Asymmetric Information and Costly State Verification

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General Idea

- Standard dsge model assumes borrowers and lenders are the same people..no conflict of interest.

- Financial friction models suppose borrowers and lenders are different people, with conflicting interests.

- Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Gale-Helwig.

- Later: integrate the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (2003,2009).

Simple Model

- There are entrepreneurs with all different levels of wealth, *N*.
  - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.

- For each value of *N*, there are many entrepreneurs.

- In what follows, we will consider the interaction between entrepreneurs with a specific amount of *N* with competitive banks.

- Later, will consider the whole population of entrepreneurs, with every possible level of *N*.

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1+R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

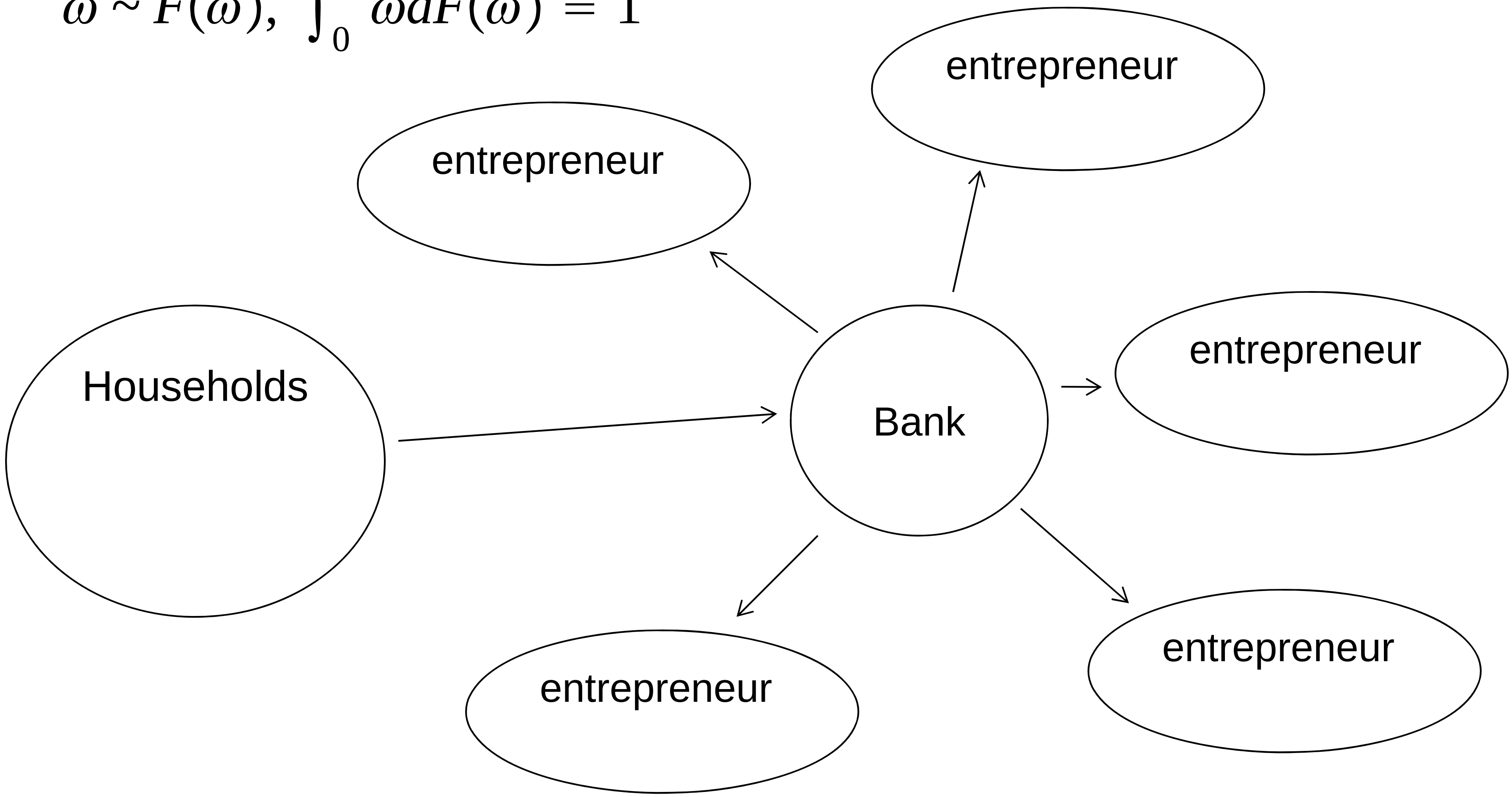
$$\int_0^\infty \omega dF(\omega) = 1$$

- The shock, ω , is privately observed by the entrepreneur.

