Simple New Keynesian Model without Capital

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What's It Good For?

- Conveying basic principles of macroeconomics -
  - Concept and measurement of *output gap*:
    - 'difference between the actual economy and where would be if policy was managed as well as possible'.
  - Importance of aggregate demand.
    - problems when it goes awry.
  - Important policy objective: assuring the right level of aggregate demand.
- What is the welfare cost of inflation?
  - Many think that the high US inflation of the 1970s was in part responsible for the poor economic performance then.
  - But, economists have not been successful at finding a mechanism that can make sense of that.
  - We will see that the simple NK model (with networks) provides such a mechanism (although this is not widely recognized).

What's It Good For?

- Thinking through the operating characteristics of policy rules:
  - Inflation targeting, Tax/spending rules, Leverage restrictions on banks.
- Can even use it to learn econometrics
  - how well do standard econometric estimators work?
  - how good is HP filter at estimating output gap?

Our Approach to NK Model

- We will derive the familiar 'three equation NK model', but they will not be our starting point.
  - Start with households, firms, technology, etc....
- Necessary to build the model from scratch -
  - need this to uncover the principles hiding inside it
  - needed to know how to 'go back to the drawing board' and modify the model so it can address interesting questions:
    - how should macro prudential policy be conducted?
    - how might currency mismatch problems affect the usual transmission of exchange rate depreciation to the economy?
    - what should the role of inflation, labor markets, credit growth, stock markets, etc., be in monetary policy?
    - how does an expansion of unemployment benefits in a recession affect the business cycle?

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \; \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$\exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced as follows:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}, \ a_t \text{ ~exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient ('First Best') allocation of resources across i.
  - simplify the discussion with $\gamma = 1$ (no materials).

Efficient Sectoral Allocation of Resources Across Sectors

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t and I_t it is optimal to set $N_{i,t} = N_{j,t}$, for all $i, j \in [0, 1]$
- In this case, final output is given by

$$Y_t = e^{a_t} N_t.$$

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \tfrac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\tfrac{1}{2}, 1\right] \end{cases}, \; 0 \le \alpha \le 1.$$

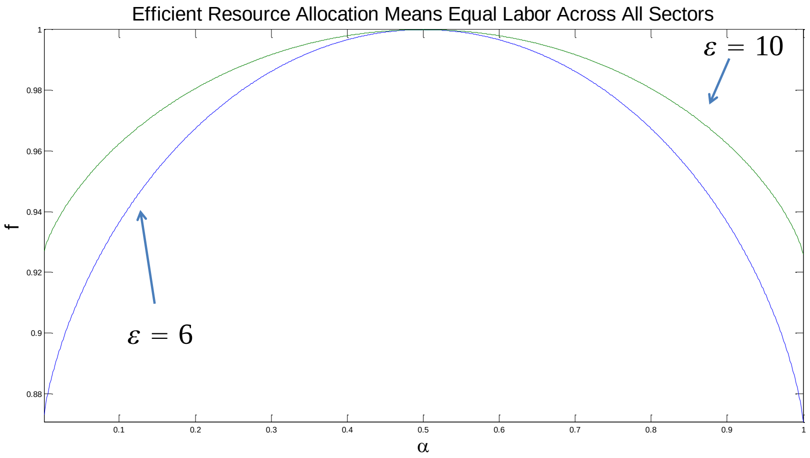
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Homogeneous Good Production

- Competitive firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Goods Production

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}, \; a_t \sim \text{exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Cost Minimization Problem

- Price setting by intermediate good firms is discussed later.
  - The intermediate good firm must produce the quantity demanded, $Y_{i,t}$, at the price that it sets.
  - Right now we take $Y_{i,t}$ as given and we investigate the cost minimization problem that determines the firm's choice of inputs.
- Cost minimization problem:

$$\min_{N_{i,t},I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \overbrace{\lambda_{i,t}}^{\text{marginal cost (money terms)}} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

with resource costs:

$$\bar{W}_t = \overbrace{(1-\nu)}^{\text{subsidy, if } \nu>0} \times \overbrace{(1-\psi_H + \psi_H R_t)\, W_t}^{\text{cost, including finance, of a unit of labor}}$$

$$\bar{P}_t = (1-\nu) \times \overbrace{(1-\psi_I + \psi_I R_t)\, P_t}^{\text{cost, including finance, of a unit of materials}}.$$

Cost Minimization Problem

- Problem:

$$\min_{N_{i,t},I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \lambda_{i,t}\left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}\right]$$

- First order conditions:

$$\bar{P}_t I_{i,t} = (1-\gamma)\,\lambda_{i,t} Y_{i,t},\ \bar{W}_t N_{i,t} = \gamma \lambda_{i,t} Y_{i,t},$$

so that,

$$\begin{aligned}
\frac{I_{it}}{N_{it}} &= \frac{1-\gamma}{\gamma}\frac{\bar{W}_t}{\bar{P}_t} = \frac{1-\gamma}{\gamma}\frac{(1-\psi_N+\psi_N R_t)}{(1-\psi_I+\psi_I R_t)}\exp(\tau_t)\, C_t N_t^{\varphi} \\
&\rightarrow \frac{I_{it}}{N_{it}} = \frac{I_t}{N_t},\ \text{for all } i.
\end{aligned}$$

Cost Minimization Problem

- Firm first order conditions imply

$$\lambda_{i,t} = \left(\frac{\bar{P}_t}{1-\gamma}\right)^{1-\gamma}\left(\frac{\bar{W}_t}{\gamma}\right)^{\gamma}\frac{1}{A_t}.$$

- Divide marginal cost by P_t :

$$\begin{aligned} s_t \equiv & \frac{\lambda_{i,t}}{P_t} = (1-\nu)\left(\frac{1-\psi_I+\psi_I R_t}{1-\gamma}\right)^{1-\gamma} \\ & \times \left(\frac{1-\psi_N+\psi_N R_t}{\gamma}\exp\left(\tau_t\right)C_t N_t^{\varphi}\right)^{\gamma}\frac{1}{A_t} \text{ (9)}, \end{aligned}$$

after substituting out for $\bar{P}_t$ and $\bar{W}_t$ and using the household's labor first order condition.

