1. Introduction

Acemoglu et al show that the patterns of amplification and propagation of industry shocks predicted by economic theory are supported by the data. They argue on this basis that macroeconomists should take networks seriously and they suggest that networks could have first order implications for macroeconomics. I agree. In my comment I describe two examples that illustrate the authors’ view that networks are important for macroeconomics. I show that in the presence of price setting frictions, network effects may imply that inflation has a much bigger cost than has previously been recognized. In addition, the presence of network effects may pose special challenges for the design of effective inflation targeting policies.

There is a general sense that the social cost of inflation is quantitatively large, and yet economists have not identified mechanisms by which these costs occur. The network structure of production combined with price setting frictions may well provide such a mechanism. The idea is that the allocative distortions generated by price frictions, distortions that increase with inflation, may be amplified by network effects in the same way that firm-level shocks are amplified. I describe this possibility using the simple approach to production networks suggested in Basu (1995).

One simple measure of the importance of networks is that gross output in the United States is roughly two times the size of Gross National Product (GDP). That is, of the gross output produced by the typical firm, about half goes to final buyers and half goes to other

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firms. Similarly, a large portion of the productive inputs used by firms are materials produced by other firms. The pattern of relationships among firms - the network of production - is what is measured by the well known (but usually ignored by macroeconomists) input-output matrices. Taking into account network effects, a one percent technology shock at the level of firms is magnified into a two percent shock to GDP. Price setting distortions have effects on gross output that resemble those of a negative technology shock (see Yun (1996)). If they show up as a one percent shock to gross output, then they are magnified into a two percent shock at the level of GDP. In effect, the combination of network effects and price level frictions provides a quantitatively substantial endogenous theory of total factor productivity.

I adopt the Calvo (1983) model of price frictions. This approach assumes that only a fraction of firms can optimize their price in a given period. The complementary fraction of firms cannot optimize their price and I assume that their price remains the same as it was in the previous period. This no-indexation assumption is motivated by the observation that at the microeconomic level, many prices remain unchanged for extended periods of time (see Eichenbaum, Jaimovich, and Rebelo 2011 and Klenow and Malin 2011). In the absence of indexation, aggregate inflation causes relative prices to deviate from what they would be if prices were flexible. This effect is greater for higher rates of inflation. That is, through network effects high inflation acts as a large, negative technology shock to GDP. Of course, this requires that the frequency of price adjustment not change too much as the level of inflation changes. But, the empirical results in Golosov and Lucas (2007, Figure 3) suggest that this is in fact the case. They present evidence that the average frequency of price adjustment changes very little for inflation rates in the range of zero to 16 percent per year.

In this setting, when steady state inflation is zero then there is no price distortion because price optimizers and non-optimizers do the same thing. As a result, there is no output lost to misallocation of resources. However, when steady state inflation reaches the levels reached in the US in the 1970s the costs of inflation reach levels that are equal to a 10 percent or greater loss in GDP, taking into account network effects. These numbers are obviously very big. The calculations require many assumptions (all presented in detail below). Apart from the assumption that production occurs in networks, the assumptions I make are standard. Still, greater scrutiny no doubt will imply that some of these assumptions deserve adjustment.

But, even if such adjustments warrant cutting the cost of inflation in half, the costs would still be very large. And, it is quite possible that adjustments would actually increase the costs assigned to inflation. For example, the modeling shortcut that I take to capture networks leaves out much of the richness and detail that Acemoglu et al describe. A consequence of the simplification - in addition to increasing the transparency and analytic tractability of the presentation - is that the number of prices in my framework is substantially less than what would appear in a more empirically realistic analysis. With such additional prices there
could come additional possibilities for distortions and more reasons for inflation to be costly when there are price setting frictions.

There is a second reason that networks in combination with price-setting frictions may be important. There is a widespread consensus that (i) inflation targeting has valuable macroeconomic stabilizing powers and that (ii) the Taylor principle represents a good basis for implementing inflation targeting. Under this principle, interest rates should be increased by more than 1 percent when inflation rises by 1 percent. Through a demand channel, such an increase in the interest rate induces a fall in GDP and thereby brings inflation back down to target. It is well known that when firms need to borrow to finance their variable inputs this opens up a second, working capital channel, by which interest rates affect inflation. In particular, by raising marginal costs, higher interest rates help to push inflation up. In principle, the working capital channel could be stronger than the demand channel, with the possibility that the Taylor principle becomes what might be called the Taylor curse. A sharp rise in the interest rate, rather than being an antidote to inflation, could actually stimulate inflation.

In standard macroeconomic models with price setting frictions, but no network effects, the working capital channel is not strong enough to overwhelm the demand channel by enough to destabilize the economy.\(^1\) So, standard models provide support to the Taylor principle. However, I show below that when network effects are taken into account, the possibility that the working capital channel is stronger than the demand channel is much greater. The analysis does not raise a question about the desirability of inflation targeting per se.\(^2\) Instead, it shows that integrating the network nature of production into monetary policy analysis is important for (ii), the design an effective strategy for implementing of inflation targeting.

These considerations are examples of why I think Acemoglu et al are right in their conjecture that network effects may have first order implications for macroeconomics.

The following section provides a rough sketch of the model used in both parts of my analysis. Details about the model and its solution are provided in the appendix. The following two sections focus on the first and second reasons that network effects may be important for macroeconomics, respectively.

\(^1\) Christiano, Eichenbaum and Evans (2005) show that the working capital is large enough that a monetary policy-induced rise in the interest rate drives inflation up. But, this effect is only transitory and not enough to wipe out the basic stabilizing effects of the Taylor principle.

