Notes on Financial Frictions Under Asymmetric Information and Costly State Verification

by

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Incorporating Financial Frictions into a Business Cycle Model

- General idea:
  - Standard model assumes borrowers and lenders are the same people..no conflict of interest
  - Financial friction models suppose borrowers and lenders are different people, with conflicting interests
  - Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Bernanke-Gertler.

- Integrating the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (2003; forthcoming, AER2014).

Simple Model

- There are entrepreneurs with all different levels of wealth, *N*.
  - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.
- For each value of *N*, there are many entrepreneurs.
- In what follows, we will consider the interaction between entrepreneurs with a specific amount of *N* with competitive banks.
- Later, will consider the whole population of entrepreneurs, with every possible level of *N*.

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1+R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

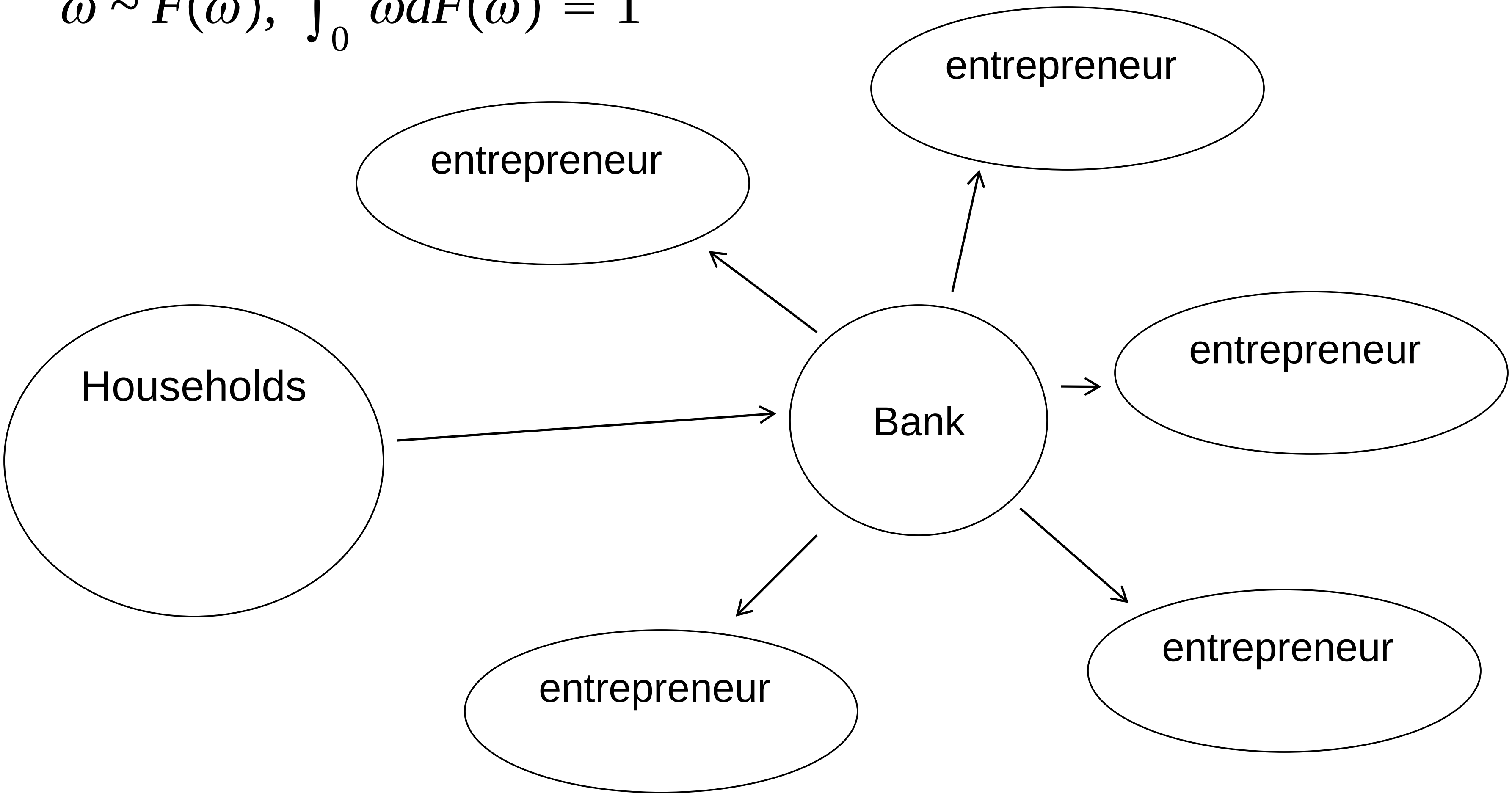
$$\int_0^\infty \omega dF(\omega) = 1$$

- The shock, ω , is privately observed by the entrepreneur.

- *F* is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega),\ \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.
  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

$$\overbrace{(1+R^k)\bar{\omega}}^{\text{gross rate of return experience by entrepreneur with 'luck', } \bar{\omega}} \times \overbrace{A}^{\text{total assets}}$$

$$= \overbrace{ZB}^{\text{interest and principle owed by the entrepreneur}}$$

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- Cutoff higher with:
  - higher leverage, *L*
  - higher $Z/(1+R^k)$

- Expected return to entrepreneur from operating risky technology, over return from depositing net worth in bank:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty} [(1+R^k)\omega A - ZB] dF(\omega)}{N(1+R)}$$

For lower values of ω, entrepreneur receives nothing 'limited liability'.

gain from depositing funds in bank ('opportunity cost of funds')

- Rewriting entrepreneur's rate of return:

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Gets smaller with *L*

Larger with *L*

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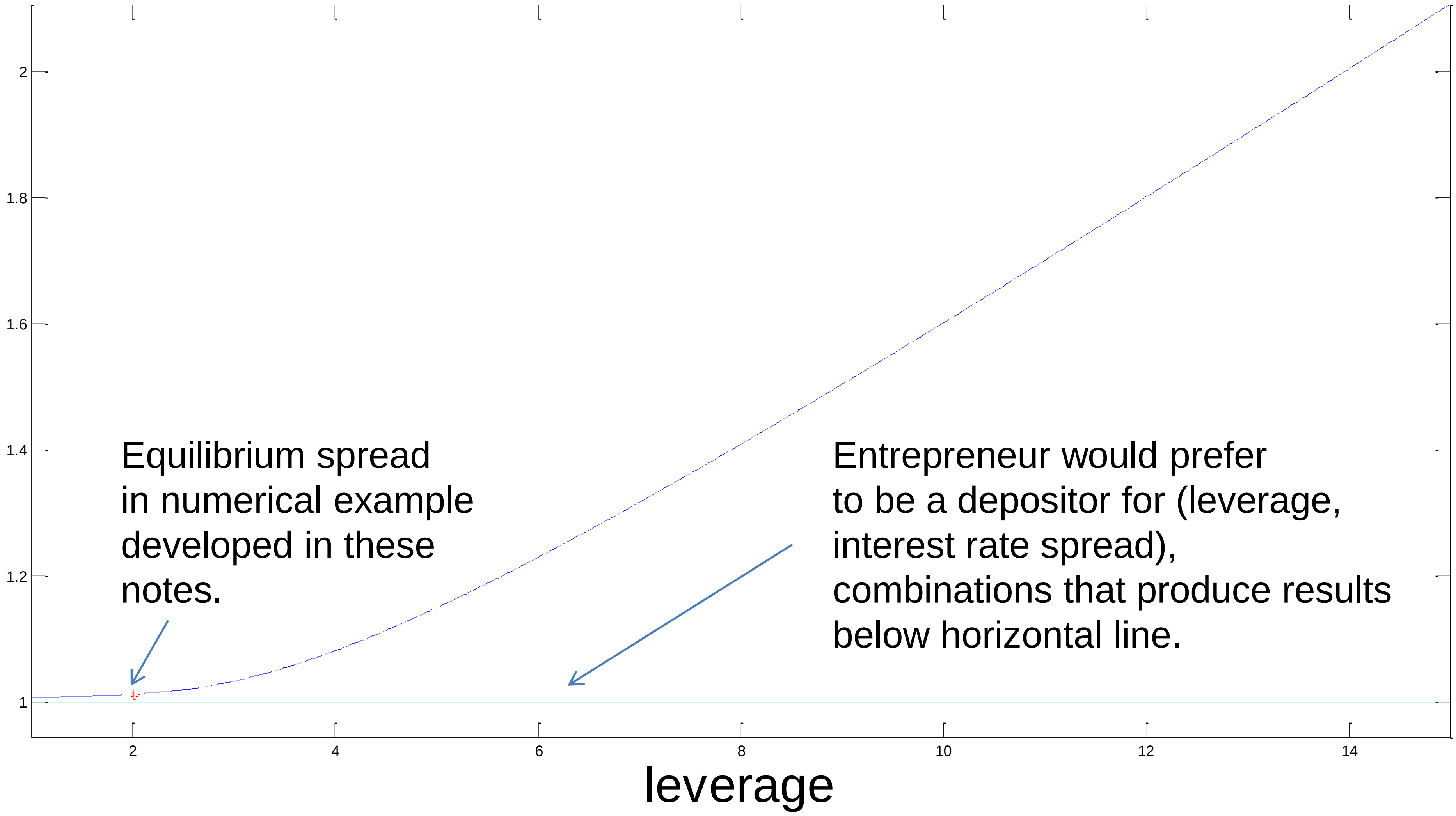
$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

Expected entrepreneurial return, over opportunity cost, N(1+R)



Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

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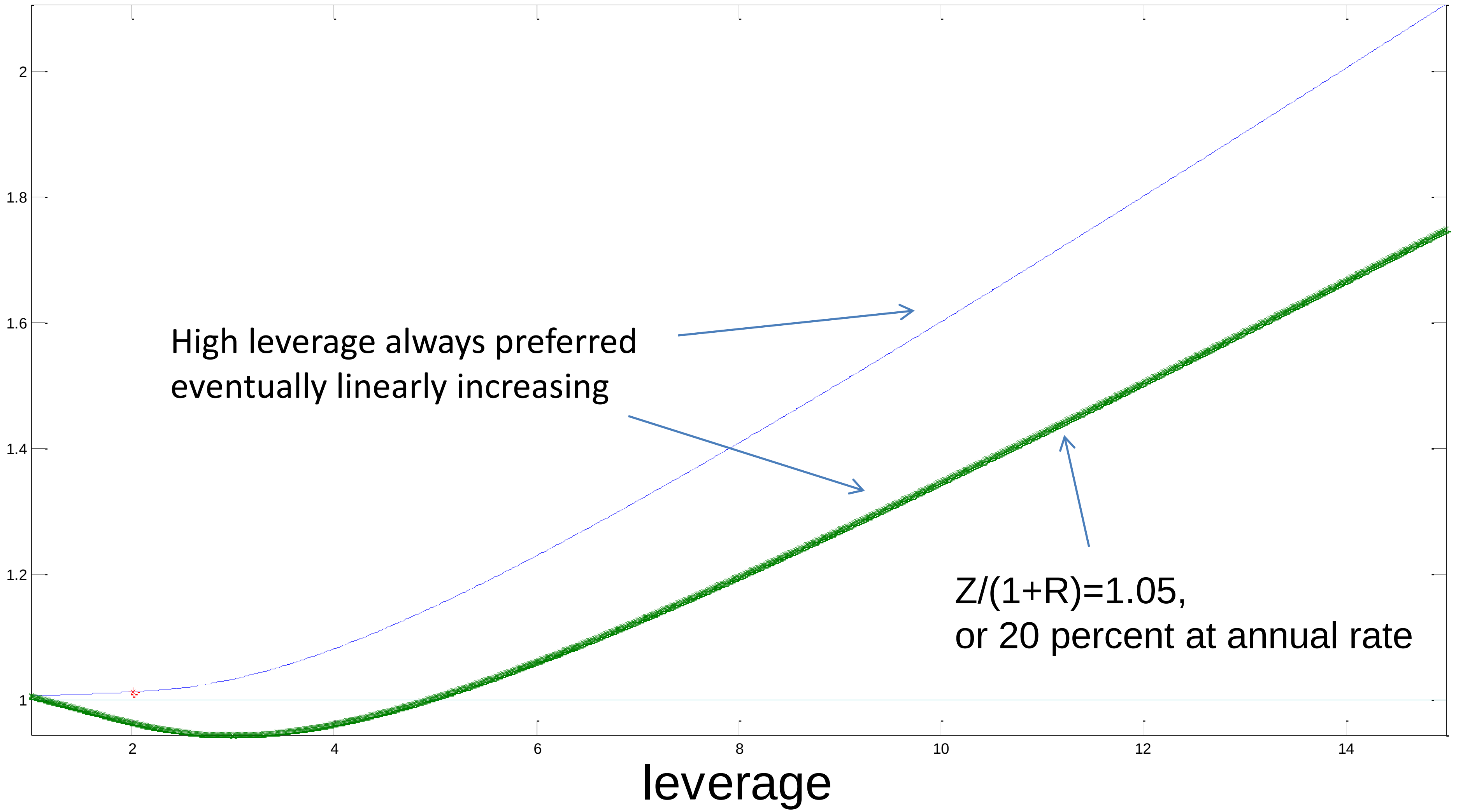

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- In equilibrium, bank can't lend an infinite amount.

- This is why a loan contract must specify *both* an interest rate, *Z*, and a loan amount, *B*.

- Need to represent preferences of entrepreneurs over *Z* and *B*.
  - Problem, possibility of local decrease in utility with more leverage makes entrepreneur indifference curves 'strange' ..

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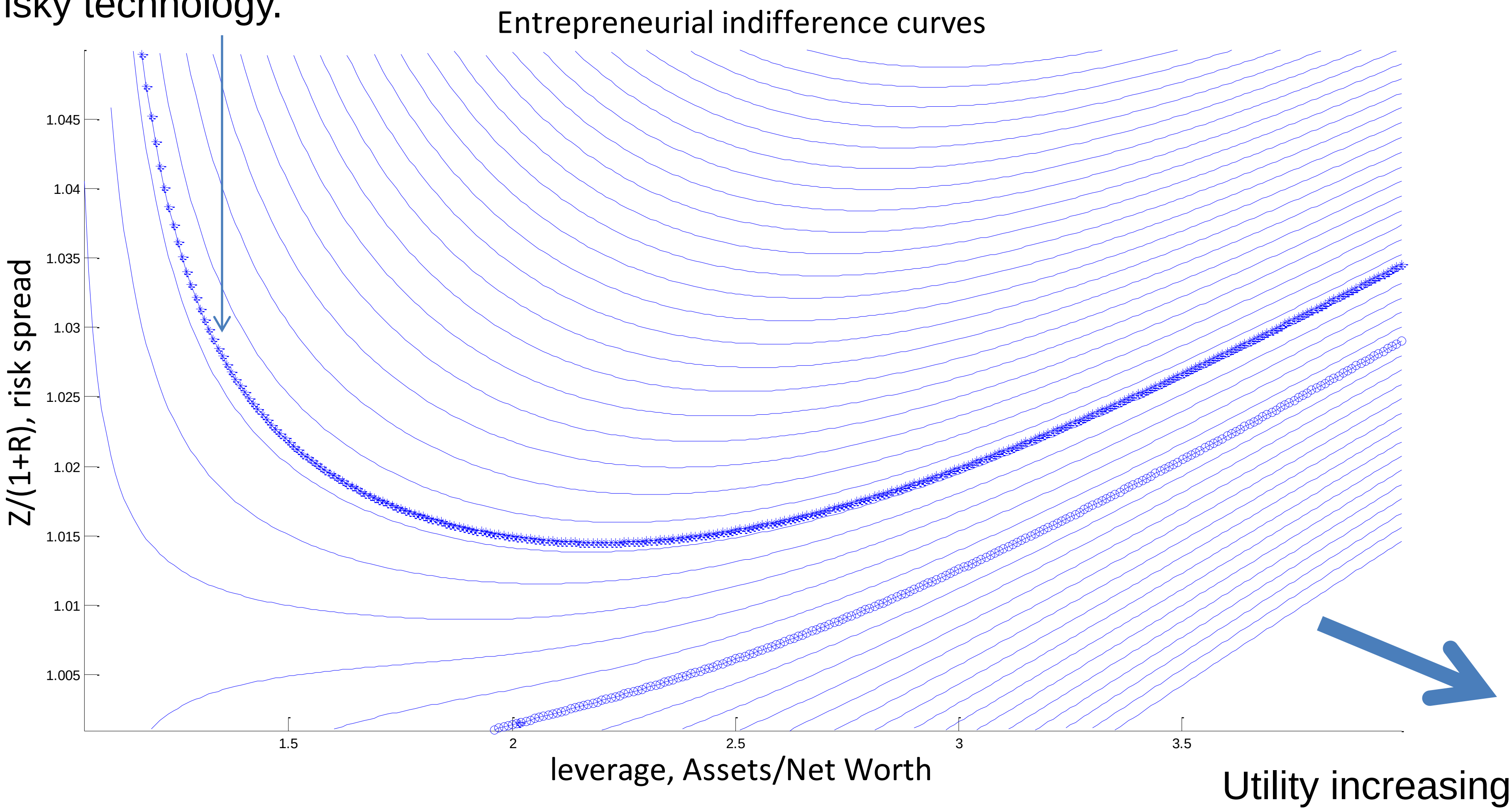
Indifference Curves Over *Z* and *B* Problematic

Utility level where entrepreneur is indifferent between depositing in bank and operating risky technology.



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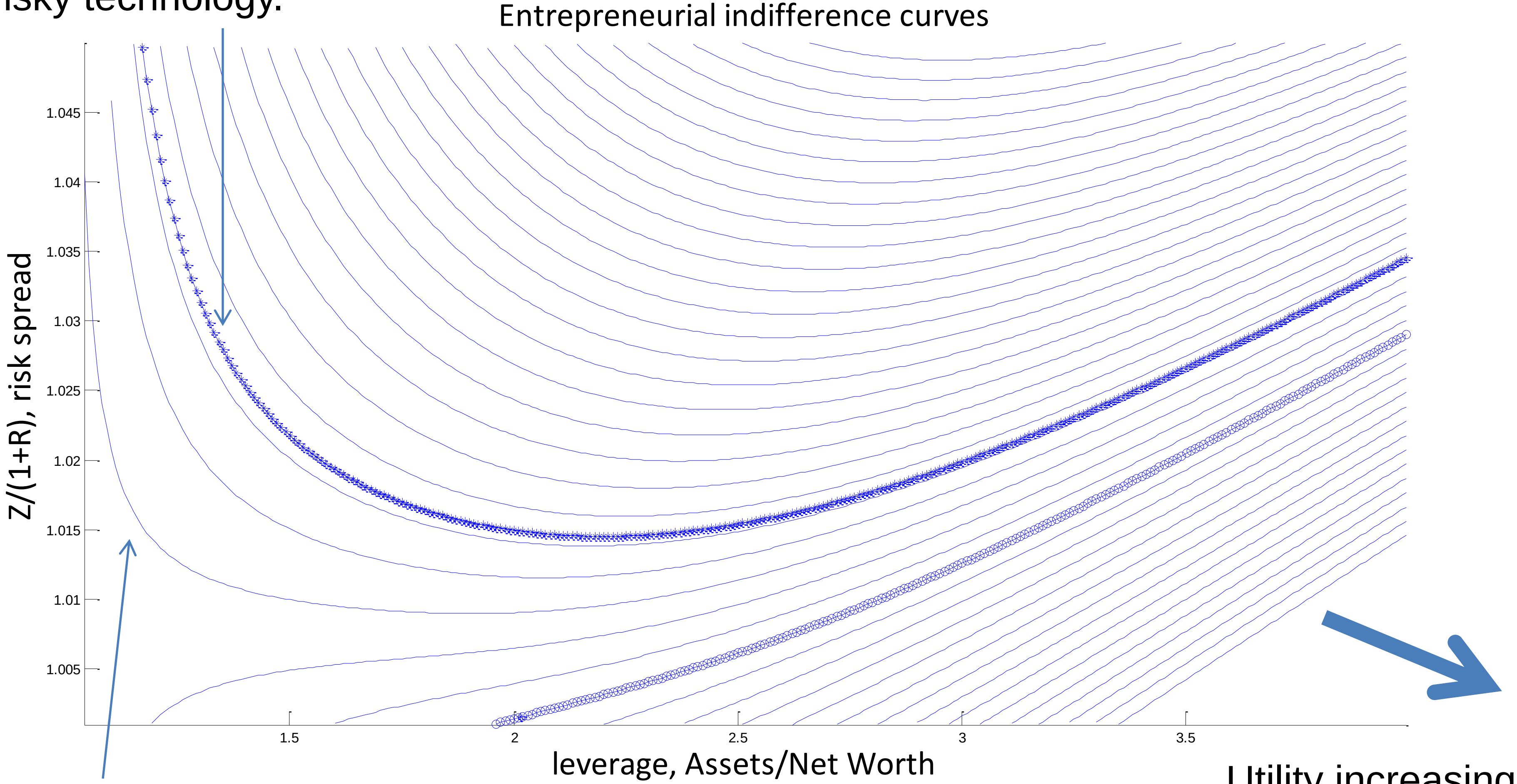

Downward-sloping indifference curves reflect local fall in net worth with rise in leverage when risk premium is high.

