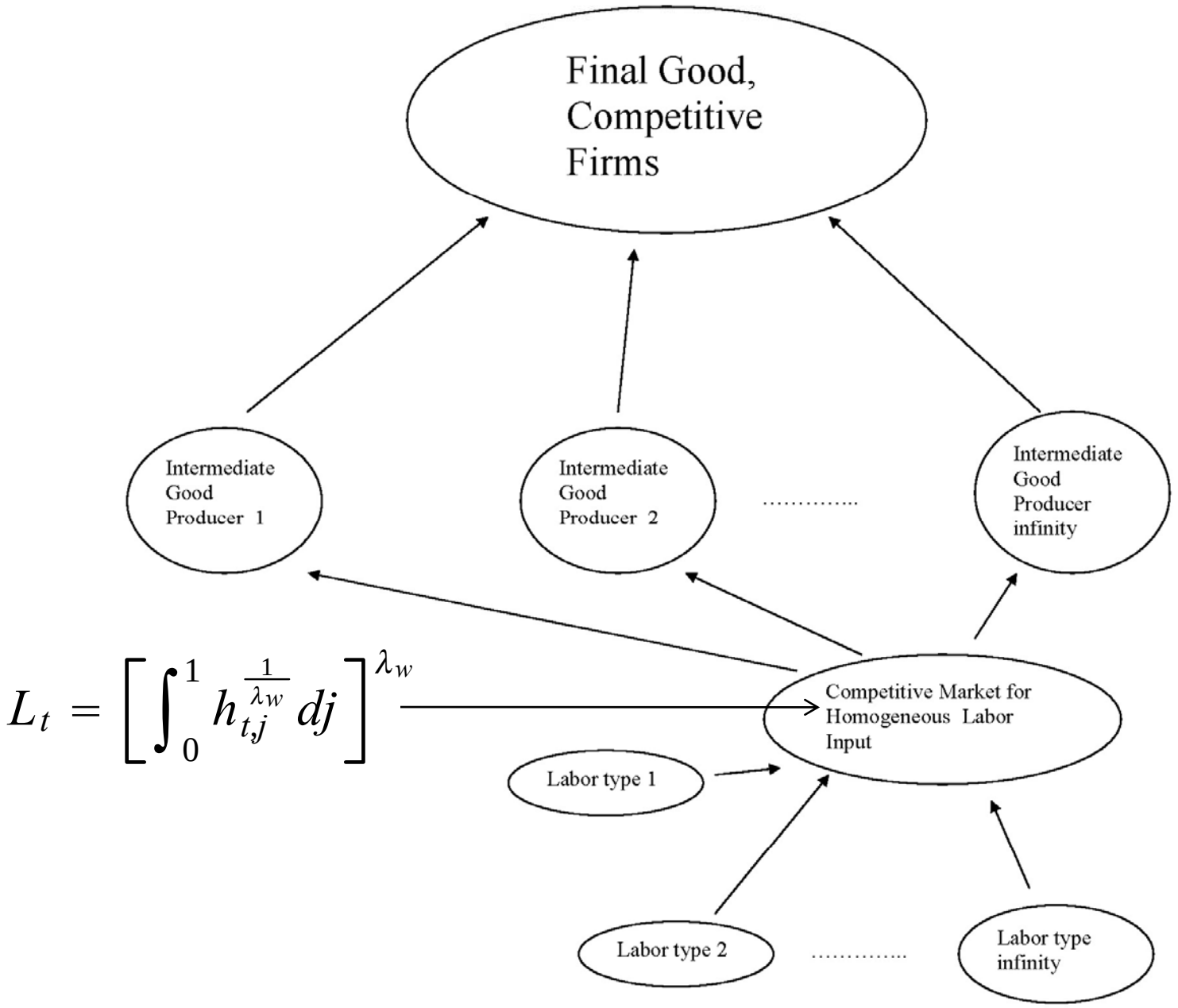
- Given the specified wage, *j*-type workers must supply whatever quantity of labor is demanded.

- Labor is demanded by competitive 'labor contractors', who aggregate different labor services into a homogeneous labor input that they rent to intermediate good producers.

- Labor contractors use the following technology:

$$L_t = \left[\int_0^1 h_{t,j}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty.$$

