Ramsey Equilibrium in the Simple New Keynesian Model with no Capital, no Networks and no Working Capital Requirement

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For a more general analysis of optimal policy which includes discussions of time Inconsistency, the effects of working capital and what happens if the monopoly distortion subsidy is not chosen optimally, see http://faculty.wcas.northwestern.edu/~lchrist/course/optimalpolicyhandout.pdf

A Model with Government Policy

- New Keynesian model has government policy: monetary policy and a subsidy to deal with monopoly distortion.
  - Could specify an equation for monetary policy (Taylor rule) and an equation that assigns a value for the subsidy.
- New Keynesian model with fiscal policy:
  - Also include an equation that characterizes the law of motion of government spending and an equation that characterizes the setting of distortionary taxes (e.g., the tax rate is a function of the debt).

Ramsey Optimal Policy

- A key question for economics is, ‘What is the *optimal* policy?’.

- The Ramsey approach to answering this question proceeds in two steps.

  - Identify the best possible equilibrium (‘identify the Ramsey equilibrium’).
  - Find a policy that supports the best possible equilibrium (‘implementation‘).
    - This a policy that is *Ramsey optimal*.

Simple NK Model

- The Ramsey approach to optimal policy is very powerful and general.

- We will apply it here only to a version of the simple New Keynesian model examined in class in which there are no networks. That is, $\Upsilon=1$.

- In these notes, we will only do the first step, identify the Ramsey equilibrium. In the 'homework', we solve the implementation problem.

Collecting Equilibrium Conditions

- Price setting:

$$K_t = (1-\nu)\frac{\varepsilon}{\varepsilon-1}\frac{\exp(\tau_t)N_t^{\varphi}C_t}{A_t} + \beta\theta E_t\bar{\pi}_{t+1}^{\varepsilon}K_{t+1} \quad (1)$$

$$F_t = 1 + \beta\theta E_t\bar{\pi}_{t+1}^{\varepsilon-1}F_{t+1} \quad (2)$$

- Intermediate good firm optimality and restriction across prices:

$$\overbrace{\frac{K_t}{F_t}}^{=\tilde{p}_t \text{ by firm optimality}} = \overbrace{\left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}}^{=\tilde{p}_t \text{ by restriction across prices}} \quad (3)$$

Equilibrium Conditions

- Law of motion of (Tack Yun) distortion:

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad (4)$$

- Household Intertemporal Condition:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

- Aggregate inputs and output:

$$C_t = p_t^* e^{a_t} N_t \quad (6)$$

- 6 equations, 8 unknowns:

$$v,\ C_t,\ p_t^*, N_t, \bar{\pi}_t, K_t, F_t, R_t$$

- System under determined!

Underdetermined System

- Not surprising: we haven’t said anything about monetary policy.
- Also, we’re counting subsidy as among the unknowns.
- Have two extra policy variables.
- One way to pin them down: compute optimal policy.

Ramsey-Optimal Policy

- 6 equations in 8 unknowns…..
  - Many configurations of the 8 unknowns that satisfy the 6 equations.
  - Look for the best configurations (Ramsey optimal)
    - Value of tax subsidy and of *R* represent optimal policy

- Finding the Ramsey optimal setting of the 6 variables involves solving a simple Lagrangian optimization problem.

Ramsey Problem

$$\max_{\nu,p_t^*,C_t,N_t,R_t,\bar{\pi}_t,F_t,K_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$+ \lambda_{1t}\left[\frac{1}{C_t} - E_t \frac{\beta}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \right]$$

$$+ \lambda_{2t}\left[\frac{1}{p_t^*} - \left((1-\theta)\left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right]$$

$$+ \lambda_{3t}[1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t]$$

$$+ \lambda_{4t}\left[(1-\nu)\frac{\varepsilon}{\varepsilon-1} \frac{C_t \exp(\tau_t) N_t^{\varphi}}{e^{a_t}} + E_t \beta\theta\bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t \right]$$

$$+ \lambda_{5t}\left[F_t \left(\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t \right]$$

$$+ \lambda_{6t}[C_t - p_t^* e^{a_t} N_t]\Bigg\}$$

Solving the Ramsey Problem (surprisingly easy in this case)

