

DSGE Models for Monetary Policy

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Outline

- Basic New Keynesian (NK) model
 - Two fundamental frictions in the model.
- Implications of NK model for monetary policy:
 - Clarifying the concepts of ‘excess and inadequate aggregate demand’.
 - The Taylor principle and inflation targeting.
 - Cases where ‘overzealous inflation targeting’ can go awry:
 - News shocks and the relationship between monetary policy and stock market volatility
 - The working capital channel and the Taylor principle.

Outline, cnt'd

- Adjustments to NK model to enable it to account for empirical estimates of the monetary transmission mechanism (the 'David Hume puzzle').
- Costly State Verification model of financial frictions: microeconomics.
- Introducing csv model into otherwise standard NK DSGE model
 - The importance of a 'risk shock' in economic fluctuations.
 - Using the model to assess a monetary policy that reacts to the interest rate premium.

Basic New Keynesian Model

- Description of the economic environment
 - Preferences and technology.
 - The best possible allocation of labor and consumption, regardless of how they are brought about.
- How ideally functioning markets can bring about the best allocations (the ‘invisible hand’).
- What happens with markets that resemble actual real world markets where prices don’t adjust instantly.

Model

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right\},$$

$$\tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iid N(0, \sigma_\varepsilon^2)$$

Production

- Final output requires lots of intermediate inputs:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

- Production of intermediate inputs:

$$Y_{i,t} = e^{a_t} N_{i,t}, \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a, \varepsilon_t^a \sim iid N(0, \sigma_a^2)$$

- Constraint on allocation of labor:

$$\int_0^1 N_{it} di = N_t$$

Efficient Allocation of Total Labor

- Suppose total labor, N_t , is fixed.
- What is the best way to allocate N_t among the various activities, $0 \leq i \leq 1$?
- Answer:
 - allocate labor equally across all the activities

$$N_{it} = N_t, \text{ all } i$$

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in [0, \frac{1}{2}] \\ 2(1 - \alpha)N_t & i \in [\frac{1}{2}, 1] \end{cases}, 0 \leq \alpha \leq 1.$$

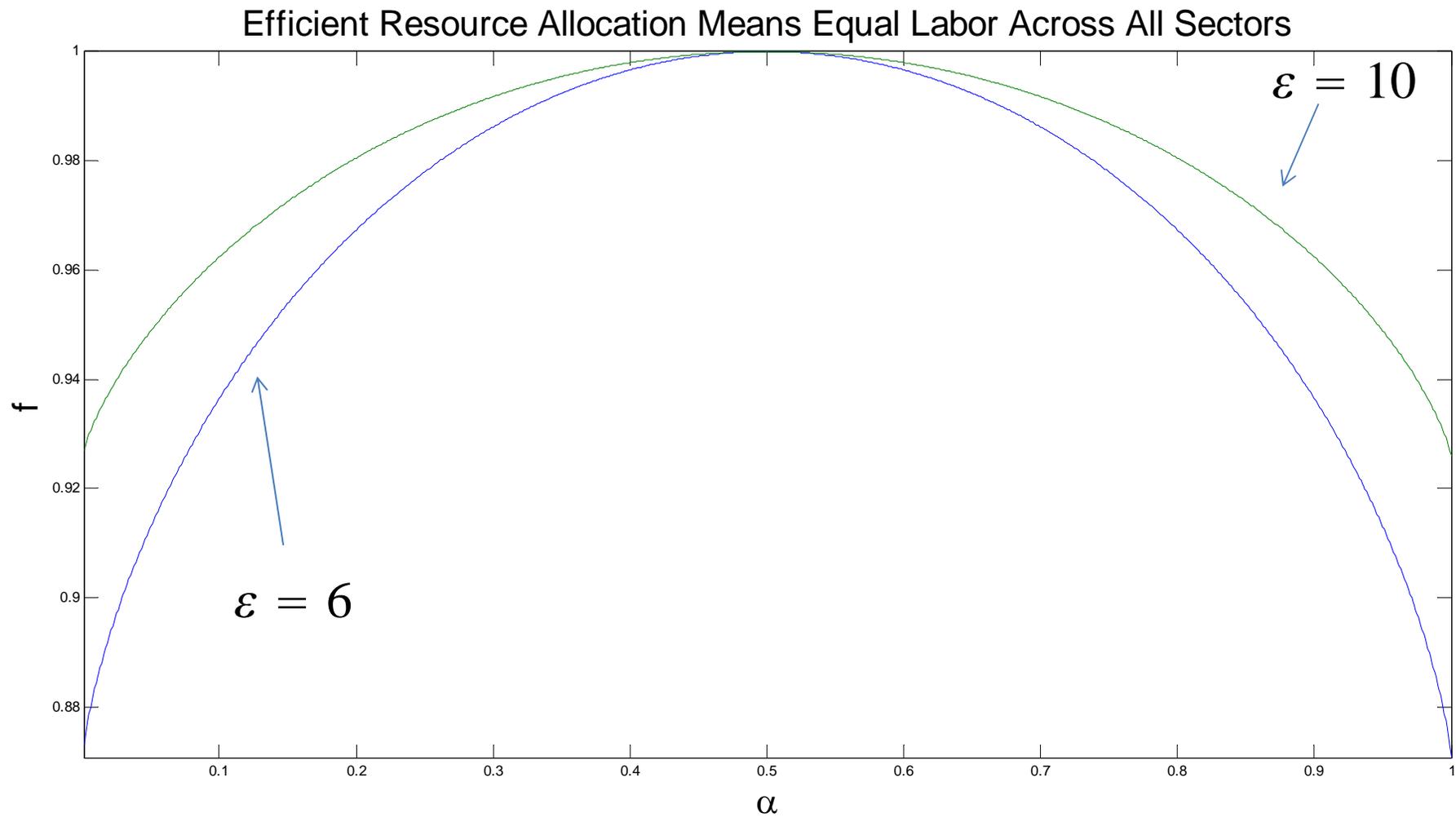
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1 - \alpha)N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$\begin{aligned} Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t f(\alpha) \end{aligned}$$

$$f(\alpha) = \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Economy with Efficient N Allocation