Firm Sector


Final Good, Competitive Firms
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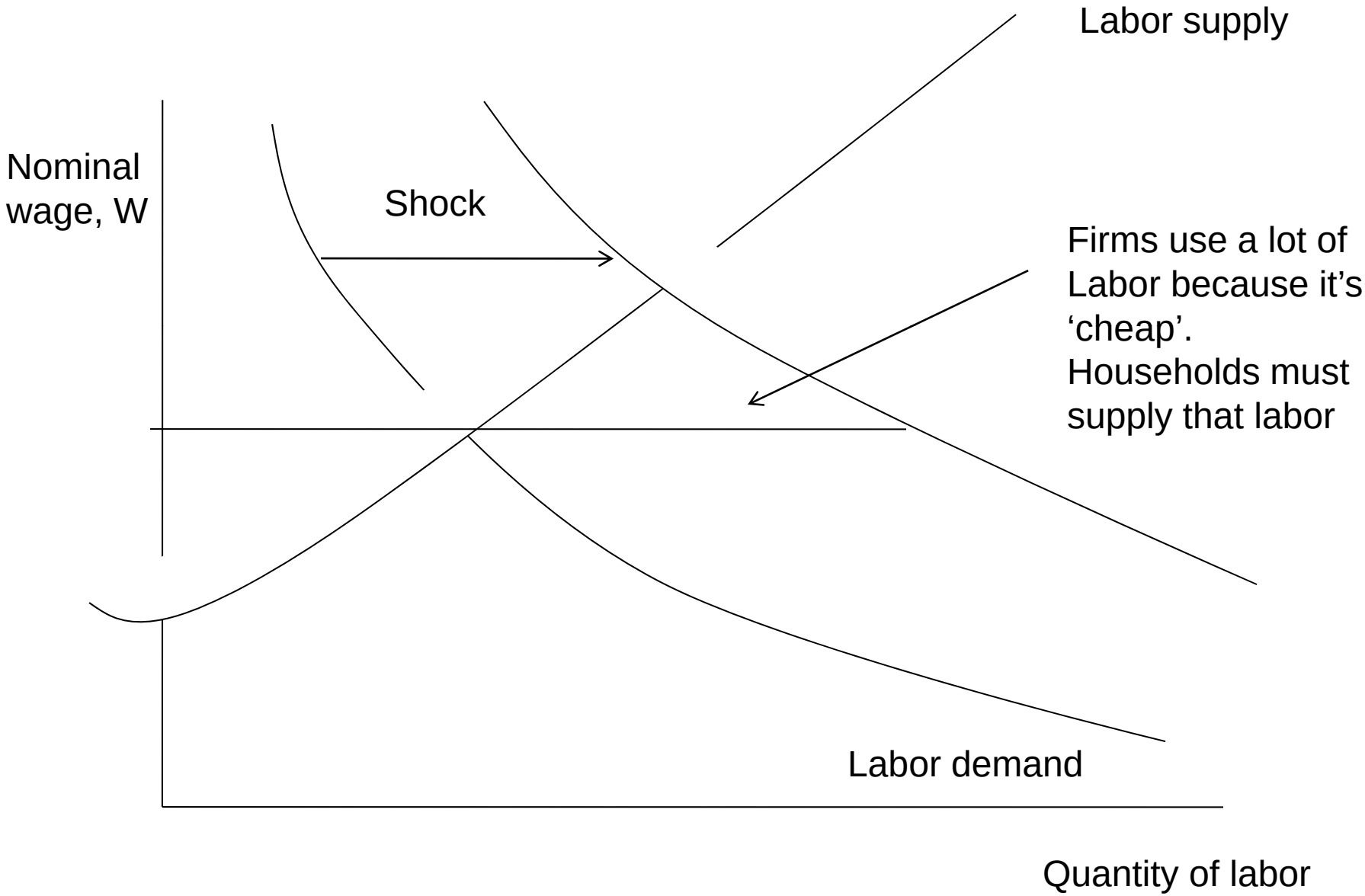
$L_t = \left[\int_0^1 h_{t,j}^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}$
Competitive Market for Homogeneous Labor Input
Labor type 1
Labor type 2
.....
Labor type infinity

What's the point of the wage setting frictions?

- They help the model account for the response of inflation and output to a monetary policy shock.
  - Sticky wage in effect makes labor supply highly elastic.
  - Positive monetary policy shock leads to:
    - Big increase in employment and output.
    - Small increase in cost and, hence, inflation.


Labor supply
Nominal wage, W
Shock
Firms use a lot of Labor because it's 'cheap'. Households must supply that labor
Labor demand
Quantity of labor

Extensions of Labor Market

- Supply of labor:
  - Theory of unemployment implicit in the EHL model of monopoly power (Gali (2010)).
  - Household search model (Christiano, Trabandt and Walentin (2010)).

- Demand for labor:
  - Gertler-Trigari, Gertler-Sala-Trigari have shown how to replace the above approach to the labor market with a Mortensen-Pissarides-style search and matching approach (also, Thomas).
  - see Christiano-Ilut-Motto-Rostagno and Christiano-Trabandt-Walentin for empirical applications to closed and small open economies.

Why Habit Persistence in Preferences?

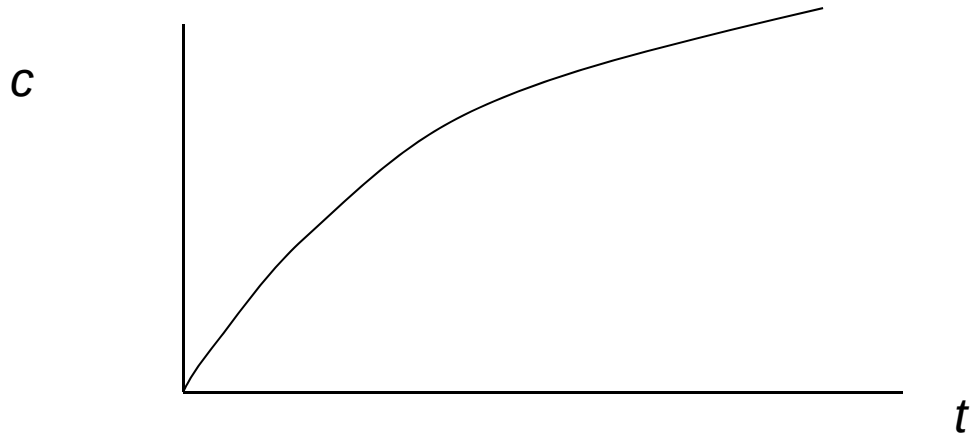
- They help resolve the ‘consumption puzzle’ in monetary economics…..
- With standard preferences, hard to understand the way consumption responds to monetary policy shock.

