

# Simple New Keynesian Model without Capital

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# Objective

- Review the foundations of the basic New Keynesian model without capital.
  - Clarify the role of money supply/demand.
- Derive the Equilibrium Conditions.
  - Small number of equations and a small number of variables, which summarize everything about the model (optimization, market clearing, gov't policy, etc.).
- Look at some data through the eyes of the model:
  - Money demand.
  - Cross-sectoral resource allocation cost of inflation.
- Some policy implications of the model will be examined.
  - Many policy implications will be 'discovered' in later computer exercises.

# Outline

- The model:
  - Individual agents: their objectives, what they take as given, what they choose.
    - Households, final good firms, intermediate good firms, gov't.
  - Economy-wide restrictions:
    - Market clearing conditions.
    - Relationship between aggregate output and aggregate factors of production, aggregate price level and individual prices.
- Properties of Equilibrium:
  - *Classical Dichotomy* - when prices flexible monetary policy irrelevant for real variables.
  - Monetary policy *essential* to determination of all variables when prices sticky.

# Households

- Households' problem.
- Concept of Consumption Smoothing.

# Households

- There are many identical households.
- The problem of the typical ('representative') household:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} + \gamma \log \left( \frac{M_{t+1}}{P_t} \right) \right),$$

s.t.  $P_t C_t + B_{t+1} + M_{t+1}$

$\leq W_t N_t + R_{t-1} B_t + M_t$

+ Profits net of government transfers and taxes $_t$ .

- Here,  $B_t$  and  $M_t$  are the beginning-of-period  $t$  stock of bonds and money held by the household.
- Law of motion of the shock to preferences:

$$\tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau$$

the preference shock is in the model for pedagogic purposes only, it is not an interest shock from an empirical point of view.

# Household First Order Conditions

- The household first order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$e^{\tau_t} C_t N_t^\varphi = \frac{W_t}{P_t}.$$

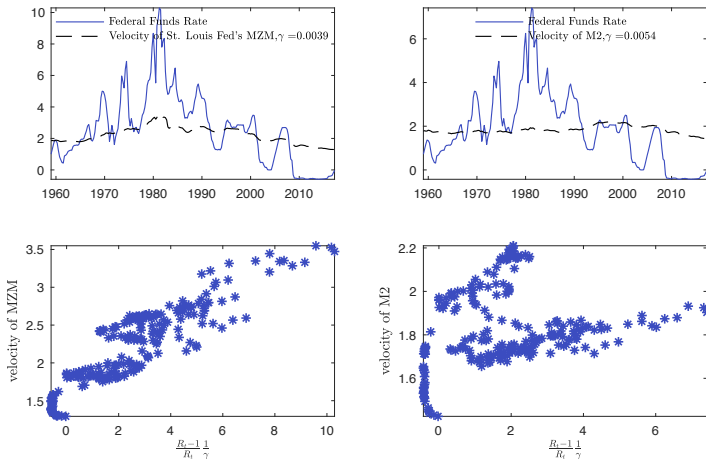
$$m_t = \left( \frac{R_t}{R_t - 1} \right) \gamma C_t \quad (7),$$

where

$$m_t \equiv \frac{M_{t+1}}{P_t}.$$

- All equations are derived by expressing the household problem in Lagrangian form, substituting out the multiplier on budget constraint and rearranging.
- The last first order condition is real money demand, increasing in  $C_t$  and decreasing in  $R_t \geq 1$ .

## Figure: Money Demand, Relative to Two Measures of Velocity



Notes: (i) velocity is GDP/M, (ii) With the MZM measure of money, the money demand equation does well qualitatively, but not quantitatively because the theory implies the scatters in the 2,1 and 2,2 graphs should be on the 45<sup>0</sup>.

# Consumption Smoothing

- Later, we'll see that *consumption smoothing* is an important principle for understanding the role of monetary policy in the New Keynesian model.
- Consumption smoothing is a characteristic of households' consumption decision when they expect a change in income and the interest rate is *not* expected to change.
  - Peoples' current period consumption increases by the amount that can, according to their budget constraint, be maintained indefinitely.