- *F* is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega),\ \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.
  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

$$\overbrace{(1+R^k)\bar{\omega}}^{\text{gross rate of return experience by entrepreneur with 'luck', } \bar{\omega}} \times \overbrace{A}^{\text{total assets}}$$

$$= \overbrace{ZB}^{\text{interest and principle owed by the entrepreneur}}$$

$$(1+R^k)\bar{\omega}A = ZB \rightarrow$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{\overbrace{\frac{A}{N}}^{\text{leverage} = L} - 1}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

- Cutoff higher with:
  - higher leverage, *L*
  - higher $Z/(1+R^k)$

- Expected return to entrepreneur, over opportunity cost of funds:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^{k})\omega A - ZB]\,dF(\omega)}{N(1+R)}$$

For lower values of ω , entrepreneur receives nothing 'limited liability'.

opportunity cost of funds

- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L}$$

Gets smaller with *L*

Larger with *L*

- Rewriting entrepreneur's rate of return:

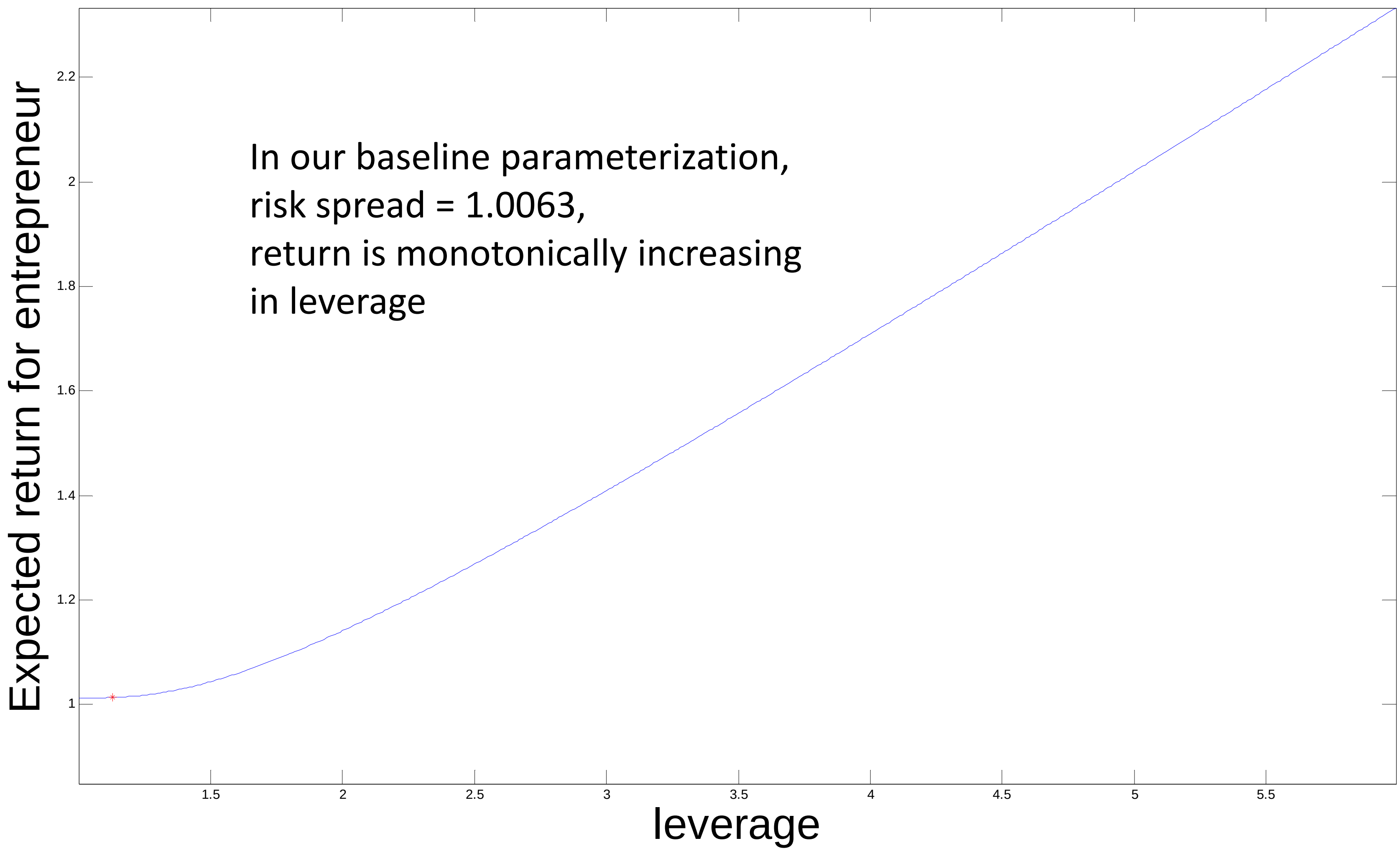
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