Consumption ‘Puzzle’

- In Estimated Impulse Responses:
  - Real Interest Rate Falls

$$R_t/\pi_{t+1}$$

  - Consumption Rises in Hump-Shape Pattern:



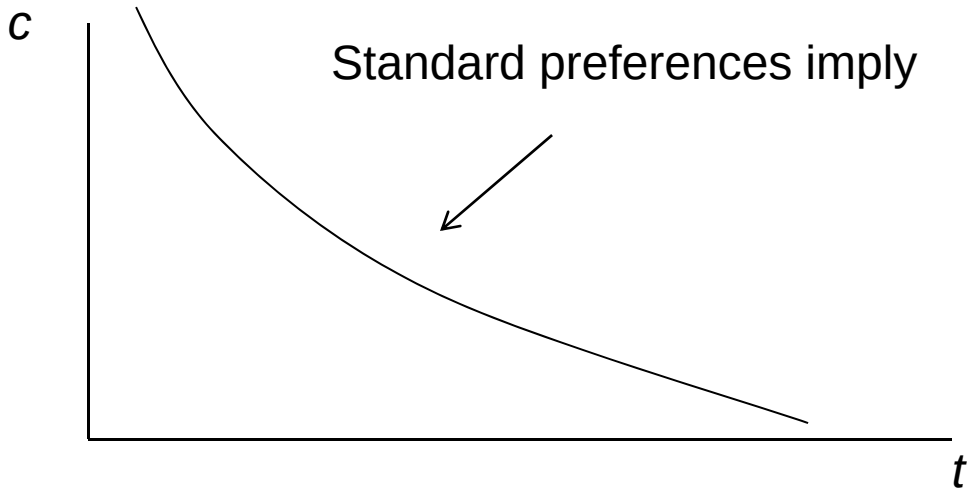
- Standard preferences inconsistent with above

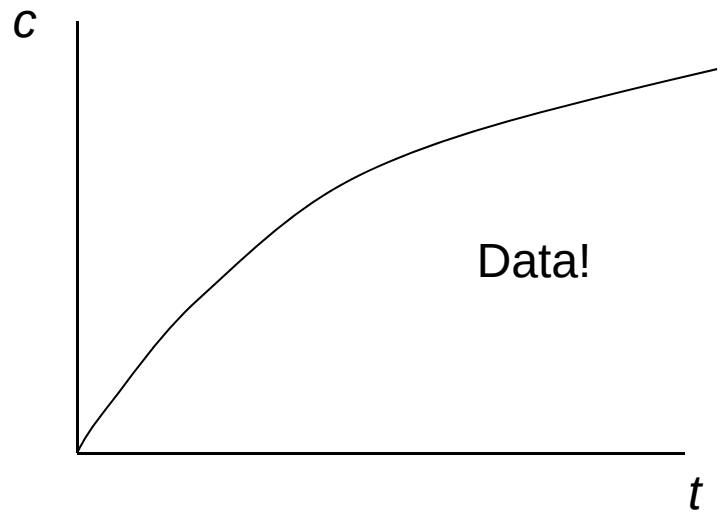
Consumption ‘Puzzle’

- Intertemporal First Order Condition:

‘Standard’ Preferences

$$\frac{c_{t+1}}{\beta c_t} = \frac{MU_{c,t}}{\beta MU_{c,t+1}} \approx R_t/\pi_{t+1}$$





A Solution to the Consumption Puzzle

- Concave Consumption Response Displays:
  - Rising Consumption (problem)
  - Falling Slope of Consumption

Habit parameter

- Habit Persistence in Consumption

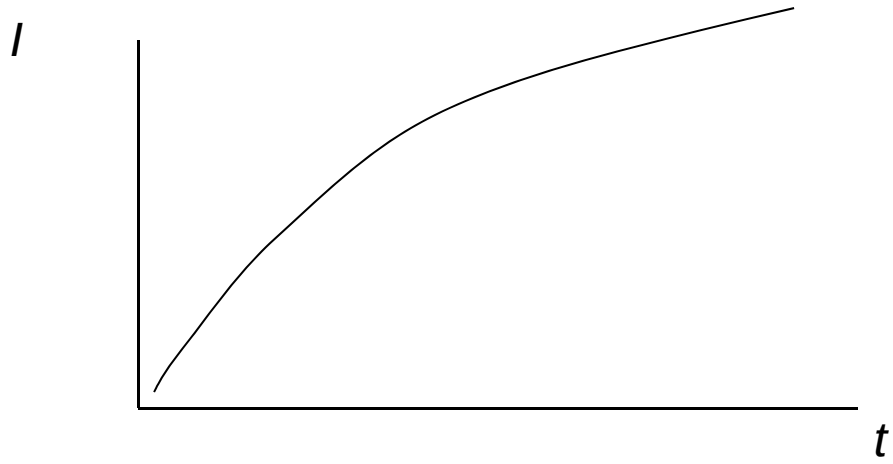
$$U(c) = \log(c - b \times c_{-1})$$

  - Marginal Utility Function of *Slope* of Consumption
  - Hump-Shape Consumption Response Not a Puzzle

- Econometric Estimation Strategy Given the Option, *b>0*

Dynamic Response of Investment to Monetary Policy Shock

- In Estimated Impulse Responses:
  - Investment Rises in Hump-Shaped Pattern:



Investment 'Puzzle'

- Rate of Return on Capital

$$R_t^k = \frac{MP_{t+1}^k + P_{k',t+1}(1-\delta)}{P_{k',t}},$$

$P_{k',t}$ ~ consumption price of installed capital

MP_t^k ~marginal product of capital

$\delta \in (0,1)$ ~depreciation rate.