- Note from (9) that i^{th} firm's marginal cost, s_t, is independent of i and $Y_{it,}$.

Share of Materials in Intermediate Good Output

- Firm i materials proportional to $Y_{i,t}$:

$$I_{i,t} = \frac{(1-\gamma)\,\lambda_{i_it} Y_{i,t}}{\bar{P}_t} = \mu_t Y_{i,t},$$

where

$$\mu_t = \frac{(1-\gamma)\,s_t}{(1-\nu)\,(1-\psi_I+\psi_I R_t)} \text{ (10).}$$

- "Share of materials in firm-level gross output", μ_t.

Decision By Firm that Can Change Its Price

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \; \upsilon_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

υ_{t+j} - Lagrange multiplier on household budget constraint

- Firm that gets to reoptimize its price is concerned only with future states in which it does not change its price:

$$\begin{aligned} & E_t^i \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\ = \;\; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] + X_t, . \end{aligned}$$

where $\tilde{P}_t$ denotes a firm's price-setting choice at time t and X_t not a function of $\tilde{P}_t$.

Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\
= \;\; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right].
\end{aligned}
$$

- Differentiate with respect to $\tilde{P}_t$:

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t \right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,
$$

  or,

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0.
$$

- When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{=\,v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0.$$

- Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0,$$

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}},\ X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\cdots\bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{cases},$$

$$X_{t,j} = X_{t+1,j-1} \frac{1}{\bar{\pi}_{t+1}},\ j > 0$$

Decision By Firm that Can Change Its Price

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solving for $\tilde{p}_t$, we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+1}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for K_t, F_t.

Decision By Firm that Can Change Its Price

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t \\
&\quad + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \overbrace{E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{Y_{t+j+1}}{C_{t+j+1}} \frac{\varepsilon}{\varepsilon - 1} s_{t+1+j}}^{\text{exactly } K_{t+1}!} \\
&= \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}
\end{aligned}
$$

For a detailed derivation, see, e.g., http://faculty.wcas.northwestern.edu/~lchrist/course/IMF2015/intro_NK_handout.pdf.

Decision By Firm that Can Change Its Price

- Conclude:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{K_t}{F_t},$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \text{ (1)}$$

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

Interpretation of Price Formula

- Note,

$$\frac{1}{P_{t+j}} = \frac{1}{P_t} X_{t,j},\ s_{t+j} = \frac{\lambda_{t+j}}{P_{t+j}} = \frac{\lambda_{t+j}}{P_t} X_{t,j},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}.$$

Multiply both sides of the expression for $\tilde{p}_t$ by P_t :

$$\tilde{P}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} \lambda_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{\varepsilon}{\varepsilon - 1} \sum_{j=0}^{\infty} E_t \omega_{t+j} \lambda_{t+j}$$

where

$$\omega_{t+j} = \frac{(\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}},\ \sum_{j=0}^{\infty} E_t \omega_{t+j} = 1.$$

Evidently, price is set as a markup over a weighted average of future marginal cost, where the weights are shifted into the future depending on how big θ is.

Restriction Between Aggregate and Intermediate Good Prices

- 'Calvo result':

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}} = \left[(1-\theta)\,\tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}.$$

- Divide by P_t :

$$1 = \left[(1-\theta)\,\tilde{p}_t^{(1-\varepsilon)} + \theta\left(\frac{1}{\bar{\pi}_t}\right)^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}.$$

- Rearrange:

$$\tilde{p}_t = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$$

Aggregate inputs and outputs

- Technically, there is no 'aggregate production function':
  - there is no exact relationship between output, Y_t, and aggregate inputs, N_t, I_t, A_t.
  - must also know the *distribution* of resources across intermediate good firms.

- Tack Yun (JME, 1996) developed a simple approach that can be used to determine the connection between N, A, I, Y and the distribution of resources.

Gross Output and Aggregate Inputs

- Define Y_t^*:

$$\begin{aligned} Y_t^* &\equiv \int_0^1 Y_{i,t} di \\ &\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t}\right)^{-\varepsilon} di \\ &= Y_t P_t^{\varepsilon} \left(P_t^*\right)^{-\varepsilon} \end{aligned}$$

where, using 'Calvo result':

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di\right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta)\,\tilde{P}_t^{-\varepsilon} + \theta\left(P_{t-1}^*\right)^{-\varepsilon}\right]^{\frac{-1}{\varepsilon}}$$

- Then

$$Y_t = p_t^* Y_t^*, \; p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon}.$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta)\,\tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then,

$$\begin{aligned} p_t^* &\equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta)\,\tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \\ &= \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \qquad (4) \end{aligned}$$

using the restriction between $\tilde{p}_t$ and aggregate inflation.