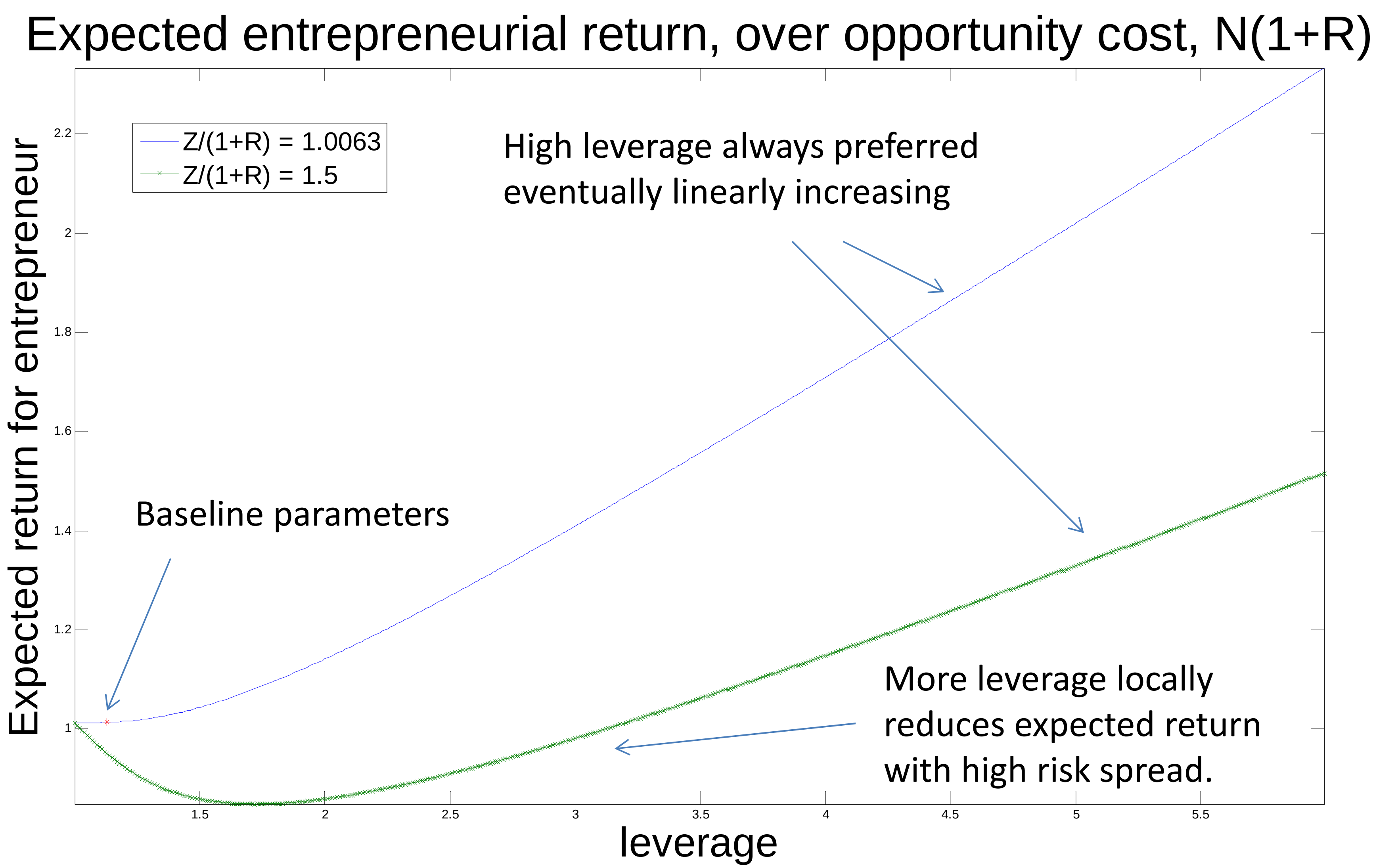
$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

Expected entrepreneurial return, over opportunity cost, N(1+R)




Expected entrepreneurial return, over opportunity cost, N(1+R)
Z/(1+R) = 1.0063
Z/(1+R) = 1.5
High leverage always preferred
eventually linearly increasing
Baseline parameters
More leverage locally
reduces expected return
with high risk spread.
Expected return for entrepreneur
leverage
2.2
2
1.8
1.6
1.4
1.2
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5