- Rough 'Arbitrage' Condition:

$$\frac{R_t}{\pi_{t+1}} \approx R_t^k.$$

- Positive Money Shock Drives Real Rate:

$$R_t^k \downarrow$$

- Problem: Burst of Investment!

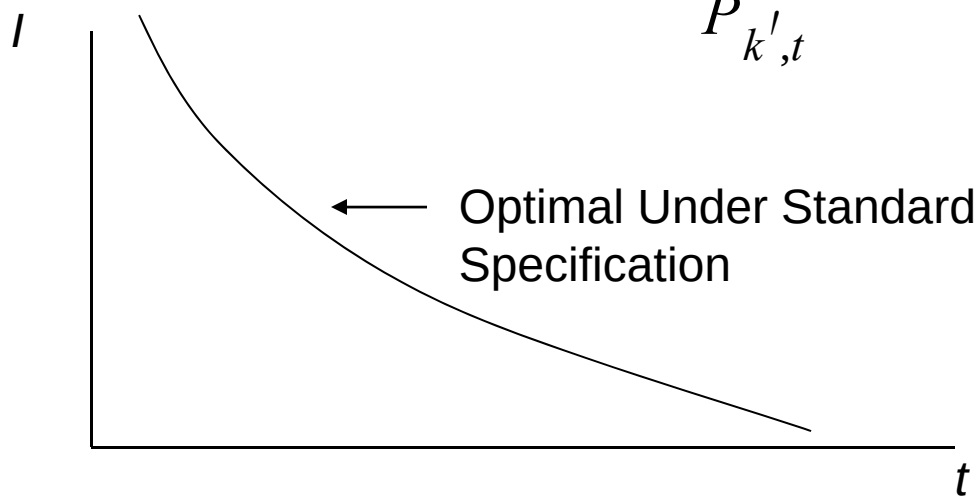
Investment Puzzle: a failed approach

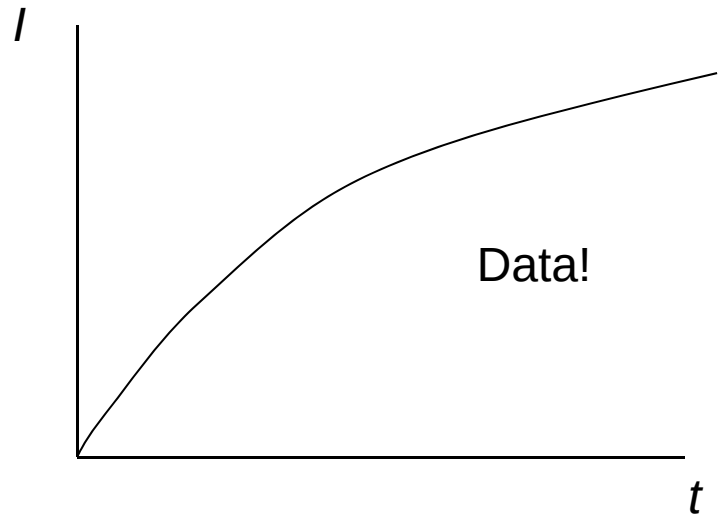
- Adjustment Costs in Investment
  - Standard Model (Lucas-Prescott)

$$k' = (1-\delta)k + F(\frac{I}{k})I.$$

  - Problem:
    - Hump-Shape Response Creates Anticipated Capital Gains

$$\frac{P_{k',t+1}}{P_{k',t}} > 1$$





A Solution to the Investment Puzzle

- Cost-of-Change Adjustment Costs:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + F(I_t, I_{t-1}) + \Delta_t$$

- This Does Produce a Hump-Shape Investment Response
  - Other Evidence Favors This Specification
  - Empirical: Matsuyama, Sherwin Rosen
  - Theoretical: Matsuyama, David Lucca

Monetary Policy

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)[r_\pi \log\left(\frac{\pi_{t+1}}{\pi}\right) + r_y \log\left(\frac{gdp_t}{gdp}\right)] + \varepsilon_{R,t}$$

$$gdp_t = \frac{G_t + C_t + I_t/\Psi_t}{z_t^+}$$

$$G_t = gz_t^+$$

Estimation

- Fixed some parameters a priori

$$\alpha, \delta, \beta, \pi, \eta_g, \lambda_w, \xi_w, \mu_z, \mu_\Psi$$

Estimation

- Fixed some parameters a priori

$$\alpha, \delta, \beta, \pi, \eta_g, \lambda_w, \xi_w, \mu_z, \mu_\Psi$$

- Econometric inference on following parameters:

$$\theta = \left(\begin{array}{cccccccccccc} \xi_p & \lambda_f & \rho_R & r_\pi & r_y & b & \phi & \sigma_a & S'' & \sigma_z & \rho_\Psi & \sigma_\Psi & \sigma_R \end{array} \right)'$$

Econometric Methodology

- Bayesian variant of impulse response matching in CEE, Rotemberg and Woodford

- Estimate impulse responses from VAR
  - Loaded into 397 by 1 vector, $\hat{\psi}$
  - 3 shocks times 9 variables times 15 responses minus 8 contemporaneous effects.

