DSGE Models for Monetary Policy

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Outline

- Basic New Keynesian model without capital
  - Two fundamental frictions in the model.
- Implications for monetary policy:
  - Clarifying the concepts of 'excess and inadequate aggregate demand'.
  - The Taylor principle and inflation targeting.
  - Cases where 'overzealous inflation targeting' can go awry:
    - News shocks and the relationship between monetary policy and stock market volatility
    - The working capital channel and the Taylor principle.

Outline, cnt’d

- Adjustments to NK model to enable it to account for empirical estimates of the monetary transmission mechanism (the ‘David Hume puzzle’).

- Costly State Verification model of financial frictions: microeconomics.

- Introducing csv model into otherwise standard NK DSGE model
  - The importance of a ‘risk shock’ in economic fluctuations.
  - Using the model to assess a monetary policy that reacts to the interest rate premium.

Outline of Discussion of NK Model

- Private economy.
  - Agents, technology, two sources of inefficiency.

- Ramsey optimal policy: ‘natural equilibrium’.

- Monetary policy.

- Solving the model.

- Impulse Response Functions.

- Private economy……

Clarida-Gali-Gertler Model

- Households maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \; \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}, \; \varepsilon_t^{\tau} \sim iid$$

- Subject to:

$$P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + T_t$$

- Intratemporal first order condition:

$$C_t \exp(\tau_t) N_t^{\varphi} = \frac{W_t}{P_t}$$

Household Intertemporal FONC

- Condition:

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{1+\pi_{t+1}}$$

 – or

$$\begin{aligned} 1 &= \beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{1+\pi_{t+1}} \\ &= \beta E_t \exp[\log(R_t) - \log(1+\pi_{t+1}) - \Delta c_{t+1}] \\ &\simeq \beta \exp[\log(R_t) - E_t\pi_{t+1} - E_t\Delta c_{t+1}],\ c_t \equiv \log(C_t) \end{aligned}$$

 – take log of both sides:

$$0 = \log(\beta) + r_t - E_t\pi_{t+1} - E_t\Delta c_{t+1},\ r_t = \log(R_t)$$

 – or

$$c_t = -\log(\beta) - [r_t - E_t\pi_{t+1}] + c_{t+1}$$

Firms

- Competitive final good firms:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di,\ \varepsilon > 1,$$

  - First order condition:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\varepsilon}.$$

- Intermediate good producer (monopolist in output, competitive in labor market):

$$Y_{i,t} = \exp(a_t) N_{i,t},\ \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,$$

  - Calvo price frictions

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t} & \text{with probability } \theta \end{cases},$$

Marginal Cost

$$\text{real marginal cost} = s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{(1-\nu)W_t/P_t}{\exp(a_t)}$$

$$\overbrace{=}^{\text{household efficiency condition}} \frac{\overbrace{(1-\nu)}^{=\frac{1}{\lambda_f}} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)}$$

- in steady state marginal cost and product of labor equal ('first-best'):

$$s = \frac{1}{\lambda_f} = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)} \rightarrow \frac{C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)} = 1$$

The Two Sources of Inefficiency in the New Keynesian Model

1. Monopoly power causes firms to restrict output and employment.

2. Allocation of labor among different intermediate goods.
    a) Efficiency requires equal allocation of labor to all activities.
    b) Any deviation from equality results in reduced total output given total employment.
    c) Price setting frictions leads to misallocation of resources among activities, with more going to the low price goods and fewer going to high price goods.

Efficient Allocation of Total Labor

- Suppose total labor, N_t , is given.

- What allocation of N_t among the various activities $0 \leq i \leq 1$ results in the highest level of final output?

- Answer:
  - allocate labor equally across all the activities

$$N_{it} = N_t, \text{ all } i$$

Suppose Labor *Not* Allocated Equally

- There are many ways in which this can happen!