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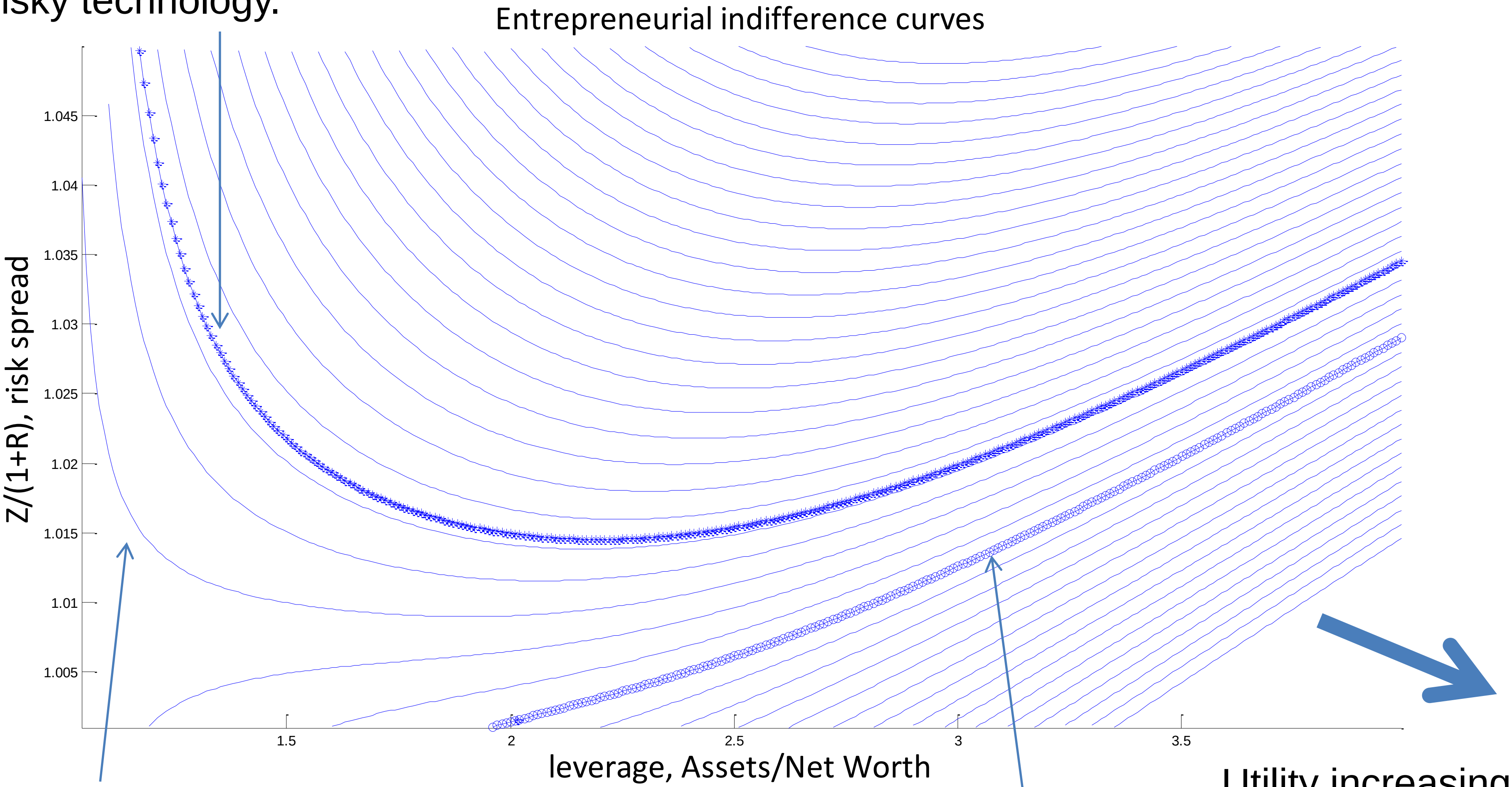

Utility increasing

Downward-sloping indifference curves reflect local fall in net worth with rise in leverage when risk premium is high.

Utility in equilibrium developed
In numerical example.

Solution to Technical Problem Posed by Result in Previous Slide

- Think of the loan contract in terms of the loan amount (or, leverage, (N+B)/N) and the cutoff, $\bar{\omega}$

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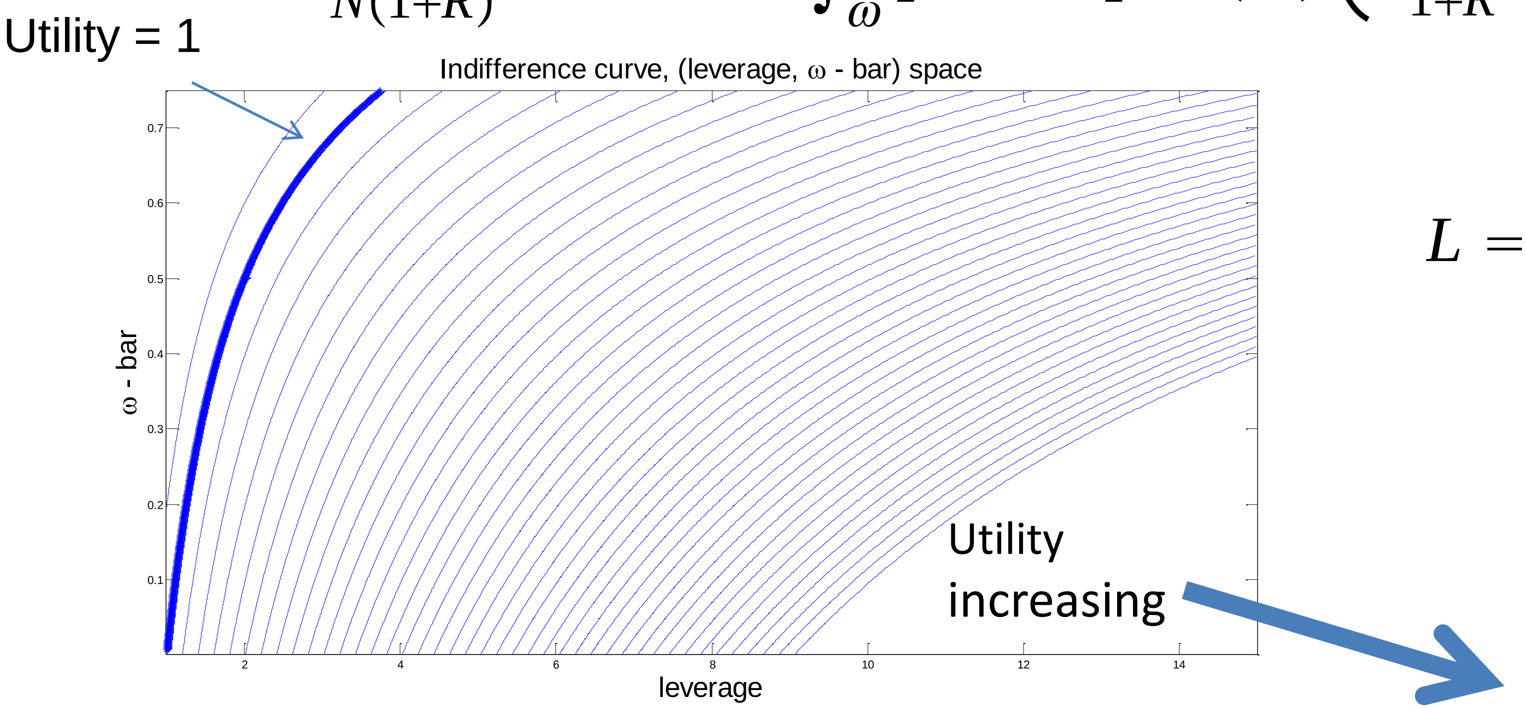
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$



$$L = \frac{A}{N} = \frac{N+B}{N}$$

Simplified Representation of Entrepreneur Utility

- Utility:

$$\int_{\bar{\omega}}^{\infty}[\omega-\bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}L$$

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$$\int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}L$$

$$= [1 - \Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L$$

- Where

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}(1 - F(\bar{\omega})) + G(\bar{\omega})$$

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega)$$

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Share of gross entrepreneurial earnings kept by entrepreneur

- Easy to show:

$$0 \leq \Gamma(\bar{\omega}) \leq 1$$

$$\Gamma'(\bar{\omega}) = 1-F(\bar{\omega}) > 0,\ \Gamma''(\bar{\omega}) < 0$$

$$\lim_{\bar{\omega}\to 0}\Gamma(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}\Gamma(\bar{\omega}) = 0$$

$$\lim_{\bar{\omega}\to 0}G(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}G(\bar{\omega}) = 1$$

Banks

- Source of funds from households, at fixed rate, *R*
- Bank borrows *B* units of currency, lends proceeds to entrepreneurs.
- Provides entrepreneurs with standard debt contract, *(Z,B)*

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1+R^k)\omega A$$

Banks, cont'd

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- Bank zero profit condition

$$\overbrace{[1-F(\bar{\omega})]}^{\text{fraction of entrepreneurs with } \omega>\bar{\omega}} \qquad \overbrace{ZB}^{\text{quantity paid by each entrepreneur with } \omega>\bar{\omega}}$$

$$+ \qquad \overbrace{(1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}^{\text{quantity recovered by bank from each bankrupt entrepreneur}}$$

$$= \qquad \overbrace{(1+R)B}^{\text{amount owed to households by bank}}$$

Banks, cont'd

- Zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

Banks, cont'd

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$$\frac{[1-F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}{B} = (1+R)$$

The risk free interest rate here is equated to the 'average return on entrepreneurial projects'.

Banks, cont'd

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The risk free interest rate here is equated to the 'average return on entrepreneurial projects'.
This is a source of inefficiency in the model. A benevolent planner would prefer that the market price observed by savers correspond to the *marginal* return on projects (Christiano-Ikeda).

Banks, cont'd

- Simplifying zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

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$$\overbrace{\left([1-F(\bar{\omega})]\bar{\omega}+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)\right)}^{\text{share of gross return, }(1+R^k)A\text{, (net of monitoring costs) given to bank}}(1+R^k)A = (1+R)B$$

Banks, cont'd

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$$[1-F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega) = \frac{1+R}{1+R^k}\frac{B/N}{A/N} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

Expressed naturally in terms of $(\bar{\omega}, L)$

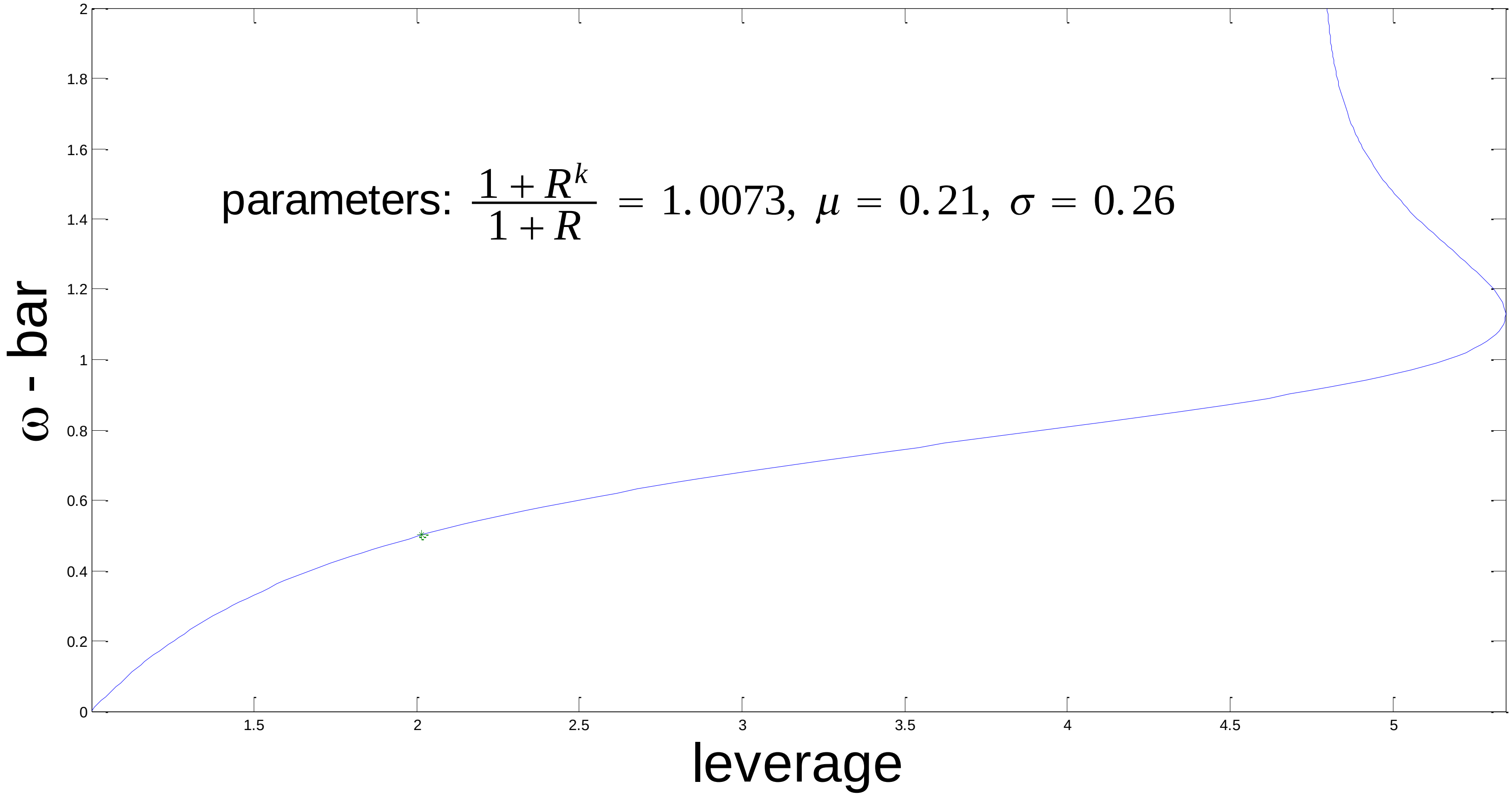
Expressing Zero Profit Condition In Terms of New Notation

$$\overbrace{(1-F(\bar{\omega}))\bar{\omega}+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)}^{\text{share of entrepreneurial profits (net of monitoring costs) given to bank}} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$\Gamma(\bar{\omega})-\mu G(\bar{\omega}) = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

Bank zero profit condition, in (leverage, ω - bar) space



Our value of $\frac{1+R^k}{1+R}$, 290 basis points at an annual rate, is a little higher than the 200 basis point value adopted in BGG (1999, p. 1368); the value of μ is higher than the one adopted by BGG, but within the range, 0.20-0.36 defended by Carlstrom and Fuerst (AER, 1997) as empirically relevant; the value of $Var(\log\omega)$ is nearly the same as the 0.28 value assumed by BGG (1999,p.1368).