Gross Output and Aggregate Input

- Relationship between aggregate inputs and outputs:

$$\begin{aligned} Y_t &= p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} di \\ &= p_t^* A_t \int_0^1 N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} di = p_t^* A_t \int_0^1 \left(\frac{N_{i,t}}{I_{i,t}} \right)^{\gamma} I_{i,t} di, \\ &= p_t^* A_t \left(\frac{N_t}{I_t} \right)^{\gamma} I_t, \end{aligned}$$

or,

$$Y_t = p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)}$$

- Tack Yun distortion p_t^*:

$$p_t^* : \begin{cases} \leq 1 & \\ = 1 & P_{i,t} = P_{j,t}, \text{ all } i,j \end{cases} .$$

Working Towards an Expression for Gross Domestic Product (Aggregate Value Added, GDP)

- Recall
$$I_{i,t} = \mu_t Y_{i,t},$$
so,
$$I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} d = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t.$$

- Then,
$$\begin{aligned} Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \\ &= p_t^* A_t N_t^{\gamma} \left(\frac{\mu_t}{p_t^*} Y_t \right)^{1-\gamma} \end{aligned}$$
$$\longrightarrow \quad Y_t = \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t$$

Gross Domestic Product (GDP)

- We have

$$
\begin{aligned}
GDP_t &= Y_t - I_t = \left(1 - \frac{\mu_t}{p_t^*}\right) Y_t \\
&= \left(1 - \frac{\mu_t}{p_t^*}\right) \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} N_t \\
&= \overbrace{\left(p_t^* A_t \left(1 - \frac{\mu_t}{p_t^*}\right)^{\gamma} \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{=\text{Total Factor Productivity}} N_t
\end{aligned}
$$

- Note how an increase in technology at the firm level, by A_t, gives rise to a bigger increase in TFP by $A_t^{1/\gamma}$.
  - In the literature on networks, $1/\gamma$ is referred to as a 'multiplier effect' (see Jones, 2011).
- The Tack Yun distortion, p_t^*, seems to be associated with the same multiplier phenomenon.

Decomposition for Total Factor Productivity

- To maximize GDP for given aggregate N_t and A_t :

$$\max_{0<p_t^* \leq 1,\ 0 \leq \lambda_t \leq 1} \left(p_t^* A_t \left(1-\lambda_t\right)^{\gamma} \left(\lambda_t\right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

$$\rightarrow \quad \lambda_t = 1-\gamma,\ p_t^* = 1.$$

- So,

$$TFP_t = \overbrace{\left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}}^{\text{Component due to market distortions} \equiv \chi_t} \times \overbrace{\left(A_t \left(\gamma\right)^{\gamma} \left(1-\gamma\right)^{1-\gamma} \right)^{\frac{1}{\gamma}}}^{\text{Technology component}}$$

Evaluating the Distortions

- The equations characterizing the TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

- Potentially, NK model provides an 'endogenous theory of TFP'.
- Standard practice in NK literature is to set $\chi_t = 1$ for all t.
  - Set $\gamma = 1$ and linearize around $\bar{\pi}_t = p_t^* = 1$.
  - With $\gamma = 1$, $\chi_t = p_t^*$, and first order expansion of p_t^* around $\bar{\pi}_t = p_t^* = 1$ is:

$$p_t^* = p^* + 0 \times \bar{\pi}_t + \theta \left(p_{t-1}^* - p^* \right), \text{ with } p^* = 1,$$

so $p_t^* \to 1$ and is invariant to shocks.

Empirical Assessment of the Distortions

- The TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

- Problem: the objects, χ_t and p_t^*, are not quite observable.
  - Still, if we assume μ_t is constant, at $1-\gamma$, we can get a feel about the magnitudes using US inflation data.
- Will consider $\gamma = 1/2$ (Basu's empirical estimate) and $\gamma = 1$ (standard assumption in NK literature).
- Will consider two values for the markup:
  - $\varepsilon / (\varepsilon - 1) = 1.20$, the baseline estimate in CEE (JPE, 2005), which corresponds to $\varepsilon = 6$,
  - $\varepsilon / (\varepsilon - 1) = 1.15$, more competition, i.e., $\varepsilon = 7.7$.

Empirical Assessment of the Distortions

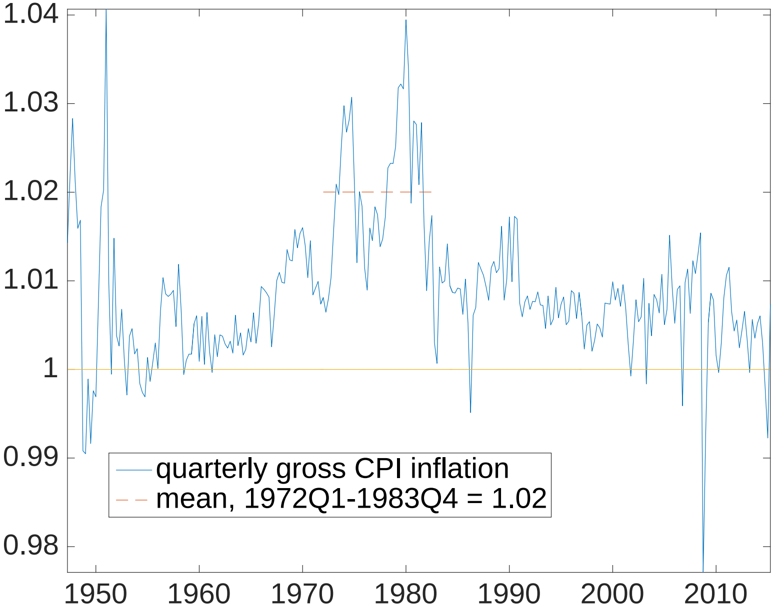
- First, do 'back of the envelope calculations in a steady state when inflation is constant and p^* is constant.

$$p^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}^{\varepsilon}}{p^*} \right]^{-1}$$

$$\rightarrow p^* = \frac{1-\theta \bar{\pi}^{\varepsilon}}{(1-\theta) \left(\frac{1-\theta \bar{\pi}^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}}}$$