\(^2\) For such a discussion, see Christiano, Ilut, Motto and Rostagno (2010).
2. A Business Cycle Model with Networks and Price Frictions

We adopt the usual Dixit-Stiglitz framework, in which there is a homogeneous good, \( Y_t \), that is produced by a representative competitive firm using the following production function:

\[
Y_t = \left[ \int_0^1 Y_{i,t}^{\varepsilon-1} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1.
\]

The representative firm takes output and input prices, \( P_t \) and \( P_{i,t} \), \( i \in (0,1) \), as given and chooses \( Y_{i,t} \), \( i \in (0,1) \) to maximize profits. The \( i^{th} \) input, \( Y_{i,t} \), is produced by a monopolist using labor, \( N_{i,t} \), and materials, \( I_{i,t} \), using the following technology:

\[
Y_{i,t} = A_t N_{i,t}^\gamma I_{i,t}^{1-\gamma},
\]

\( 0 < \gamma \leq 1 \). The monopolist sets its price, \( P_{i,t} \), subject to Calvo price-setting frictions. In particular, with probability \( \theta \) the \( i^{th} \) monopolist must set \( P_{i,t} = P_{i,t-1} \), i.e., there is no price indexation. The assumption that non-optimizing firms do not index their price was discussed in the introduction. With probability \( \theta \), the monopolist can set \( P_{i,t} \) optimally.

The \( i^{th} \) monopolist is competitive in the market for materials and labor. It acquires materials by purchasing the homogeneous good and converting it one-for-one into \( I_{i,t} \). In addition, it buys output from other firms. In effect, the \( i^{th} \) firm is embedded in a network, in which some of its output is sold to other firms for their use as materials and some of its inputs are materials acquired from other firms.

My analysis is drastically simplified by adopting the device suggested in Basu (1995) of assuming that the relationship between intermediate good firms operates via the homogeneous goods market. In Acemoglu et al firms interact directly with each other. I conjecture that the basic principles emphasized in my discussion carry over to the more empirically relevant framework in Acemoglu et al. It is less clear how the quantitative properties carry over, and this would be a very interesting issue to investigate more closely. For example, a consequence of the simplicity of my environment is that the market price of materials, the price of gross output and the price of aggregate value added all coincide with \( P_t \). When, as is appropriate in an empirically realistic setting, all these prices are allowed to differ then the possibilities of misallocations due to price distortions are increased. At the same time, my model implicitly assumes that the price of materials is subject to the same frictions as the price of final goods produced by firms. It is possible that this assumption overstates the implications of price frictions, in case the prices of materials are more flexible than the price of final goods. This is certainly something worth investigating. However, the outcome is not so obvious. Firms’ relationship with their suppliers resembles the relationship between firms and workers and all the factors that support long-term relationships and contracts between workers and firms also apply to firms and their suppliers.
The effective price of labor and materials for intermediate good firms is given by $\bar{W}_t$ and $\bar{P}_t$, respectively, where

$$\bar{W}_t = (1 - \nu) [1 - \psi + \psi R_t] W_t, \quad \bar{P}_t = (1 - \nu) [1 - \psi + \psi R_t] P_t.$$  

Here, $W_t$ denotes the competitively determined price of labor; $\psi$ is the fraction of input costs that must be paid in advance; $R_t$ is the gross nominal rate of interest; and $\nu$ is a lump sum tax-financed subsidy designed to extinguish the distortions of monopoly power in steady state, so that $(1 - \nu) \varepsilon / (\varepsilon - 1) = 1$.

In equilibrium, aggregate gross output, $Y_t$, is related to aggregate employment, $N_t$, and the aggregate use of materials, $I_t$, by the following gross output production function:

$$Y_t = p^*_t A_t N_t^\gamma I_t^{1-\gamma}, \tag{2.1}$$

where $p^*_t$ is a measure of dispersion among sectoral prices, $P_{i,t}$, $i \in (0, 1)$. The variable, $p^*_t$, captures the amount of homogeneous output that is lost because of the misallocation of aggregate employment and materials across sectors due to price-setting frictions. I refer to $p^*_t$ as the Tack Yun distortion because it is described in Yun (1996). The law of motion of the Tack Yun distortion is given by:

$$p^*_t = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{1}{\varepsilon}} + \frac{\theta \bar{\pi}_t}{p^*_{t-1}} \right]^{-1}. \tag{2.2}$$

To understand this expression, note that when prices are flexible (i.e., $\theta = 0$), then $p^*_t = 1$ and there is no price dispersion. The upper bound on $p^*_t$ is attained when $\theta = 0$ because all firms face the same marginal cost and demand elasticity, so that if they could all flexibly set their prices, then they would all set the same price. Producers of $Y_t$ would in this case demand an equal amount $Y_{i,t}$ from each of the $i \in (0, 1)$ intermediate good producers. For a given level of aggregate resources, $N_t$ and $I_t$, the greatest value of $Y_t$ is achieved by distributing labor and materials in equal amounts across sectors and this explains the result in (2.2) that $p^*_t = 1$ when $\theta = 0$. When there are price setting frictions, $\theta > 0$, then price dispersion is possible, giving rise to a loss in output, $p^*_t < 1$. Aggregate inflation naturally generates price dispersion when $\theta > 0$ as non-optimizing firms hold prices fixed while on average prices are changing.