- First, substitute out consumption everywhere

$$\max_{\nu, p_t^*, N_t, R_t, \bar{\pi}_t, F_t, K_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\log N_t + \log p_t^* - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$+ \lambda_{1t} \left[\frac{1}{p_t^* N_t} - E_t \frac{e^{a_t} \beta}{p_{t+1}^* e^{a_{t+1}} N_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \right]$$

$$+ \lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta) \left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right]$$

$$+ \lambda_{3t} [1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} - F_t]$$

$$+ \lambda_{4t} \left[(1-\nu) \frac{\varepsilon}{\varepsilon-1} \exp(\tau_t) N_t^{1+\varphi} p_t^* + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t \right]$$

$$+ \lambda_{5t} \left[F_t \left(\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t \right] \Bigg\}$$

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defines R

$$+ \lambda_{1t} \left[\frac{1}{p_t^* N_t} - E_t \frac{e^{a_t}\beta}{p_{t+1}^* e^{a_{t+1}} N_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \right]$$

$$+ \lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta)\left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right]$$

defines F

$$+ \lambda_{3t} [1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t]$$

defines tax

$$+ \lambda_{4t} \left[(1-\nu) \frac{\varepsilon}{\varepsilon - 1} \exp(\tau_t) N_t^{1+\varphi} p_t^* + E_t \beta\theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t \right]$$

$$+ \lambda_{5t} \left[F_t \left(\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t \right] \}$$

defines K

Solving the Ramsey Problem, cnt'd

- Simplified problem:

$$\max_{\bar{\pi}_t, p_t^*, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(\log N_t + \log p_t^* - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$+ \lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta) \left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right] \}$$

- First order conditions with respect to $p_t^*,\ \bar{\pi}_t,\ N_t$

$$p_t^* + \beta \lambda_{2,t+1} \theta \bar{\pi}_{t+1}^{\varepsilon} = \lambda_{2t},\ \bar{\pi}_t = \left[\frac{(p_{t-1}^*)^{\varepsilon-1}}{1-\theta+\theta(p_{t-1}^*)^{\varepsilon-1}} \right]^{\frac{1}{\varepsilon-1}},\ N_t = \exp\left(-\frac{\tau_t}{\varphi+1} \right)$$

- Substituting the solution for inflation into law of motion for price distortion:

$$p_t^* = \left[(1-\theta) + \theta (p_{t-1}^*)^{(\varepsilon-1)} \right]^{\frac{1}{(\varepsilon-1)}}.$$

Solution to Ramsey Problem

Eventually, price distortions eliminated, regardless of shocks

$$p_t^* = \left[(1-\theta) + \theta (p_{t-1}^*)^{(\varepsilon-1)} \right]^{\frac{1}{(\varepsilon-1)}}$$

When price distortions gone, so is inflation.