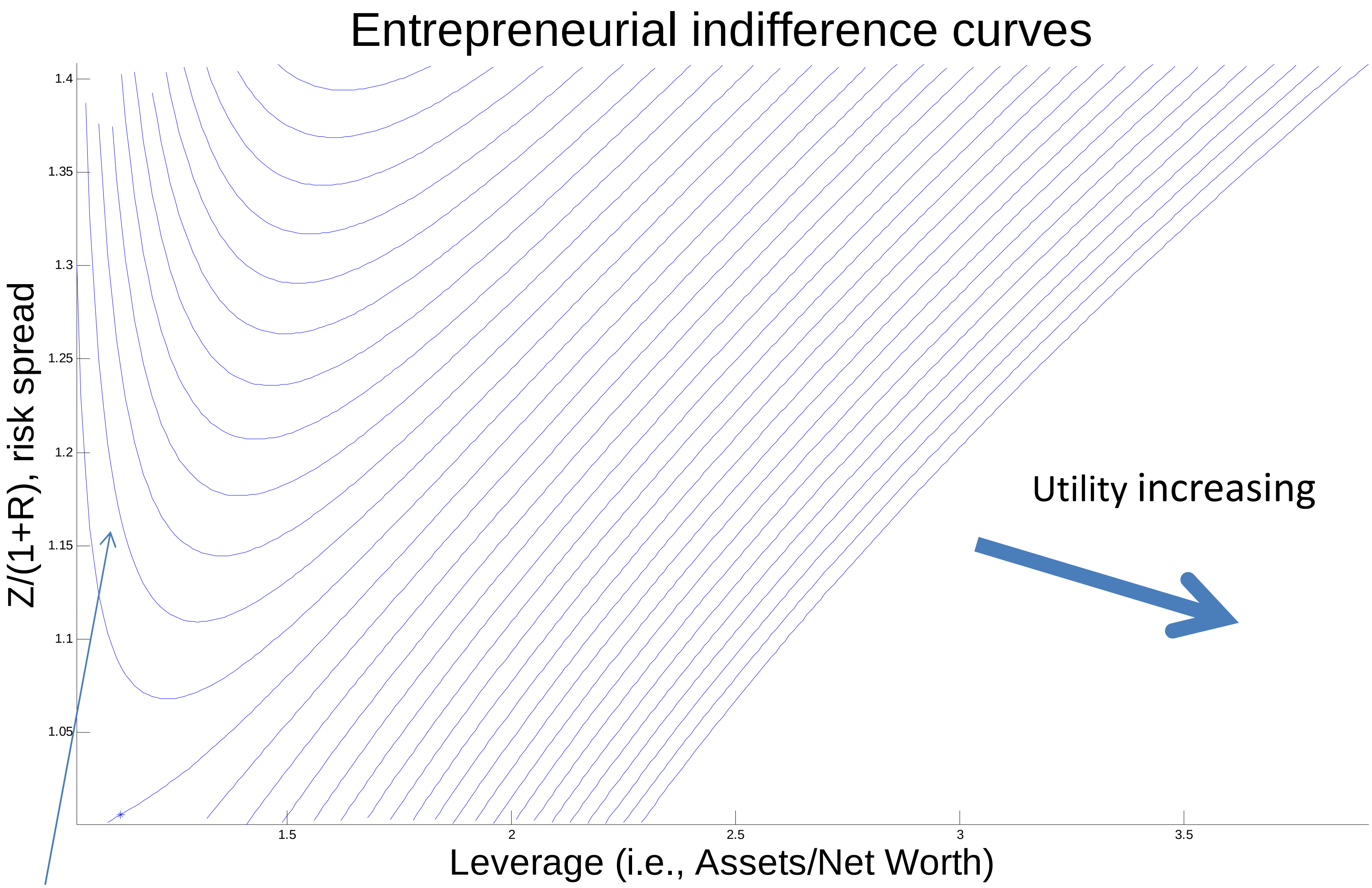
- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

- In equilibrium, bank can’t lend an infinite amount.

- This is why a loan contract must specify *both* an interest rate, Z, and a loan amount, B.

- Need to represent preferences of entrepreneurs over Z and B.
  - Problem, possibility of local decrease in utility with more leverage makes entrepreneur indifference curves ‘strange’ ..

Indifference Curves Over *Z* and *B* Problematic



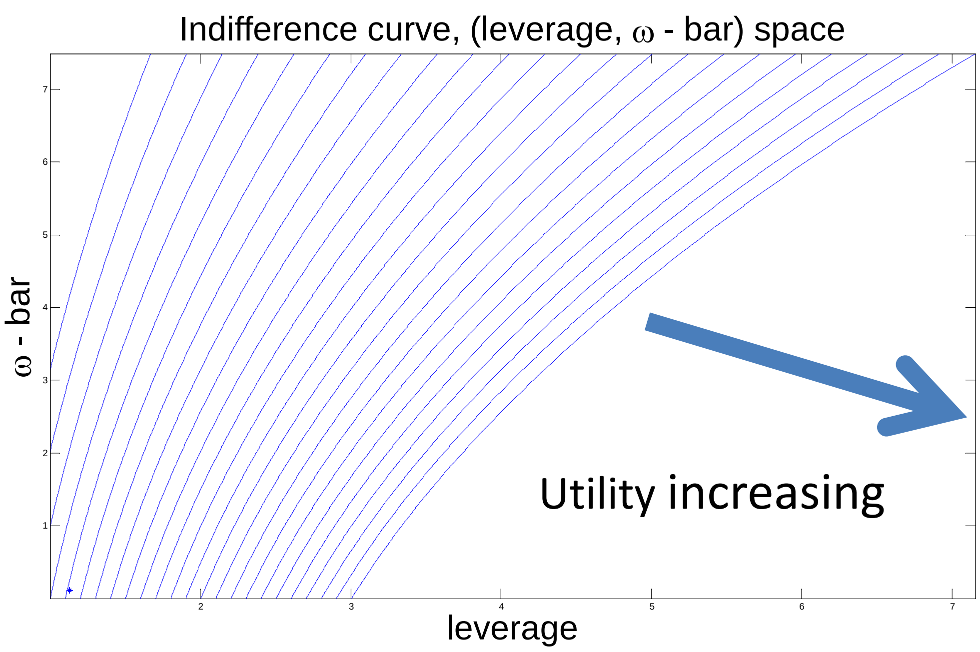
Downward-sloping indifference curves reflect local fall in net worth with rise in leverage when risk premium is high.