# Consumption Smoothing: Example

- Problem:

$$\begin{aligned} & \max_{c_1, c_2} \log(c_1) + \beta \log(c_2) \\ \text{subject to : } & c_1 + B_1 \leq y_1 + rB_0 \\ & c_2 \leq rB_1 + y_2. \end{aligned}$$

- where  $y_1$  and  $y_2$  are (given) income and, after imposing equality (optimality) and substituting out for  $B_1$ ,

$$\begin{aligned} c_1 + \frac{c_2}{r} &= y_1 + \frac{y_2}{r} + rB_0, \\ \frac{1}{c_1} &= \beta r \frac{1}{c_2}, \end{aligned}$$

second equation is fonic for  $B_1$ .

- Suppose  $\beta r = 1$  (this happens in 'steady state', see later):

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0$$

# Consumption Smoothing: Example, cnt'd

- Solution to the problem:

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0.$$

- Consider three polar cases:
  - *temporary change in income*:  $\Delta y_1 > 0$  and  $\Delta y_2 = 0 \implies \Delta c_1 = \Delta c_2 = \frac{\Delta y_1}{1 + \frac{1}{r}}$
  - *permanent change in income*:  
 $\Delta y_1 = \Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \Delta y_1$
  - *future change in income*:  $\Delta y_1 = 0$  and  $\Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \frac{\frac{\Delta y_2}{r}}{1 + \frac{1}{r}}$
- Common feature of each example:
  - When income rises, then - assuming  $r$  does not change -  $c_1$  increases by an amount that can be maintained into the second period: **consumption smoothing**.

# Goods Production

- We turn now to the technology of production, and the problems of the firms.
- The technology requires allocating resources across sectors.
  - We describe the *efficient* cross-sectoral allocation of resources.
  - With price setting frictions, the market may not achieve efficiency.

# Final Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

- Each intermediate good,  $Y_{i,t}$ , is produced by a monopolist using the following production function:

$$Y_{i,t} = e^{a_t} N_{i,t}, \quad a_t \sim \text{exogenous shock to technology.}$$

- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient allocation of resources across  $i$ .

# Efficient Sectoral Allocation of Resources

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities,  $Y_{i,t}$ .
- It is optimal to run them all at the same rate, *i.e.*,  $Y_{i,t} = Y_{j,t}$  for all  $i, j \in [0, 1]$ .
- For given  $N_t$ , *allocative efficiency*:  $N_{i,t} = N_{j,t} = N_t$ , for all  $i, j \in [0, 1]$ .

In this case, final output is given by

$$Y_t = \left[ \int_0^1 (e^{a_t} N_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} = e^{a_t} N_t.$$

- One way to understand allocated efficiency result is to suppose that labor is *not* allocated equally to all activities.
- Explore one simple deviation from  $N_{i,t} = N_{j,t}$  for all  $i, j \in [0, 1]$ .

# Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in [0, \frac{1}{2}] \\ 2(1 - \alpha)N_t & i \in [\frac{1}{2}, 1] \end{cases}, 0 \leq \alpha \leq 1.$$

- Note that this is a particular distribution of labor across activities:

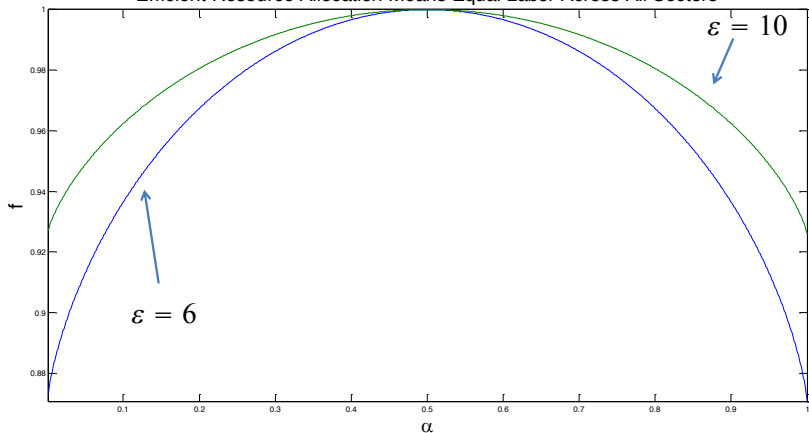
$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1 - \alpha)N_t = N_t$$

## Labor *Not* Allocated Equally, cnt'd

$$\begin{aligned} Y_t &= \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[ \int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} \left[ \int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} \left[ \int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t \left[ \int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t \left[ \frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= e^{a_t} N_t f(\alpha) \end{aligned}$$

$$f(\alpha) = \left[ \frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Efficient Resource Allocation Means Equal Labor Across All Sectors