Cost minimization by firms leads to the following relationship between aggregate materials and gross output:

$$I_t = \frac{\mu_t}{p^*_t} Y_t. \tag{2.3}$$

In the flexible price version of the model with no working capital channel, $\mu_t = 1 - \gamma$. In the
version of the model with price frictions and a working capital, \( \mu_t \) fluctuates with changes in the price markup and with changes in the interest rate, \( R_t \).\(^3\)

Using (2.3) to substitute out for \( I_t \) in (2.1), and solving the latter for \( Y_t \) we obtain:

\[
Y_t = \left[ \frac{p_t^* A_t \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma}}{\gamma} \right]^{\frac{1}{\gamma}} N_t.
\] (2.4)

Gross Domestic Product (GDP) in this economy is just consumption:

\[
C_t = Y_t - I_t = \left( 1 - \frac{\mu_t}{p_t^*} \right) Y_t.
\]

Making use of (2.4), we obtain the value-added aggregate production function:

\[
C_t = TFP_t \times N_t.
\]

Here, Total Factor Productivity, \( TFP \), is given by

\[
TFP_t = \left[ A_t \gamma^\gamma (1 - \gamma)^{1-\gamma} \right]^{\frac{1}{\gamma}} \chi_t,
\] (2.5)

where

\[
\chi_t \equiv \left( \frac{p_t^*}{\gamma} \right)^\gamma \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \left( \frac{1}{1 - \gamma} \right)^\gamma.
\] (2.6)

I refer to \( \chi_t \) as the allocative distortion. In the flexible price version of the model with no working capital, \( \chi_t = 1 \), so that there is no allocative distortion.

There are two things to observe about TFP in this economy. First, from (2.5) we see that, in the presence of networks, the disturbance in the value-added production function is a magnified version of the shock, \( A_t \), in the gross output production function (recall, \( 1/\gamma > 1 \)). This is the well-known ‘multiplier effect’ in the literature on networks (see, e.g., Jones (2013)). When \( A_t \) is treated as an unobserved variable, then this multiplier effect is of limited substantive interest. Given observations on \( C_t \) and \( N_t \), all we observe is \( TFP \) and whether we think of the whole of \( TFP \) as a shock, or of \( TFP \) as a smaller shock that has been magnified is immaterial. Second, we can see that price dispersion, \( p_t^* \), is also magnified in the value-added production function. This is potentially of substantial significance because, according to (2.2), \( p_t^* \) is a function of inflation, which is observable.

In the version of the model with flexible prices and no working capital channel, we have enough equilibrium conditions to determine all the equilibrium variables of interest, including \( C_t, N_t, I_t, p_t^* \) and \( \chi_t \).\(^4\) In the presence of price setting frictions, we are short one equation

\(^3\) See equation (A.9) in the Appendix.

\(^4\) The fact that the aggregate variables can be determined without reference to monetary policy reflects that this version of the model exhibits the ‘classical dichotomy’ in which the private sector conditions determine real variables and monetary policy determines nominal variables.
for determining these variables. I fill this gap in the standard way, with a Taylor rule for setting the nominal rate of interest:

\[ R_t / R = (R_{t-1} / R)^{\rho} \exp \left[ (1 - \rho) 1.5(\bar{\pi}_t - 1.025^{1/4}) \right], \tag{2.7} \]

where \( R \) denotes the value of \( R_t \) in the non-stochastic steady state. We posit an inflation target of 2.5 percent per year and that the weight on inflation is 1.5, a value that easily satisfies the well-known Taylor principle which requires that the weight on inflation exceed unity.

3. Networks, Price Frictions and the Welfare Cost of Inflation

According to the New Keynesian model described in the previous section, inflation reduces social welfare by creating relative price distortions which in turn lead to a misallocation of resources. The resulting loss of output is measured by \( \chi_t \) in (2.6). This section shows that the losses are quantitatively large, particularly when I take into account the network nature of production.

It is particularly interesting to assess the cost of the high inflation in 1970s and early 1980s because there is a widespread view that that inflation was at least in part responsible for the poor performance of output and productivity during the period.\(^5\) At the same time, economists have not been successful at identifying a mechanism by which inflation can have quantitatively large, negative economic effects. I show here that when networks are combined with price-setting frictions as in the previous section, we do have such a mechanism.

The first subsection examines the distortions using steady state calculations. The steady state has the advantage that it is characterized by simple, transparent expressions. The second subsection reports a time series on the distortions. Both sets of distortions convey the same message: the distortions from inflation are quantitatively large, especially when network effects are taken into account.

\(^5\)The American public appears to believe that the 1970s inflation is at least partially responsible for the poor economic performance of the 1970s. This can be inferred from revealed preference. It is widely believed that the severe recession of the early 1980s was brought about by Federal Reserve Chairman Paul Volcker as a side-effect of his strategy for ending the high inflation of the 1970s. Despite the perceived high cost of Volcker’s anti-inflation policy, his public reputation is very high. I infer that the public views the benefit of Volcker’s strategy - ridding the economy of high inflation - is preferred to the cost - the punishing recession of the early 1980s. That is, the public assigns a high cost to inflation.
3.1. Steady State Distortions

Consumer price inflation from 1972Q1 to 1982Q4 averaged 8 percent, at an annual rate.\(^6\) One way to evaluate implications of this level of inflation for \(\chi_t\) is to compute the steady state value of \(p^*\) when inflation is 2 percent per quarter. The steady state value of \(p^*\) is, according to (2.2),

\[
p^* = \frac{1 - \theta \bar{\pi}^\varepsilon}{(1 - \theta) \left( \frac{1-\theta\bar{\pi}^\varepsilon(\varepsilon-1)}{1-\theta} \right)^\frac{\varepsilon}{\varepsilon-1}}.
\] (3.1)

The calculation relevant for the period, 1972Q1-1982Q4 uses \(\bar{\pi} = 1.02\). In addition, I set \(\theta = 3/4\), which implies that prices are constant on average for one year. I also require a value for the elasticity of demand, \(\varepsilon\). The results are dependent on the value assigned to the elasticity of demand, \(\varepsilon\), and this is not surprising. The greater that elasticity is, the greater is the response of resource allocations to a given distortion in relative prices.\(^7\) As a baseline, I follow Christiano, Eichenbaum and Evans (2005) by using \(\varepsilon = 6\), which implies a steady state price markup of 20 percent. I also allow for a greater amount of competition by considering a price markup of 15 percent, which requires \(\varepsilon = 7.7\).