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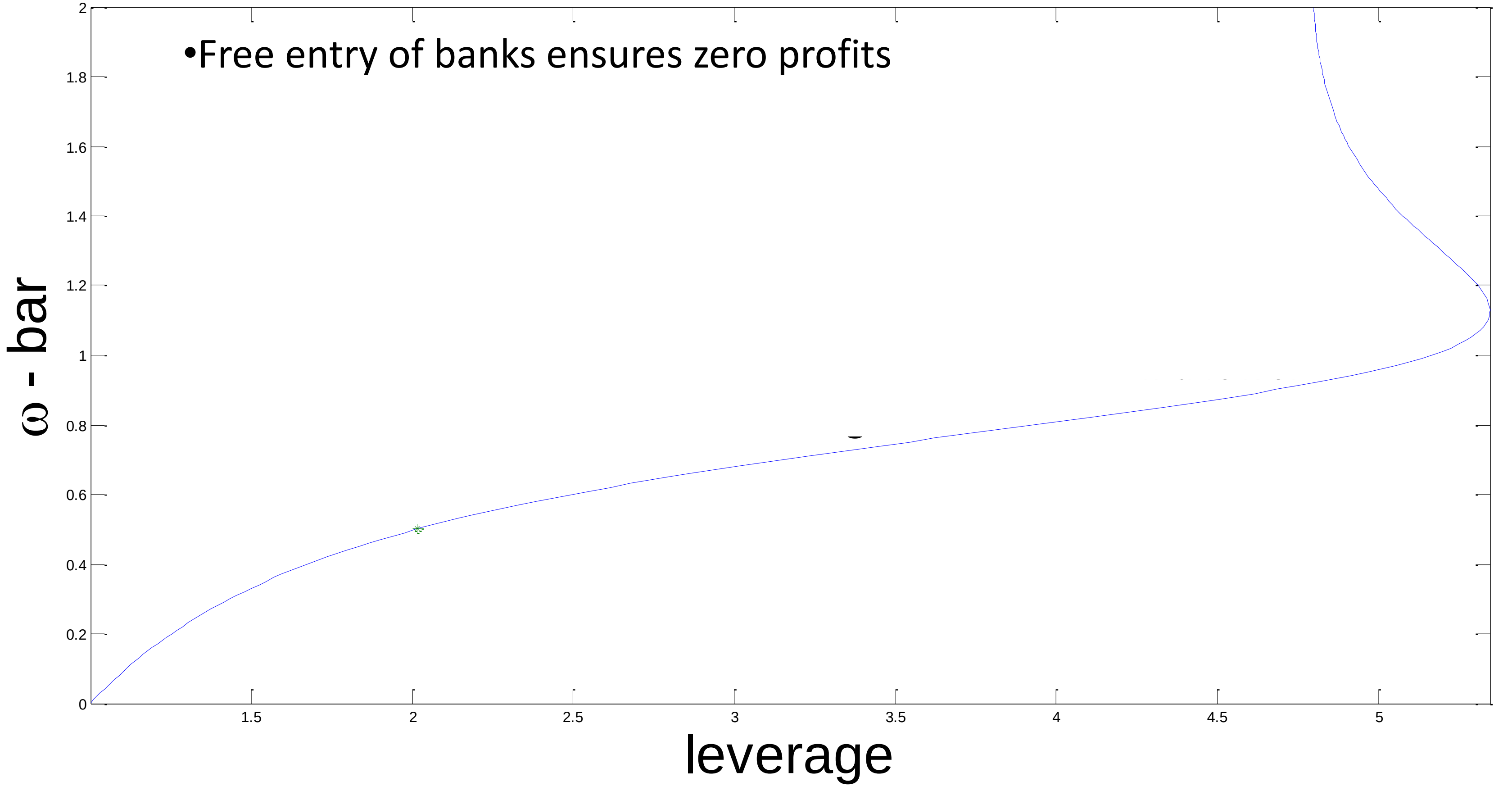

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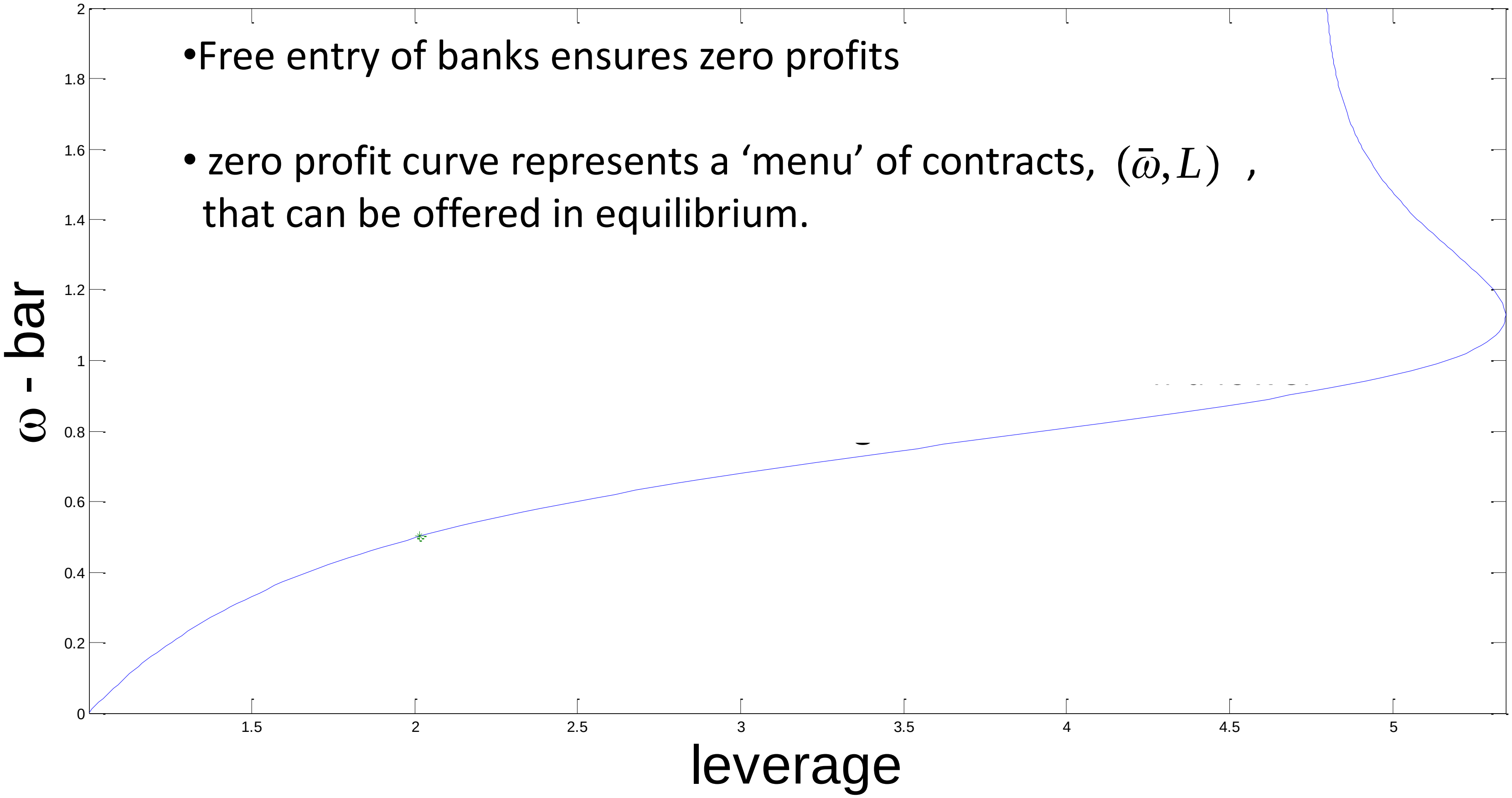
- Free entry of banks ensures zero profits
- zero profit curve represents a 'menu' of contracts, $(\bar{\omega}, L)$, that can be offered in equilibrium.



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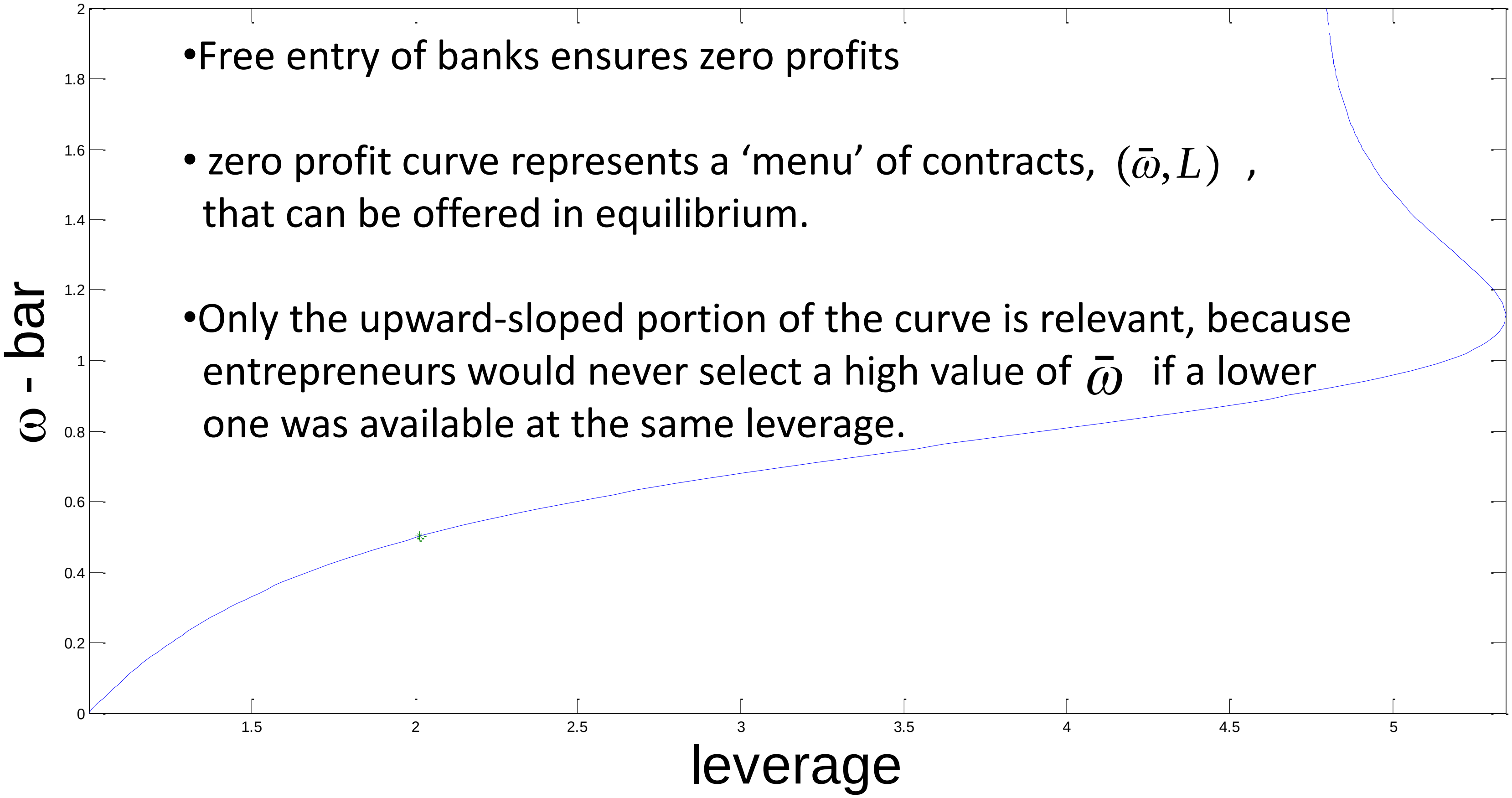

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- zero profit curve represents a 'menu' of contracts, $(\bar{\omega}, L)$, that can be offered in equilibrium.

- Only the upward-sloped portion of the curve is relevant, because entrepreneurs would never select a high value of $\bar{\omega}$ if a lower one was available at the same leverage.

Deriving the Basic Shape of the Zero Profit Function Analytically

- First, some simple notation.

- Then, the results.

Some Notation and Results

- Let:

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- Result:

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$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) - \bar{\omega} F'(\bar{\omega}) + G'(\bar{\omega}) = 1 - F(\bar{\omega}) \geq 0$$

$$\Gamma''(\bar{\omega}) = -F'(\bar{\omega}) < 0$$

Some Notation and Results

- Let:

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega) = \left[\int_0^{\bar{\omega}} \omega \overbrace{\frac{dF(\omega)}{F(\bar{\omega})}}^{\text{density of } \omega \text{, conditional on } \omega \leq \bar{\omega}} \right] F(\bar{\omega}) = \overbrace{E[\omega | \omega < \bar{\omega}]}^{\text{expected value of } \omega \text{, conditional on } \omega < \bar{\omega}} F(\bar{\omega})$$

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega) = \bar{\omega}[1 - F(\bar{\omega})] + E[\omega | \omega < \bar{\omega}] F(\bar{\omega})$$

- Result:

$$G'(\bar{\omega}) = \frac{d}{d\omega} \int_0^{\bar{\omega}} \omega dF(\omega) \overbrace{=}^{\text{Leibniz's rule}} \bar{\omega} F'(\bar{\omega})$$

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) - \bar{\omega} F'(\bar{\omega}) + G'(\bar{\omega}) = 1 - F(\bar{\omega}) \geq 0$$

$$\Gamma''(\bar{\omega}) = -F'(\bar{\omega}) < 0$$

$\rightarrow \Gamma(\bar{\omega})$ increasing and concave

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

$$= 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]$$

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

$$= 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] - \bar{\omega}[1 - F(\bar{\omega})]$$

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

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- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

$$= 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= (E[\omega|\omega > \bar{\omega}] - \bar{\omega})[1 - F(\bar{\omega})] \geq 0$$

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

$$= 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= (E[\omega|\omega > \bar{\omega}] - \bar{\omega})[1 - F(\bar{\omega})] \geq 0$$

- Conclude: $0 \leq \Gamma(\bar{\omega}) \leq 1$, for all $\bar{\omega} \geq 0$.

Limiting Properties

- According to our previous result:

$$0 \leq \overbrace{\bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega)}^{\Gamma(\bar{\omega})} \leq 1, \text{ for all } \bar{\omega} \geq 0$$

- So that,

$$\bar{\omega}[1 - F(\bar{\omega})] \leq 1 - \int_0^{\bar{\omega}} \omega dF(\omega) \rightarrow 0, \text{ as } \bar{\omega} \rightarrow \infty$$

- But, $0 \leq \bar{\omega}[1 - F(\bar{\omega})] \leq 1 - \int_0^{\bar{\omega}} \omega dF(\omega)$, so

$$\lim_{\bar{\omega} \rightarrow \infty} \bar{\omega}[1 - F(\bar{\omega})] = 0.$$

- Conclude:

$$\lim_{\bar{\omega} \rightarrow \infty} \Gamma(\bar{\omega}) = \lim_{\bar{\omega} \rightarrow \infty} \bar{\omega}[1 - F(\bar{\omega})] + \lim_{\bar{\omega} \rightarrow \infty} G(\bar{\omega})$$

$$= 0 + 1 = 1.$$

More Limiting Properties

- Obvious results:

$$\lim_{\bar{\omega}\to\infty} G(\bar{\omega}) = 1,\ \lim_{\bar{\omega}\to 0} G(\bar{\omega}) = 0,\ \text{where } G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega)$$

$$\lim_{\bar{\omega}\to 0} \Gamma(\bar{\omega}) = \lim_{\bar{\omega}\to 0}\left(\bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega)\right) = 0$$

- Finally,

$$\lim_{\bar{\omega}\to 0}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = 0$$

$$\lim_{\bar{\omega}\to\infty}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = 1 - \mu$$

- Formula for *L* indicates that we want to know about $q(\bar{\omega}) \equiv \Gamma(\bar{\omega}) - \mu G(\bar{\omega})$
- The hazard function is increasing for log normal *F* (see BGG):

$$h(\bar{\omega}) = \frac{\bar{\omega}F'(\bar{\omega})}{1-F(\bar{\omega})}$$

- Differentiate $q(\bar{\omega})$:

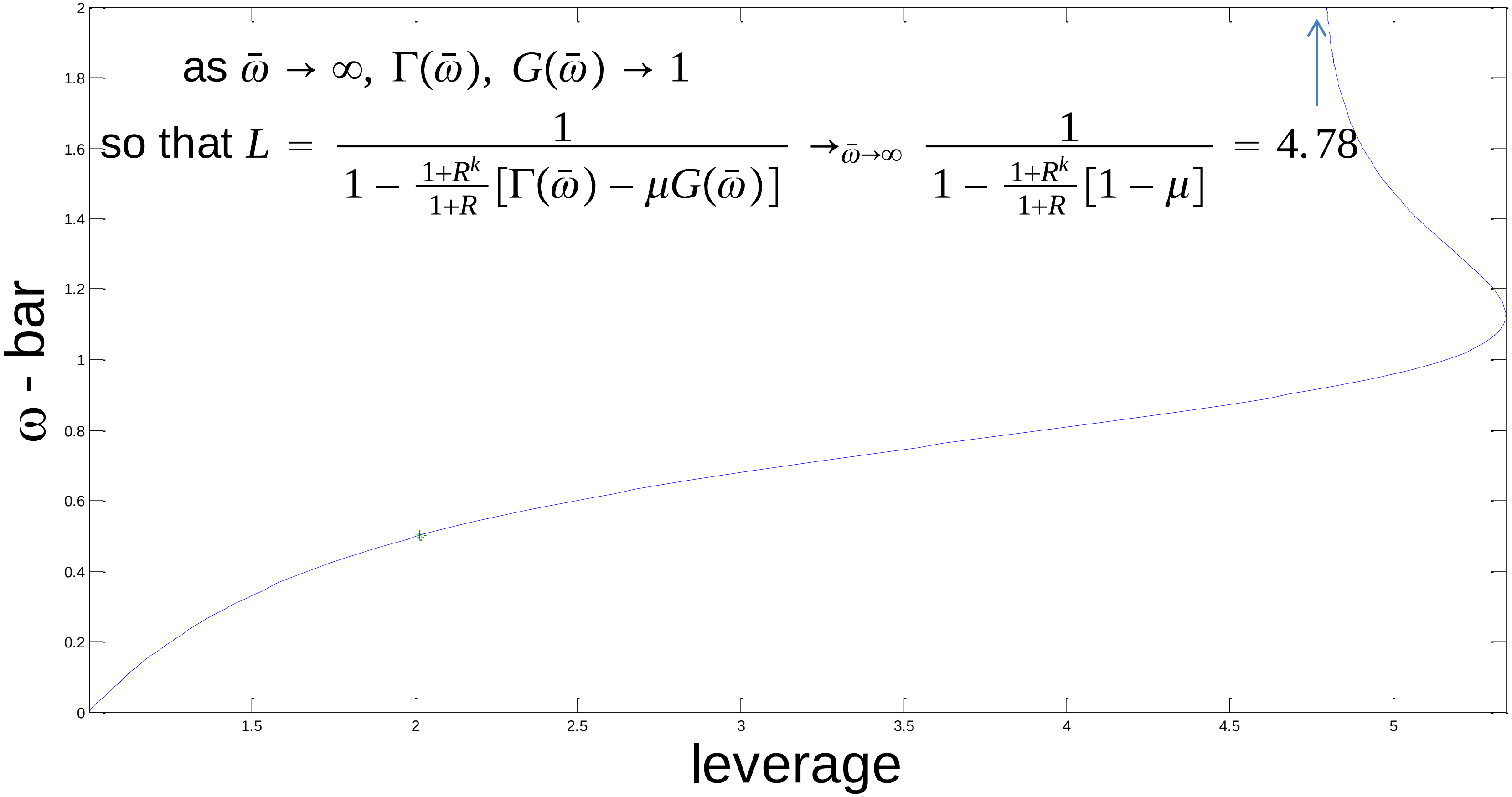
$$\begin{aligned} q'(\bar{\omega}) &= 1 - F(\bar{\omega}) - \mu\bar{\omega}F'(\bar{\omega}) \\ &= 1 - F(\bar{\omega}) - \mu h(\bar{\omega})(1 - F(\bar{\omega})) \\ &= [1 - F(\bar{\omega})][1 - \mu h(\bar{\omega})] \end{aligned}$$

- So, $q(\bar{\omega})$ initially rises and then falls. *L* does too, explaining the basic shape of the zero profit function (see BGG(1999, p. 1382)).