# Final Good Producers

- Competitive firms:
  - maximize profits

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to  $P_t, P_{i,t}$  given, all  $i \in [0, 1]$ , and the technology:

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Foncs:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon \rightarrow P_t = \overbrace{\left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

# Intermediate Good Producers

- The  $i^{th}$  intermediate good is produced by a monopolist.
- Demand curve for  $i^{th}$  monopolist:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon .$$

- Production function:

$$Y_{i,t} = e^{a_t} N_{i,t}, \quad a_t \sim \text{exogenous shock to technology.}$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases} .$$

# Marginal Cost of Production

- An important input into the monopolist's problem is its marginal cost:

$$s_t = \frac{dCost}{dOutput} = \frac{\frac{dCost}{dWorker}}{\frac{dOutput}{dWorker}} = \frac{(1 - \nu) \frac{W_t}{P_t}}{e^{a_t}}$$
$$= \frac{(1 - \nu) e^{\tau_t} C_t N_t^\varphi}{e^{a_t}}$$

after substituting out for the real wage from the household intratemporal Euler equation.

- The tax rate,  $\nu$ , represents a subsidy to hiring labor, financed by a lump-sum government tax on households.
- Firm's job is to set prices whenever it has the opportunity to do so.
  - It must always satisfy whatever demand materializes at its posted price.

# Present Discounted Value of Intermediate Good Revenues

- $i^{\text{th}}$  intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} \overbrace{\left[ \overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} S_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

$v_{t+j}$  - Lagrange multiplier on household budget constraint

- Here,  $E_t^i$  denotes the firm's expectation over future variables, including the future probability that the firm gets to reset its price.

## Firms that Can Change Price at $t$

- Let  $\tilde{P}_t$  denote the price set by the  $1 - \theta$  firms who optimize at time  $t$ .
- Expected value of future profits sum of two parts:
  - future states in which price is still  $\tilde{P}_t$ , so  $\tilde{P}_t$  matters.
  - future states in which the price is not  $\tilde{P}_t$ , so  $\tilde{P}_t$  is irrelevant.
- That is,

$$E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} [P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}]$$
$$= E_t \underbrace{\sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} [\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}]}_{Z_t} + X_t,$$

where

- $Z_t$  is the present value of future profits over all future states in which the firm's price is  $\tilde{P}_t$ .
- $X_t$  is the present value over all other states, so  $dX_t/d\tilde{P}_t = 0$ .

# Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$\begin{aligned} E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} [\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}] \\ = E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[ \tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right]. \end{aligned}$$

- Differentiate with respect to  $\tilde{P}_t$  :

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[ (1 - \varepsilon) (\tilde{P}_t)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,$$

or,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[ \frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

- When  $\theta = 0$ , get standard result - price is fixed markup over marginal cost.

# Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'(C_{t+j})}{P_{t+j}}}_{= v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[ \frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0.$$

- Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \left[ \tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0,$$
$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \quad \tilde{\pi}_t \equiv \frac{P_t}{P_{t-1}}, \quad X_{t,j} = \begin{cases} \frac{1}{\tilde{\pi}_{t+j}\tilde{\pi}_{t+j-1}\dots\tilde{\pi}_{t+1}}, & j \geq 1 \\ 1, & j = 0. \end{cases},$$

'recursive property':  $X_{t,j} = X_{t+1,j-1} \frac{1}{\tilde{\pi}_{t+1}}, j > 0$

# Decision By Firm that Can Change Its Price

- Want  $\tilde{p}_t$  in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \left[ \tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0$$

- Solving for  $\tilde{p}_t$ , we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} \frac{Y_{t+j}}{C_{t+j}} (\beta\theta)^j (X_{t,j})^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for  $K_t$ ,  $F_t$ .



# Decision By Firm that Can Change Its Price

- Recall,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{1-\varepsilon}} = \frac{K_t}{F_t}$$

The numerator has the following simple representation:

$$\begin{aligned} K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\ &= \frac{\varepsilon}{\varepsilon-1} \frac{(1-\nu) e^{\tau_t} Y_t N_t^\varphi}{e^{a_t}} + \beta\theta E_t \left( \frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} K_{t+1} \quad (1), \end{aligned}$$

after using  $s_t = (1-\nu) e^{\tau_t} C_t N_t^\varphi / e^{a_t}$ .