Results are reported in Table 1. When I evaluate (3.1) in the baseline case, I obtain \(p^* = 0.97\). In the case of greater competition, I obtain \(p^* = 0.96\). Thus, the output loss is 3 percent and 4 percent, respectively, in the baseline and higher competition cases, when we ignore network effects, i.e., \(\gamma = 1\). To take into account network effects, I measure the distortion by \((p^*)^{1/\gamma}\).\(^8\) To see the impact of raising \(p^*\) to the power, \(1/\gamma\), express \(p^*\) as \(p^* = 1 - \omega\). Then, to a first order approximation,

\[
(p^*)^{\frac{1}{\gamma}} \approx 1 - \frac{1}{\gamma} \omega.
\] (3.2)

From this, we see that the effect of introducing networks, i.e., increasing \(\gamma\) from 1 to 2, roughly doubles the magnitude of the distortion measured in terms of \(\omega\). Thus, when there are network effects, \(\gamma = 2\), the loss of output is 5 percent and 9 percent, respectively, in the baseline and higher competition cases. These are big numbers, hard to ignore.

A possible source of concern about the preceding analysis has to do with the value that I used for the frequency of price changes, \(\theta\). That value is based on studies of US data under moderate inflation. A natural question is, “was the frequency of price changes in the 1970s

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\(^6\)Here and below, I use data on the consumer price index, CPIAUCSL, taken from the FRED database, maintained by the Federal Reserve Bank of St. Louis.

\(^7\)There is another effect that is dominated in practice by the one cited in the text. In particular, the greater is the elasticity of substitution between goods, the less severe are the consequences of a given degree of misallocation.

\(^8\)I have ignored another channel by which \(p^*\) affects, \(\chi\) (see (3.1)). However, \(p^*\) has both a positive and negative effect on \(\chi\) by that channel and the tworoughly cancel.
much higher than it was during periods of more moderate inflation in the US?” The empirical results reported in Golosov and Lucas (2007) suggest that the answer to the question is ‘no’. They summarize empirical evidence on the frequency of price adjustment (see their Figure 3) which indicates that a change in inflation from 2.5 percent per year to 10 percent per year is associated with virtually no change in the frequency of price changes.

3.2. Dynamic Distortions

Next, we turn to the time series evidence on $p^*_t$ and $\chi_t$ displayed in Figure 1. The objects that determine these variables are not fully observable. In the case of $p^*_t$ I require an initial condition (see (2.2)). However, I found that the initial condition has a negligible dynamic effect on $p^*_t$. Figure 1 reports the time series on $p^*_t$ when the initial value of $p^*_t$ (in 1947Q3) is set to unity. The results after 1960 are essentially unchanged if I instead choose a far smaller value of 0.80 for the initial $p^*_t$.

To compute $\chi_t$ I need time series data on $\mu_t$ in addition to $p^*_t$ (see (2.6)). In Figure 1 I simply approximate $\mu_t$ by $1 - \gamma$, its value when prices are flexible and there is no working capital channel. To investigate the quantitative magnitude of the distortions implied by this approximation, I did stochastic simulations of the model (see the model details in the appendix) and found that replacing the actual value of $\mu_t$ with $1 - \gamma$ has virtually no impact on $\chi_t$. The reason is that $\mu_t$ enters $\chi_t$ with both a positive and negative sign (see (2.6)).

Figure 1 has two panels. Figure 1a displays results for the baseline parameterization reported above, in which the markup is 20 percent. Figure 2b reports results for slightly greater competition, with a markup of 15 percent. Each panel displays three time series: quarterly observations on quarterly gross inflation in the consumer price index; quarterly observations on $p^*_t$ and quarterly observations on $\chi_t$. Consider the top panel first. There are several notable results. First, the output lost implied by the results ($\omega_t$ in (3.2)) is roughly twice as large when the network structure of production is taken into account (i.e., $\gamma = 2$). Second, the output losses are quantitatively large. The average value of $\chi_t$ is 0.98 so that on average 2 percent of GDP is lost. Also, 10% of the time, the amount of output lost exceeds 4.5 percent of GDP. Finally, the losses during the high inflation of the 1970s are quantitatively large. Indeed, they reach a maximum value of 14 percent in 1979 (see Figure 1).

In the case of increased competition the losses are even larger. Clearly, this model has no trouble rationalizing the view that high inflation imposes a big cost on society. If anything,
one is suspicious that the cost suggested by the model is implausibly high.

| Table 1: Fraction of GDP Lost Due to 8 Percent (APR) Inflation, 100(1 − χ) |
|-----------------------------|-----------------------------|
| Without networks, γ = 1 | With networks, γ = 2 |
| Steady state lost output$^1$ | 2.61 (4.34)$^3$ percent | 5.16 (8.50) percent |
| Mean, 1972Q1-1982Q4, Figure 1$^2$ | 3.13 (5.22) percent | 6.26 (10.44) percent |

Note 1: steady state formula - χ = ($p^*$)$^{1/γ}$, where $p^*$ is defined in (3.1).

Note 2: output lost measure defined in (2.6).

Note 3: number not in parentheses assumes a markup of 20 percent and number not in parentheses assumes a markup of 15 percent.