Solution to Technical Problem Posed by Result in Previous Slide

- Think of the loan contract in terms of the loan amount (or, leverage, (N+B)/N) and the cutoff, $\bar{\omega}$

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$L = \frac{A}{N} = \frac{N+B}{N}$$



Banks

- Source of funds from households, at fixed rate, R
- Bank borrows B units of currency, lends proceeds to entrepreneurs.
- Provides entrepreneurs with standard debt contract, (Z,B)

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1+R^k)\omega A$$

- Bank zero profit condition

$$\overbrace{[1-F(\bar{\omega})]}^{\text{fraction of entrepreneurs with } \omega>\bar{\omega}} \qquad \overbrace{ZB}^{\text{quantity paid by each entrepreneur with } \omega>\bar{\omega}}$$

$$+ \quad \overbrace{(1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}^{\text{quantity recovered by bank from each bankrupt entrepreneur}}$$

$$= \quad \overbrace{(1+R)B}^{\text{amount owed to households by bank}}$$

Banks, cont'd

- Simplifying zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$[1 - F(\bar{\omega})]\bar{\omega}(1+R^k)A + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\overbrace{\left\{[1 - F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)\right\}}^{\text{share of entrepreneurial returns given to bank, net of monitoring}} \times \overbrace{(1+R^k)A}^{\text{average return, per entrepreneur}} = (1+R)B$$

$$[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1+R^k)A = (1+R)B$$

Share of entrepreneur return going to bank.

$$\Gamma(\bar{\omega}) \equiv [1 - F(\bar{\omega})]\bar{\omega} + G(\bar{\omega}),$$

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega)$$

Banks, cont'd

- Simplifying zero profit condition:

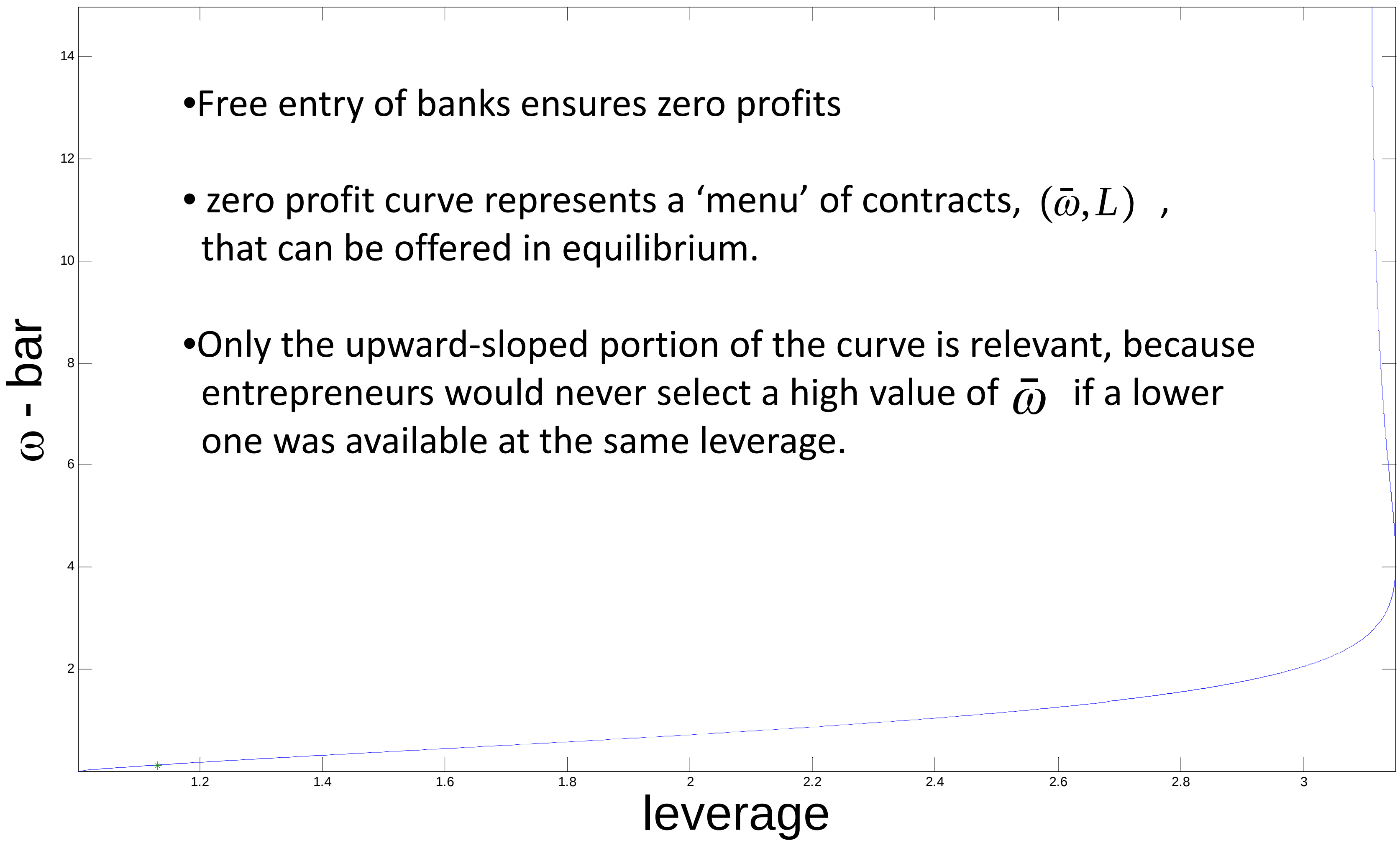
$$[1 - F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$[1 - F(\bar{\omega})]\bar{\omega}(1+R^k)A + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\begin{aligned}\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) &= \frac{1+R}{1+R^k}\frac{B/N}{A/N} \\ &= \frac{1+R}{1+R^k}\frac{L-1}{L}\end{aligned}$$