- Approximate TFP distortion, χ :

$$\chi_t = \left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \simeq (p^*)^{1/\gamma}$$



1.04
1.03
1.02
1.01
1
0.99
0.98
1950
1960
1970
1980
1990
2000
2010
quarterly gross CPI inflation
mean, 1972Q1-1983Q4 = 1.02

Cost of Average Inflation in 1970s Using Steady State Formulas

- Formulas:

$$p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{(1-\theta)\left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}},\ \chi = (p^*)^{1/\gamma}$$

- Results

Table 1: Fraction of GDP Lost Due to Inflation, $100(1-\chi)$		
	No networks, $\gamma = 1$	Networks, $\gamma = 2$
Steady state lost output	2.61 (4.34)*	5.16 (8.50)
Note * number not in parentheses - markup of 20 percent (i.e., $\varepsilon = 6$)		
number in parentheses - markup of 15 percent. (i.e., $\varepsilon = 7.7$)		

Next: Assess Costs of Inflation Using Non-Steady State Formulas

Figure 1a: Graph of Quarterly, Gross US CPI inflation, p-star and chi, assumed markup is 1.2

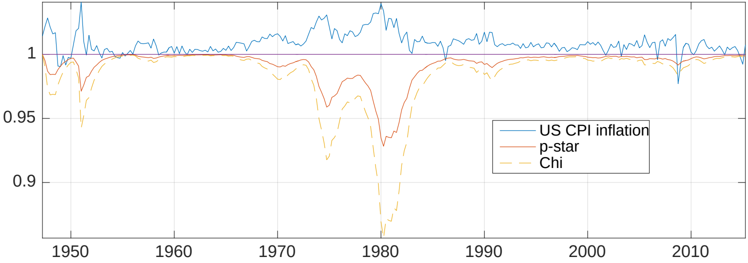
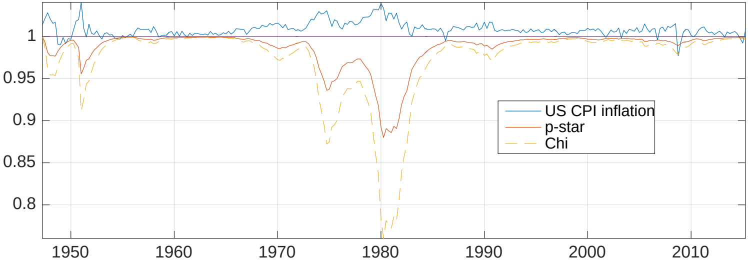


Figure 1b: Graph of Quarterly, Gross US CPI inflation, p-star and chi, assumed markup is 1.15



Inflation Distortions Displayed are Big

- With $\varepsilon = 6$,
  - mean$(\chi_t) = 0.98$, a 2% loss of GDP.
  - frequency, $\chi_t < 0.955$, is 10% (i.e., 10% of the time, the output loss is greater than 4.5 percent).
- With more competition (i.e., ε higher), the losses are greater.
  - with higher elasticity of demand, given movements in inflation imply much greater substitution away from high priced items, thus greater misallocation (caveat: this intuition is incomplete since with greater ε the consequences of a given amount of misallocation are smaller).
- Distortions with $\gamma = 1/2$ are roughly twice the size of distortions in standard case, $\gamma = 1$.
  - To see this, let $p^* = 1 - \omega$. Then,

$$\chi_t \simeq (p^*)^{\frac{1}{\gamma}} \simeq 1 - \frac{1}{\gamma}\omega.$$

Comparison of Steady State and Dynamic Costs of Inflation in 1970s

- Results

Table 1: Fraction of GDP Lost Due to Inflation, $100(1-\chi)$		
	No networks, $\gamma = 1$	Networks, $\gamma = 2$
Steady state lost output	2.61 (4.34)*	5.16 (8.50)
Mean, 1972Q1-1982Q4	3.13 (5.22)	6.26 (10.44)
Note * number not in parentheses - markup of 20 percent (i.e., $\varepsilon = 6$)		
number in parentheses - markup of 15 percent. (i.e., $\varepsilon = 7.7$)		

- Evidently, distortions increase rapidly in inflation,

$$E\left[distortion\left(\text{inflation}\right)\right] > distortion\left(E\text{inflation}\right)$$

Next

- Summarize the equilibrium conditions.
- Compare flexible price and sticky price equilibria
  - sticky price equilibrium incomplete.
    - One equation short because real allocations in private economy co-determined along with the nominal quantities.
  - flexible price equilibrium (at least, the one without working capital) dichotomizes.
    - real allocations in flexible price model are determined and monetary policy only delivers inflation and the nominal interest, things that do not affect utility.
- Evaluate distortions in steady state.

Summarizing the Equilibrium Conditions

- Break up the equilibrium conditions into three sets:
  - Conditions (1)-(4) for prices: $K_t, F_t, \bar{\pi}_t, p_t^*, s_t$
  - Conditions (6)-(10) for: $C_t, Y_t, N_t, I_t, \mu_t$
  - Conditions (5) and (11) for R_t and χ_t.
- Consider
  - conditions for the model as is.
  - conditions pertaining to the case of flexible prices, no working capital and efficient subsidy for monopoly power:

$$\theta = 0,\ \psi_I = \psi_N = 0,\ \frac{\varepsilon}{\varepsilon - 1}(1 - \nu) = 1.$$

    - equilibrium supports 'first best' allocations: those that would occur if a benevolent planner chose the allocations rather than the market.