4. Networks and the Taylor Principle

There is a consensus that inflation targeting is a monetary policy with excellent operating characteristics, at least in ‘normal’ times when the zero lower bound on the interest rate is not binding. Inflation targeting can be operationalized by applying a rule like (2.7) with a coefficient on inflation that is substantially greater than unity, i.e., that satisfies the ‘Taylor principle’. The idea is that the Taylor principle serves two important objectives. One is the achievement of low average inflation. By raising the interest rate when inflation is above target, the central bank reigns in the demand for goods and services. Working through the demand channel, this policy generates a slowdown in economic activity, which brings inflation back down to target by reducing marginal costs.

The second objective of the Taylor principle is to anchor inflation expectations. Unanchored inflation expectations can be a source of instability in inflation as well as in aggregate output and employment. To see this, suppose that for some reason there is a jump in inflation expectations. The resulting lower real rate of interest stimulates spending and output. By producing a rise in marginal cost, the increase in output contributes to a rise in inflation. In this way, the initial jump in inflation expectations is self-fulfilling and so inflation is without an anchor.

The Taylor principle helps to short-circuit this loop from higher inflation expectations to higher actual inflation. This is also accomplished by working through the demand channel. When the monetary authority raises the interest rate vigorously in response to inflation, the demand channel produces a fall in spending which reduces output and hence inflation. As people become aware of the lower actual inflation, the inflation expectations that initiated the loop would evaporate before it could have much of an effect on the macroeconomy. Under rational expectations, the initial jump in inflation expectations would not occur in the first place.

The stabilizing effects of the Taylor principle depend on the demand channel being the primary avenue through which monetary policy operates. When firms have to borrow to pay for their variable inputs, then the interest rate is a part of marginal cost and monetary policy
also operates through a working capital channel. If the working capital channel is sufficiently important then, instead of curbing inflation, a jump in the nominal rate of interest could actually ignite inflation.

Whether the working capital or demand channel dominate has been studied extensively in the type of model described in the previous section (see, for example, CTW). The general finding is that when $\gamma = 1$ the demand channel dominates the working capital channel and the Taylor principle achieves the two objectives described above. This is so, even when the working capital channel is strongest, with $\psi = 1$. When gross output and value added coincide, there is not enough borrowing for the working capital channel to overwhelm the demand channel. However, when we take into account the network nature of production (i.e., $\gamma = 2$), then the amount of borrowing for working capital purposes is potentially much greater. As a result, when $\psi = 1$ and there is no interest rate smoothing in the Taylor rule, i.e., $\rho = 0$, the non-stochastic steady state equilibrium is indeterminate. Even though monetary policy satisfies the Taylor principle, there are many equilibria. These equilibria can be characterized in terms of the loop from higher expected inflation to higher actual inflation discussed above. When there is no working capital channel, then the Taylor principle short-circuits this loop by raising the interest rate and preventing the rise in expected inflation from occurring. But, when the working capital channel is sufficiently strong, then the rise in the interest rate simply reinforces the loop from higher expected inflation to higher actual inflation. In this way the Taylor principle could become the ‘Taylor curse’ referred to in the introduction.

It is interesting that the Taylor principle works as hoped for when there is substantial interest rate smoothing in monetary policy, i.e., $\rho$ is large. Presumably, the intuition for this is that demand responds most strongly to long term interest rates rather than to short term interest rates. As a result, the strength of the demand channel is increasing in $\rho$ while the working capital channel, which is only a function of the short term interest rate, remains unaffected. The example highlights how the integration of network effects could be important for the design of monetary policy. This is a topic that deserves further study. It is important to know, for example, how pervasive working capital is in the data.\(^9\) It is also important to understand better how the working capital channel works in network environments that are closer to the more realistic one advocated in the Acemoglu et al paper, in which firms buy materials directly from other firms.

\(^9\)For one revelant study, see Barth and Ramey (2002).
A. Appendix: Model Used in the Analysis

This comment makes use of a standard New Keynesian model, extended to include network effects following the suggestion of Basu (1995). At the level of detail, the model corresponds to the one analyzed in CTW. I include a description of the model here for completeness and because CTW do not describe all the connections between gross output and value-added that I need for my discussion. Stochastic simulations of the model are used in the discussion in section 3 and section 4 studies the determinacy properties of the model non-stochastic steady state.

A.1. Households

There are many identical, competitive households who maximize utility,

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( u(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad u(C_t) \equiv \log C_t,
\]

subject to the following budget constraint:

\[
\text{s.t. } P_tC_t + B_{t+1} \leq W_tN_t + R_{t-1}B_t + \text{Profits net of taxes}_t.
\]

Here, \(C_t\) denotes consumption; \(W_t\) denotes the nominal wage rate; \(N_t\) denotes employment; \(P_t\) denotes the nominal price of consumption; \(B_{t+1}\) denotes a nominal one period bond purchased in period \(t\) which pays off a gross, nominal non-state contingent return, \(R_t\), in period \(t+1\). Also, the household earns lump sum profits and pays lump sum taxes to the government. Optimization by households implies:

\[
\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \quad \text{(A.1)}
\]

\[
C_tN_t^{\varphi} = \frac{W_t}{P_t}. \quad \text{(A.2)}
\]

A.2. Goods Production

The structure of production has the Dixit-Stiglitz structure that is standard in the New Keynesian literature, extended to consider network effects.