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left( \frac{1}{\bar{\pi}_{t+1}} \right)^{1-\varepsilon} F_{t+1} \quad (2)$$

# Moving On to Aggregate Restrictions

- Link between aggregate price level,  $P_t$ , and  $P_{i,t}$ ,  $i \in [0, 1]$ .
  - Potentially complicated because there are MANY prices,  $P_{i,t}$ ,  $i \in [0, 1]$ .
- Link between aggregate output,  $Y_t$ , and  $N_t$ .
  - Potentially complicated because of earlier example with  $f(\alpha)$ .
  - Analog of  $f(\alpha)$  will be a function of degree to which  $P_{i,t} \neq P_{j,t}$ .
- Market clearing conditions.
  - Money and bond market clearing.
  - Labor and goods market clearing.

# Aggregate Price Index

- Important Calvo result:

$$\begin{aligned} P_t &= \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} \\ &= \left( (1-\theta) \tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \end{aligned}$$

- Divide by  $P_t$  :

$$1 = \left( (1-\theta) \tilde{p}_t^{1-\varepsilon} + \theta \left( \frac{1}{\bar{\pi}_t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Rearrange:  $\tilde{p}_t = \left[ \frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$

# Aggregate Output vs Aggregate Labor and Tech (Tack Yun, JME1996)

- Define  $Y_t^*$ :

$$Y_t^* \equiv \int_0^1 Y_{i,t} di \quad \left( = \int_0^1 e^{at} N_{i,t} di = e^{at} N_t \right)$$
$$\underbrace{\quad}_{\text{demand curve}} \equiv Y_t \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t P_t^\varepsilon \int_0^1 (P_{i,t})^{-\varepsilon} di$$
$$= Y_t P_t^\varepsilon (P_t^*)^{-\varepsilon}$$

where, using 'Calvo result':

$$P_t^* \equiv \left[ \int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[ (1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta (P_{t-1}^*)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then

$$Y_t = p_t^* Y_t^*, \quad p_t^* = \left( \frac{P_t^*}{P_t} \right)^\varepsilon.$$

# Gross Output vs Aggregate Labor

- Relationship between aggregate inputs and outputs:

$$Y_t = p_t^* Y_t^*$$

or,

$$Y_t = p_t^* e^{a_t} N_t.$$

- Note that  $p_t^*$  is a function of the ratio of two averages (with different weights) of  $P_{i,t}$ ,  $i \in (0, 1)$
- So, when  $P_{i,t} = P_{j,t}$  for all  $i, j \in (0, 1)$ , then  $p_t^* = 1$ .
- But, what is  $p_t^*$  when  $P_{i,t} \neq P_{j,t}$  for some (measure of)  $i, j \in (0, 1)$ ?

# Tack Yun Distortion

- Consider the object,

$$p_t^* = \left( \frac{P_t^*}{P_t} \right)^\varepsilon ,$$

where

$$P_t^* = \left( \int_0^1 P_{i,t}^{-\varepsilon} di \right)^{\frac{-1}{\varepsilon}} , P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$

- In following slide, use Jensen's inequality to show:

$$p_t^* \leq 1.$$

# Tack Yun Distortion

- Let  $f(x) = x^4$ , a convex function. Then,

$$\text{convexity: } \alpha x_1^4 + (1 - \alpha) x_2^4 > (\alpha x_1 + (1 - \alpha) x_2)^4$$

for  $x_1 \neq x_2$ ,  $0 < \alpha < 1$ .

- Applying this idea:

$$\begin{aligned} \text{convexity: } \int_0^1 \left( P_{i,t}^{(1-\varepsilon)} \right)^{\frac{\varepsilon}{\varepsilon-1}} di &\geq \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \\ &\iff \left( \int_0^1 P_{i,t}^{-\varepsilon} di \right) \geq \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \\ &\iff \overbrace{\left( \int_0^1 P_{i,t}^{-\varepsilon} di \right)^{\frac{-1}{\varepsilon}}}^{P_t^*} \leq \overbrace{\left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{P_t} \end{aligned}$$

# Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[ (1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta (P_{t-1}^*)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Dividing by  $P_t$ :

$$\begin{aligned} p_t^* &\equiv \left( \frac{P_t^*}{P_t} \right)^\varepsilon = \left[ (1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \\ &= \left( (1 - \theta) \left[ \frac{1 - \theta (\bar{\pi}_t)^{\varepsilon-1}}{1 - \theta} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right)^{-1} \end{aligned} \quad (4)$$

using the restriction between  $\tilde{p}_t$  and aggregate inflation developed earlier.