Equilibrium Conditions for Prices

$$K_t = \frac{\varepsilon}{\varepsilon - 1}\frac{Y_t}{C_t}s_t + \beta\theta E_t\left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \quad (1)$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t\left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \quad (2)$$

$$\frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad (4)$$

- When $\theta = 0$, these boil down to (i) zero price dispersion and (ii) everyone sets price as markup, $\varepsilon/\left(\varepsilon - 1\right)$, over marginal cost:

$$p_t^* = 1,\ \frac{\varepsilon}{\varepsilon - 1}s_t = 1,\ K_t = F_t = C_t/Y_t,\ \text{no restriction on } \bar{\pi}_t$$

Other, Static, Equilibrium Conditions

- Variables:

$$C_t, Y_t, N_t, I_t, \mu_t$$

- Equations:

$$\begin{aligned}
Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)}, \ C_t + I_t = Y_t \text{ (7)}, \ I_t = \mu_t \frac{Y_t}{p_t^*} \text{ (8)} \\
s_t &= (1-\nu)\left(\frac{1-\psi_I+\psi_I R_t}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1-\psi_N+\psi_N R_t}{\gamma}\exp(\tau_t)\, C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)} \\
\mu_t &= \frac{(1-\gamma)\, s_t}{(1-\nu)(1-\psi_I+\psi_I R_t)} \text{ (10)},
\end{aligned}$$

Other Variables in Flexible Price, no Working Capital Case

- Suppose $\varepsilon(1-\nu)/(\varepsilon-1)=1$, $\theta=\psi_I=\psi_N=0$,

$$
\begin{aligned}
Y_t &= \left[A_t \mu_t^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6)}, \; C_t = \left[A_t (1-\mu_t)^{\gamma} \mu_t^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6,7,8)} \\
1 &= \frac{\varepsilon}{\varepsilon-1}(1-\nu)\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\exp(\tau_t) C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)} \\
\mu_t &= \frac{\varepsilon-1}{\varepsilon}\frac{(1-\gamma)}{(1-\nu)} = 1-\gamma \text{ (10)},
\end{aligned}
$$

- Combining (6,7,8) and (10):

$$
C_t = \left[A_t \gamma^{\gamma}(1-\gamma)^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6,7,8,10)}
$$

- Consumption maximized, conditional on aggregate employment, N_t.

Other Variables in Flexible Price, no Working Capital Case (cnt'd)

- Suppose $\varepsilon\left(1-\nu\right)/\left(\varepsilon-1\right)=1$, $\theta=\psi_I=\psi_N=0$.
- Solve equation (9) for cost of working, $\exp\left(\tau_t\right)C_tN_t^{\varphi}$,

$$\overbrace{\exp\left(\tau_t\right)C_tN_t^{\varphi}}^{\text{cost of working}} = \overbrace{\left[A_t\left(\gamma\right)^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}^{\text{benefit of working}} \quad (9)$$

- Conditions (6,7,8,10) and (9) imply that first-best levels of consumption and employment occur:

$$\begin{aligned} N_t &= \exp\left(-\frac{\tau_t}{1+\varphi}\right) \\ C_t(&= GDP_t) = \left[A_t\left(\gamma\right)^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\exp\left(-\frac{\tau_t}{1+\varphi}\right) \end{aligned}$$

Last Equilibrium Conditions

- Distortion:

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \quad (11)$$

in $\varepsilon(1-\nu)/(\varepsilon-1) = 1$, $\theta = \psi_I = \psi_N = 0$ case,

$$\chi_t = 1, \text{ for all } t.$$

- Intertemporal equation

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

Real Interest Rate in Flex P Equilibrium

- The real interest rate, $R_t/\bar{\pi}_{t+1}$.
  - Absent uncertainty, $R_t/\bar{\pi}_{t+1}$ determined uniquely:
$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}.$$
  - With uncertainty, household intertemporal condition simply places a single linear restriction across all the period $t+1$ values for $R_t/\bar{\pi}_{t+1}$ that are possible given period t.
- The real interest rate, $\tilde{r}_t$, on a risk free one-period bond that pays in $t+1$ *is* uniquely determined:
$$\frac{1}{C_t} = \tilde{r}_t \beta E_t \frac{1}{C_{t+1}}.$$
- By no-arbitrage, only the following weighted average of $R_t/\bar{\pi}_{t+1}$ across period $t+1$ states of nature is determined:
$$\tilde{r}_t = \frac{E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}}{E_t \frac{1}{C_{t+1}}} = E_t \frac{\frac{1}{C_{t+1}}}{E_t \frac{1}{C_{t+1}}} \frac{R_t}{\bar{\pi}_{t+1}} = E_t \nu_{t+1} \frac{R_t}{\bar{\pi}_{t+1}}.$$

Classical Dichotomy and New Keynesian Economics

- Captured by flexible price, no working capital, no monopoly distortion version of model.
  - Real variables determined independent of monetary policy.
  - The things that matter - consumption, employment - are first best and there is no constructive role for monetary policy.
  - Monetary policy irrelevant. Money is a veil.
- With price frictions.
  - Now, all aspects of the system are interrelated and jointly determined.
  - Whole system depends on the nature of monetary policy.
  - Within the context of a market system, monetary policy has an essential role as a potential 'lubricant', to help the economy to get as close as possible to the first best.
  - Monetary policy:
    - has the potential to do a good job.
    - or, if mismanaged, could get very bad outcomes.