- Expressed naturally in terms of $(\bar{\omega}, L)$

Bank zero profit condition, in (leverage, ω - bar) space



- Free entry of banks ensures zero profits
- zero profit curve represents a 'menu' of contracts, $(\bar{\omega}, L)$, that can be offered in equilibrium.
- Only the upward-sloped portion of the curve is relevant, because entrepreneurs would never select a high value of $\bar{\omega}$ if a lower one was available at the same leverage.

Some Notation and Results

- Recall:

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega), \ \Gamma(\bar{\omega}) \equiv [1 - F(\bar{\omega})]\bar{\omega} + G(\bar{\omega})$$

- Results:

$$G'(\bar{\omega}) = \frac{d}{d\bar{\omega}} \int_0^{\bar{\omega}} \omega dF(\omega) \overbrace{=}^{\text{by Leibniz's rule}} \bar{\omega}F'(\bar{\omega})$$

$$\Gamma'(\bar{\omega}) \equiv 1 - F(\bar{\omega}) - \bar{\omega}F'(\bar{\omega}) + G'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0$$

Moving Towards Equilibrium Contract

- Entrepreneurial utility:

$$\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R} L$$

$$= (1 - G(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]) \frac{1+R^k}{1+R} L$$

$$= \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{share of entrepreneur return going to entrepreneur}} \frac{1+R^k}{1+R} L$$

Moving Towards Equilibrium Contract, cn't

- Bank profits:

$$\overbrace{(1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)}^{\text{share of entrepreneurial profits (net of monitoring costs) given to bank}} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$L = \frac{1}{1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

Equilibrium Contract

- Entrepreneur selects the contract is optimal, given the available menu of contracts.

- The solution to the entrepreneur problem is the $\bar{\omega}$ that solves:

$$\log\left\{ \overbrace{\int_{\bar{\omega}}^{\infty}[\omega-\bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

$$=\log \overbrace{[1-\Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log\frac{1+R^k}{1+R} \overbrace{-\log\left(1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]\right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\overbrace{\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R}\left[1-F(\bar{\omega})-\mu F'(\bar{\omega})\right]}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R}\bar{\omega}\frac{L}{L-1}$$

Result

- Leverage, *L*, and entrepreneurial rate of interest, *Z*, **not a function of net worth**, *N*.

- Quantity of loans proportional to net worth:

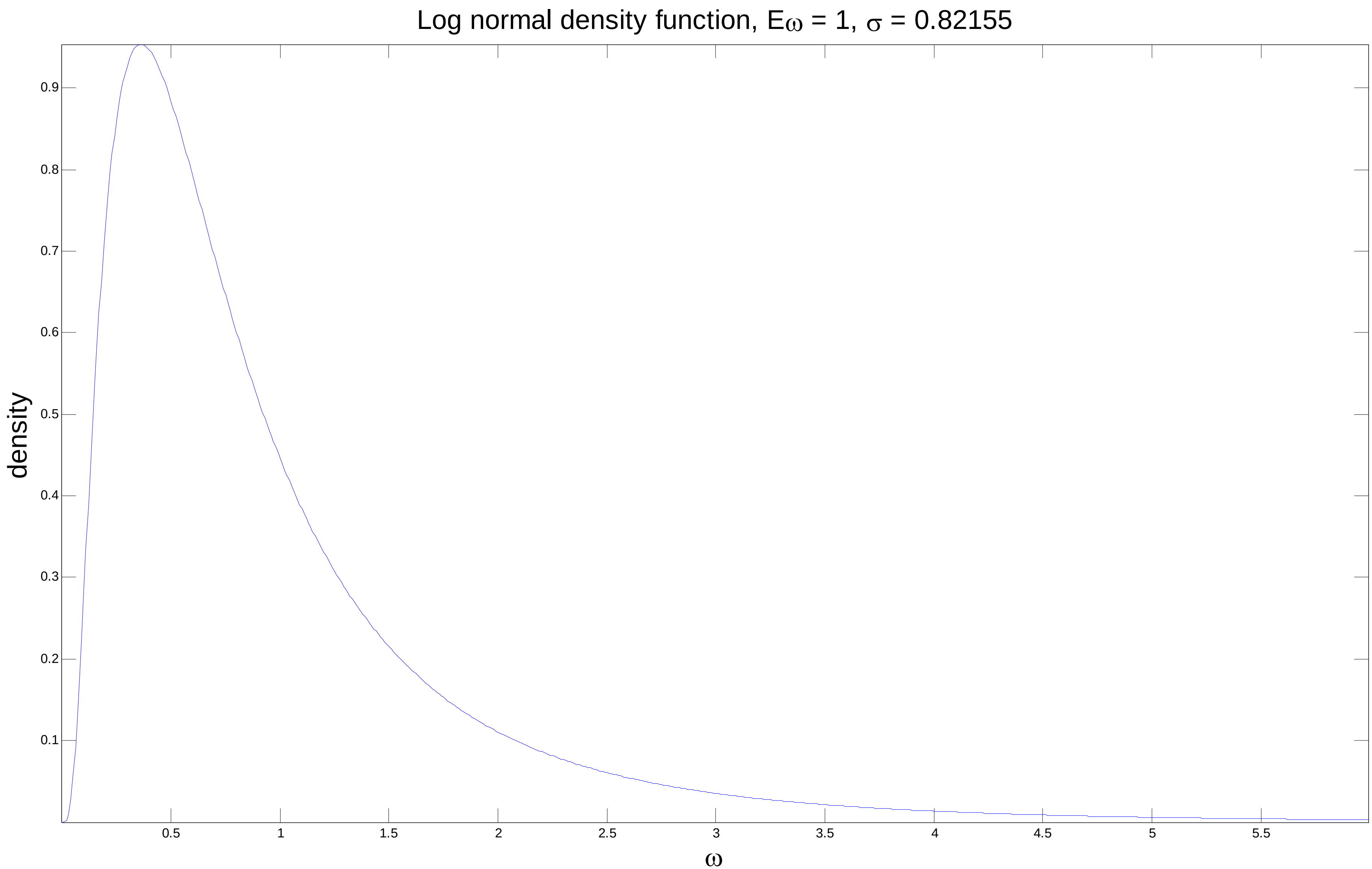
$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