- Efficiency dictates

$$N_{it} = N_t \text{ all } i$$

- So, with efficient production:

$$Y_t = e^{a_t} N_t$$

- Resource constraint:

$$C_t \leq Y_t$$

- Preferences:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iid,$$

Efficient Determination of Labor

- Lagrangian:

$$\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} = \log C_t - \exp(\tau_t) \frac{N_t^{1+\phi}}{1+\phi} \\ \underbrace{u(C_t, N_t, \tau_t)} + \lambda_t [e^{a_t} N_t - C_t] \end{array} \right\}$$

- First order conditions:

$$u_c(C_t, N_t, \tau_t) = \lambda_t, \quad u_n(C_t, N_t, \tau_t) + \lambda_t e^{a_t} = 0$$

- or:

$$u_{n,t} + u_{c,t} e^{a_t} = 0$$

marginal cost of labor in consumption units = $-\frac{du}{dN_t} = \frac{dC_t}{dN_t}$

$$\frac{\overbrace{-u_{n,t}}}{u_{c,t}} = \text{marginal product of labor} \quad \underbrace{e^{a_t}}$$

Efficient Determination of Labor, cont'd

- Solving the fonic's:

$$\frac{-u_{n,t}}{u_{c,t}} = e^{a_t}$$

$$C_t \exp(\tau_t) N_t^\varphi = e^{a_t}$$

$$e^{a_t} N_t \exp(\tau_t) N_t^\varphi = e^{a_t}$$

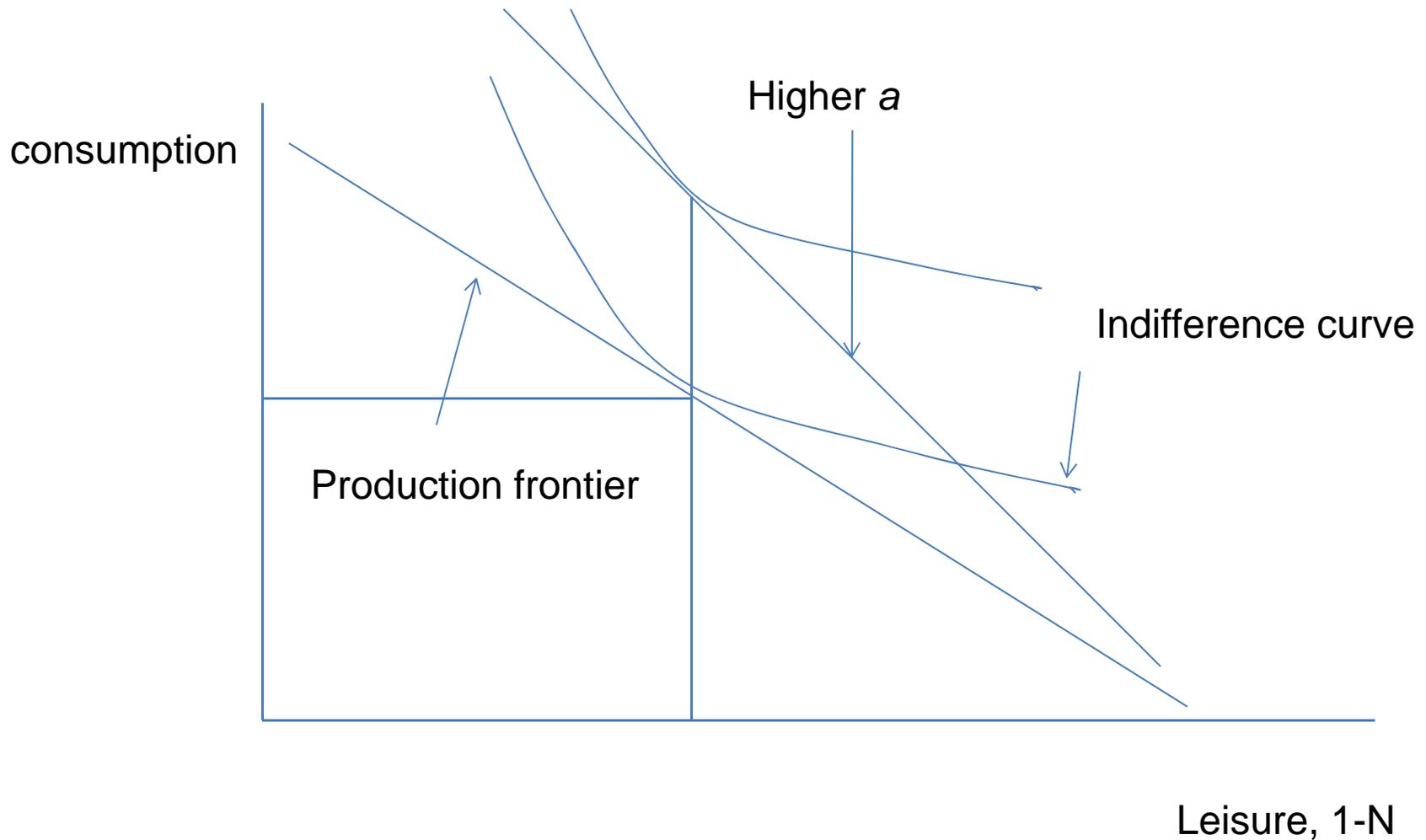
$$\rightarrow N_t = \exp\left(\frac{-\tau_t}{1 + \varphi}\right)$$

$$\rightarrow C_t = \exp\left(a_t - \frac{\tau_t}{1 + \varphi}\right)$$

- Note:

– Labor responds to preference shock, *not* to tech shock

Response to a Jump in a



Case Where Markets Work Beautifully (triumph of the ‘invisible hand’)

- Give households budget constraints, put them in markets and let them pursue their individual interests.
- Give the production functions to firms and suppose that they seek to maximize profits.
- There is monopoly power....extinguish the effects of that by a subsidy.
- Let markets work perfectly: prices and wages adjust instantly all the time, to clear markets.

Households

- Solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right\},$$

- Subject to:

Profits, *net of government taxes*

bonds purchases in t wage rate profits (real) interest on bonds

$$C_t + \underbrace{B_{t+1}}_{\text{bonds purchases in } t} \leq \underbrace{w_t}_{\text{wage rate}} N_t + \underbrace{\pi_t}_{\text{profits}} + \underbrace{r_{t-1}}_{\text{(real) interest on bonds}} B_t$$

- First order conditions:

$$\frac{-u_{n,t}}{u_{c,t}} = C_t \exp(\tau_t) N_t^\varphi = w_t \quad \text{'marginal cost of working equals marginal benefit'}$$

$$u_{c,t} = \beta E_t u_{c,t+1} r_t \quad \text{'marginal cost of saving equals marginal benefit'}$$

Final Good Firms

- Final good firms buy $Y_{i,t}$, $i \in (0, 1)$, at given prices, $P_{i,t}$, to maximize profits:

$$Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

- Subject to

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Foc's:

$$P_{i,t} = \left(\frac{Y_t}{Y_{i,t}} \right)^{\frac{1}{\varepsilon}}$$

$$\rightarrow Y_{i,t} = P_{i,t}^{-\varepsilon} Y_t, \quad 1 = \int_0^1 P_{i,t}^{1-\varepsilon} di$$

Intermediate Good Firms

- For each $Y_{i,t}$ there is a single producer who is a monopolist in the product market and hires labor, $N_{i,t}$ in competitive labor markets.

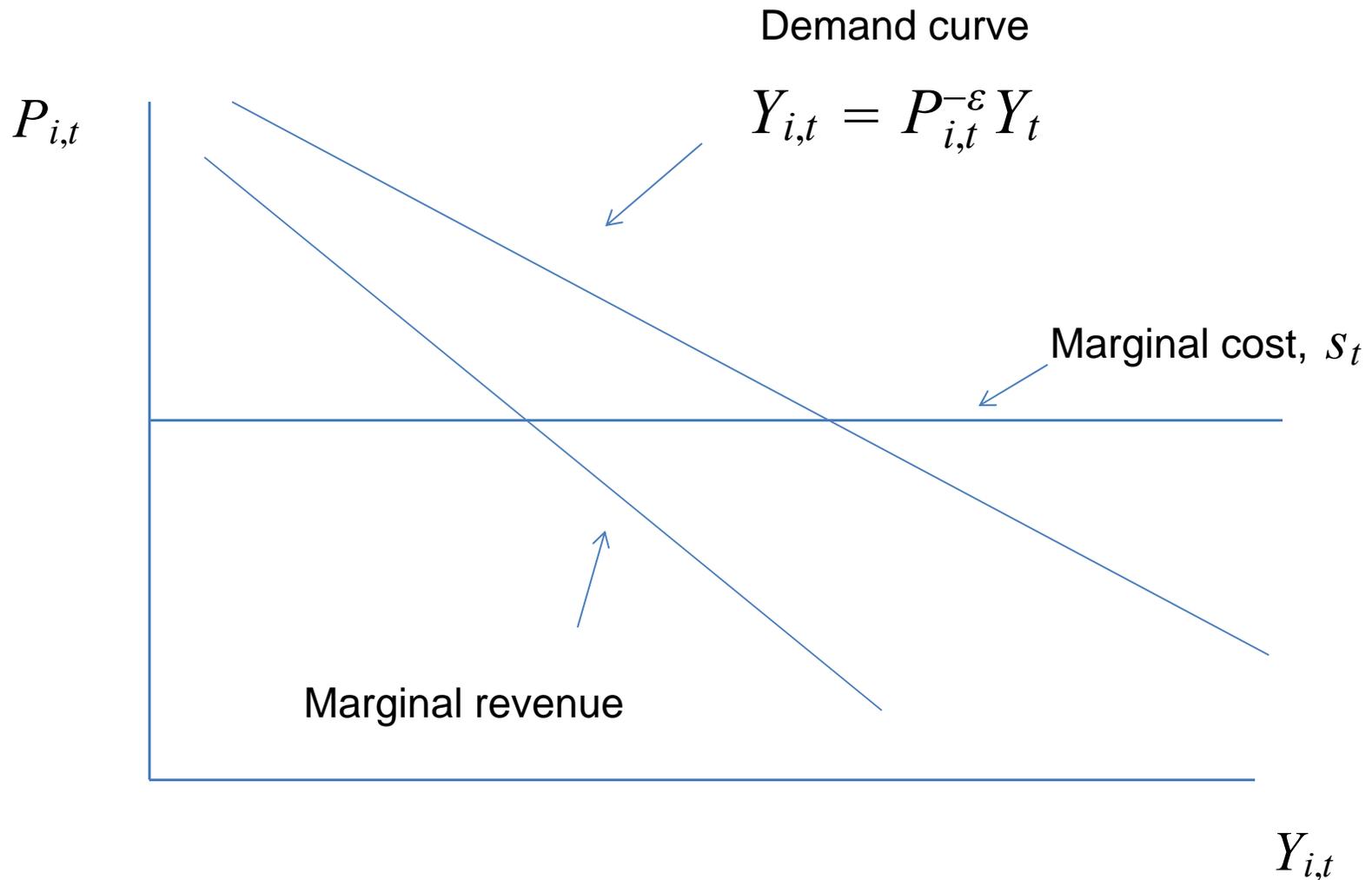
- Marginal cost of production:

$$\text{(real) marginal cost} = s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{\left(1 - \underbrace{\quad}_{\nu}\right) w_t}{\exp(a_t)}$$

subsidy payment to firm

- Subsidy will be required to ensure markets work efficiently.

Intermediate Good Firms



ith Intermediate Good Firm

- Problem: $\max_{N_{it}} P_{it} Y_{it} - s_t Y_{it}$
- Subject to demand for $Y_{i,t}$: $Y_{i,t} = P_{i,t}^{-\varepsilon} Y_t$

- Problem:

$$\max_{N_{it}} P_{it} P_{i,t}^{-\varepsilon} Y_t - s_t P_{i,t}^{-\varepsilon} Y_t$$

$$\text{fonc : } (1 - \varepsilon) P_{it}^{-\varepsilon} Y_t + \varepsilon s_t P_{i,t}^{-\varepsilon-1} Y_t = 0$$

$$P_{it} = \frac{\varepsilon}{\varepsilon - 1} s_t \text{ 'price is markup over marginal cost'}$$

- Note: all prices are the same, so resources allocated efficiently across intermediate good firms.

$$P_{i,t} = P_{j,t} = 1, \text{ because } 1 = \int_0^1 P_{i,t}^{1-\varepsilon} di$$

Equilibrium

- Pulling things together:

$$1 = \frac{\varepsilon}{\varepsilon - 1} s_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1 - \nu)w_t}{\exp(a_t)}$$

household fnc
 $\underbrace{\hspace{10em}}_{=}$

$$\frac{\varepsilon(1 - \nu)}{\varepsilon - 1} \frac{\frac{-u_{n,t}}{u_{c,t}}}{\exp(a_t)}$$

if $\frac{\varepsilon(1-\nu)}{\varepsilon-1} = 1$
 $\underbrace{\hspace{10em}}_{=}$

$$\frac{-u_{n,t}}{u_{c,t}} \cdot \frac{1}{\exp(a_t)}$$

If proper subsidy is provided to monopolists, employment is efficient:

$$\text{if } 1 - \nu = \frac{\varepsilon - 1}{\varepsilon}, \text{ then } \frac{-u_{n,t}}{u_{c,t}} = \exp(a_t)$$

Equilibrium Allocations

- With efficient subsidy,

$$\frac{-u_{n,t}}{u_{c,t}} \quad \underbrace{\quad}_{\text{functional form}} \quad C_t \exp(\tau_t) N_t^\varphi \quad \underbrace{\quad}_{\text{resource constraint}} \quad \exp(a_t) \exp(\tau_t) N_t^{1+\varphi} = \exp(a_t)$$
$$\rightarrow N_t = \exp\left(\frac{-\tau_t}{1+\varphi}\right)$$
$$C_t = e^{a_t} N_t = \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)$$

- Bond market clearing implies:

$$B_t = 0 \text{ always}$$

Interest Rate in Equilibrium

- Interest rate backed out of household intertemporal Euler equation:

$$u_{c,t} = \beta E_t u_{c,t+1} r_t \rightarrow \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} r_t$$

$$\rightarrow r_t = \frac{1}{\beta E_t \frac{C_t}{C_{t+1}}} = \frac{1}{\beta E_t \exp[c_t - c_{t+1}]} = \frac{1}{\beta E_t \exp\left[a_t - a_{t+1} - \frac{\tau_t - \tau_{t+1}}{1+\varphi}\right]}$$

$$= \frac{1}{\beta \exp\left[E_t\left(-\Delta a_{t+1} - \frac{\tau_t - \tau_{t+1}}{1+\varphi}\right) + \frac{1}{2} V\right]}, \quad V = \sigma_a^2 + \left(\frac{1}{1+\varphi}\right)^2 \sigma_\lambda^2$$

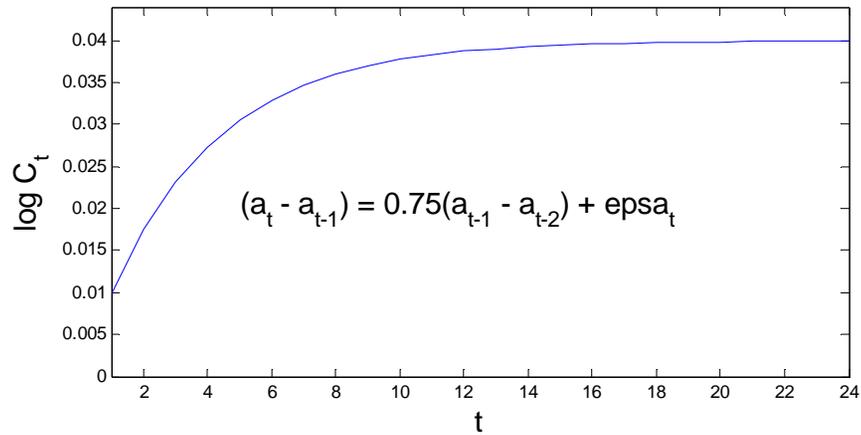
$$\log r_t = -\log \beta + E_t \left(\overbrace{\Delta a_{t+1} - \frac{\tau_{t+1} - \tau_t}{1+\varphi}}^{c_{t+1} - c_t} \right) + \frac{1}{2} V$$