# Evaluating the Distortions

- Tack Yun distortion:

$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}.$$

- Potentially, NK model provides an 'endogenous theory of TFP'.
- Standard practice in NK literature is to set  $p_t^* = 1$  for all  $t$ .
  - First order expansion of  $p_t^*$  around  $\bar{\pi}_t = p_t^* = 1$  is:

$$p_t^* = p^* + 0 \times \bar{\pi}_t + \theta (p_{t-1}^* - p^*), \text{ with } p^* = 1,$$

so  $p_t^* \rightarrow 1$  and is invariant to shocks.

# Empirical Assessment of Tack Yun Distortion

- First, do 'back of the envelope' calculations in a steady state when inflation is constant and  $p^*$  is constant.
- Then, use

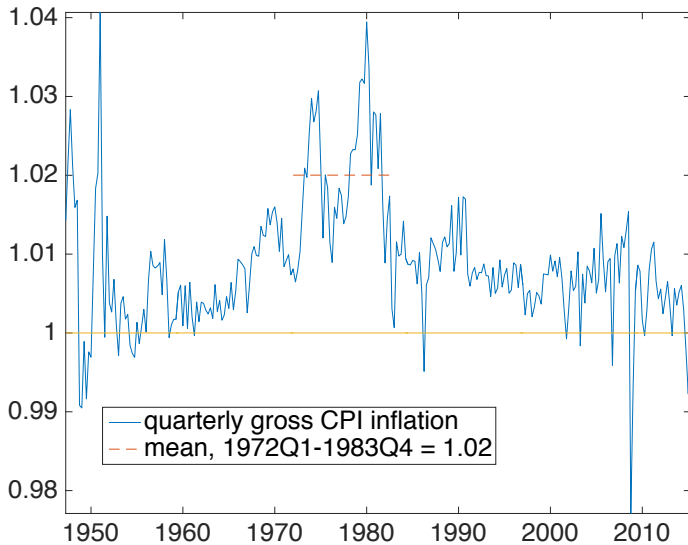
$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} .$$

to compute times series estimate of  $p_t^*$ .

# Three Inflation Rates:

- Average inflation in the 1970s, 8 percent APR.
- Suggestion: raise inflation target to 4 percent so that nominal rate of interest is higher, and less likely to hit lower bound.
  - <http://www.voxeu.org/article/case-4-inflation>
- Two percent inflation is the average in the recent (pre-2008) low inflation environment.

**Figure: US Quarterly Gross Inflation**



# Cost of Three Alternative Permanent Levels of Inflation

$$p^* = \frac{1 - \theta \bar{\pi}^\varepsilon}{1 - \theta} \left( \frac{1 - \theta}{1 - \theta \bar{\pi}^{(\varepsilon-1)}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

**Table:** Percent of GDP Lost Due to Inflation,  $100(1 - p_t^*)$

steady state inflation	markup, $\frac{\varepsilon}{\varepsilon-1}$		
	1.20	1.15	1.10
8%	2.41	3.92	10.85
4%	0.46	0.64	1.13
2%	0.10	0.13	0.21

# Costs of Inflation, Dynamic Formula

Figure 1a: Percent loss of GDP due to Inflation, assumed markup is 1.2

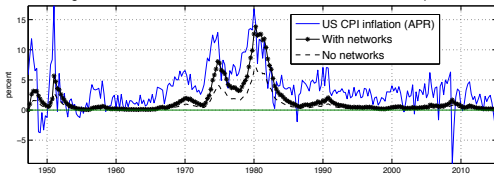
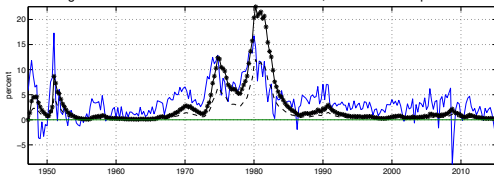


Figure 1b: Percent loss of GDP due to Inflation, assumed markup is 1.15



Notes: (i) the figure reports the percent loss of output,  $100(1 - p_t^*)$ , due to cross-sectional resource misallocation; (ii) losses are for the model in these notes as well as for the version of the model with networks, the annualized percent inflation **networks**; the inflation rate is expressed in annual, percent terms.