Steady State

The steady state may found by implementing the following calculations in sequence, for given $\bar{\pi}$:

$$
\begin{aligned}
R &= \frac{\bar{\pi}}{\beta},\ K_f \equiv \frac{K}{F} = \left[\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}},\ s = K_f\frac{\varepsilon-1}{\varepsilon}\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{1-\beta\theta\bar{\pi}^{\varepsilon-1}} \\
p^* &= \frac{1-\theta\bar{\pi}^{\varepsilon}}{(1-\theta)\left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}},\ \mu = \frac{(1-\gamma)\,s}{(1-\nu)\,(1-\psi_I+\psi_I R)}, \\
C_Y &\equiv \frac{C}{Y} = 1-\frac{\mu}{p^*},\ Y = \left[p^*\left(\frac{\mu}{p^*}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} N, \\
C &= \overbrace{\left[p^*\left(1-\frac{\mu}{p^*}\right)^{\gamma}\left(\frac{\mu}{p^*}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}^{Q} N,
\end{aligned}
$$

Steady State, Continued

$$N = \left[\frac{s}{(1-\nu)\left(\frac{1-\psi_I+\psi_I R}{1-\gamma}\right)^{1-\gamma}\left(\frac{1-\psi_N+\psi_N R}{\gamma}Q\right)^{\gamma}} \right]^{\frac{1}{(1+\varphi)\gamma}}$$

$$C = QN,\ Y = \frac{C}{1-\frac{\mu}{p^*}},\ I = \mu\frac{Y}{p^*},\ F = \frac{1/C_Y}{1-\beta\theta\bar{\pi}^{1-\varepsilon}},\ K = K_f \times F$$

Now, Move to the Standard Three Equation Model

- Model described above with
  - no network effects, $\gamma = 1$.
  - price-setting frictions, $\theta > 0$.
  - no working capital, $\psi_I = \psi_N = 0$.

The Linearized Private Sector Equilibrium Conditions of Standard Model

- $\gamma = 1$ and No Working Capital Channel.
- Derive, as a benchmark, best possible equilibrium:
  - Ramsey or 'natural' equilibrium.
- Study 'actual equilibrium': equilibrium in which monetary policy is government by a Taylor rule.
  - as is standard in literature, Taylor rule forces inflation to be zero in steady state.
  - in long run, market economy functions well.
  - in short run, it could get off track.
- Derive classic IS curve as difference between log-linear intertemporal Euler equation in actual and natural equilibrium.
- Display linearized Phillips curve.

The Linearized Private Sector Equilibrium Conditions of Standard Model

- $\gamma = 1$, no working capital:

$$C_t = Y_t.$$

- Can show that best possible equilibrium (i.e., Ramsey or Natural equilibrium) satisfies:

$$\begin{aligned}
\bar{\pi}_t &= 1, \\
p_t^* &= 1, \\
\log C_t^* &= a_t - \frac{\tau_t}{1+\varphi} \\
\log N_t^* &= -\frac{\tau_t}{1+\varphi}
\end{aligned}$$

- See http://faculty.wcas.northwestern.edu/~lchrist/course/IMF2015/intro_NK_handout.pdf

The Linearized Private Sector Equilibrium Conditions of Standard Model

- Intertemporal First Order Condition:

$$\frac{1}{C_t} = R_t E_t \frac{\beta}{C_{t+1}\bar{\pi}_{t+1}}.$$

or, in Ramsey

$$\begin{aligned} -\log C_t^* &= \log\beta + \log R_t + \log E_t \frac{1}{C_{t+1}^*} \\ &= \log\beta + \log R_t + \log E_t \exp\left[-\log C_{t+1}^*\right] \\ &\simeq \log\beta + \log R_t + \log\exp\left[-E_t \log C_{t+1}^*\right] \end{aligned}$$

or

$$\log C_t^* = -\log\beta - \overbrace{\log R_t}^{r_t^*} + E_t \log C_{t+1}^*$$

so, Ramsey (Natural) rate of interest:

$$r_t^* = -\log\beta + E_t\left[\log C_{t+1}^* - \log C_t^*\right]$$

The Linearized Private Sector Equilibrium Conditions of Standard Model

- Intertemporal First Order Condition:

$$\frac{1}{C_t} = E_t \frac{\beta}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}.$$

or, in actual (not necessarily Ramsey) equilibrium:

$$\begin{aligned}
\log C_t &= -\log \beta - \overbrace{\log R_t}^{=r_t} - \log E_t \frac{1}{C^*_{t+1}} \frac{1}{\bar{\pi}_{t+1}} \\
&= -\log \beta - r_t - \log E_t \exp \left[-\log C_{t+1} - \overbrace{\log \bar{\pi}_{t+1}}^{=\pi_{t+1}} \right]
\end{aligned}$$

or, approximately

$$\log C_t = -\log \beta - (r_t - E_t \pi_{t+1}) + E_t \log C^*_{t+1}$$

The IS Equation

- Ramsey and actual intertemporal conditions:

$$\begin{aligned}\log C_t &= -\log\beta - (r_t - E_t\pi_{t+1}) + E_t \log C^*_{t+1} \\ \log C^*_t &= -\log\beta - r^*_t + E_t \log C^*_{t+1}\end{aligned}$$

- Subtract second from first to obtain IS equation:

$$x_t = -\left(r_t - E_t\pi_{t+1} - r^*_t\right) + E_t x_{t+1}$$

where x_t is the 'output gap':

$$x_t = \log\left(C_t\right) - \log\left(C^*_t\right)$$

Standard Linearized Analysis About Steady State With No Price and Monopoly Distortions

- The linearized equations of the model (interpreting r_t and r_t^* as deviations from steady states):

$x_t = E_t x_{t+1} - \left[r_t - E_t \pi_{t+1} - r_t^* \right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t + \beta E_t \pi_{t+1}$
$\hat{s}_t = (\varphi + 1)\, x_t$
$r_t^* = E_t \left[a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi} \right]$

- Monetary policy rule:

$$r_t = \alpha r_{t-1} + (1-\alpha) \left[\phi_\pi \pi_t + \phi_x x_t \right]$$

- See: http://faculty.wcas.northwestern.edu/~lchrist/course/CIED_2014/NK_model_handout.pdf for a formal derivation.