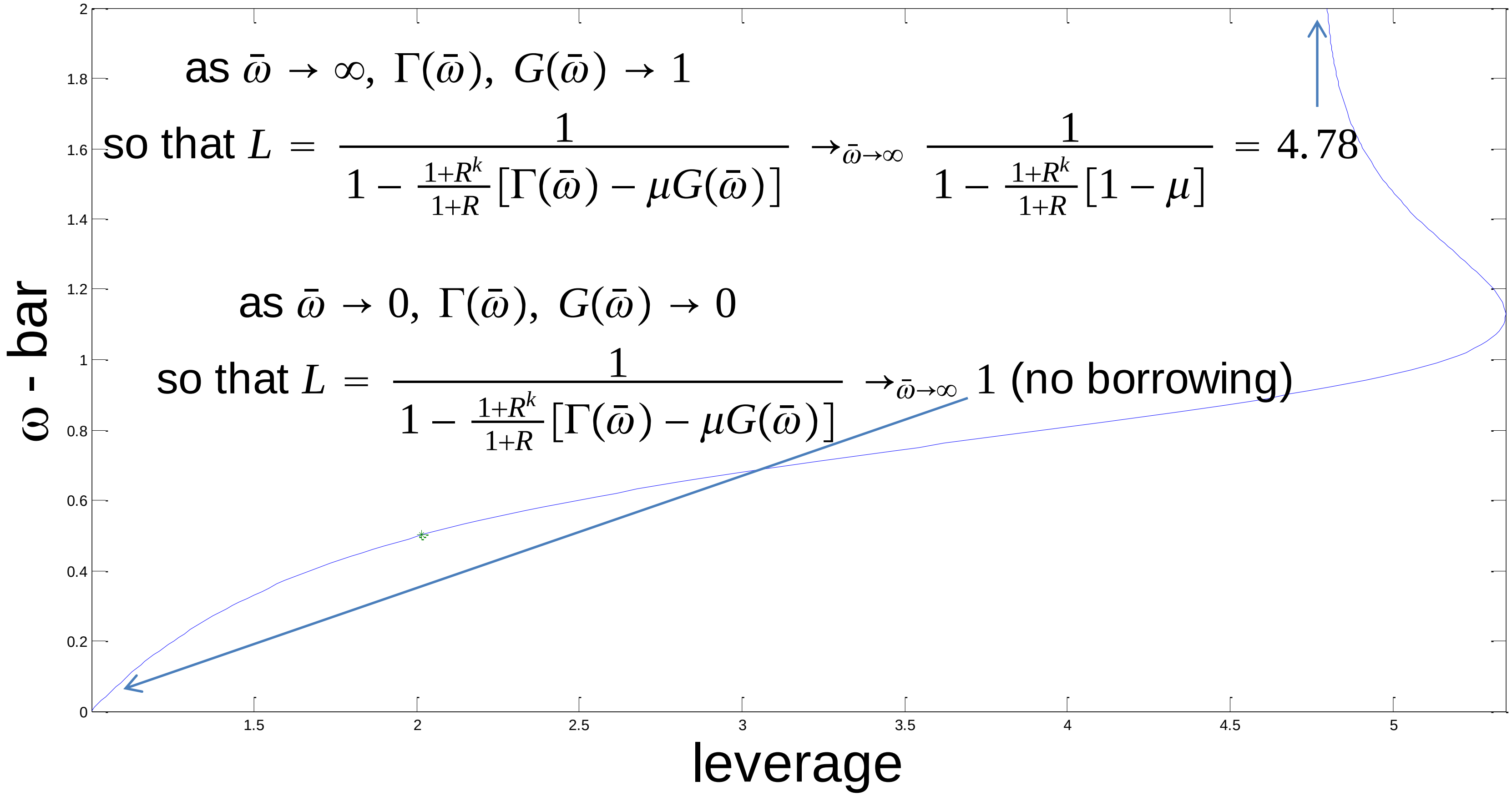
Bank zero profit condition, in (leverage, ω - bar) space

as $\bar{\omega} \to \infty,\ \Gamma(\bar{\omega}),\ G(\bar{\omega}) \to 1$

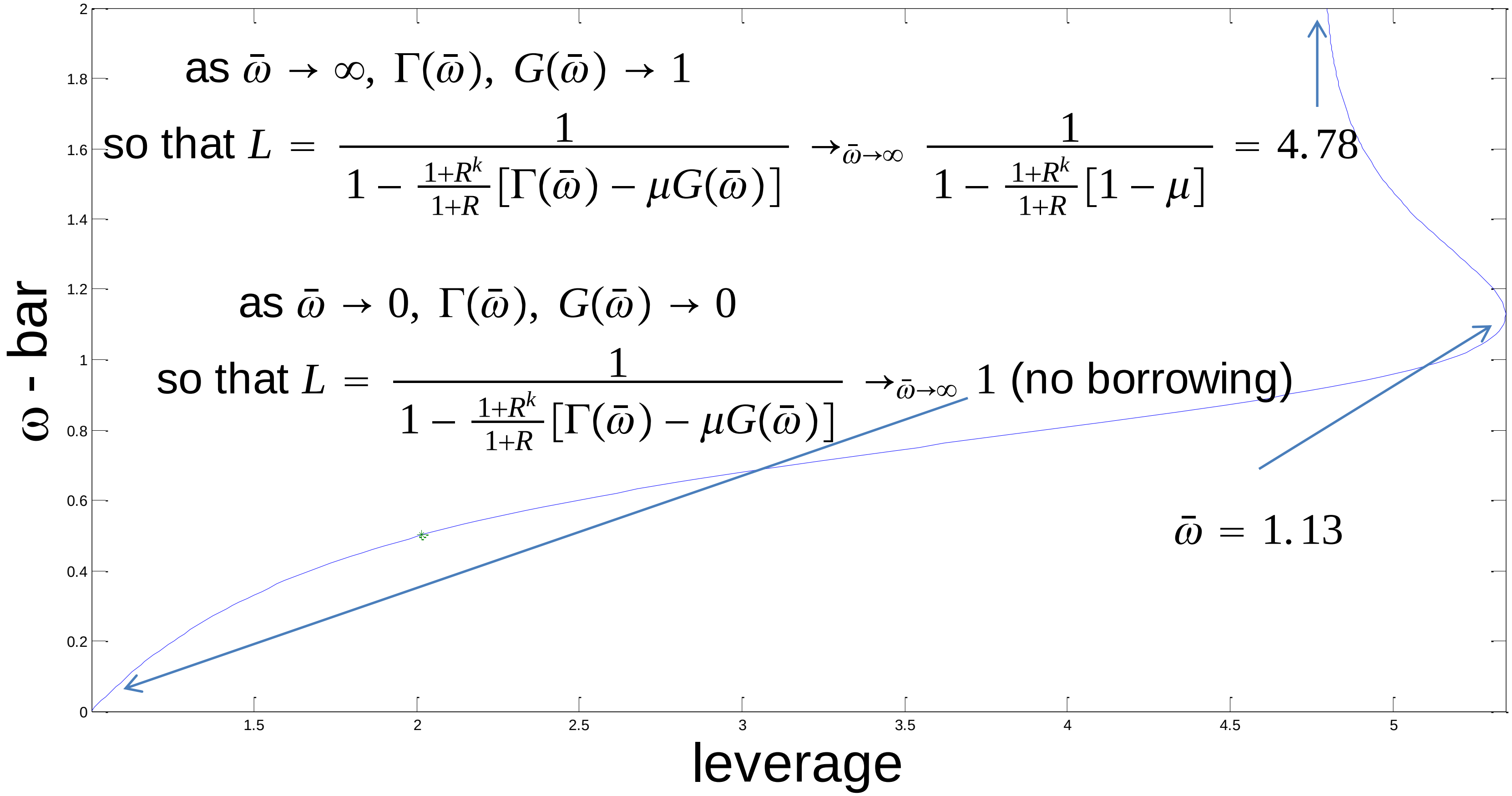
so that $L = \dfrac{1}{1 - \frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right]} \to_{\bar{\omega}\to\infty} \dfrac{1}{1 - \frac{1+R^k}{1+R}\left[1 - \mu\right]} = 4.78$



Bank zero profit condition, in (leverage, ω - bar) space



Bank zero profit condition, in (leverage, ω - bar) space



conclude: possible equilibrium $\bar{\omega}$'s, $[0, 1.13]$

Entrepreneurial utility in the New Notation

- Expected gain from operating investment project, divided by gain from depositing net worth in bank:

$$\int_{\bar{\omega}}^{\infty}[\omega-\bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}L$$

$$=\left(1-G(\bar{\omega})-\bar{\omega}[1-F(\bar{\omega})]\right)\frac{1+R^k}{1+R}L$$

$$=\overbrace{[1-\Gamma(\bar{\omega})]}^{\text{share of entrepreneur return going to entrepreneur}}\frac{1+R^k}{1+R}L$$

Equilibrium Contract

- Entrepreneur selects the contract is optimal, given the available menu of contracts.

- The solution to the entrepreneur problem is the $\bar{\omega}$ that maximizes, over the relevant domain (i.e., $\bar{\omega} \in [0, 1.13]$ in the example):

$$\log\left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

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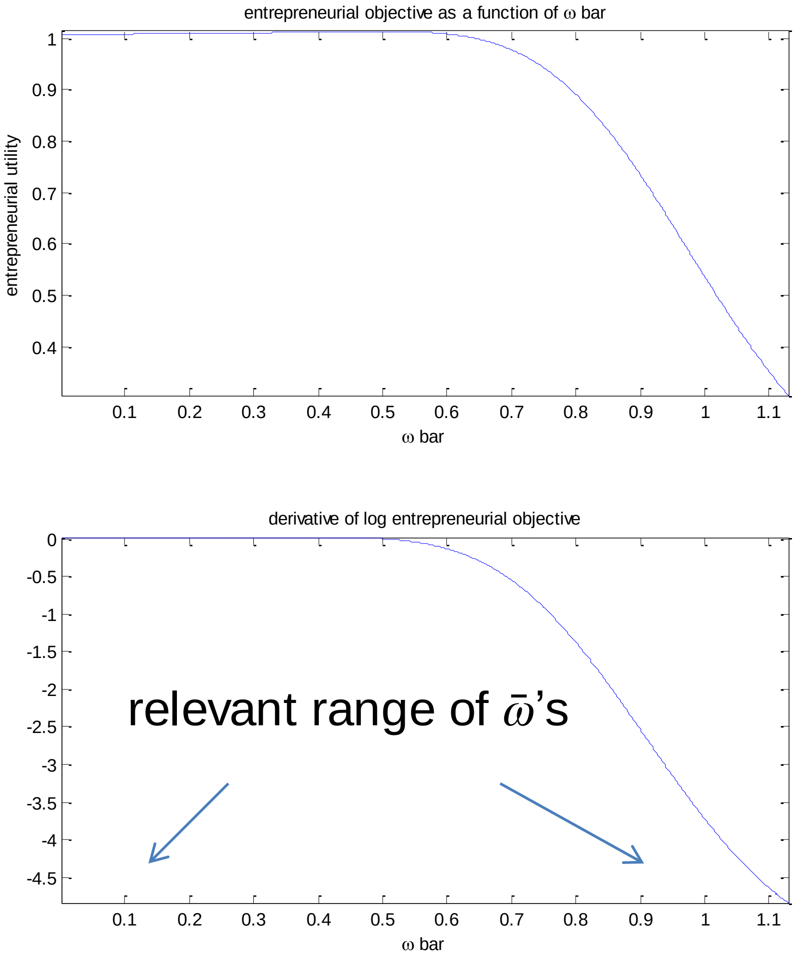
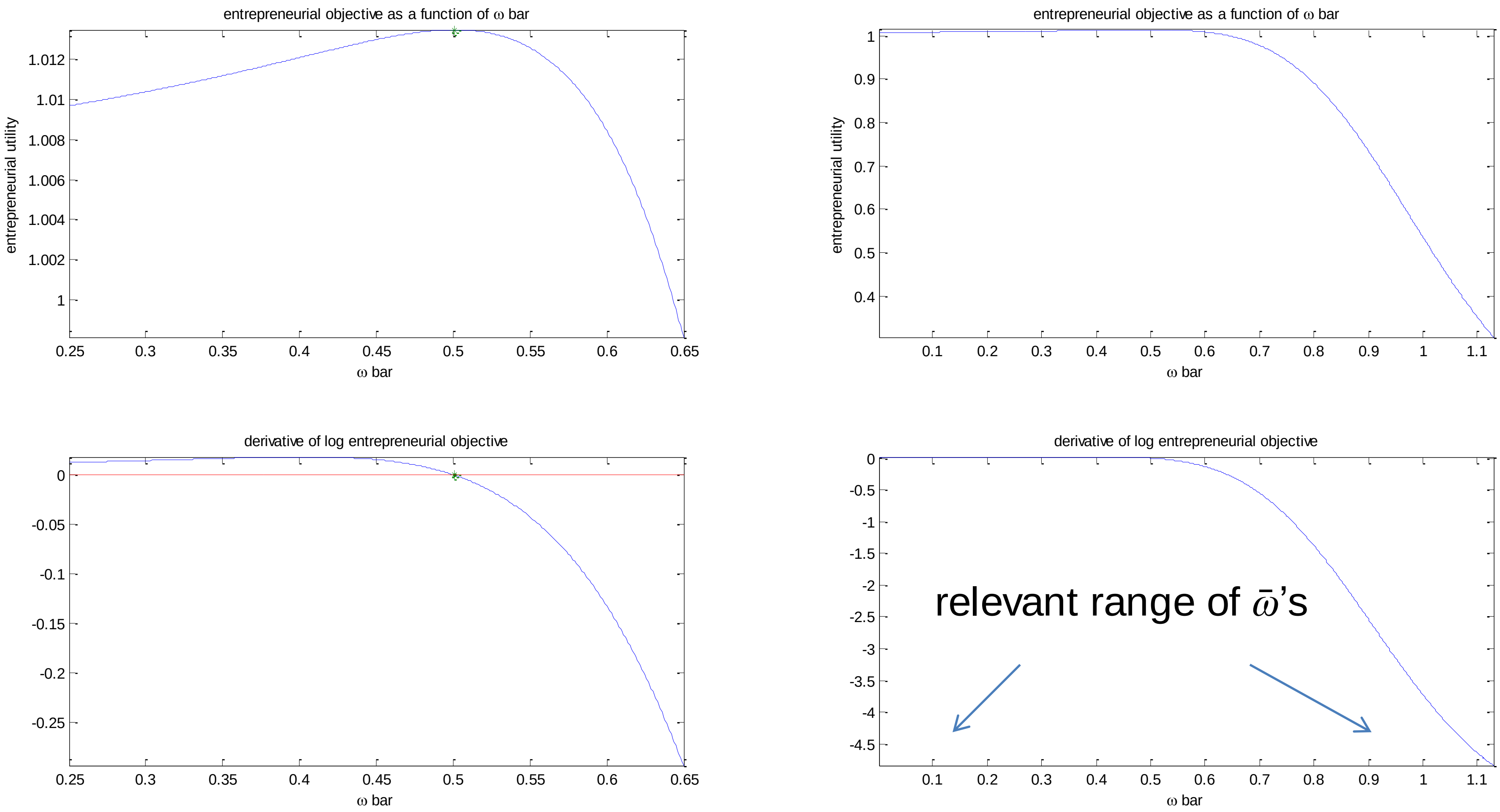
$$= \log \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1+R^k}{1+R} \overbrace{-\log\left(1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Entrepreneur Objective



Entrepreneur Objective



Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\overbrace{\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R}\left[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})\right]}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R}\bar{\omega}\frac{L}{L-1}$$

Result

- Leverage, *L*, and entrepreneurial rate of interest, *Z*, **not a function of net worth**, *N*.

- Quantity of loans proportional to net worth:

$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

$$B = (L-1)N$$

- To compute *L*, *Z/(1+R)*, must make assumptions about *F* and parameters.

$$\frac{1+R^k}{1+R},\ \mu,\ F$$

Formulas Needed to do the Computations

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega),\ F'(\omega)$

Can get these from the pdf and the cdf of the standard normal distribution.

These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.

Results for log-normal

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega),\ F'(\omega)$

$$\int_0^{\bar{\omega}} \omega dF(\omega) \overbrace{=}^{\text{change of variables, } x=\log\omega} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x e^{\frac{-(x-Ex)^2}{2\sigma_x^2}} dx$$

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$$\underbrace{=}_{E\omega=1 \text{ requires } Ex=-\frac{1}{2}\sigma_x^2} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x e^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

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$$\underbrace{=}_{\text{combine powers of } e \text{ and rearrange}} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^{\frac{-\left(x-\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

Results for log-normal

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$$\int_0^{\bar{\omega}} \omega dF(\omega) \underbrace{=}^{\text{change of variables, } x=\log\omega} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x e^{\frac{-(x-Ex)^2}{2\sigma_x^2}} dx$$

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$$\underbrace{=}^{\text{combine powers of } e \text{ and rearrange}} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^{\frac{-\left(x-\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx$$

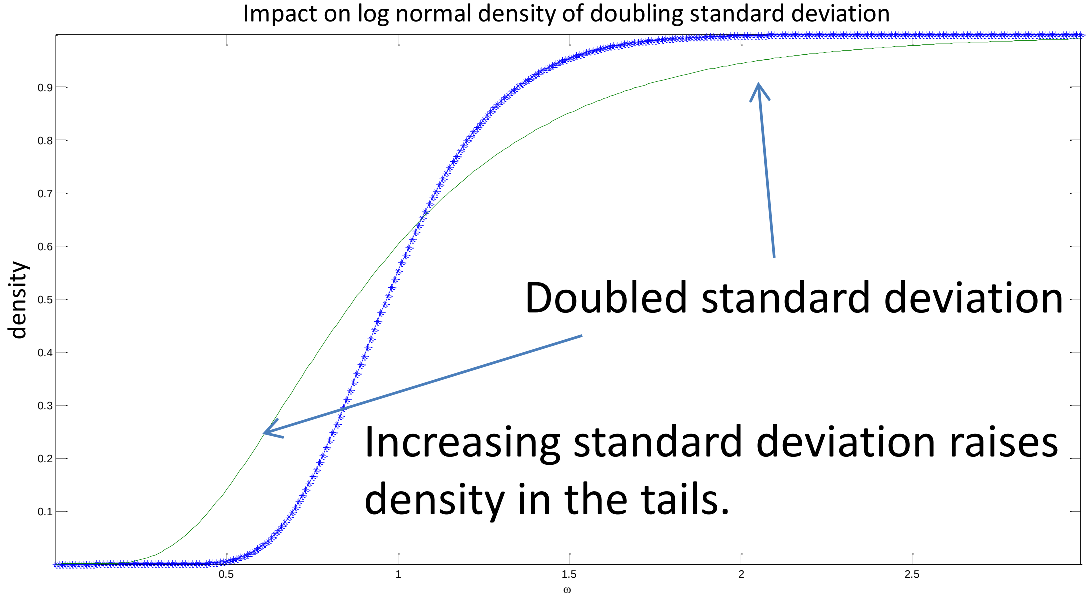
$$\underbrace{=}^{\text{change of variables, } v=\frac{x-\frac{1}{2}\sigma_x^2}{\sigma_x}} \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} \sigma_x dv$$

Effect of Increase in Risk, σ

- Keep

$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.