# Tack Yun Distortion

- The magnitude of the distortion is typically small.
  - Explains why standard literature abstracts from the distortion by linearizing about zero inflation.
  - To first order approximation,  $p^* = 1$  at zero inflation (see later).
  - Could have  $p^* = 1$  to first order around positive inflation if price indexation is assumed, as in CEE.
    - But, prices don't appear to be indexed.
- Caution: distortion may be small because of simplicity of the model.
  - Distortions at least two times bigger when production occurs in networks of firms. See this and this.
  - Distortions may be bigger when there are intermediate good firm-specific idiosyncratic shocks to demand and supply of intermediate good firms.

# Government

- Government budget constraint: expenditures = receipts

$$\begin{array}{ccccccc}
 \text{purchases of final goods} & & \text{subsidy payments} & & \text{gov't bonds (lending, if positive)} & & \\
 \underbrace{P_t G_t} & + & \underbrace{v W_t N_t} & + & \underbrace{B_{t+1}^g} & + & \\
 & & & & \text{transfer payments to households} & & \\
 & & & & \underbrace{T_t^{trans}} & + & \\
 & & \text{money injection, if positive} & & \text{tax revenues} & & \\
 = & & \underbrace{M_t \mu_t} & + & \underbrace{T_t^{tax}} & + & R_{t-1} B_t^g
 \end{array}$$

where  $\mu_t$  denotes money growth rate.

- Then,

$$T_t^{tax} - T_t^{trans} = v W_t N_t + B_{t+1}^g + P_t G_t - M_t \mu_t - R_{t-1} B_t^g$$

- Government's choice of  $\mu_t$  determines evolution of money supply:

$$M_{t+1} = (1 + \mu_t) M_t, \mu_t \sim \text{money growth rate.}$$



# Government

- The law of motion for money places restrictions on  $m_t$ :

$$m_t \equiv \frac{M_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} \frac{M_t}{P_{t-1}} \frac{P_{t-1}}{P_t}$$

$$\rightarrow m_t = \left( \frac{1 + \mu_t}{\bar{\pi}_t} \right) m_{t-1} \quad (8),$$

for  $t = 0, 1, \dots$  .

# Market Clearing

- We now summarize the market clearing conditions of the model.
  - Money, labor, bond and goods markets.

# Money Market Clearing

- We temporarily use the bold notation,  $\mathbf{M}_t$ , to denote the per capita supply of money at the start of time  $t$ , for  $t = 0, 1, 2, \dots$ .
- The supply of money is determined by the actions,  $\mu_t$ , of the government:

$$\mathbf{M}_{t+1} = \mathbf{M}_t + \mu_t \mathbf{M}_t,$$

for  $t=0,1,2,\dots$

- Households being identical means that in period  $t = 0$ ,

$$\mathbf{M}_0 = M_0,$$

where  $M_0$  denotes beginning of time  $t = 0$  money stock of the representative household.

- Money market clearing in each period,  $t = 0, 1, \dots$ , requires

$$\mathbf{M}_{t+1} = M_{t+1},$$

where  $M_{t+1}$  denotes the representative household's time  $t$  choice of money.

- From here on, we do not distinguish between  $\mathbf{M}_t$  and  $M_t$ .

# Other Market Clearing Conditions

- Bond market clearing:

$$B_{t+1} + B_{t+1}^g = 0, \quad t = 0, 1, 2, \dots$$

- Labor market clearing:

$$\underbrace{N_t}_{\text{supply of labor}} = \underbrace{\int_0^1 N_{i,t} di}_{\text{demand for labor}}$$

- Goods market clearing:

$$\underbrace{C_t + G_t}_{\text{demand for final goods}} = \underbrace{Y_t}_{\text{supply of final goods}},$$

and, using relation between  $Y_t$  and  $N_t$ :

$$C_t + G_t = p_t^* e^{a_t} N_t \quad (6)$$

## Next

- Collect the equilibrium conditions associated with private sector behavior.
- Comparison of NK model with RBC model (i.e.,  $\theta = 0$ )
  - *Classical Dichotomy*: In flexible price version of model real variables determined independent of monetary policy.
  - Fiscal policy still matters, because equilibrium depends on how government deals with the monopoly power, i.e., selects value for subsidy,  $\nu$ .
  - In NK model, markets don't necessarily work well and good monetary policy essential.
- To close model with  $\theta > 0$  must take a stand on monetary policy.