Solving the Model

- Vision about evolution of actual data:
  - Nature draws the exogenous shocks.
  - The economy transforms exogenous shocks into realization of endogenous variables, inflation, output, unemployment, etc.
- 'Solving the model':
  - Using the computer to imitate nature - drawing shocks from random number generator and transforming these into movements in the endogenous variables.
  - Problem: equilibrium conditions cannot be used for this pupose
    - In equilibrium conditions current variables are functions of past data and *expected future value of endogenous variables*.
- One strategy for solving a model:
  - Find a representation ('policy rule') of the endogenous variables, z_t, in terms of current and past data only:

$$z_t = Az_{t-1} + Bs_t$$

    such that the (linearized) equilibrium conditions are satisfied.

- Exogenous shocks:

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^\tau \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

- Equilibrium Conditions:

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r_{t+1}^* \end{pmatrix} + \begin{bmatrix} -1 & \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi) & 0 & 0 \\ 0 & -1 & -1 & 1 \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r_t^* \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & \frac{1}{1+\varphi} \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{1+\varphi} \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

- Collecting:

$$\begin{aligned} E_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] &= 0 \\ s_t - P s_{t-1} - \epsilon_t &= 0. \end{aligned}$$

- Policy rule:

$$z_t = A z_{t-1} + B s_t$$

- As before, want A such that

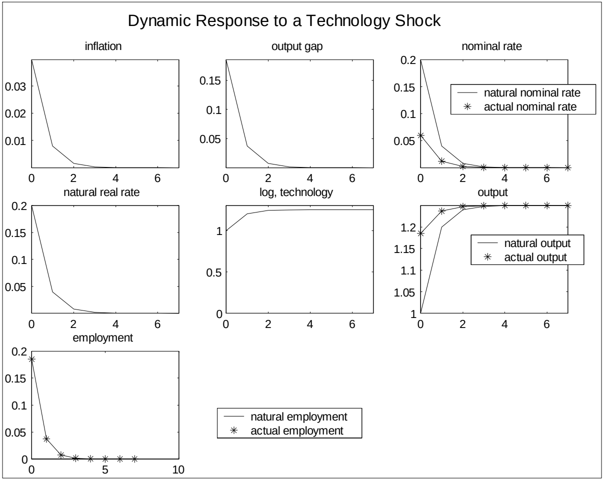
$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

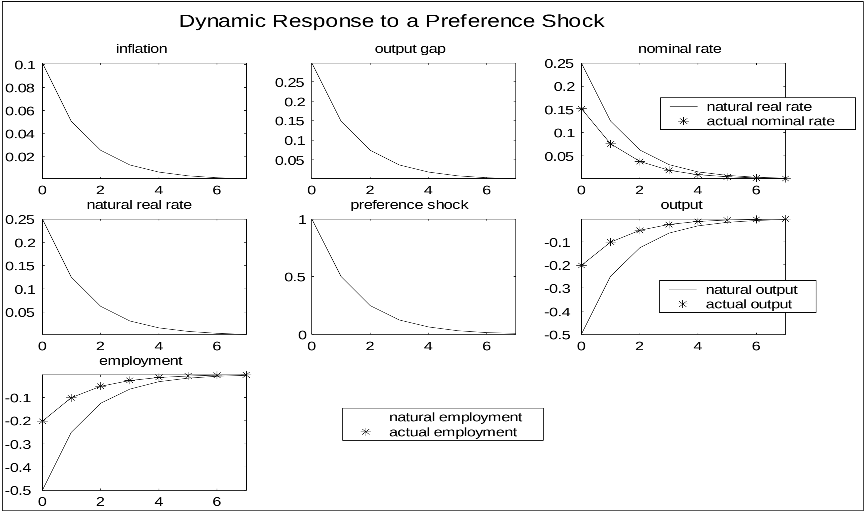
- Want B such that:

$$(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

- Note: if $\alpha = 0$, then $A = 0$ is one solution (there is another one!).

$$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$$




Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment
0.1
0.08
0.06
0.04
0.02
0.25
0.2
0.15
0.05
0.5
1
-0.1
-0.2
-0.3
-0.4
-0.5
0
2
4
6

Next, to Assignment 9.....

Next, Analysis of Bigger Model with Networks and Working Capital Channel

- See how the nonlinear equilibrium conditions of the model are input into Dynare.
- Use the Dynare to solve and simulate the model with first and second order perturbation method.
  - Resuts suggest that for plausible model parameterization, there is little difference between the two methods, suggesting that linearization is ok, at least for US-sized fluctuations.
- See the impact of working capital on the stabilizing properties of the Taylor principle.