$$B = (L-1)N$$

- To compute *L*, *Z/(1+R)*, must make assumptions about *F* and parameters.

$$\frac{1+R^k}{1+R},\ \mu,\ F$$

The Distribution, *F*



Results for log-normal

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega),\ F'(\omega)$

Can get these from the pdf and the cdf of the standard normal distribution.

These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.

Results for log-normal

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega),\ F'(\omega)$

$$\int_0^{\bar{\omega}} \omega dF(\omega) \underbrace{=}_{\text{change of variables, } x=\log\omega} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x e^{\frac{-(x-Ex)^2}{2\sigma_x^2}} dx$$

$$\underbrace{=}_{E\omega=1 \text{ requires } Ex=-\frac{1}{2}\sigma_x^2} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x e^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

$$\underbrace{=}_{\text{combine powers of } e \text{ and rearrange}} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^{\frac{-\left(x-\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

$$\underbrace{=}_{\text{change of variables, } v=\frac{x-\frac{1}{2}\sigma_x^2}{\sigma_x}} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} \sigma_x dv$$

$$= prob\left[v < \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \right]$$

cdf for standard normal

Results for log-normal, cnt'd

- The log-normal cumulative density:

$$F(\bar{\omega}) = \int_0^{\bar{\omega}} dF(\omega) = \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

- Differentiating (using Leibniz's rule):

$$F_{\bar{\omega}}(\omega;\sigma) = \frac{1}{\bar{\omega}\sigma}\frac{1}{\sqrt{2\pi}}\exp^{\frac{-\left[\frac{\log(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}\right]^2}{2}}$$

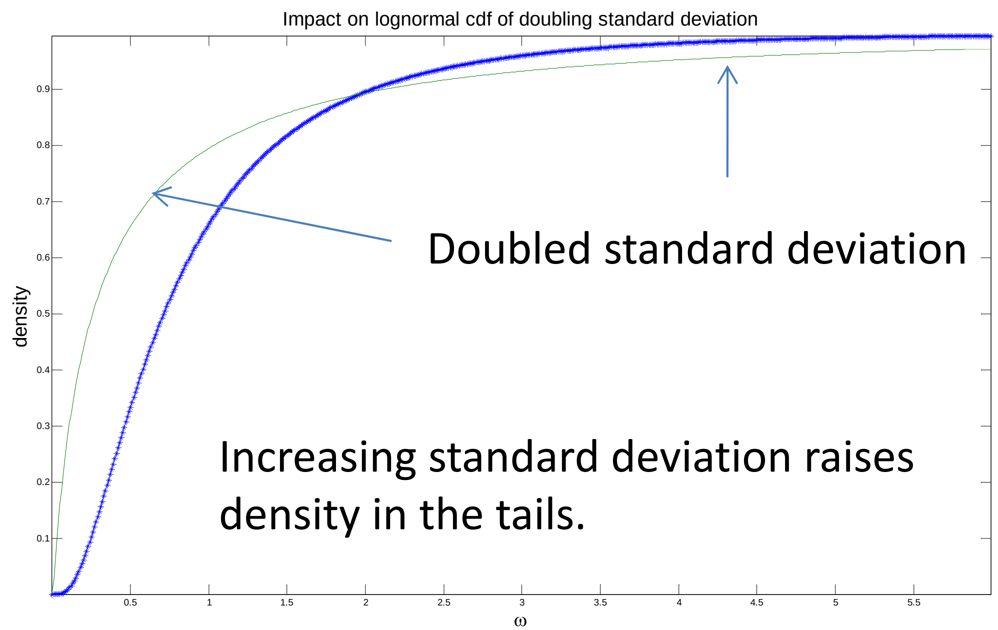
$$= \frac{1}{\bar{\omega}\sigma}\text{Standard Normal pdf}\left(\frac{\log(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}\right)$$