A.2.1. Homogeneous Goods

A representative, homogenous good firm produces output, \(Y_t\), using the following technology:

\[
Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon}{\varepsilon-1}} dj \right]^{\frac{\varepsilon-1}{\varepsilon}}, \quad \varepsilon > 1. \quad \text{(A.3)}
\]
The firm takes the price of homogeneous goods, $P_t$, and the prices of intermediate goods, $P_{i,t}$, as given and maximizes profits,

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} d\gamma,$$

subject to (A.3). Optimization leads to the following first order condition:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^{\varepsilon}, \quad \text{(A.4)}$$

for all $i \in (0,1)$. Combining the first order condition with the production function, we obtain the following equilibrium condition:

$$P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} d\gamma \right)^{\frac{\varepsilon}{1-\varepsilon}}. \quad \text{(A.5)}$$

A.2.2. Intermediate Goods

The intermediate good, $i \in (0,1)$, is produced by a monopolist using the following technology:

$$Y_{i,t} = A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}, \quad 0 < \gamma \leq 1.$$

Here, $N_{i,t}$ and $I_{i,t}$ denotes the quantity of labor and materials, respectively, used by the $i^{th}$ producer. The producer obtains $I_{i,t}$ by purchasing the homogenous good, $Y_t$, and converting it one-for-one into materials. Both $N_{i,t}$ and $I_{i,t}$ are acquired in competitive markets.

Firms experience Calvo-style frictions in setting their price. That is, the $i^{th}$ firm sets its period $t$ price, $P_{i,t}$, as follows:

$$P_{i,t} = \begin{cases} \hat{P}_t & \text{with probability } 1 - \theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}, \quad 0 \leq \theta < 1. \quad \text{(A.6)}$$

Here, $\hat{P}_t$ denotes the price selected in the probability $1 - \theta$ event that it can choose its price. Firms that cannot optimize their price must simply set it to whatever value it took on in the previous period.

Given its current price (however arrived at), the firm must satisfy the $Y_{i,t}$ that is implied by (A.4). Linear homogeneity of its technology and our assumption that the $i^{th}$ firm acquires materials and labor in competitive markets implies that marginal cost is independent of $Y_{i,t}$. By studying its cost minimization problem we find that $s_t$, the $i^{th}$ firm’s marginal cost (scaled by $P_t$) is

$$s_t = \left( \frac{\hat{P}_t/P_t}{1 - \gamma} \right)^{1-\gamma} \left( \frac{\hat{W}_t/P_t}{\gamma} \right)^{\gamma} \frac{1}{A_t}. \quad \text{(A.7)}$$
Here, $\bar{P}_t$ and $\bar{W}_t$ denote the net price, after taxes and interest rate costs, of materials and labor, respectively. In particular,

$$\bar{W}_t = (1 - \nu)(1 - \psi + \psi R_t) W_t$$
$$\bar{P}_t = (1 - \nu)(1 - \psi + \psi R_t) P_t,$$

where, $\nu$ is a subsidy selected by the government to extinguish the distortions due to monopoly power in steady state:\footnote{The fiscal authority levies lump sum taxes in the amount, $\nu \left[ (1 - \psi + \psi R_t) W_t N_t + (1 - \psi + \psi R_t) P_t I_t \right]$, on households.}

$$\frac{(1 - \nu)}{\varepsilon - 1} = 1.$$ 

Also, $\psi$ represents the fraction of input costs that must be financed in advance so that, for example, one unit of labor used during period $t$ costs $\psi W_t R_t$ units of currency at the end of the period.\footnote{We assume that banks create credits which they provide in the amount, $\left( W_t N_t + P_t I_t \right)$, to firms at the beginning of the period. At the end of the period they receive $R_t \left( \psi W_t N_t + \psi P_t I_t \right)$ back from firms and the profits, $(R_t - 1) \left( \psi W_t N_t + \psi P_t I_t \right)$, are transferred to households in lump sum form.} Another implication of the $i^{th}$ firm’s cost minimization problem is that cost of materials, $\bar{P}_t I_{i,t}$, as a fraction of total cost, is equal to $1 - \gamma$. This implies,

$$I_{i,t} = \mu_t Y_{i,t}, \tag{A.8}$$

where $\mu_t$ is the share of materials in gross output, and:

$$\mu_t = \frac{(1 - \gamma) s_t}{(1 - \nu)(1 - \psi + \psi R_t)}. \tag{A.9}$$

I now turn to the problem of one of the $1 - \theta$ randomly selected firms that has an opportunity to select its price, $\hat{P}_t$, in period $t$. Such a firm is concerned about the value of its cash flow (i.e., revenues net of costs) in period $t$ and in later periods:

$$E_t \sum_{j=0}^{\infty} (\beta \theta)^j v_{t+j} [\hat{P}_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}] + \Phi_t. \tag{A.10}$$

The objects in square brackets are the cash flows in current and future states in which the firm does not have an opportunity to reset its price. The expectation operator in (A.10) integrates over aggregate uncertainty, while the firm-level idiosyncratic uncertainty is manifest in the presence of $\theta$ in the discounting. In (A.10), cash flows in each period are weighed by the associated date and state-contingent value that the household assigns to cash. The firm takes these weights as given and

$$v_{t+j} = \frac{u'(C_{t+j})}{P_{t+j}}. \tag{A.11}$$
The second term in (A.10), $\Phi_t$, represents the value of cash flow in future states in which the firm is able to reset its price. Given the structure of our environment, $\Phi_t$ is not affected by the choice of $\tilde{P}_t$. The problem of a firm that is able to choose its price is to select a value of $\tilde{P}_t$ that maximizes (A.10) subject to (A.4), taking $\tilde{P}_t$, $\tilde{W}_t$, $P_t$ and $W_t$ as given.

To solve the firm problem I first substitute out for $Y_{i,t}$ and $\nu_t$ using (A.4) and (A.11), respectively. I then differentiate (A.10) taking into account that $\Phi_t$ is independent of $\tilde{P}_t$.