using assumptions about Δa_t and τ_t

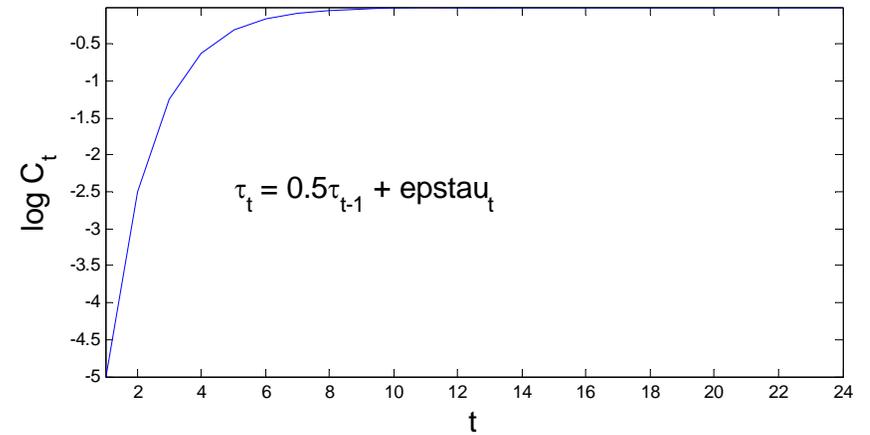
$$\underbrace{\hspace{1.5cm}} = -\log \beta + \rho \Delta a_t - \frac{(\lambda-1)\tau_t}{1+\varphi} + \frac{1}{2} V$$

Dynamic Properties of the Model

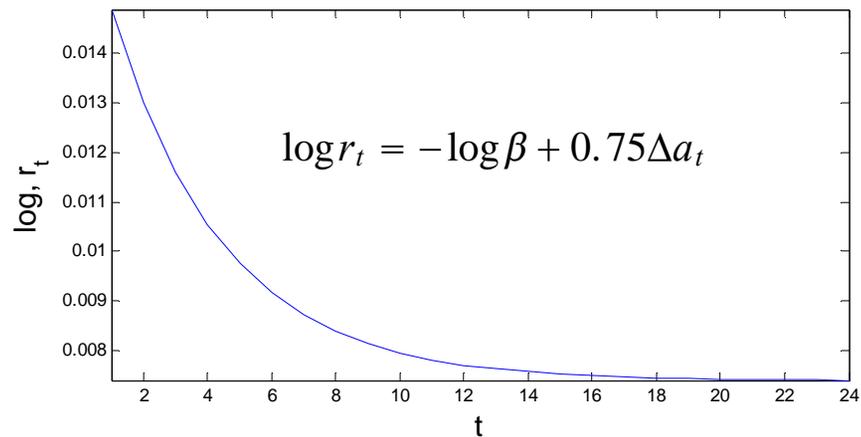
Response to .01 Technology Shock in Period 1



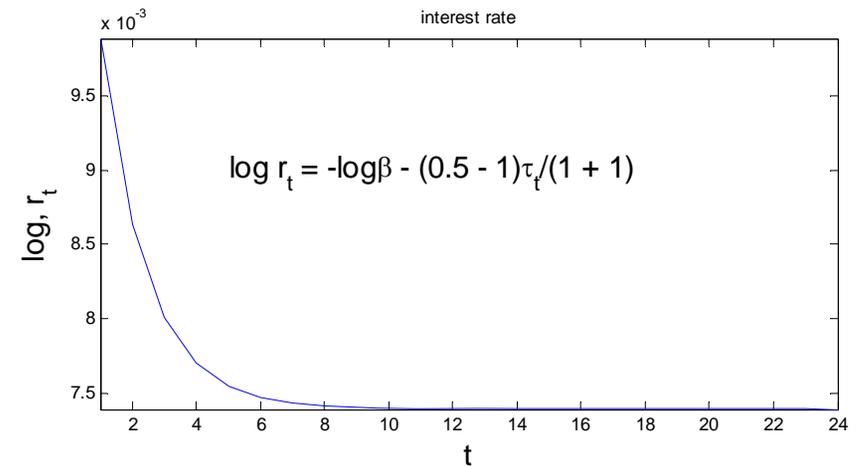
$\times 10^{-3}$ Response to .01 Preference Shock in Period 1



interest rate



interest rate



Key Features of Equilibrium Allocations

- Allocations *efficient* (as long as monopoly power neutralized)
- Employment does not respond to technology
 - Improvement in technology raises marginal product of labor and marginal cost of labor by same amount.
- First best consumption not a function of intertemporal considerations
 - Discount rate irrelevant.
 - Anticipated future values of shocks irrelevant.
- Natural rate of interest steers consumption and employment towards their natural levels, ‘as if guided by an invisible hand’.