Jump in Risk

- σ replaced by $\sigma \times 3$

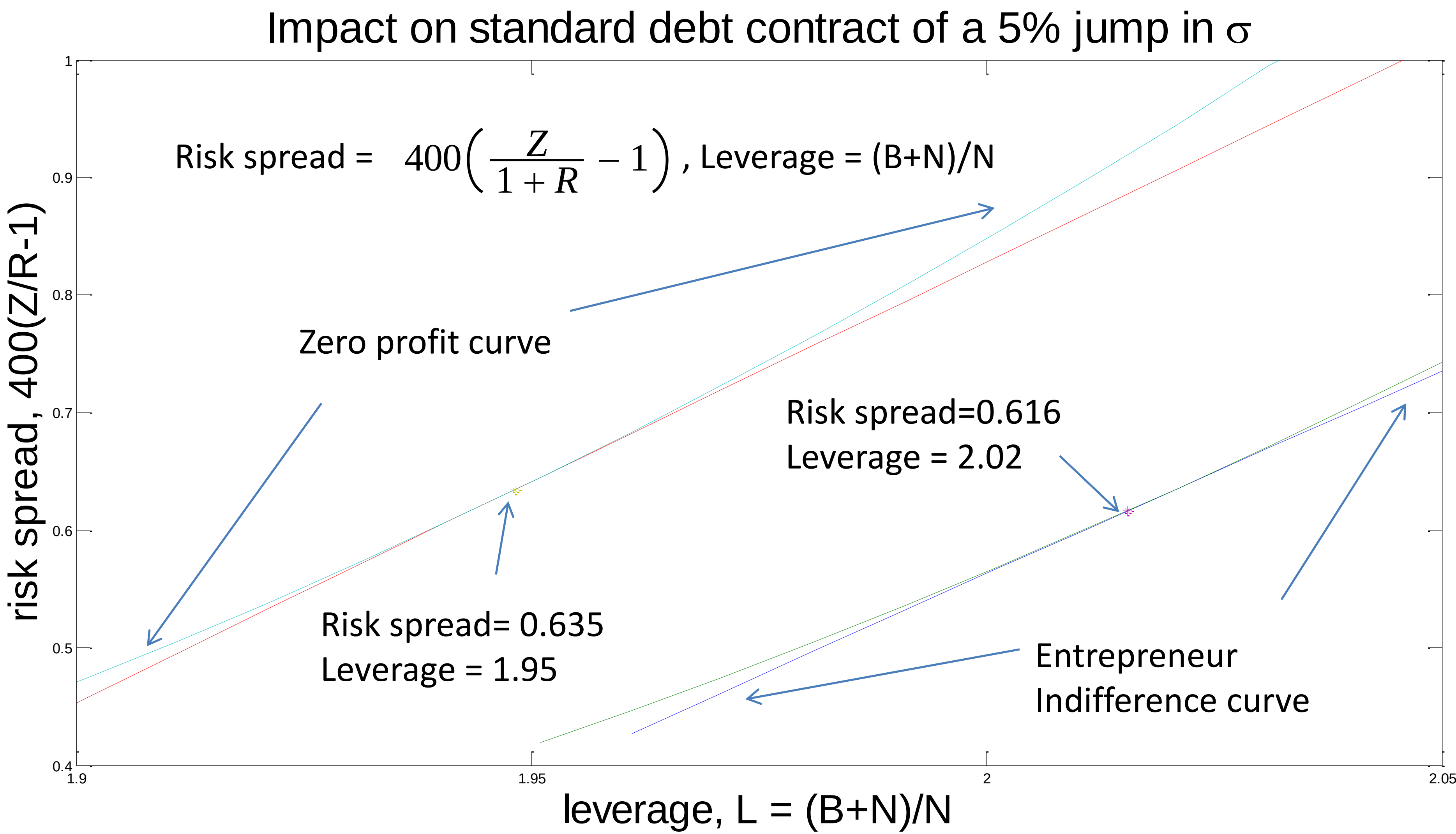
$$\overbrace{\bar{\omega} = 0.12}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.12}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0108}^{\text{bankruptcy rate: 1.08\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0011}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 1.1418}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0041}^{1.66 \text{ (APR)}}, \quad \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0080}^{\text{avg earnings of entrepreneur, per unit of net worth}} > 1$$

- Comparison with benchmark:

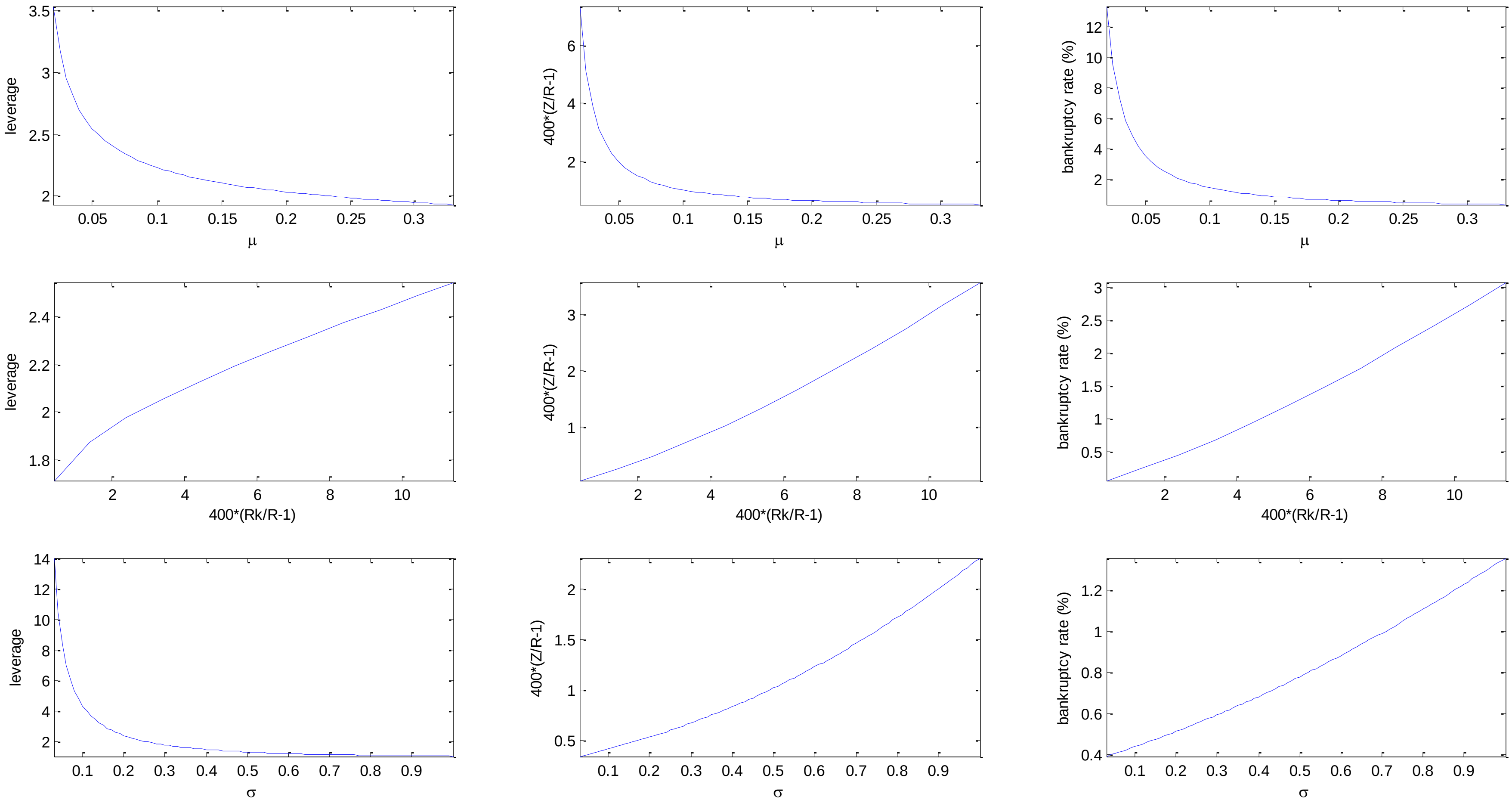
$$\overbrace{\bar{\omega} = 0.50}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.5008}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0056}^{\text{bankruptcy rate: 0.56\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0026}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 2.02}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0015}^{0.62 \text{ (APR)}}, \quad \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0135}^{\text{avg earnings of entrepreneur, divided by opportunity cost}} > 1$$



Impact on standard debt contract of a 5% jump in σ
Risk spread = $400\left(\frac{Z}{1+R}-1\right)$, Leverage = (B+N)/N
Zero profit curve
Risk spread=0.616
Leverage = 2.02
Risk spread= 0.635
Leverage = 1.95
Entrepreneur
Indifference curve
risk spread, 400(Z/R-1)
leverage, L = (B+N)/N
1
0.9
0.8
0.7
0.6
0.5
0.4
1.9
1.95
2
2.05

Leverage and interest rate spread, for alternative parameter settings



Higher monitoring costs: shifts the menu of contracts up; entrepreneurs choose contracts with lower leverage and lower interest rate; bankruptcy rate falls.

Higher return projects: shifts menu of contracts down; entrepreneurs choose contracts with higher leverage, higher interest rate and higher bankruptcy rates.

Higher risk: shifts menu of contracts up; entrepreneurs choose contracts with lower leverage, higher interest rate, higher bankruptcy rates.

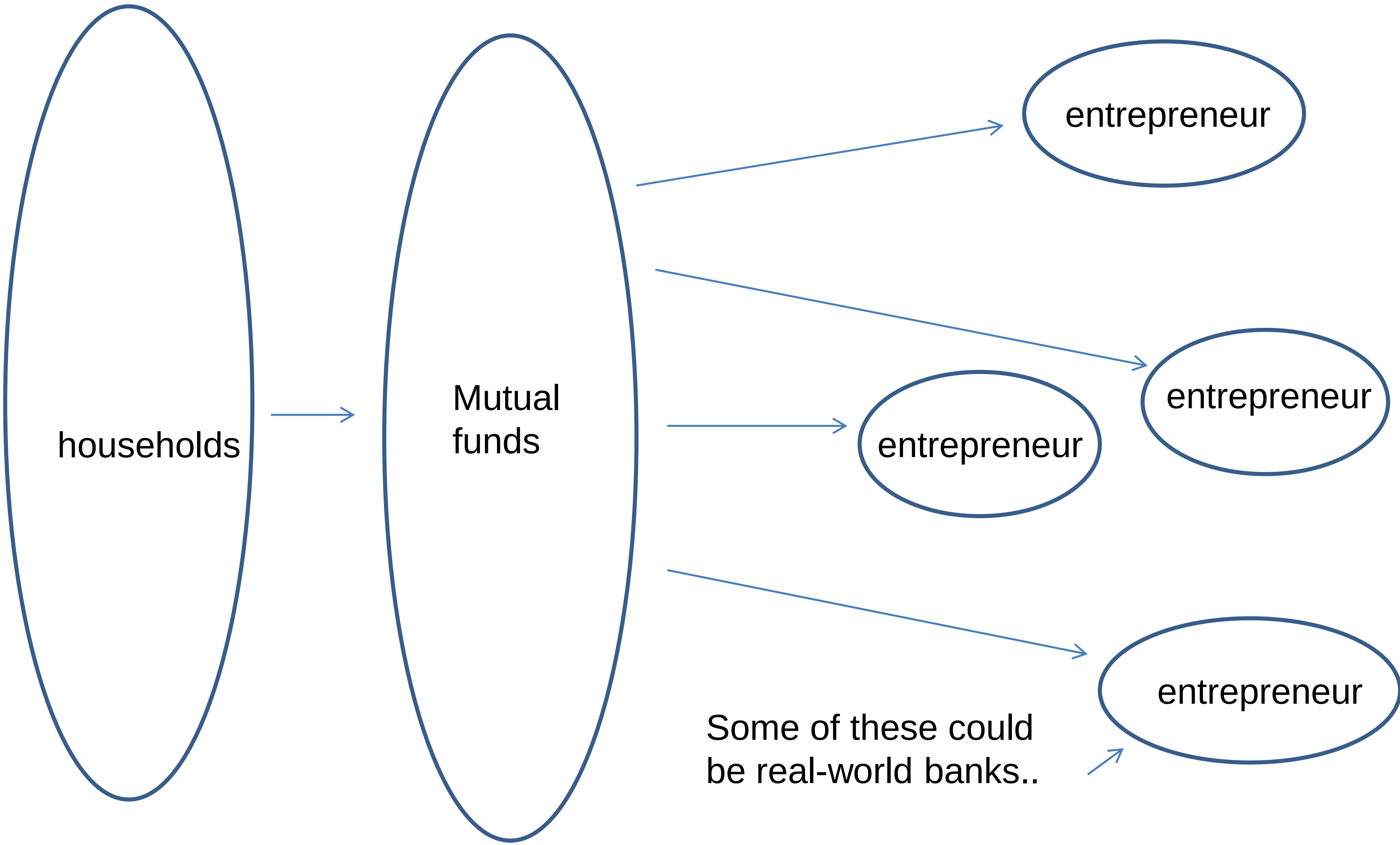
Possible Issues With the Model

- Strictly speaking, applies only to ‘mom and pop grocery stores’: entities run by entrepreneurs who are bank dependent for outside finance.
  - Not clear how to apply this to actual firms with access to equity markets.
- Assume no long-run connections with banks.
- Entrepreneurial returns independent of scale.
- Overly simple representation of entrepreneurial utility function (assumes entrepreneurs behave as though they are risk neutral)
- Ignores alternative sources of risk spread (risk aversion, liquidity)
- Seems not to allow for bankruptcies in banks.

The issue about ‘bankruptcies in banks’

- We have referred to the sources of funds for the entrepreneur as banks.
- Banking is risk-free in the model.
- Real-world banks seem risky….
- Could assume some entrepreneurs in the model are actually banks (like banks, entrepreneurs have assets, net worth and debt).
  - The assets could be the investment projects of borrowers from the banks.
- Next slide illustrates the relationship between entrepreneurs and households, intermediated through ‘mutual funds’.

Flow of Funds Through Financial Markets



Incorporating CSV Financial Frictions into Neoclassical Model

- Bernanke, Gertler and Gilchrist, 1999 Handbook of Macroeconomics Chapter.

- Outline
  – Broad overview of model.
  – Details of entrepreneur
    - Must take into account the heterogeneity of entrepreneurs by net worth, N.
    - Describe relationship of entrepreneur to household
  – Describe household and general equilibrium.

Standard, Neoclassical Model

$$\max_{\{c_t, B_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c_t) = \log(c_t)$$

subject to:

$$c_t + B_{t+1} + K_{t+1} - (1-\delta)K_t \leq w_t l_t + r_t K_t + (1 + R_{t-1})B_t$$

$$0 \leq l_t \leq 1$$

Standard, Neoclassical Model

$$\max_{\{c_t, B_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c_t) = \log(c_t)$$

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$$0 \leq l_t \leq 1$$

Optimization:

$$u'(c_t) = \beta u'(c_{t+1})(r_{t+1} + 1 - \delta),$$

$$u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$$

$$l_t = 1$$

Standard, Neoclassical Model

$$\max_{\{c_t, B_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \; u(c_t) = \log(c_t)$$

subject to:

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$$0 \le l_t \le 1$$

Optimization:

$$u'(c_t) = \beta u'(c_{t+1})(r_{t+1} + 1 - \delta),$$

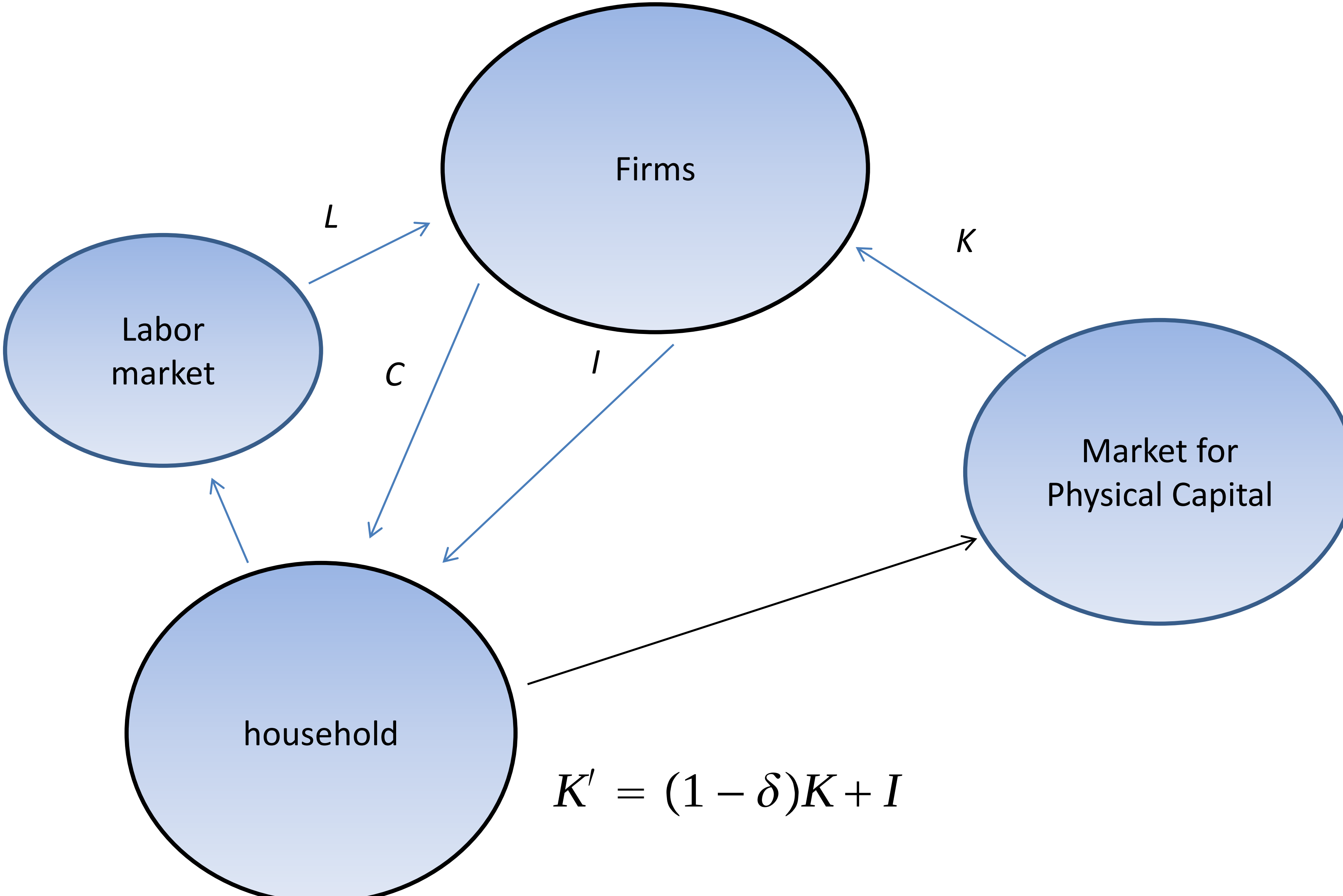
$$u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$$

$$l_t = 1$$

Firms and market clearing:

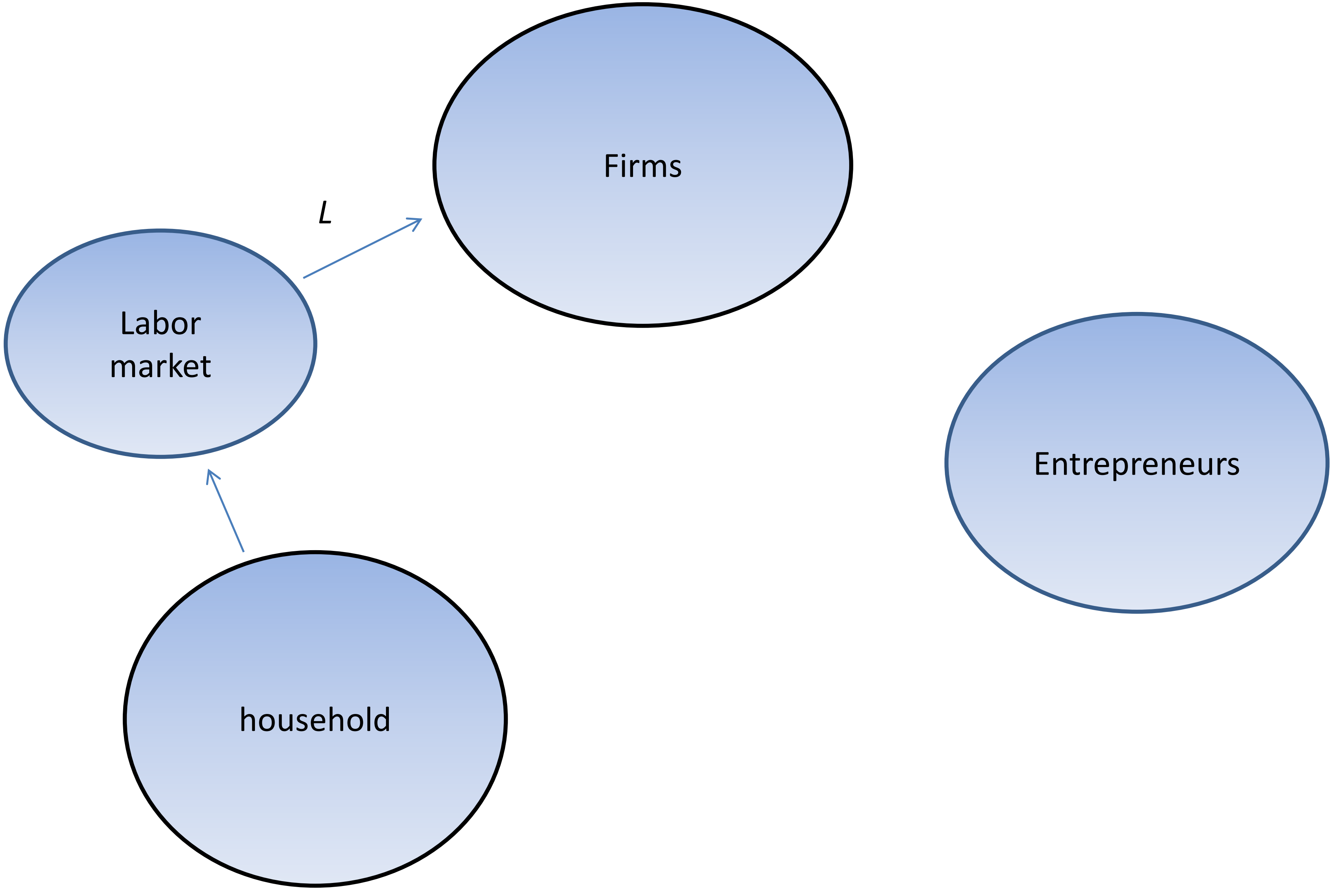
$$c_t + I_t = K_t^{\alpha} l_t^{1-\alpha}, \; B_{t+1} = 0, \; w_t = (1-\alpha)K_t^{\alpha}, \; r_t = \alpha K_t^{\alpha-1}$$

Standard Model

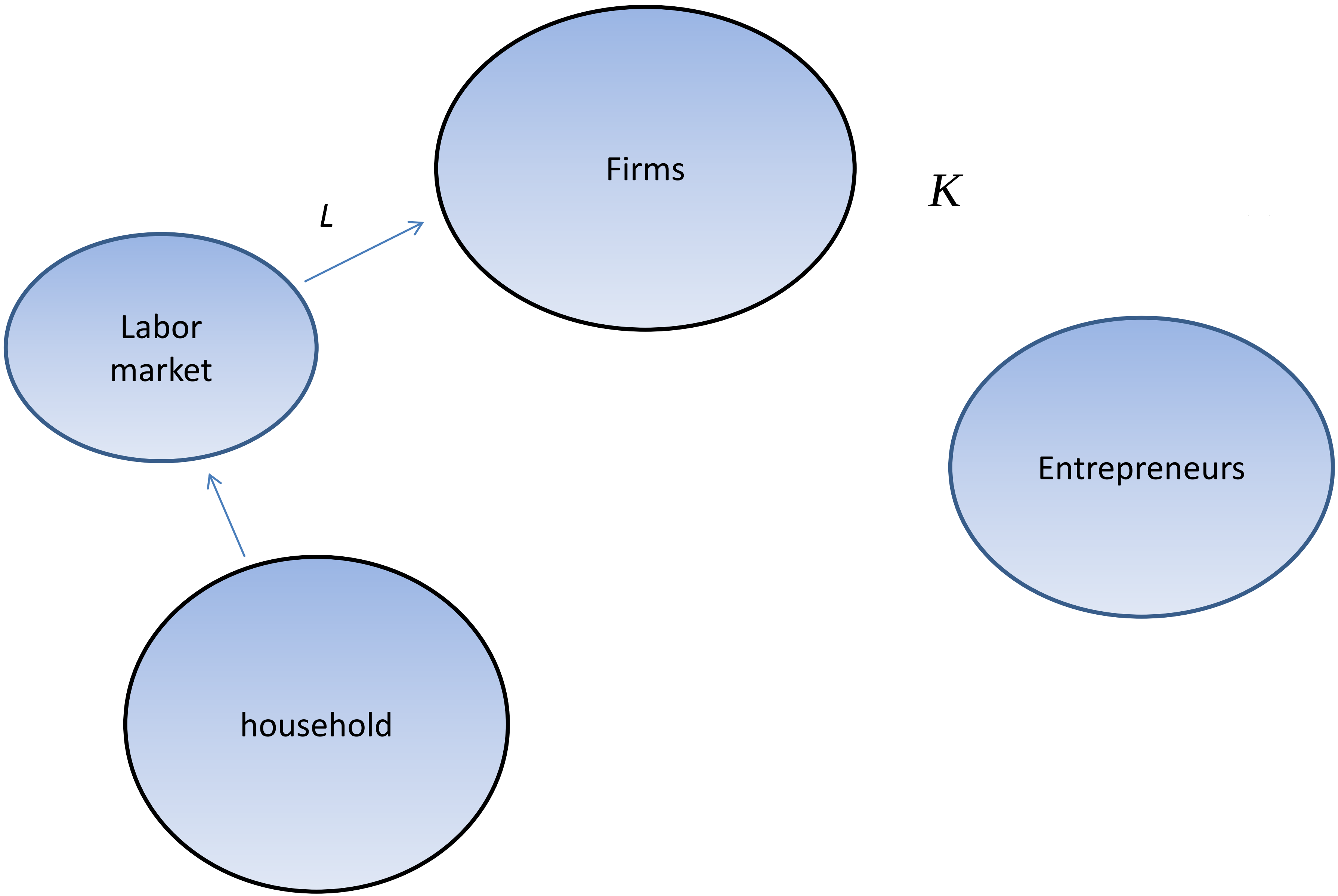


$$K' = (1-\delta)K + I$$

Standard Model with CSV



Standard Model with CSV



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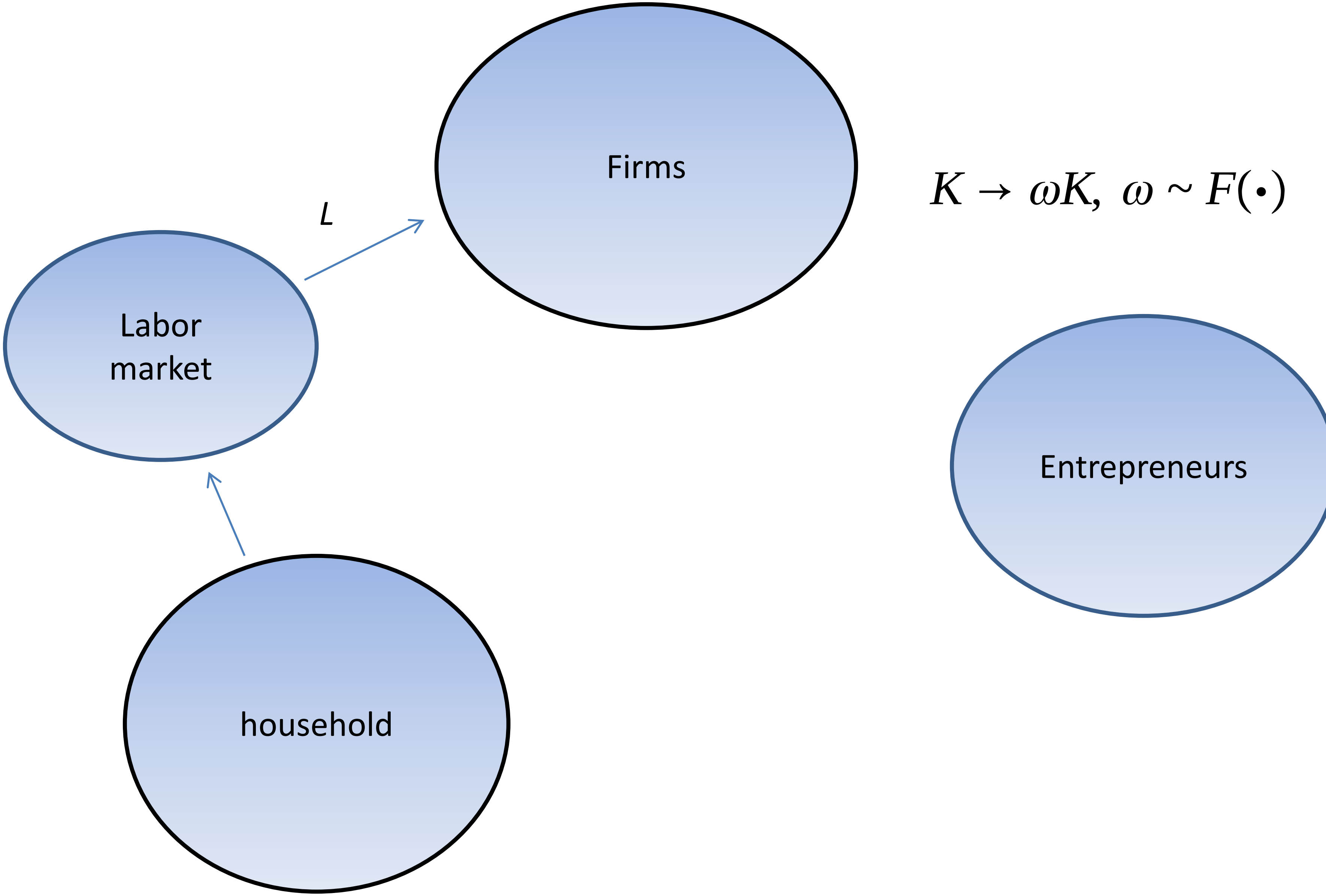

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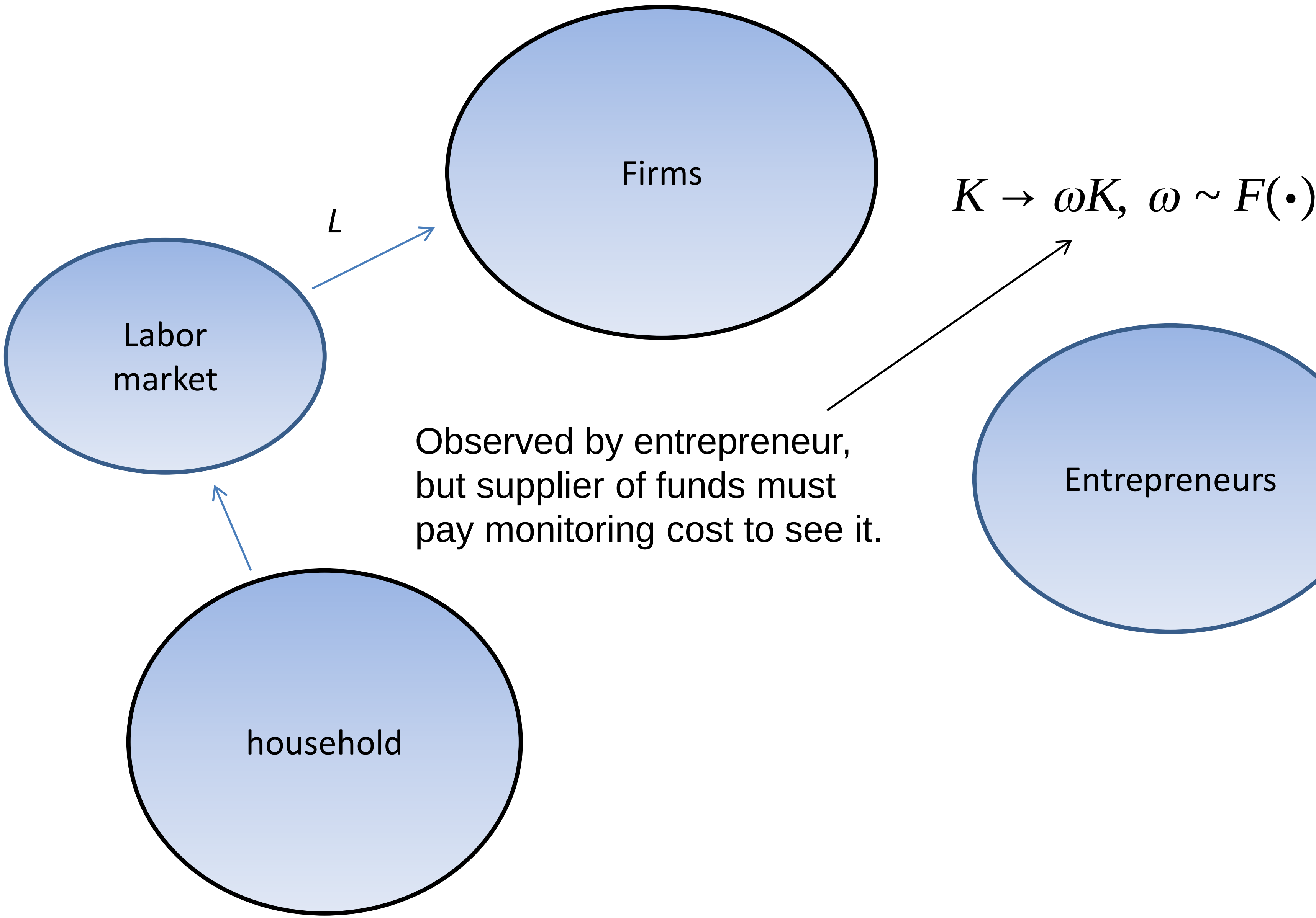

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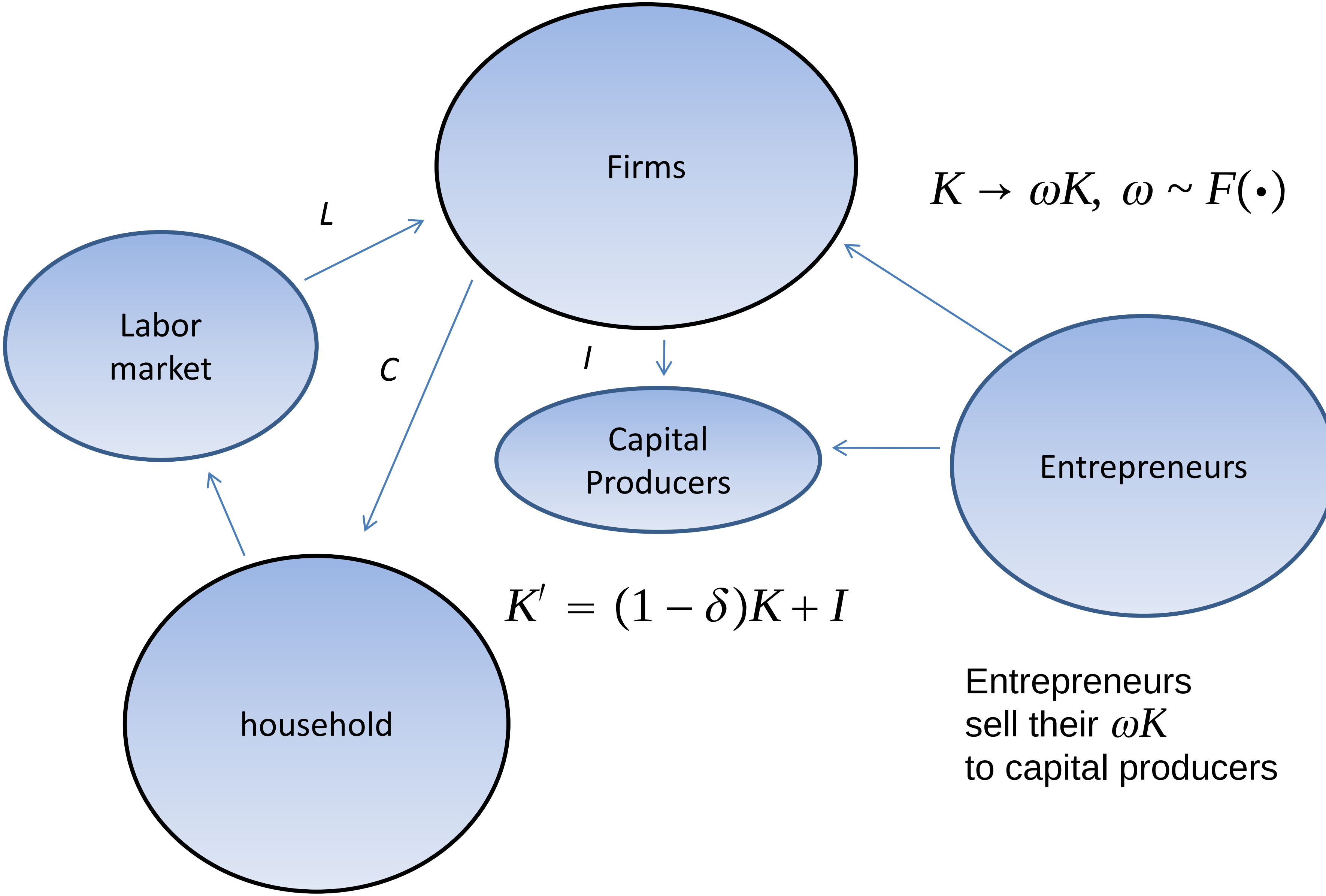
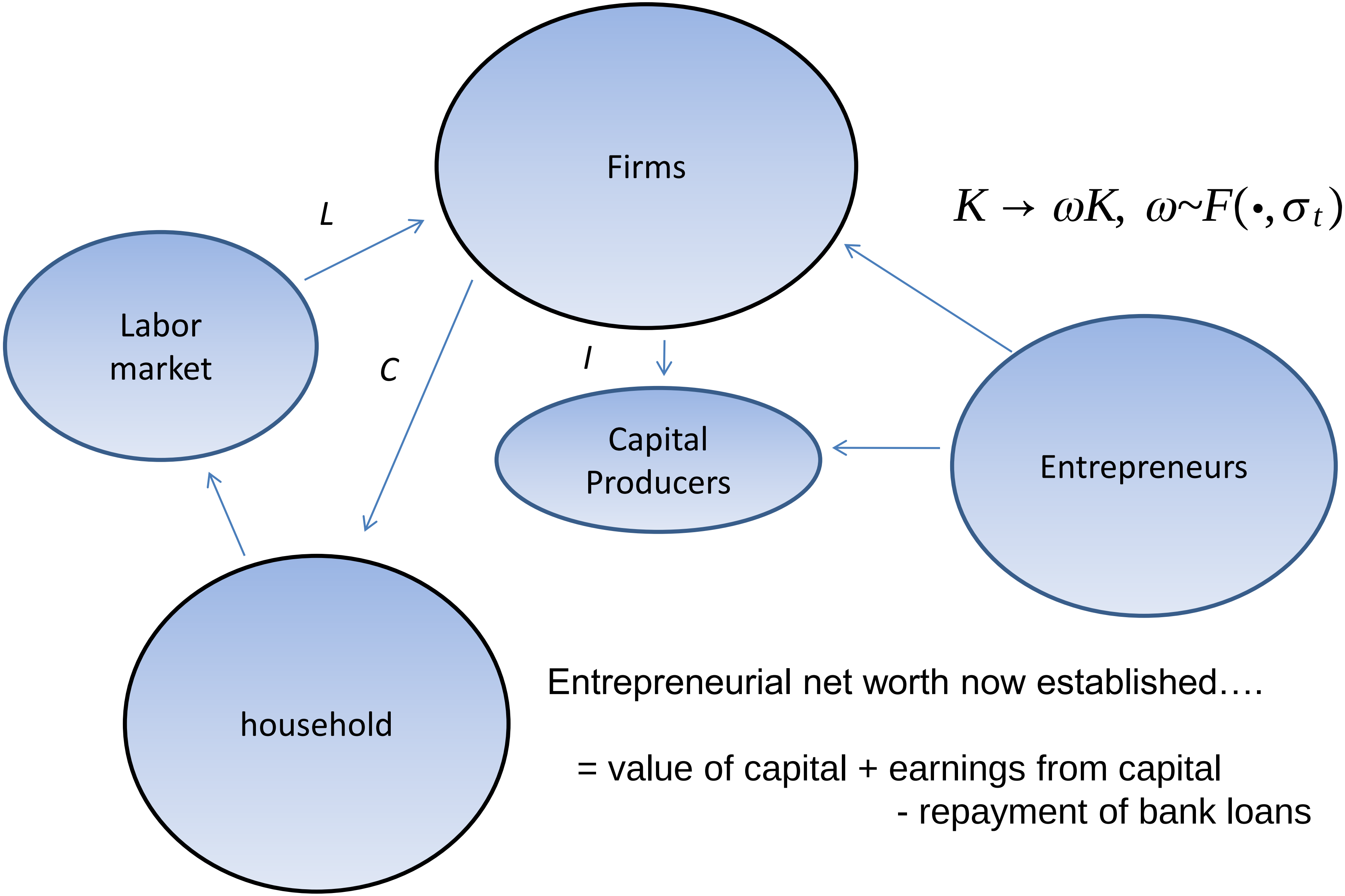