$$\bar{\pi}_t = \frac{p_{t-1}^*}{p_t^*}$$

$$N_t = \exp\left(-\frac{\tau_t}{1+\varphi} \right)$$

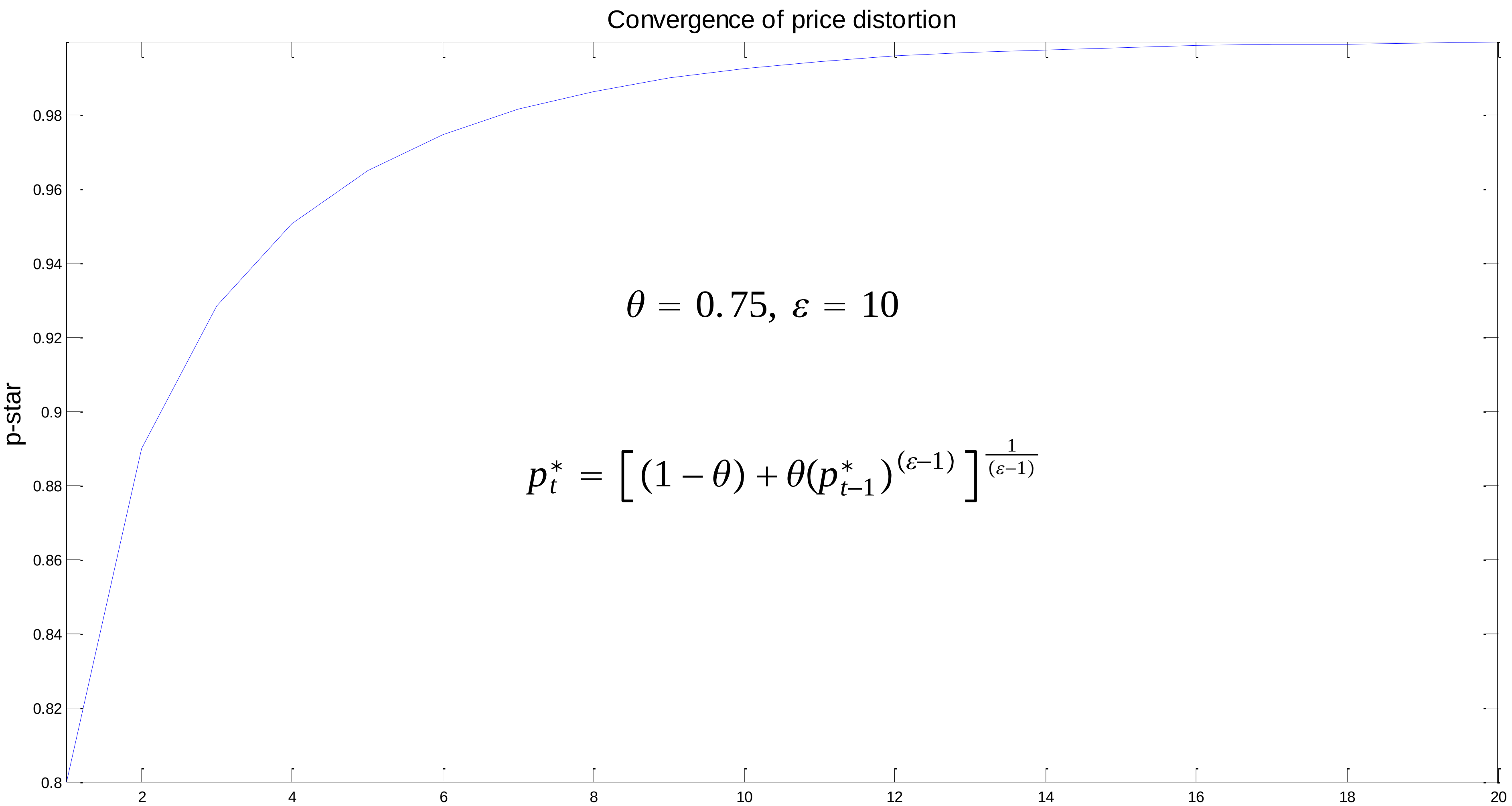
Efficient ('first best') allocations in real economy

$$1-\nu = \frac{\varepsilon - 1}{\varepsilon}$$

$$C_t = p_t^* e^{a_t} N_t.$$

Consumption corresponds to efficient allocations in real economy, eventually when price distortions gone

Eventually, Optimal (Ramsey) Equilibrium and Efficient Allocations in Real Economy Coincide



- The Ramsey allocations are eventually the best allocations in the economy without price frictions (i.e., ‘first best allocations’)

- Refer to the Ramsey allocations as the ‘natural allocations’….
  – Natural consumption, natural rate of interest, etc.