Magnitude of TFP Distortion Stochastic Simulations

- Parameter values

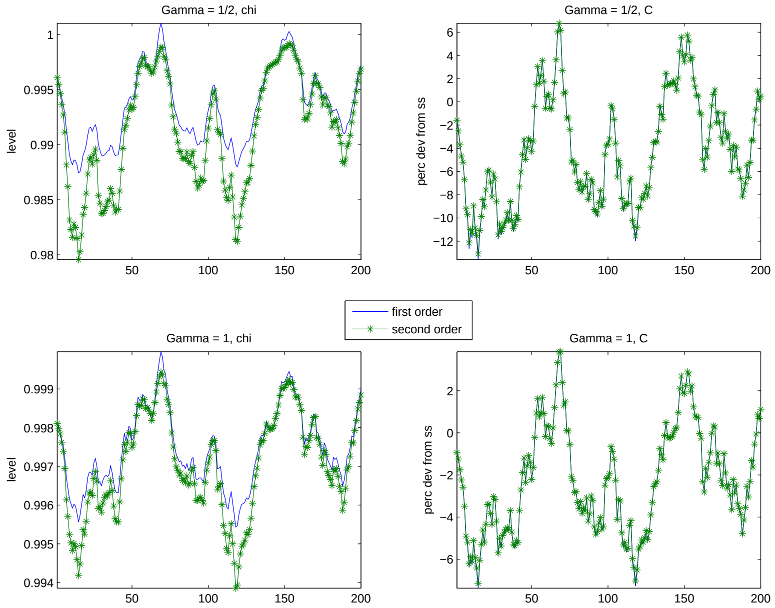
$$
\begin{aligned}
\bar{\pi} &= 1.025^{\frac{1}{4}},\ \psi_I = \psi_N = 1,\ \gamma = \frac{1}{2},\ \beta = 1.03^{-0.25}, \\
\theta &= 0.75,\ \varepsilon = 6\ \left(\frac{\varepsilon}{\varepsilon - 1} = 1.2\right),\ \varphi = 1,\ \nu = \frac{1}{\varepsilon}, \\
\sigma_a &= 0.01,\ \sigma_\tau = 0.01,\ \rho_a = 0.95,\ \rho_\tau = 0.90.
\end{aligned}
$$

Technology shock:

$$
\begin{aligned}
a_t &= (\rho_1 + \rho_2)\, a_{t-1} - \rho_1 \rho_2 a_{t-2} + \varepsilon_t,\ E\varepsilon_t^2 = 0.01^2, \\
\rho_1 &= 0.99 \text{ and } \rho_2 = 0.3.
\end{aligned}
$$

- Monetary policy rule:

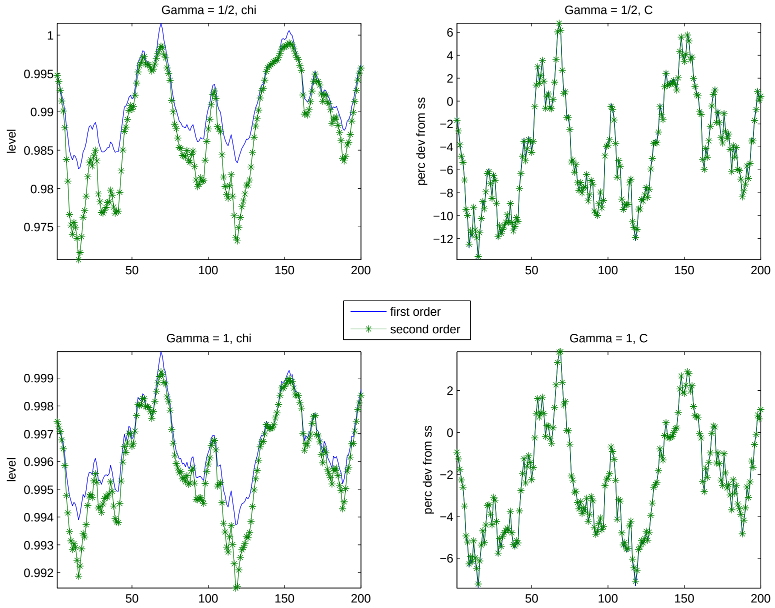
$$
R_t / R = (R_{t-1}/R)^{0.8} \exp\left[(1 - 0.8)\, 1.5(\bar{\pi}_t - 1.0062)\right]
$$



Gamma = 1/2, chi
level
Gamma = 1/2, C
perc dev from ss
first order
second order
Gamma = 1, chi
level
Gamma = 1, C
perc dev from ss

Results in Previous Graph

- Differences between first and second order perturbations
  - Negligible for consumption, and small for distortion, χ.
- Effect of reducing γ to $1/2$.
  - Volatility of consumption rises noticeably, consistent with the 'multiplier' discussed in the input-output literature.
  - Distortion, χ_t, not as great as the emprical estimate.
    - this is because the model does not generate the high inflation of the 1970s.
- The overall volatility of GDP in the example is somewhat higher than in the data. Prescott (1986) reports the standard deviation of log, HP filtered GDP to be around 2 percent. For the model, the standard deviation of log consumption is around 2.5 percent $(\gamma = 1)$ and around 4.7 percent $(\gamma = 1/2)$.
- The US data calculations suggest that the distortions are increased when the degree of competition is increased, as one can see in the next figure where ε was increased from 6 to 7.7.



Gamma = 1/2, chi
level
Gamma = 1/2, C
perc dev from ss
Gamma = 1, chi
level
Gamma = 1, C
perc dev from ss
first order
second order

Conclusion

- Some evidence of misallocation distortions from price setting frictions when production done in networks.
  - The evidence is very substantial when measured from the data using minimal restrictions from the model.
  - The evidence is less dramatic (though still non-negligible) when based on all the restrictions of the model using stochastic simulation.
- An extensive discussion of the implications for the Taylor principle appears in my 2011 handbook chapter.
  - When the smoothing parameter is set to zero and $\psi_I = \psi_N = 1$, then the model has indeterminacy, even when the coefficient on inflation is 1.5. So, the likelihood of the Taylor principle breaking down goes up when γ is reduced, consistent with intuition.
  - When the smoothing parameter is at its empirically plausible value of 0.8, then the solution of the model does not display indeterminacy.