$$K \rightarrow \omega K, \ \omega \sim F(\bullet)$$

$$K' = (1-\delta)K + I$$

Entrepreneurs
sell their ωK
to capital producers

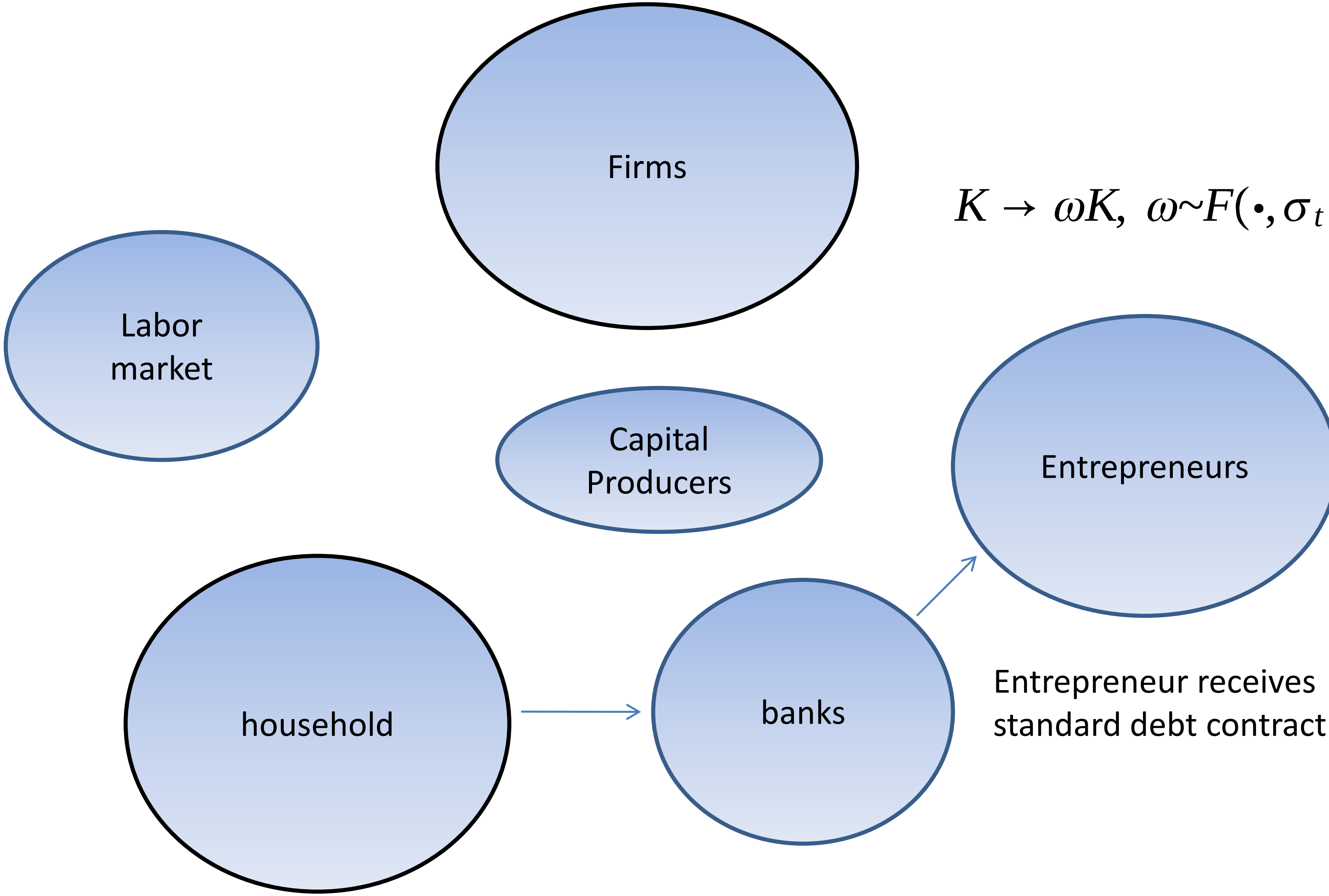
Standard Model with CSV



Entrepreneurial net worth now established….

= value of capital + earnings from capital
- repayment of bank loans

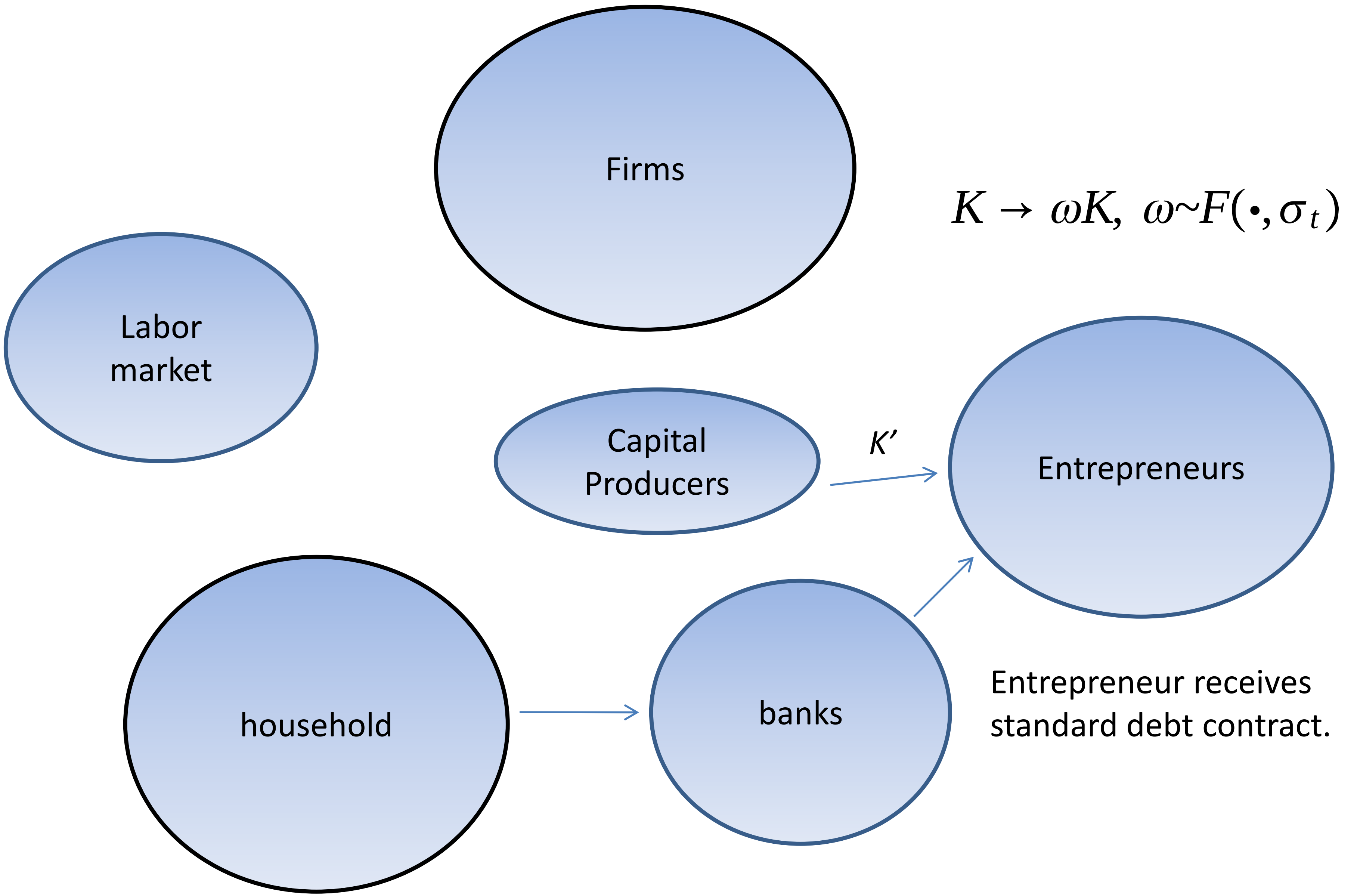
Standard Model with CSV



$$K \rightarrow \omega K, \ \omega \sim F(\bullet, \sigma_t)$$

Entrepreneur receives standard debt contract.

Standard Model with CSV



Details About the Entrepreneur

- Begin after period t production, when entrepreneurial net worth is known, and they go to banks for loans.

- End at the point in $t+1$ when net worth in $t+1$ is determined.

Entrepreneur at end of *t*

- Let $f_t(N)$ denote the number (density, actually) of entrepreneurs with net worth, N, $N \geq 0$.

- An entrepreneur with net worth, N, goes to the bank, receives a loan, B^N_{t+1}, and buys raw, physical capital:

$$K^N_{t+1} = N + B^N_{t+1}$$

- After purchasing the capital, the entrepreneur experiences an idiosyncratic shock, ω, so that physical capital is transformed into effective capital as follows:

$$\omega K^N_{t+1},\ \omega \sim F(\bullet, \sigma_t)$$

Entrepreneur in *t+1*

- The entrepreneur with net worth *N* in *t* rents

$$\omega K_{t+1}^{N}$$

  in a competitive capital rental market for rental rate, r_{t+1}.
- After goods production, entrepreneur sells undepreciated capital, $\omega K_{t+1}^{N}(1-\delta)$, at price unity.
- So, rate of return on capital for entrepreneur is:

$$\omega(1+R_{t+1}^{k}),\ 1+R_{t+1}^{k} \equiv r_{t+1}+1-\delta$$

Constant rate of return project, just like in micro example

Entrepreneur in *t+1,* cnt'd

- Period *t+1* net worth of entrepreneur with net worth *N* in period *t* and who experiences shock ω:

Gross rate of interest on loan

$$N' = \max\{0, (1+R_{t+1}^{k})K_{t+1}^{N}\omega - Z_{t+1}B_{t+1}^{N}\}$$

Entrepreneurial loan contract in *t*

- The banking system in period *t* is competitive.
  - The zero profit condition must be satisfied:

$$\text{Leverage} \equiv \frac{K_{t+1}^N}{N} = \frac{1}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})]}$$

  - Here:

$$(1+R_{t+1}^k)K_{t+1}^N\bar{\omega}_{t+1} = Z_{t+1}B_{t+1}^N$$

- Entrepreneurs treat the zero profit condition as a 'menu' of contracts. They select the contract at time *t* that maximizes expected *N'* given *N*.
- Entrepreneurial efficiency condition:

$$\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t}[1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})]}$$

Aggregate Net Worth, Loans and Capital

- The total amount of net worth held by all entrepreneurs at the end of time t:

$$N_{t+1} = \int_0^\infty N f_t(N) dN$$

Aggregate Net Worth, Loans and Capital

- The total amount of net worth held by all entrepreneurs at the end of time *t*:

$$N_{t+1} = \int_0^{\infty} N f_t(N) dN$$

- Total effective capital supplied by all entrepreneurs during *t+1*:

$$\int_0^{\infty} \int_0^{\infty} K_{t+1}^{N} \omega f_t(N) dF(\omega) dN$$

Must aggregate over all *N* types and all ω types

Aggregate Net Worth, Loans and Capital

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$$= \int_0^\infty \overbrace{\left[\int_0^\infty K_{t+1}^N \omega dF(\omega) \right]}^{\text{total effective capital of all entrepreneurs with net worth, } N} f_t(N) dN$$

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total effective capital of all entrepreneurs with net worth, N

$$= \int_0^\infty \overbrace{\left[\int_0^\infty K_{t+1}^N \omega dF(\omega) \right]} f_t(N) dN$$

This is just K^N_{t+1} because an entrepreneur with net worth, *N*, observes *ω* after selecting the loan contract, and, hence, the quantity of capital purchased.

Aggregate Net Worth, Loans and Capital

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$$N_{t+1} = \int_0^\infty N f_t(N) dN$$

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Aggregates, cont'd

- Aggregate capital:

$$K_{t+1} = \int_0^\infty \int_0^\infty K_{t+1}^N \omega f_t(N) dF(\omega) dN$$

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Aggregates, cont'd

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$$
\begin{aligned}
K_{t+1} &= \int_0^\infty \int_0^\infty K_{t+1}^N \omega f_t(N) dF(\omega) dN = \int_0^\infty K_{t+1}^N f_t(N) dN \\
&= \frac{1}{1 - \frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\right]} \int_0^\infty N f_t(N) dN
\end{aligned}
$$

Aggregates, cont'd

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&= \frac{1}{1 - \frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\right]} N_{t+1}
\end{aligned}
$$

Aggregates, cont'd

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- Aggregate net worth in *t+1*:

$$\int_0^\infty \left[\int_0^\infty \max\{0, (1 + R_{t+1}^k) K_{t+1}^N \omega - Z_{t+1} B_{t+1}^N\} dF(\omega) \right] f_t(N) dN$$

Aggregates, cont'd

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$$= [1 - \Gamma(\bar{\omega}_{t+1})](1 + R_{t+1}^k) K_{t+1}$$

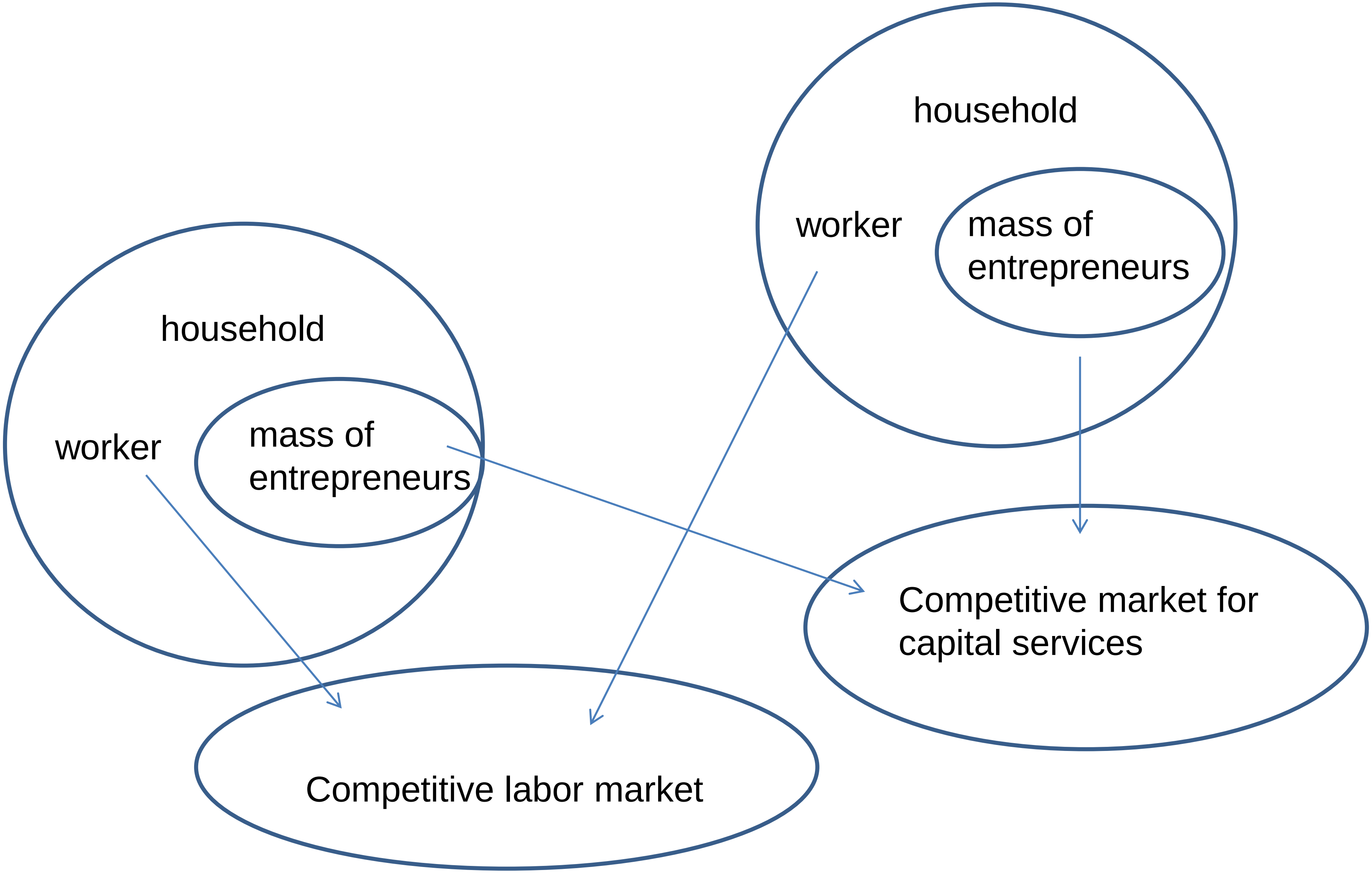
Relationship of Entrepreneur to Household

- Simplest assumption, which avoids a lot of complications, is the ‘large family assumption’.

- There are many identical households

  – Each has a worker.
  – Each has many entrepreneurs….enough so that average net worth in the representative family is always equal to average net worth in the economy as a whole.
  – Entrepreneurs receive perfect consumption insurance from the household:
    - Entrepreneurs and the worker all consume the same amount, C_t.

Large Family Assumption



Entrepreneurs and Households, cnt'd

- Aggregate net worth of all entrepreneurs, after accounting for all their income in *t+1*.

$$[1-\Gamma(\bar{\omega}_{t+1})](1+R^k_{t+1})K_{t+1}$$

- Then,
  - a fraction, $1-\Upsilon$, of each entrepreneur's net worth is transferred to the household as a lump-sum.
  - the household transfers resources, W^e_{t+1}, as a lump sum to each entrepreneur.
- So, net worth of all entrepreneurs at the end of *t+1* is:

$$N_{t+2} = \gamma[1-\Gamma(\bar{\omega}_{t+1})](1+R^k_{t+1})K_{t+1} + W^e_{t+1}$$

- The entrepreneurs as a whole take this net worth to the bank in *t+1*, get loans and buy capital, and so on….