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \ 0 \leq \alpha \leq 1.$$

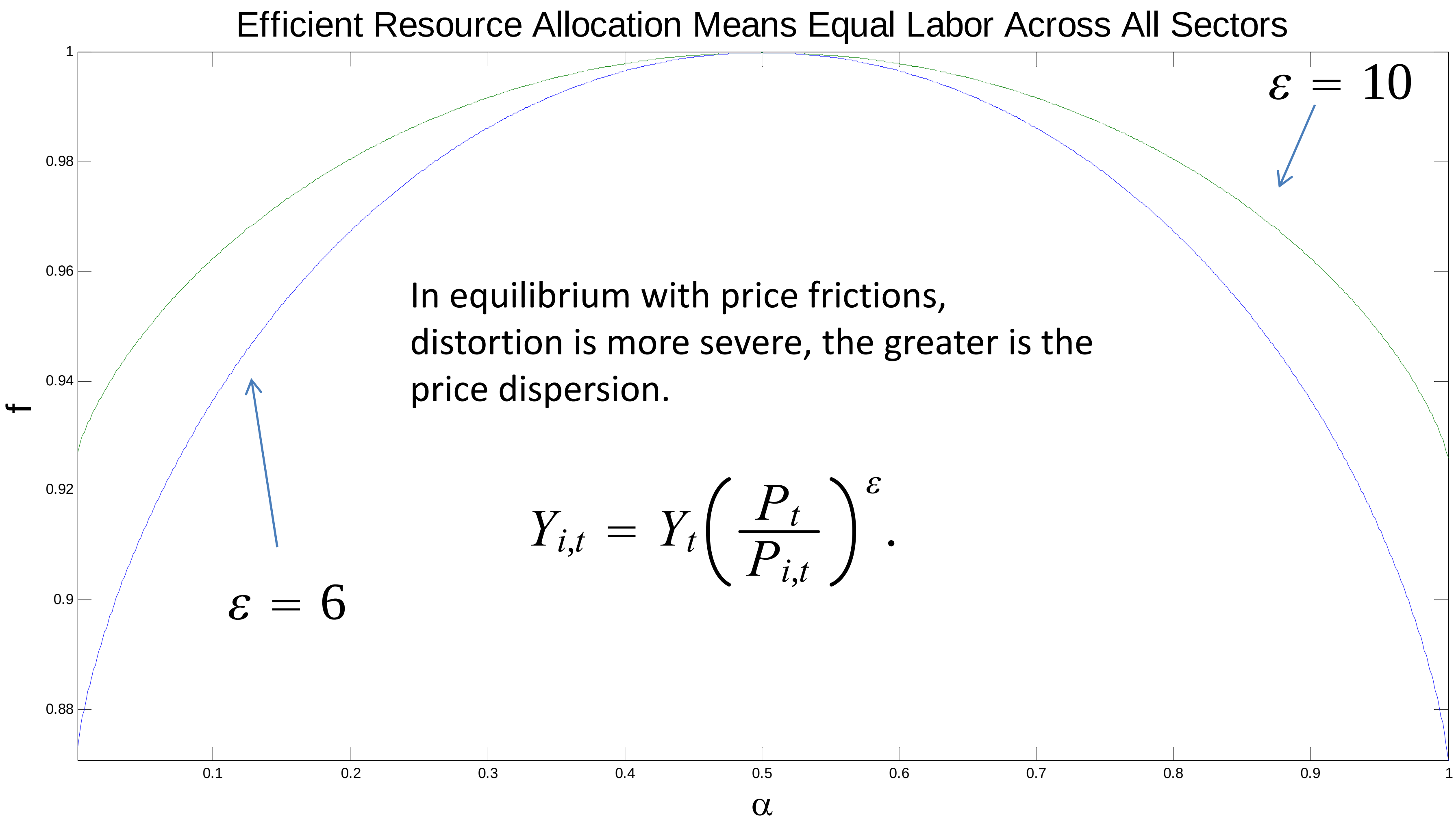
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha)N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}\left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}\left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t\left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t\left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t f(\alpha) \longleftarrow \text{'Tak Yun distortion'}
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



- Ramsey optimal policy: ‘natural equilibrium’.

Optimal Monetary Policy

- Properties of (Ramsey-) optimal monetary policy in CGG model when effects of monopoly power are extinguished with an employment subsidy to monopolists:
  - After a period of transition, inflation is zero for all *t* and for all realizations of shocks.
  - Allocations coincide with allocations in first-best ('natural') equilibrium.
  - Proof: see, among other places, http://faculty.wcas.northwestern.edu/~lchrist/course/optimalpolicyhandout.pdf

First Best Allocations

- Maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \ \varepsilon_t^\tau \sim iid$$

- Subject to $C_t = \exp(a_t) N_t$
- Intratemporal first order condition:

$$\frac{\text{marginal utility of leisure}}{\text{marginal utility of consumption}} = C_t \exp(\tau_t) N_t^\varphi = \exp(a_t) = \text{marginal product of labor}$$

  – natural employment and consumption:

$$\log(N_t^*) = -\tfrac{\tau_t}{1+\varphi}, \ \log(C_t^*) = a_t - \tfrac{\tau_t}{1+\varphi}$$

Natural Rate of Interest

- Given natural consumption, intertemporal Euler equation defines natural rate of interest

$$1 = \beta E_t \frac{u^*_{c,t+1}}{u^*_{c,t}} \frac{R^*_t}{1+\pi^*_{t+1}}$$

- Applying the same log as before:

= Zero

$$c^*_t = -\log(\beta) - [r^*_t - E_t\pi^*_{t+1}] + c^*_{t+1}$$

$$c^*_t = \log(C^*_t),\ r^*_t = \log R_t,\ \pi^*_t = 0$$

- The natural rate:

$$\boxed{r^*_t = -\log(\beta) + E_t[c^*_{t+1} - c^*_t]}$$

Key Features of First-Best

- Employment does not respond to technology
  - Improvement in technology raises marginal product of labor and marginal cost of labor by same amount.
- First best consumption not a function of intertemporal considerations
  - Discount rate irrelevant.
  - Anticipated future values of shocks irrelevant.
- Natural rate of interest steers consumption and employment towards their natural levels.