- Asymptotic theory:

true values of model parameters

$$\hat{\psi} \overset{a}{\sim} N(\psi(\theta_0), V(\theta_0, \zeta_0, T))$$

$$V(\theta_0, \zeta_0, T) \equiv \frac{W(\theta_0, \zeta_0)}{T}$$

Parameters of non-modeled shocks

Econometric Methodology

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- Estimate impulse responses from VAR
  - Loaded into 397 by 1 vector, $\hat{\psi}$
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$$\hat{\psi} \overset{a}{\sim} N(\psi(\theta_0), V(\theta_0, \zeta_0, T))$$

$$V(\theta_0, \zeta_0, T) \equiv \frac{W(\theta_0, \zeta_0)}{T}$$

Can estimate consistently

Econometric Methodology

- (Approximate) likelihood, $f(\hat{\psi}|\theta)$, of data, $\hat{\psi}$ as a function of parameters, θ :

$$f(\hat{\psi}|\theta) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V(\theta_0, \zeta_0, T)|^{-\frac{1}{2}}$$
$$\times \exp\left[-\frac{1}{2}(\hat{\psi} - \psi(\theta))' V(\theta_0, \zeta_0, T)^{-1} (\hat{\psi} - \psi(\theta))\right].$$

treated as known

dsge model's implication for impulse responses, given model parameters

Econometric Methodology

- Bayes' rule:

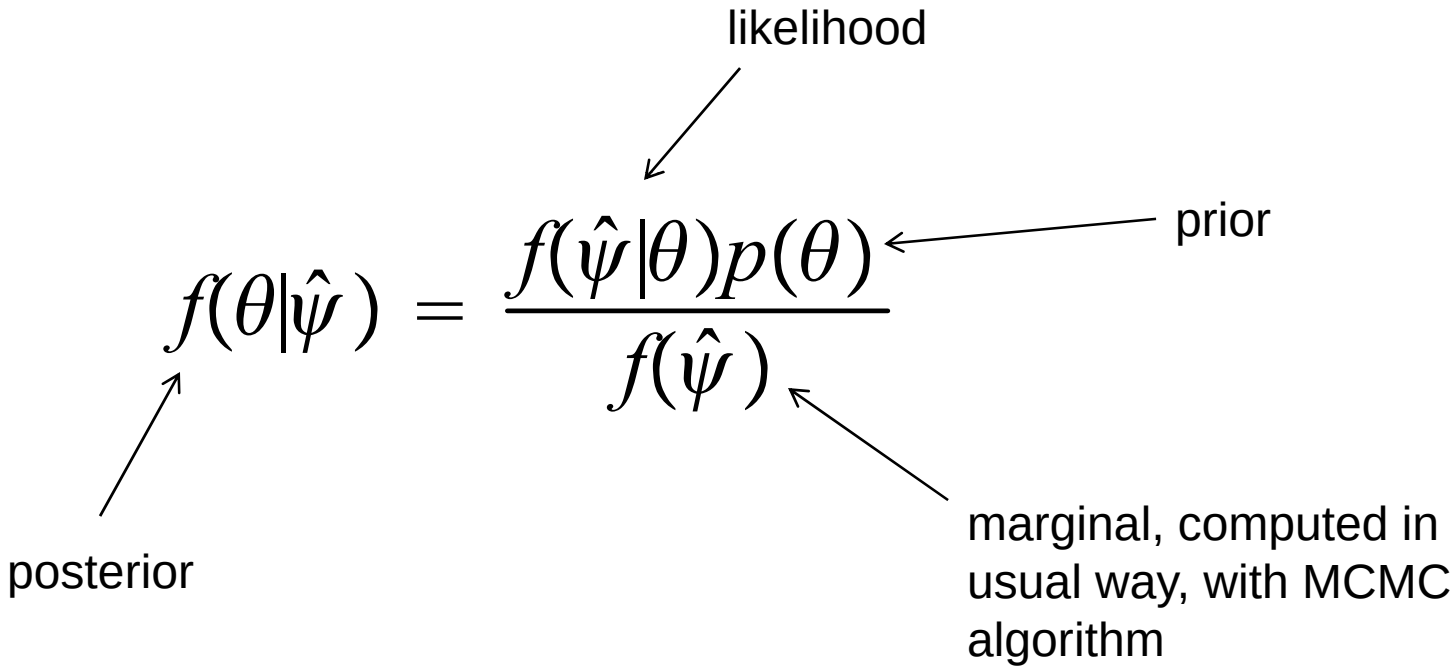
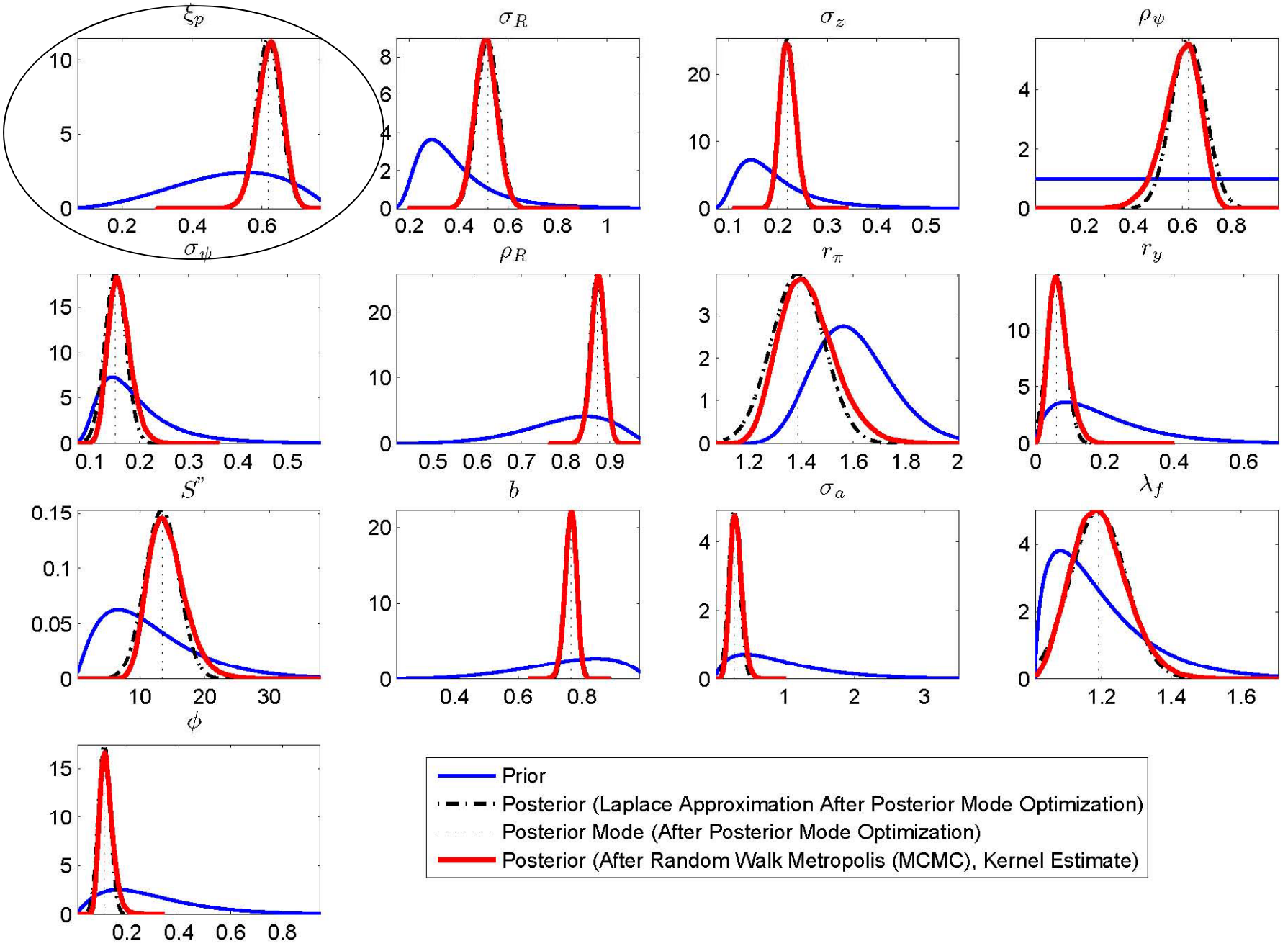