Introducing Price Setting Frictions (Clarida-Gali-Gertler Model)

- Households maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iid,$$

- Subject to:

Profits, net of taxes raised by
Government to finance subsidies.

$$P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + T_t$$

- Intratemporal first order condition:

$$C_t \exp(\tau_t) N_t^\varphi = \frac{W_t}{P_t}$$

Household Intertemporal FONC

- Condition:

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{1 + \pi_{t+1}}$$

– or

$$\begin{aligned} 1 &= \beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{1 + \pi_{t+1}} \\ &= \beta E_t \exp[\log(R_t) - \log(1 + \pi_{t+1}) - \Delta c_{t+1}] \\ &\simeq \beta \exp[\log(R_t) - E_t \pi_{t+1} - E_t \Delta c_{t+1}], \quad c_t \equiv \log(C_t) \end{aligned}$$

– take log of both sides:

$$0 = \log(\beta) + r_t - E_t \pi_{t+1} - E_t \Delta c_{t+1}, \quad r_t = \log(R_t)$$

– or

$$c_t = -\log(\beta) - [r_t - E_t \pi_{t+1}] + c_{t+1}$$

NK IS Curve

- Euler equation in two equilibria:

Output equals consumption


$$\text{Actual equilibrium: } y_t = -[r_t - E_t \pi_{t+1} - rr] + E_t y_{t+1}$$

$$\text{Natural equilibrium: } y_t^* = -[r_t^* - rr] + E_t y_{t+1}^*$$

- Subtract:

Output gap


$$x_t = -[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1}$$

Final Good Firms

- Buy $Y_{i,t}$, $i \in [0, 1]$ at prices $P_{i,t}$ and sell Y_t for P_t
- Take all prices as given (competitive)

- Profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

- Production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1,$$

- First order condition:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \quad \rightarrow \quad P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$

Intermediate Good Firms

- Each i th good produced by a single monopoly producer.
- Demand curve:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon}$$

- Technology:

$$Y_{i,t} = \exp(a_t) N_{i,t}, \quad \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,$$

- Calvo Price-setting Friction

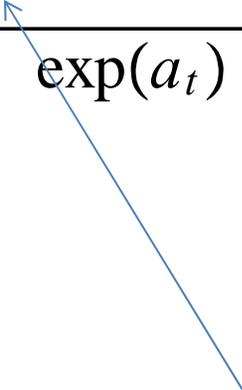
$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}$$

Marginal Cost

$$\begin{aligned} \text{real marginal cost} = S_t &= \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{Output}}{d\text{worker}}} = \frac{(1 - \nu)W_t/P_t}{\exp(a_t)} \\ &= \frac{\overbrace{(1 - \nu)}^{=\frac{\varepsilon-1}{\varepsilon} \text{ in efficient setting}}}{\exp(a_t)} C_t \exp(\tau_t) N_t^\varphi \end{aligned}$$

Marginal Cost

- Marginal cost:

$$s_t = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^\varphi}{\exp(a_t)}$$


$\frac{1}{\lambda_f} = \frac{\varepsilon - 1}{\varepsilon} \sim$ tax subsidy, $1 - \nu$, to 'trick' firms into
setting price equal to marginal cost

Marginal Cost

- Marginal cost:

$$s_t = \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^\varphi}{\exp(a_t)}$$

$C_t = \exp(a_t) N_t$ holds only as a first order approximation $\underbrace{\hspace{10em}}_{\simeq} \frac{\frac{1}{\lambda_f} C_t \exp(\tau_t) \left[\frac{C_t}{\exp(a_t)} \right]^\varphi}{\exp(a_t)}$

$$= \frac{1}{\lambda_f} C_t^{1+\varphi} \left[\frac{\exp\left(\frac{\tau_t}{1+\varphi}\right)}{\exp(a_t)} \right]^{(1+\varphi)} = \frac{1}{\lambda_f} \left(\frac{C_t}{C_t^*} \right)^{1+\varphi}$$

- Then,

a hat indicates log-deviation from steady state

$$\hat{s}_t$$

$$\underbrace{\hspace{1em}}_{\equiv}$$

$$\begin{aligned} \log(s_t \lambda_f) &= (1 + \varphi)(c_t - c_t^*) \\ &= (1 + \varphi)x_t \end{aligned}$$

=1/s

The Intermediate Firm's Decisions

- *ith* firm is required to satisfy whatever demand shows up at its posted price.
- Its only real decision is to adjust price whenever the opportunity arises.
- It sets its price as a function of current and expected future marginal cost.
 - Inflation evolves (to first order approximation) as follows:

$$\pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} [\hat{s}_t + \beta E_t \hat{s}_{t+1} + \beta^2 E_t \hat{s}_{t+2} + \dots]$$

$$\hat{s}_t \equiv \frac{s_t - s}{s} \simeq \log\left(\frac{s_t}{s}\right), \quad \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

NK Phillips Curve

- Repeating from previous slide:

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} [\hat{s}_t + \beta E_t \hat{s}_{t+1} + \beta^2 E_t \hat{s}_{t+2} + \dots]$$

- Also,

$$\beta E_t \pi_{t+1} = \frac{(1-\theta)(1-\beta\theta)}{\theta} [\beta E_t \hat{s}_{t+1} + \beta^2 E_t \hat{s}_{t+2} + \beta^3 E_t \hat{s}_{t+3} + \dots]$$

- Then,

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t$$

Back to the Phillips Curve

- Phillips curve summarizes price setting by intermediate good firms:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t$$

- or, substituting out for marginal cost:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} (1 + \varphi) x_t$$