Effect of Increase in Risk, σ

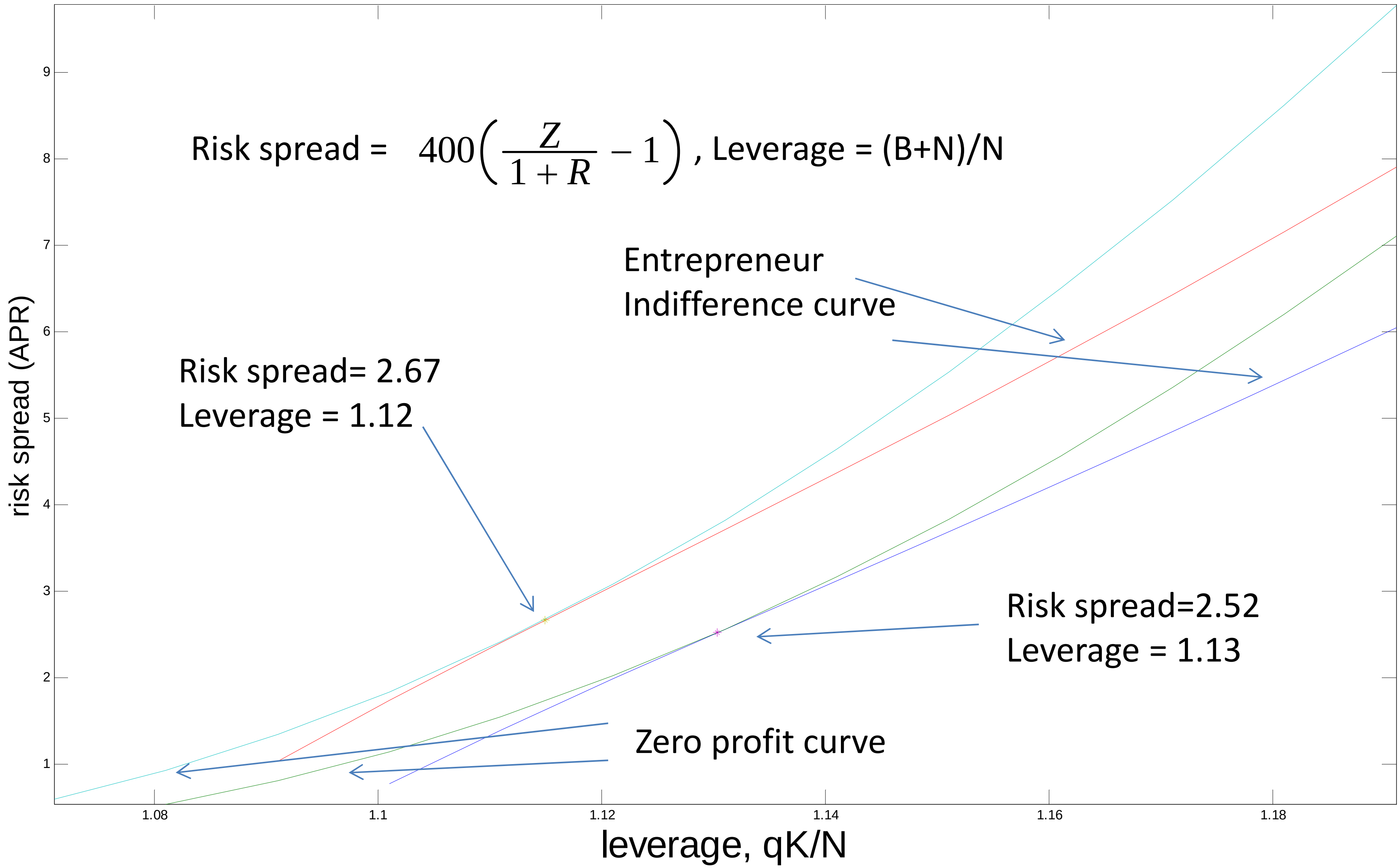
- Keep

$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.



Effect of a 5% jump in σ



Issues With the Model

- Strictly speaking, applies only to 'mom and pop grocery stores': entities run by entrepreneurs who are bank dependent for outside finance.
  - Not clear how to apply this to actual firms with access to equity markets.
- Assume no long-run connections with banks.
- Entrepreneurial returns independent of scale.
- Overly simple representation of entrepreneurial utility function.
- Ignores alternative sources of risk spread (risk aversion, liquidity)
- Seems not to allow for bankruptcies in banks.