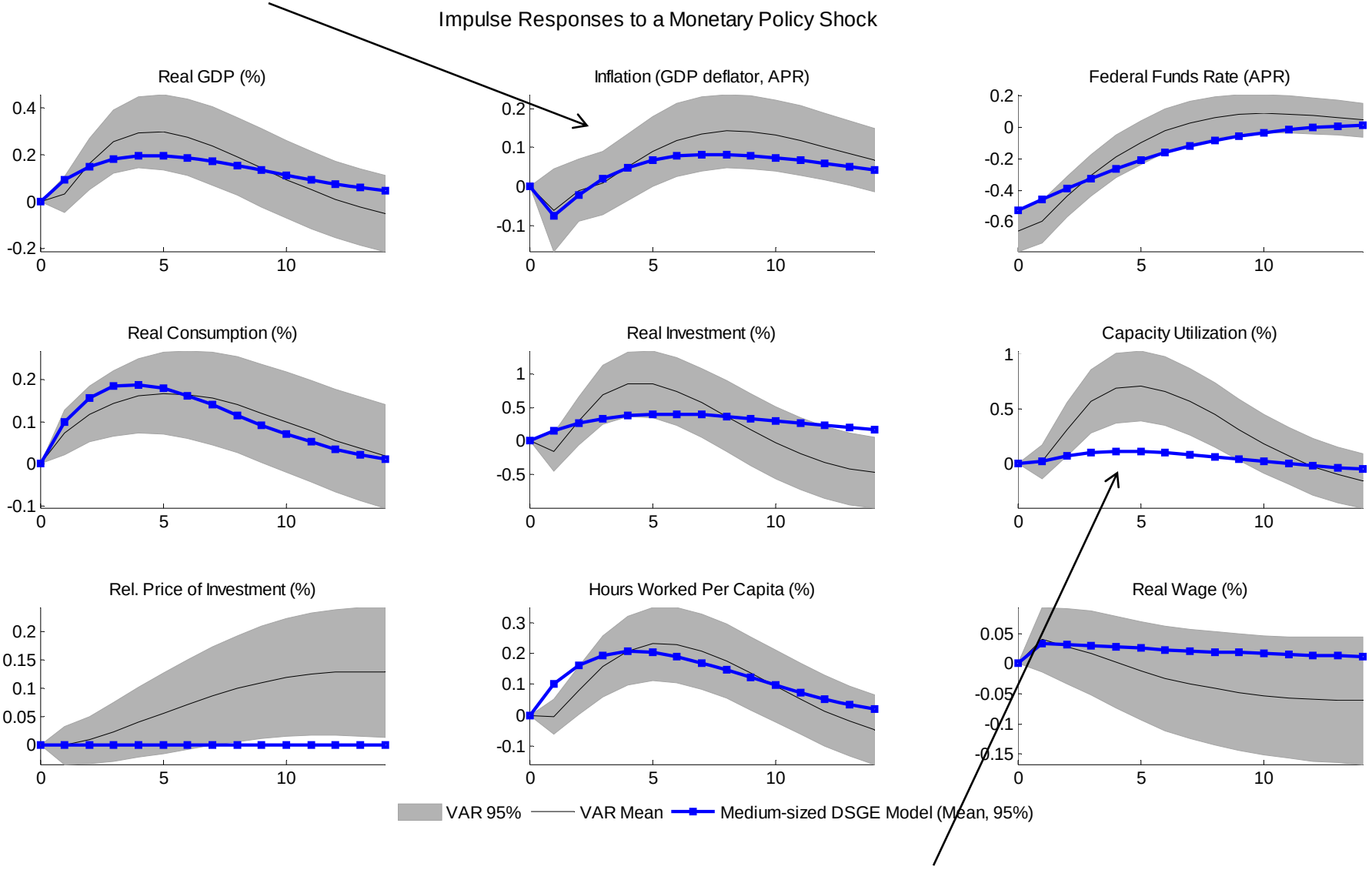
Figure 16: Priors and Posteriors of Estimated Parameters of the Medium−Sized DSGE Model



- How well does the estimated model match the VAR-based impulse responses?

- Is there a macro-micro puzzle?

Inflation response no problem – micro/macro puzzle resolved!