The solution to this problem is obtained by a standard set of manipulations. In particular, let

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \quad \tilde{\pi}_t \equiv \frac{P_t}{P_{t-1}}, \quad X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j-1}\pi_{t+1}}, & j \geq 1 \\ 1, & j = 0 \end{cases},$$

$$X_{t,j} = X_{t+1,j-1} \frac{1}{\pi_{t+1}}, \quad j > 0$$

Then, the (scaled by $P_t$) solution to the firm problem is:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta \theta)^j (X_{t,j})^{\frac{1}{1-\varepsilon}} Y_{t+j} \frac{\varepsilon}{C_{t+j}} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \theta)^j (X_{t,j})^{1-\varepsilon} Y_{t+j} \frac{\varepsilon}{C_{t+j}}} = \frac{K_t}{F_t}, \quad (A.12)$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1} Y_t C_t + \beta \theta E_t \left( \frac{1}{\pi_{t+1}} \right)^{-\varepsilon} K_{t+1} \quad (A.13)$$

$$F_t = \frac{Y_t}{C_t} + \beta \theta E_t \left( \frac{1}{\pi_{t+1}} \right)^{1-\varepsilon} F_{t+1}. \quad (A.14)$$

### A.3. Economy-wide Variables and Equilibrium

By the usual result associated with Calvo-sticky prices, we have the following cross-price restriction:

$$P_t = \left( \int_0^1 P_t^{(1-\varepsilon)} \, di \right)^{\frac{1}{1-\varepsilon}} = \left[ (1 - \theta) \tilde{P}_t^{(1-\varepsilon)} + \theta P_t^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \quad (A.15)$$

Dividing by $P_t$ and rearranging,

$$\tilde{p}_t = \left[ \frac{1 - \theta \pi_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \quad (A.16)$$

Combining this expression with (A.12), we obtain a useful equilibrium condition:

$$\frac{K_t}{F_t} = \left[ \frac{1 - \theta \pi_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \quad (A.17)$$
It is convenient to express real marginal cost, (A.7), in terms that do not involve prices:

\[
s_t = (1 - \nu) \left( \frac{1 - \psi + \psi R_t}{1 - \gamma} \right)^{1-\gamma} \times \left( \frac{1 - \psi + \psi R_t}{\gamma} C_t N_t^\phi \right)^\gamma \frac{1}{A_t},
\]

where (A.2) has been used to substitute out for \( W_t/P_t \).

We now seek the relationship between Gross Domestic Product (GDP) and aggregate employment, \( N_t \). To this end, we first compute the equilibrium relationship between aggregate inputs, \( I_t \) and \( N_t \), and gross output, \( Y_t \). We do this by adapting an argument suggested in Yun (1996). We then adapt a version of the argument in Jones (2013) to obtain the mapping from aggregate employment to GDP.

Let \( Y_t^* \) denote the unweighted sum of \( Y_{i,t} \) and then substitute out for \( Y_{i,t} \) in terms of prices using (A.4):

\[
Y_t^* = \int_0^1 Y_{i,t} \, di = Y_t \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} \, di = Y_t \left( \frac{P_t}{P_t^*} \right)^{\epsilon},
\]

where

\[
P_t^* = \left[ \int_0^1 P_{i,t}^{-\epsilon} \, di \right]^{\frac{1}{-\epsilon}} = \left( (1 - \theta) \frac{\hat{P}_t^{-\epsilon}}{P_t} + \theta (P_{t-1}^*)^{-\epsilon} \right)^{\frac{1}{-\epsilon}},
\]

using the analog of the result in (A.15). In this way, we obtain the following expression for \( Y_t \):

\[
Y_t = p_t^* Y_t^* = p_t^* \left( \frac{P_t^*}{P_t} \right)^{\epsilon},
\]

\[
p_t^* = \begin{cases} 
\leq 1 & \text{not } P_{i,t} = P_{j,t}, \text{ all } i, j \\
1 & P_{i,t} = P_{j,t}, \text{ all } i, j 
\end{cases}
\]

We refer to \( p_t^* \) as the Tack Yun distortion.

By a standard calculation, I obtain the law of motion for \( p_t^* \) by dividing (A.19) by \( P_t^* \), using (A.16) and rearranging,

\[
p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{P}_t^{(\epsilon-1)}}{1 - \theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \frac{\bar{P}_t^{(\epsilon-1)}}{p_{t-1}^*} \right]^{-1}. \tag{A.20}
\]

Then,

\[
Y_t = p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} \, di = p_t^* A_t \int_0^1 N_{i,t} I_{i,t}^{1-\gamma} \, di = p_t^* A_t \left( \frac{N_t}{I_t} \right)^{\gamma} I_t,
\]

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where I have used the fact that all firms have the same materials to labor ratio. In this way, we obtain

\[ Y_t = p_t^* A_t N_t^\gamma t_t^{1-\gamma}. \]  (A.21)

I substitute out for \( I_t \) in (A.21) by noting from (A.8) that

\[ I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} di = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t. \]  (A.22)

Use this to solve out for \( I_t \) in (A.21):

\[ Y_t = \left( p_t^* A_t \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t. \]  (A.23)

**GDP** for this economy is simply

\[ C_t = Y_t - I_t. \]  (A.24)

I conclude,

\[ GDP_t = Y_t - I_t = \left( 1 - \frac{\mu_t}{p_t^*} \right) Y_t = TFP_t \times N_t, \]  (A.25)

after some rearranging. Here, \( TFP \) denotes total factor productivity and is given by:

\[ TFP_t = \left( p_t^* A_t \left( 1 - \frac{\mu_t}{p_t^*} \right)^{\gamma} \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}. \]  (A.26)

There are two features of (A.26) worth stressing. First, note that \( A_t \) is raised to the power, \( 1/\gamma \), so that when \( \gamma < 1 \) the effect of a technology shock is magnified by the network effects. This is an illustration of the ‘multiplier effect’ emphasized in that literature (see, e.g., Jones, 2013).