Equations of Actual Equilibrium, with Monetary Policy Rule

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \text{ (Calvo pricing equation)}$$

$$x_t = -[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} \text{ (intertemporal equation)}$$

$$r_t = \alpha r_{t-1} + (1 - \alpha)[\phi_\pi \pi_t + \phi_x x_t] + u_t \text{ (policy rule)}$$

$$r_t^* = E_t(y_{t+1}^* - y_t^*) = E_t\left(\Delta a_{t+1} - \frac{1}{1 + \varphi} \Delta \tau_{t+1}\right) \text{ (natural rate)}$$

$$y_t^* = a_t - \frac{1}{1 + \varphi} \tau_t \text{ (natural output),} \quad x_t = y_t - y_t^* \text{ (output gap)}$$

also, time series representations for shocks listed above

Solving the Model

- Express the equations in matrix form:

$$z_t = \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r_t^* \end{pmatrix}, s_t = \begin{pmatrix} \tau_t \\ a_t \\ u_t \end{pmatrix}, \varepsilon_t = \begin{pmatrix} \varepsilon_t^\tau \\ \varepsilon_t^a \\ \varepsilon_t^u \end{pmatrix}$$

$$(*) \alpha_0 E_t z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 E_t s_{t+1} + \beta_1 s_t = 0$$

$$s_t = P s_{t-1} + \varepsilon_t$$

- Solution: A and B matrices such that $(*)$ is satisfied and

$$z_t = A z_{t-1} + B s_t$$

Solving the model....

- Iterating forward one period on the solution:

$$\begin{aligned} z_{t+1} &= Az_t + Bs_{t+1} \\ &= A^2z_{t-1} + ABs_t + Bs_{t+1} \end{aligned}$$

- Then,

Finding solution for A is only hard part.

Choose A that solves problem if there is only one with eigenvalues less than unity in absolute value.

If more than one A like this: multiple solutions.

$$\begin{aligned} &E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] \\ &= E_t[\alpha_0(A^2 z_{t-1} + ABs_t + Bs_{t+1}) + \alpha_1(Az_{t-1} + Bs_t) + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] \\ &= [\alpha_0 A^2 + \alpha_1 A + \alpha_2] z_{t-1} + E_t[(\alpha_0 B + \beta_0) s_{t+1} + (\alpha_0 AB + \alpha_1 B + \beta_1) s_t] \end{aligned}$$

$$= \overbrace{[\alpha_0 A^2 + \alpha_1 A + \alpha_2]}^{=0} z_{t-1} + \overbrace{[(\alpha_0 B + \beta_0)P + (\alpha_0 AB + \alpha_1 B + \beta_1)]}^{=0} s_t = 0, \text{ all } s_t, z_{t-1}$$

Impulse Response Function

- Law of motion of endogenous and exogenous variables:

$$z_t = Az_{t-1} + Bs_t$$

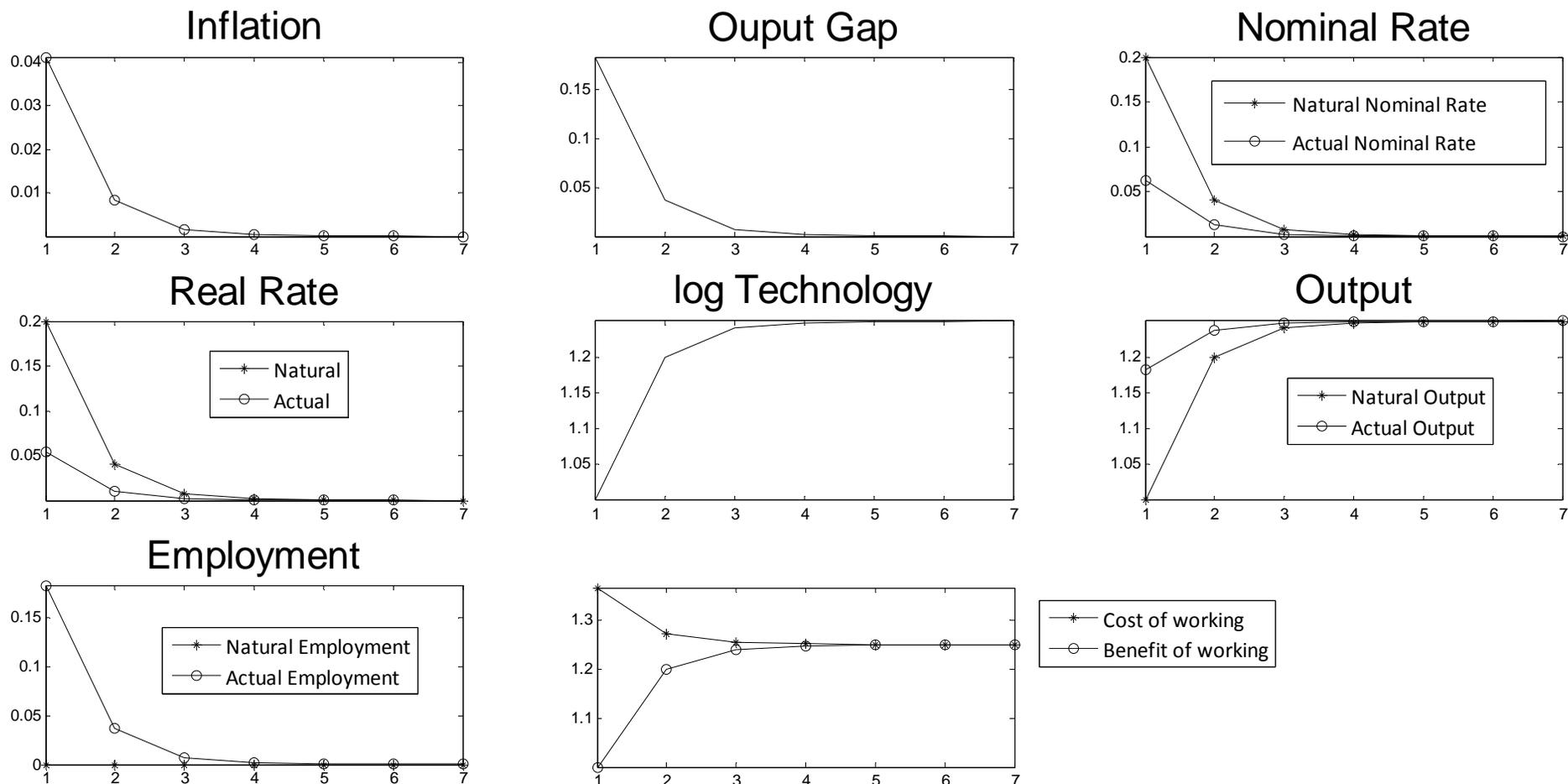
$$s_t = Ps_{t-1} + \varepsilon_t$$

- Set one element of ε_0 non-zero and let $\varepsilon_t = 0, t > 0$
- Set $s_{-1} = z_{-1} = 0$ and compute

$$s_0, s_1, s_2, \dots$$

$$z_0, z_1, z_2, \dots$$

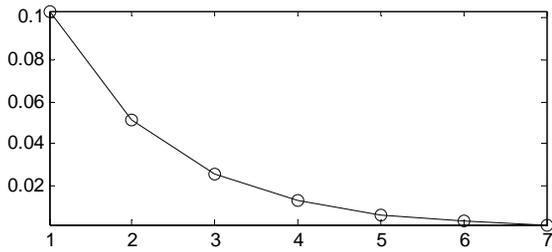
Dynamic Response to a Technology Shock



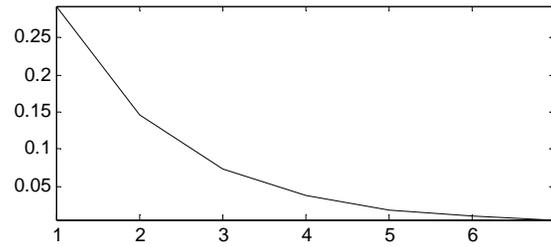
$$\phi_x = 0, \phi_\pi = 1.5, \beta = 0.97, \varphi = 1, \rho = 0.2, \theta = 0.75, \alpha = 0, \lambda = 0.5.$$

Dynamic Response to a Preference Shock

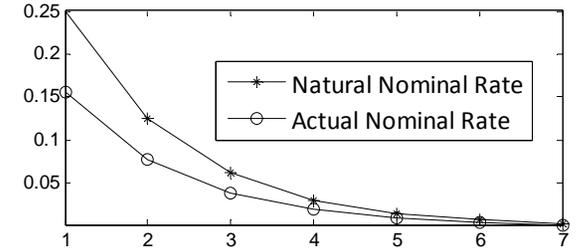
Inflation



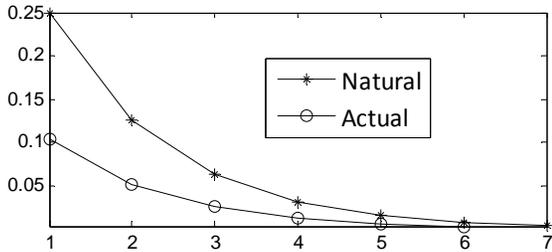
Output Gap



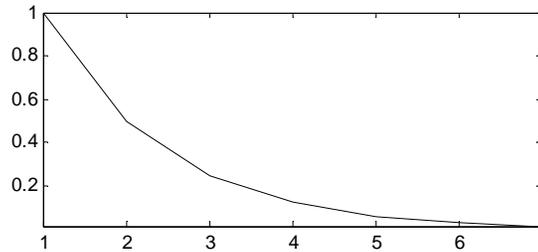
Nominal Rate



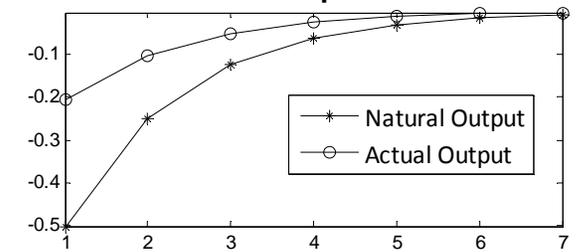
Real Rate



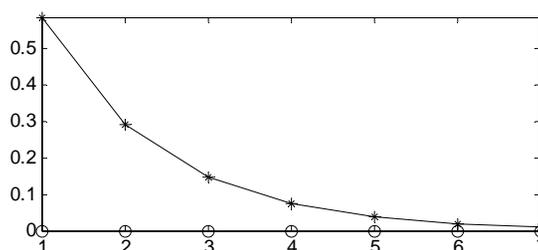
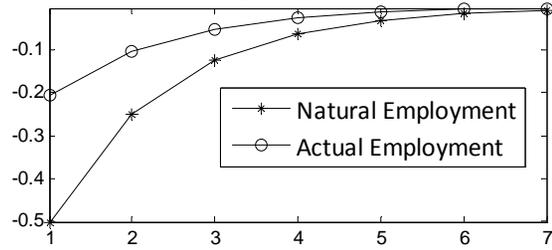
Preference Shock



Output



Employment



* Cost of working
 ○ Benefit of working