Did not make much use of variable capital utilization

No problem with the big drop in inflation

Figure 4: Dynamic Responses of Variables to a Neutral Technology Shock

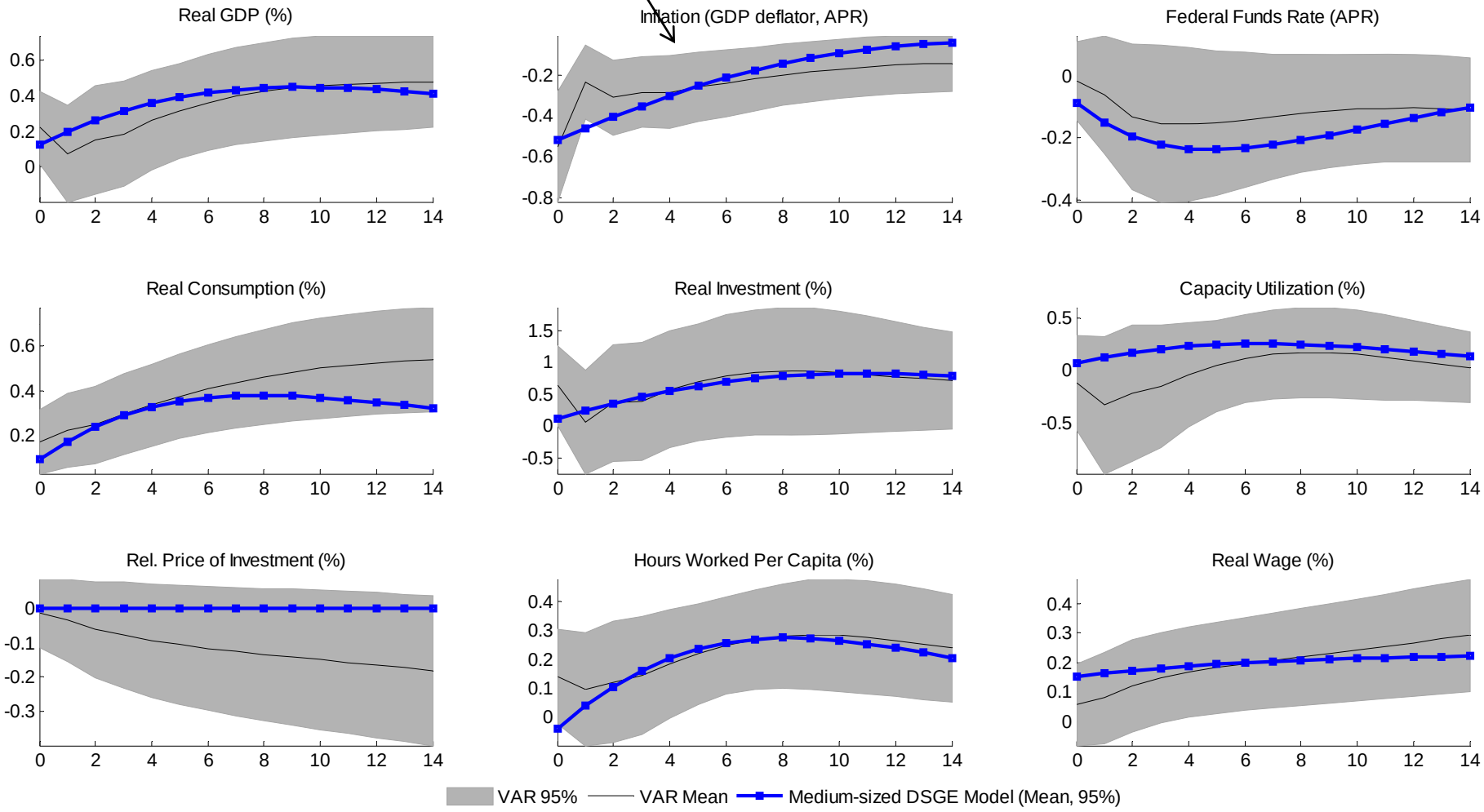
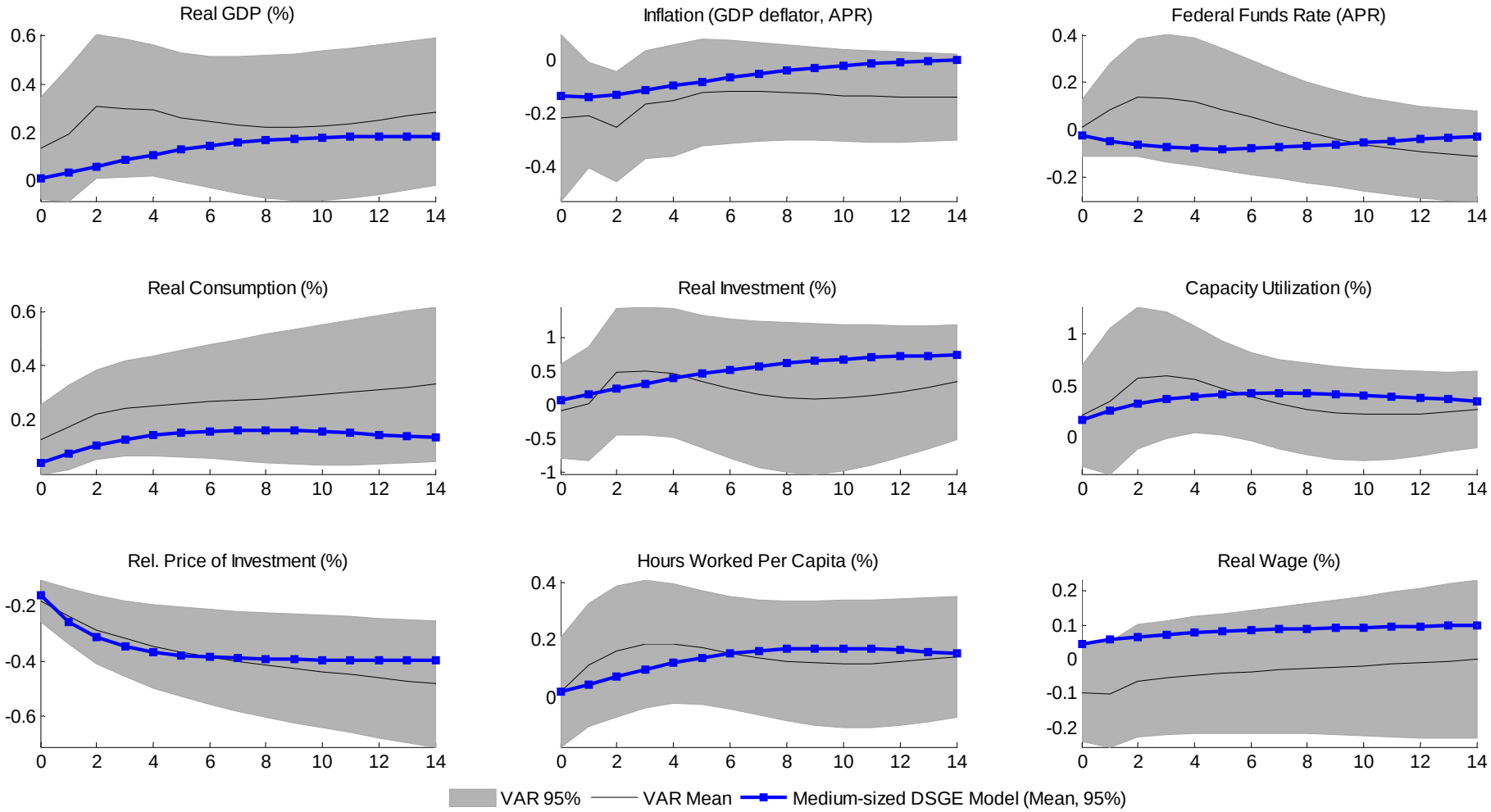


Figure 5: Dynamic Responses of Variables to an Investment Specific Technology Shock



Conclusion

- Simple model with various frictions is capable of accounting well for key features of economic responses to monetary and technology shocks.
- No evidence of a macro/micro puzzle.
- Model is a platform on which to build financial/labor market frictions.