One day in life of entrepreneur

Period *t*

Period *t+1*

* End of period *t*: Using net worth, N_{t+1}, and loans, entrepreneur purchases new, end-of-period stock of capital from capital goods producers. Entrepreneur observes idiosyncratic disturbance to its newly purchased capital.

Entrepreneur supplies capital to capital rental market

Entrepreneur sells undepreciated capital to capital producers

Entrepreneur pays off debt to bank, determines current net worth.

Fraction, 1-γ, of each entrepreneurs net worth transferred to households. Each entrepreneur receives a lump-sum transfer, W^e, from household.

Density of entrepreneurs having net worth, *N*:

$$f_{t+1}(N)$$

Entrepreneurs go to *.

Other Equations of the Model

- Goods market clearing (resource constraint):

$$\overbrace{c_t}^{\text{goods bought by households to feed their entrepreneurs and workers}}$$

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- Capital producers' technology:

$$K_{t+1} = (1-\delta)K_t + I_t$$

Other Equations…

- Household problem:

$$\max_{\{c_t,B_{t+1},l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c_t) = \log(c_t)$$

subject to:

$$c_t + B_{t+1} \leq w_t l_t + (1 + R_{t-1})B_t + \overbrace{\text{transfers from entrepreneurs}}^{=(1-\gamma)[1-\Gamma(\bar{\omega}_t)]\left(1+R_t^k\right)K_t - W_t^e}$$

$$0 \leq l_t \leq 1$$

Optimization: $u'(c_t) = \beta u'(c_{t+1})(1 + R_t),\ l_t = 1$

- Bond market clearing:

$$\overbrace{K_{t+1} - N_{t+1}}^{\text{borrowing by entrepreneurs}} = \overbrace{B_{t+1}}^{\text{lending by households}}$$

Result

- There are seven aggregate variables among the unknowns:

$$c_t, I_t, \bar{\omega}_t, R_t^k, K_t, N_t, R_t$$

- There are seven dynamic equilibrium conditions that can be used to pin the down (see next slide).

- Solution does not require knowing $f_t(N)$
  - Reflects constant returns to scale in entrepreneurial projects and entrepreneurial objective.
  - Otherwise, leverage and interest rate in standard debt contract would be a function of *N*. In that case, what entrepreneurs as a group do depends on the distribution of *N*.
  - Constant return to scale assumptions massively simplify the analysis. Whether this entails significant distortions in conclusions is an interesting issue to explore.

Summary of Equations

Equilibrium conditions familiar from standard neoclassical model	
resource constraint:	$c_t + I_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_t^k)K_t = K_t^\alpha$
household fonc:	$u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$
Capital accumulation:	$K_{t+1} = (1 - \delta)K_t + I_t$
Rate of return on capital:	$1 + R_t^k = \alpha K_t^{\alpha - 1} + 1 - \delta$

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entrepreneur fonc:	$\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t}[1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})]}$
zero profit condition:	$K_t = \frac{1}{1-\frac{1+R_t^k}{1+R_{t-1}}[\Gamma(\bar{\omega}_t)-\mu G(\bar{\omega}_t)]} N_t$
Aggregate Net Worth:	$N_{t+1} = \gamma[1-\Gamma(\bar{\omega}_t)](1+R_t^k)K_t + W_t^e$

In practice, monitoring costs small

Summary of Equations

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In neoclassical model these are the same. Financial frictions introduce a wedge.

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Large family assumption has consequence that model focuses exclusively on the implications of financial frictions for distortions in the intertemporal margin, abstracting from all other implications, such as for distribution of income between entrepreneurs and others.

Financial Friction as Intertemporal Wedge

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resource constraint:	$c_t + I_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_t^k)K_t = K_t^\alpha$
household fonc:	$u'(c_t) = \beta u'(c_{t+1})(1 + R_{t+1}^k)(1 - \tau_{t+1})$
Capital accumulation:	$K_{t+1} = (1 - \delta)K_t + I_t$
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wedge	$\tau_{t+1} = 1 - \frac{1+R_t}{1+R_{t+1}^k} = 1 - \frac{[1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})}} - \Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})$
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Looks just like neoclassical model with a particular tax on capital income.

Solving the Model

- Perturbation methods can be applied.
  - First, require steady state of the model.
  - Then, linearize equilibrium conditions about the steady state and solve.

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- Parameterization:

$$u(c) = \log c,\ \sigma = 0.26,\ \mu = 0.21,\ \gamma = 0.97,\ \alpha = 1/3,\ \delta = 0.02,\ \beta = 1.03^{-1/4},\ w^e = 0.01$$

$$\log(\sigma_t/\sigma) = 0.97 \log(\sigma_{t-1}/\sigma) + \varepsilon_t$$

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$$\log(\sigma_t/\sigma) = 0.97 \log(\sigma_{t-1}/\sigma) + \varepsilon_t$$

- Cross-sectional dispersion evolves according to a first order autoregressive process.

Solving the Model

- Endogenous variables in steady state:

$$\bar{\omega} = 0.5286,\ G(\bar{\omega}) = 0.0049,\ F(\bar{\omega}) = 0.010,\ \Gamma(\bar{\omega}) = 0.5282,\ n = 11.65$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 0.5271,\ \frac{c}{c+i} = 0.77,\ \frac{k-n}{c+i} = 4.60,\ \frac{k}{n} = 2.14,\ \frac{k}{c+i} = 8.62$$

$$400\left(\frac{Z}{1+R} - 1\right) = 1.11,\ \gamma[1-\Gamma(\bar{\omega})](1+R^k)\frac{k}{n} = 0.9991,\ \text{wedge} = 0.0114$$

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- Financial distortion:

without financial frictions: $K = \left[\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right]^{\frac{1}{1-\alpha}} = 42.4,\ c = 2.64$

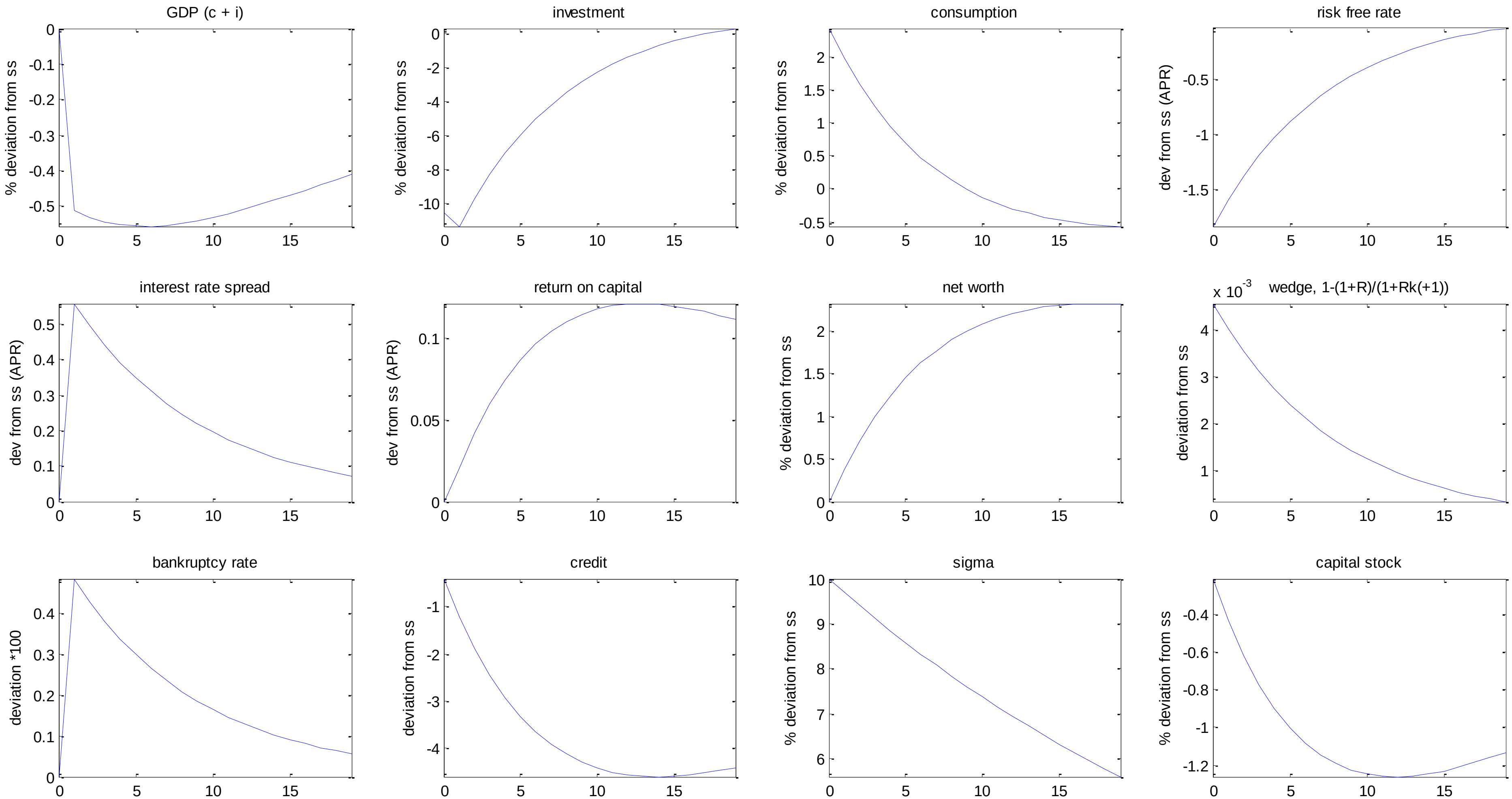
with financial frictions: $K = \left[\frac{\alpha}{\frac{1}{\beta(1-\text{wedge})} - (1-\delta)}\right]^{\frac{1}{1-\alpha}} = 25.0,\ c = 2.42$

steady state welfare cost of the financial frictions: 8.9 percent of consumption (=100(2.64-2.42)/2.42)

10% jump

Response to 0.1 jump in ε_t, where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26,\ \mu = 0.21,\ \gamma = 0.97,\ \alpha = 1/3,\ \delta = 0.02,\ \beta = 1.03^{-1/4},\ \eta = 1.$

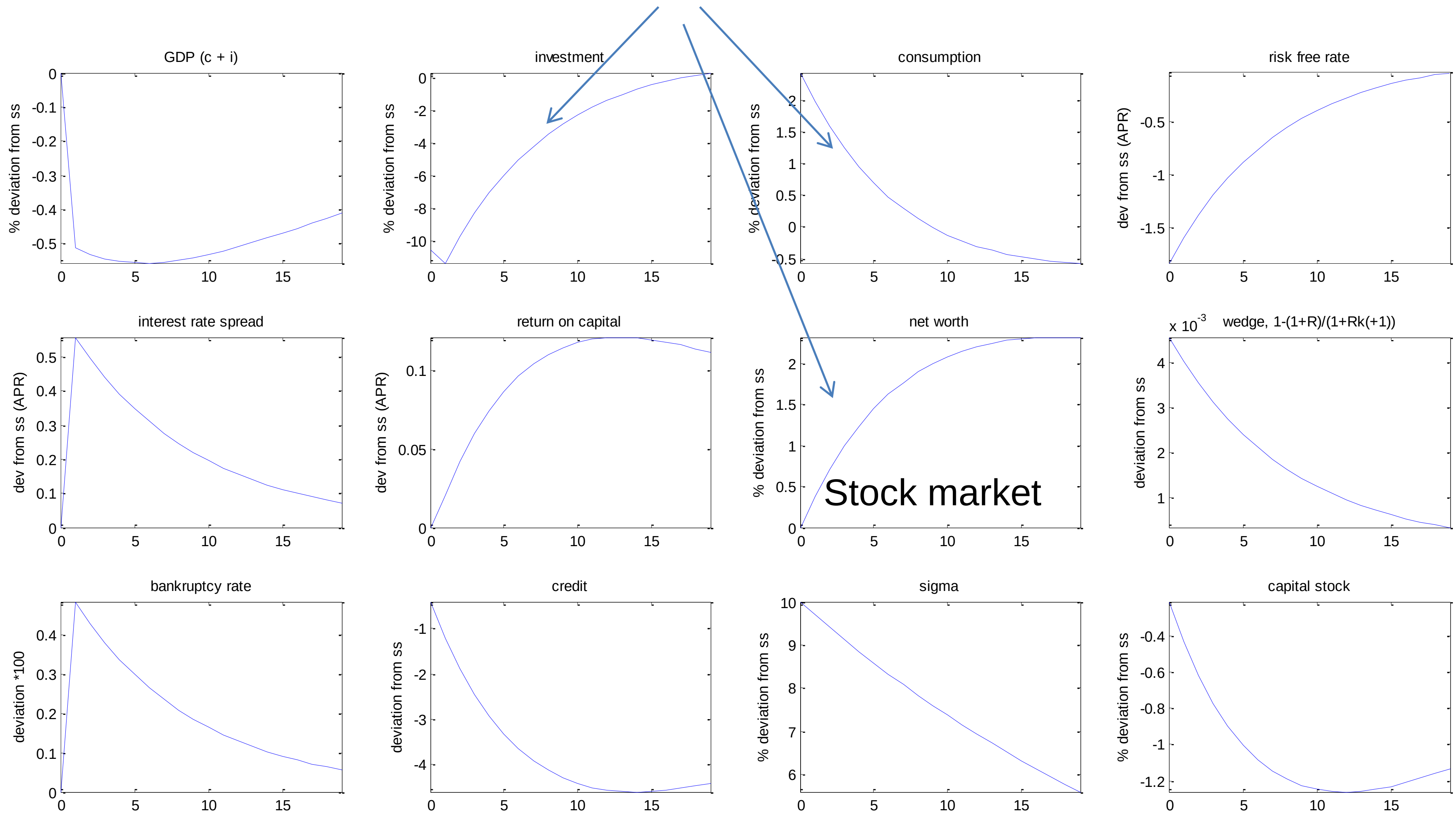


10% jump

Response to 0.1 jump in ε_t, where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26,\ \mu = 0.21,\ \gamma = 0.97,\ \alpha = 1/3,\ \delta = 0.02,\ \beta = 1.03^{-1/4},\ \eta = 1.$

Suggests (but see later slide) that risk not likely to be important in business cycles

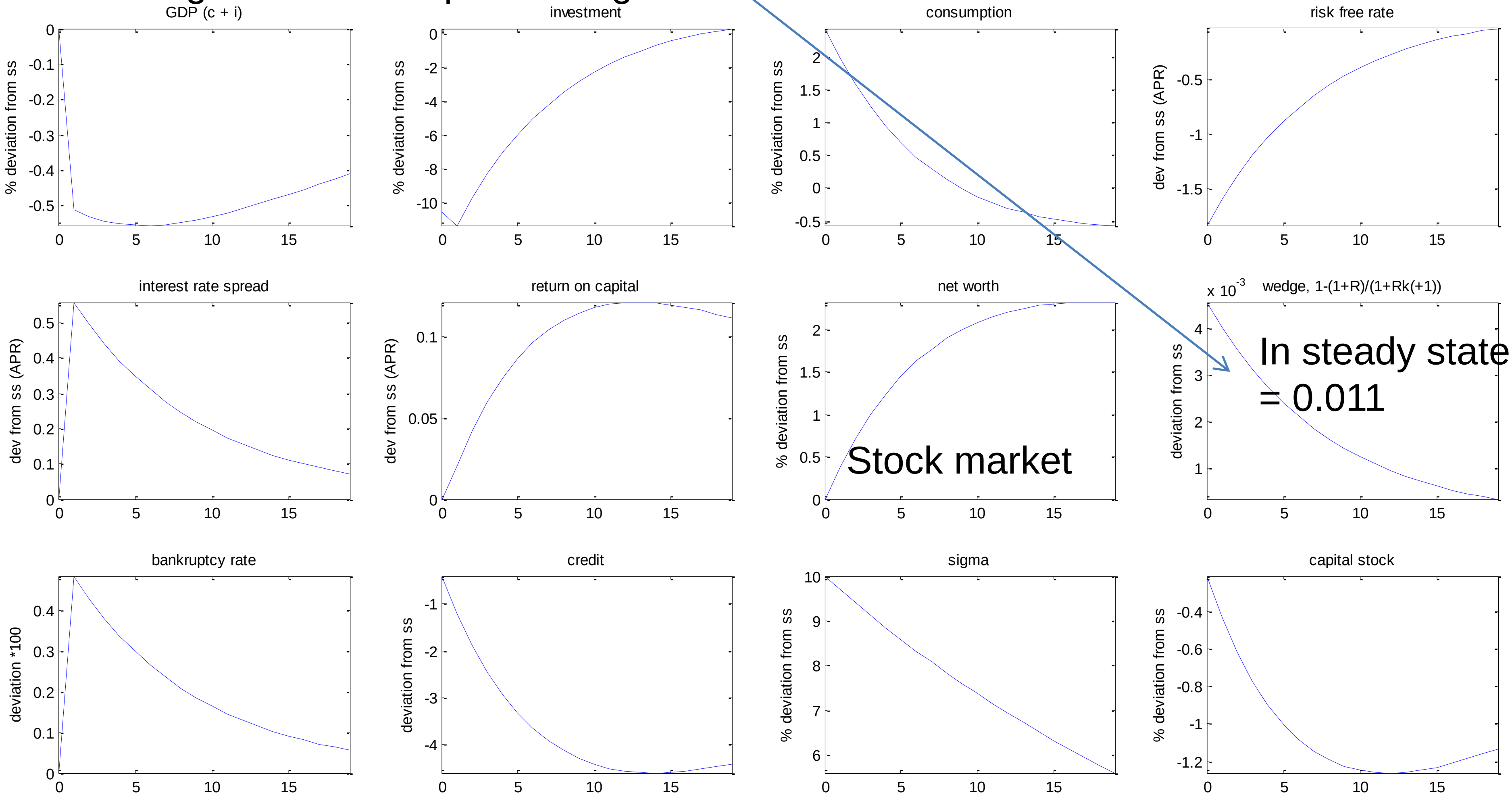


10% jump

Response to 0.1 jump in ε_t, where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26,\ \mu = 0.21,\ \gamma = 0.97,\ \alpha = 1/3,\ \delta = 0.02,\ \beta = 1.03^{-1/4},\ \eta = 1.$

The mechanism by which the shock moves economic variables operates through the intertemporal wedge.



Comparison of results to Monetary Models that Fit the Data Well

- The model incorporates the neoclassical model's property that the price of capital is fixed at unity.
  – In more elaborate models, there is curvature in production function for capital, and this causes its price to drop when less is produced.
  – In those models, net worth falls with a jump in risk.
- Rise in consumption.
  – This is induced in part by the sharp drop in the risk free interest rate, which reduces the household's incentive to save.
  – In more elaborate models, the risk free interest rate does not fall so much.

$$1 + R = \frac{1 + R^{\$}}{1 + \pi^{e}}$$

Monetary policy controls nominal rate, and does not move it much (actually, not at all when it is stuck at zero) in empirical representations of monetary policy.

Expected inflation 'sluggish' because empirical representations of monetary policy assume central bank committed to low inflation

Conclusion

- We’ve reviewed one interesting model of financial frictions.

- There are others!