## Equilibrium Conditions

- 8 equations in 8 unknowns:  $m_t, C_t, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$ , and 3 policy variables:  $\nu, \mu_t, G_t$ .

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1 - \nu) e^{\tau_t} Y_t N_t^\varphi}{A_t} + \beta \theta E_t \bar{\pi}_{t+1}^\varepsilon K_{t+1} \quad (1)$$

$$F_t = \frac{Y_t}{C_t} + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \quad (2), \quad \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \quad (4)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5), \quad C_t + G_t = p_t^* e^{a_t} N_t \quad (6)$$

$$m_t = \frac{\gamma C_t}{\left(1 - \frac{1}{R_t}\right)} \quad (7), \quad m_t = \left( \frac{1 + \mu_t}{\bar{\pi}_t} \right) m_{t-1} \quad (8)$$

# Classical Dichotomy Under Flexible Prices

- *Classical Dichotomy*: when prices flexible,  $\theta = 0$ , then real variables determined regardless of the rule for  $\mu_t$  (i.e., monetary policy).
  - Equations (2),(3) imply:

$$F_t = K_t = \frac{Y_t}{C_t},$$

which, combined with (1) implies

$$\frac{\varepsilon(1-\nu)}{\varepsilon-1} \times \overbrace{e^{\tau_t} C_t N_t^\varphi}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

- Expression (6) with  $p_t^* = 1$  (since  $\theta = 0$ ) is

$$C_t + G_t = e^{a_t} N_t.$$

- Thus, we have two equations in two unknowns,  $N_t$  and  $C_t$ .

# Classical Dichotomy: No Uncertainty

- Real interest rate,  $R_t^* \equiv R_t / \bar{\pi}_{t+1}$ , is determined:

$$R_t^* = \frac{\frac{1}{C_t}}{\beta \frac{1}{C_{t+1}}}.$$

- So, with  $\theta = 0$ , the following are determined:

$$R_t^*, C_t, N_t, t = 0, 1, 2, \dots$$

- What about the nominal variables?

- Suppose the monetary authority wants a given sequence of inflation rates,  $\bar{\pi}_t, t = 0, 1, \dots$ .
- Then,

$$R_t = \bar{\pi}_{t+1} R_t^*, t = 0, 1, 2, \dots$$

- What money growth sequence is required?
  - From (7), obtain  $m_t, t = 0, 1, 2, \dots$ . Also,  $m_{-1}$  is given by initial  $M_0$  and  $P_{-1}$ .
  - From (8)

$$1 + \mu_t = \frac{m_t}{m_{t-1}} \bar{\pi}_t, t = 0, 1, 2, \dots$$



# Classical Dichotomy versus New Keynesian Model

- When  $\theta = 0$ , then the Classical Dichotomy occurs.
- In this case, monetary policy (i.e., the setting of  $\mu_t$ ,  $t = 0, 1, 2, \dots$ ) cannot affect the real interest rate, consumption and employment.
  - Monetary policy simply affects the split in the real interest rate between nominal and real rates:

$$R_t^* = \frac{R_t}{\bar{\pi}_{t+1}}.$$

- For a careful treatment when there is uncertainty, [see](#).
- When  $\theta > 0$  (NK model) then real variables are not determined independent of monetary policy.
  - In this case, monetary policy matters.

# Monetary Policy in New Keynesian Model

- Suppose  $\theta > 0$ , so that we're in the NK model and monetary policy matters.
- The standard assumption is that the monetary authority sets  $\mu_t$  to achieve an interest rate target, and that that target is a function of inflation:

$$R_t/R = (R_{t-1}/R)^\alpha \exp [(1 - \alpha) \phi_\pi (\bar{\pi}_t - \bar{\pi}) + \phi_x x_t] \quad (7)',$$

where  $x_t$  denotes the log deviation of actual output from target (more on this later).

- This is a *Taylor rule*, and it satisfies the *Taylor Principle* when  $\phi_\pi > 1$ .
- Smoothing parameter:  $\alpha$ .
  - Bigger is  $\alpha$  the more persistent are policy-induced changes in the interest rate.