Back to Actual Economy

- Output gap, x_t

$$x_t = c_t - c_t^*$$

- Intertemporal conditions in natural and actual equilibrium:

$$c_t = -\log(\beta) - [r_t - E_t\pi_{t+1}] + E_t c_{t+1}$$
$$c_t^* = -\log(\beta) - r_t^* + E_t c_{t+1}^*$$

- Subtract, to obtain familiar IS equation:

$$\boxed{x_t = E_t x_{t+1} - [r_t - E_t\pi_{t+1} - r_t^*]}$$

Actual Economy

- Marginal cost:

$$s_t = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)} \overbrace{\simeq}^{C_t = \exp(a_t) N_t \text{ holds only as a first order approximation}} \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) \left[\frac{C_t}{\exp(a_t)} \right]^{\varphi}}{\exp(a_t)}$$

$$= \frac{1}{\lambda_f} C_t^{1+\varphi} \left[\frac{\exp\left(\frac{\tau_t}{1+\varphi} \right)}{\exp(a_t)} \right]^{(1+\varphi)} = \frac{1}{\lambda_f} \left(\frac{C_t}{C_t^*} \right)^{1+\varphi}$$

- Then,

$$\hat{s}_t \overbrace{\equiv}^{\text{a hat indicates log-deviation from steady state}} \log(s_t \lambda_f) = (1+\varphi)(c_t - c_t^*)$$

$$= (1+\varphi) x_t$$

Actual Economy

- Phillips curve summarizes price setting by intermediate good firms:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\hat{s}_t$$

- or, substituting from previous slide

$$\boxed{\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}(1+\varphi)x_t}$$

Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \text{ (Calvo pricing equation)}$$

$$x_t = -[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} \text{ (intertemporal equation)}$$

$$r_t = \alpha r_{t-1} + (1-\alpha)[\phi_\pi \pi_t + \phi_x x_t] + u_t \text{ (policy rule)}$$

$$r_t^* = E_t(y_{t+1}^* - y_t^*) = E_t\left(\Delta a_{t+1} - \frac{1}{1+\varphi}\Delta \tau_{t+1}\right) \text{ (natural rate)}$$

$$y_t^* = a_t - \frac{1}{1+\varphi}\tau_t \text{ (natural output)}, \qquad x_t = y_t - y_t^* \text{ (output gap)}$$

also, time series representations for shocks listed above

Solving the Model

- Express the equations in matrix form:

$$z_t = \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r_t^* \end{pmatrix}, \ s_t = \begin{pmatrix} \tau_t \\ a_t \\ u_t \end{pmatrix}, \ \varepsilon_t = \begin{pmatrix} \varepsilon_t^\tau \\ \varepsilon_t^a \\ \varepsilon_t^u \end{pmatrix}$$

$$(*)\ \alpha_0 E_t z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 E_t s_{t+1} + \beta_1 s_t = 0$$

$$s_t = P s_{t-1} + \varepsilon_t$$

- Solution: *A* and *B* matrices such that (*) is satisfied and

$$z_t = A z_{t-1} + B s_t$$

Solving the model....

- Iterating forward one period on the solution:

$$
\begin{aligned}
z_{t+1} &= Az_t + Bs_{t+1} \\
&= A^2 z_{t-1} + ABs_t + Bs_{t+1}
\end{aligned}
$$

- Then,

Finding solution for A is only hard part.
Choose A that solves problem if there is only one with eigenvalues less than unity in absolute value.
If more than one A like this: multiple solutions.

$$
\begin{aligned}
& E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] \\
&= E_t[\alpha_0 (A^2 z_{t-1} + ABs_t + Bs_{t+1}) + \alpha_1 (Az_{t-1} + Bs_t) + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] \\
&= [\alpha_0 A^2 + \alpha_1 A + \alpha_2] z_{t-1} + E_t[(\alpha_0 B + \beta_0) s_{t+1} + (\alpha_0 AB + \alpha_1 B + \beta_1) s_t]
\end{aligned}
$$

$$
= \overbrace{[\alpha_0 A^2 + \alpha_1 A + \alpha_2]}^{=0} z_{t-1} + \overbrace{[(\alpha_0 B + \beta_0)P + (\alpha_0 AB + \alpha_1 B + \beta_1)]}^{=0} s_t = 0, \text{ all } s_t, z_{t-1}
$$

Impulse Response Function

- Law of motion of endogenous and exogenous variables:

$$z_t = Az_{t-1} + Bs_t$$
$$s_t = Ps_{t-1} + \varepsilon_t$$

- Set one element of ε_0 non-zero and let $\varepsilon_t = 0,\ t > 0$
- Set $s_{-1} = z_{-1} = 0$ and compute

$$s_0, s_1, s_2, \ldots$$
$$z_0, z_1, z_2, \ldots$$

Implications for Policy

- These will be pursued in computer exercises in the afternoon sessions.