Second, note that network effects also magnify the effects of the Tack Yun distortion. In standard analyses of the New Keynesian model, which ignore networks (i.e., \( \gamma = 1 \)), \( p_t^* \) is often assumed to be unity. This is justified by the usual practice of setting \( \gamma = 1 \) and linearizing the model about a steady state with no price distortions, so that \( p_t^* = 1 \). Given my assumption of zero price indexation, this requires that there be no inflation in steady state. It is easy to verify that the first order Taylor series expansion of \( p_t^* \) about \( p_{t-1}^* = 1 \) and \( \pi_t = 1 \) is given by:

\[ p_t^* = 1 + 0 \times \pi_t + \theta \left( p_{t-1}^* - 1 \right). \]

Evidently, to a first order approximation \( p_t^* \) converges to unity and shocks (which can only impact \( p_t^* \) via their effect on \( \pi_t \)) have no effect on \( p_t^* \). The practice of linearizing about a steady state undistorted by price dispersion presumably reflects the conjecture that if the
steady state were distorted then the fluctuations in \( p_t^* \) would be negligibly small. But, even if the distortions were small in the production of gross output, they may nevertheless be non-negligible for value-added because of the multiplier effects associated with networks. Thus, if we express \( p_t^* = 1 - \omega_t \), where \( \omega_t \) is a small positive number, then

\[
(p_t^*)^{1/\gamma} \simeq 1 - \frac{1}{\gamma} \omega_t.
\]

Evidently, \( p_t^* \) enters (A.26) not just by way of \( (p_t^*)^{1/\gamma} \), but also via \( \mu_t / p_t^* \). However, stochastic simulation of the model suggest that \( p_t^* \) has only a small effect via that avenue, essentially because \( p_t^* \) has both a positive and negative effects on it.

To explore the multiplier effect of misallocation due to price frictions, \( p_t^* \), it is convenient to identify the biggest that \( TFP \) can feasibly be for given \( N_t \). This is the value of \( TFP \) associated with \( p_t^* = 1 \) and \( \mu_t / p_t^* = \gamma \). Multiplying and dividing (A.26) by this maximum, I obtain the following decomposition of TFP:

\[
TFP_t = \chi_t \left( A_t \gamma (1 - \gamma)^{1-\gamma} \right)^{1/\gamma},
\]

where

\[
\chi_t \equiv \left( \frac{p_t^*}{\gamma} \left( \frac{1 - \mu_t}{\mu_t} \right)^{-1/\gamma} \left( 1 - \frac{\mu_t}{\gamma} \right)^{1-\gamma} \right)^{1/\gamma}.
\]  

(A.27)

There are 12 variables to be determined for the model:

\[ K_t, F_t, \bar{\pi}_t, p_t^*, s_t, C_t, Y_t, N_t, I_t, \mu_t, R_t, \chi_t. \]

There are 11 equilibrium conditions implied by private sector decisions: (A.13), (A.14), (A.17), (A.20), (A.1), (A.21), (A.24), (A.23), (A.18), (A.9), (A.27). In the special case of flexible prices, no working capital and an efficient subsidy for monopoly power:

\[ \theta = 0, \; \psi_I = \psi_N = 0, \; \frac{\varepsilon}{\varepsilon - 1} (1 - \nu) = 1, \]

then the model dichotomizes. In this case, equilibrium consumption and employment solve the following two equations:

\[ N_t = 1, \; C_t = \left[ A_t (\gamma)^{\gamma} (1 - \gamma)^{1-\gamma} \right]^{1/\gamma}. \]

In the case that is of interest in this discussion, we need an additional equation to solve the model variables. To this end, I adopt the following specification of monetary policy:

\[ R_t / R = (R_{t-1} / R)^{0.8} \exp \left[ (1 - 0.8) 1.5(\bar{\pi}_t - 1.025^{1/4}) \right], \]

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where $R$ denotes the steady state value of $R_t$. According to this specification, the target inflation rate is 1.0062, or 2.5 at an annual rate. The smoothing parameter, 0.8, is large as it is generally found in the empirical literature. Under my specification, the Taylor principle is satisfied because the coefficient on inflation, 1.5, is well above unity. It can be verified that the steady state of the model is determinate when the smoothing parameter is 0.8 but indeterminate when the smoothing parameter is zero. The reasons for this are discussed in the text.

To compute the dynamic properties of the model I solve the model using second order perturbation (with pruning), using Dynare.\footnote{The code is available on my website.} For this, I require the model steady state in which the shocks, $a_t$, are held at their steady state values of zero. This steady state is easy to solve in closed form.

The parameter values I assign to the model are as follows:

\[
\bar{\pi} = 1.025^{1/4}, \quad \psi = 1, \quad \gamma = \frac{1}{2}, \quad \beta = 1.03^{-0.25},
\]
\[
\theta = 0.75, \quad \varepsilon = 6 \left( \frac{\varepsilon}{\varepsilon - 1} = 1.2 \right), \quad \varphi = 1, \quad \nu = \frac{1}{\varepsilon}.
\]

The time series representation I use for $a_t$ is that it is roughly a first order autoregression in its first difference. In particular,

\[
a_t = (\rho_1 + \rho_2) a_{t-1} - \rho_1 \rho_2 a_{t-2} + \varepsilon_t, \quad E \varepsilon_t^2 = 0.01^2,
\]

where $\rho_1 = 0.99$ and $\rho_2 = 0.3$.

References


Figure 1a: Graph of Quarterly, Gross US CPI inflation, p-star and chi, assumed markup is 1.2

Figure 1b: Graph of Quarterly, Gross US CPI inflation, p-star and chi, assumed markup is 1.15