# Equilibrium Conditions of NK Model with Taylor Rule

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1 - \nu) e^{\tau_t} Y_t N_t^\varphi}{A_t} + \beta \theta E_t \bar{\pi}_{t+1}^\varepsilon K_{t+1} \quad (1)$$

$$F_t = \frac{Y_t}{C_t} + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \quad (2), \quad \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \quad (4)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5), \quad C_t + G_t = p_t^* e^{a_t} N_t \quad (6)$$

$$R_t/R = (R_{t-1}/R)^\alpha \exp [(1 - \alpha) \phi_\pi (\bar{\pi}_t - \bar{\pi}) + \phi_x x_t] \quad (7)'$$

Conditions (7) and (8) have been replaced by (7)'.

# Equilibrium Conditions of NK Model

- The model represents 7 equations in 7 unknowns:

$$C, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$$

- After this system has been solved for the 7 variables, equations (7) and (8) can be used to solve for  $\mu_t$  and  $m_t$ .
  - This is rarely done, because researchers are uncertain of the exact form of money demand and because  $m_t$  and  $\mu_t$  are in practice not of direct interest.

# Natural Equilibrium

- When  $\theta = 0$ , then

$$\frac{\varepsilon(1-\nu)}{\varepsilon-1} \times \overbrace{e^{\tau_t} C_t N_t^\varphi}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

so that we have a form of efficiency when  $\nu$  is chosen so that  $\varepsilon(1-\nu) / (\varepsilon-1) = 1$ .

- In addition, recall that we have allocative efficiency in the flexible price equilibrium.
- So, the flexible price equilibrium with the efficient setting of  $\nu$  represents a natural benchmark for the New Keynesian model, the version of the model in which  $\theta > 0$ .
  - We call this the *Natural Equilibrium*.
- To simplify the analysis, from here on we set  $G_t = 0$ .

# Natural Equilibrium

- With  $G_t = 0$ , equilibrium conditions for  $C_t$  and  $N_t$ :

$$\overbrace{e^{\tau_t} C_t N_t^\varphi}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

aggregate production relation:  $C_t = e^{a_t} N_t$ .

- Substituting,

$$e^{\tau_t} e^{a_t} N_t^{1+\varphi} = e^{a_t} \rightarrow N_t = \exp\left(\frac{-\tau_t}{1+\varphi}\right)$$

$$C_t = \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)$$

$$R_t^* = \frac{\frac{1}{C_t}}{\beta E_t \frac{1}{C_{t+1}}} = \frac{1}{\beta E_t \frac{C_t}{C_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1+\varphi}\right)}$$

# Natural Equilibrium, cnt'd

- Natural rate of interest:

$$R_t^* = \frac{\frac{1}{\bar{C}_t}}{\beta E_t \frac{1}{\bar{C}_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1+\varphi}\right)}$$

- Two models for  $a_t$  :

$$DS : \Delta a_{t+1} = \rho \Delta a_t + \varepsilon_{t+1}^a$$

$$TS : a_{t+1} = \rho a_t + \varepsilon_{t+1}^a$$

- Model for  $\tau_t$  :

$$\tau_{t+1} = \lambda \tau_t + \varepsilon_{t+1}^\tau$$

## Natural Equilibrium, cnt'd

- Suppose the  $\varepsilon_t$ 's are Normal. Then,

$$E_t \exp \left( -\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1 + \varphi} \right) = \exp \left( -E_t \Delta a_{t+1} + E_t \frac{\Delta \tau_{t+1}}{1 + \varphi} + \frac{1}{2} V \right)$$

where

$$V = \sigma_a^2 + \frac{\sigma_\tau^2}{(1 + \varphi)^2}$$

- Then, with  $r_t^* \equiv \log R_t^*$

$$r_t^* = -\log \beta + E_t \Delta a_{t+1} - E_t \frac{\Delta \tau_{t+1}}{1 + \varphi} - \frac{1}{2} V.$$

- Useful: consider how natural rate responds to  $\varepsilon_t^a$  shocks under DS and TS models for  $a_t$  and how it responds to  $\varepsilon_t^\tau$  shocks.
  - To understand how  $r_t^*$  responds, consider implications of consumption smoothing in absence of change in  $r_t^*$ .
  - Hint: in natural equilibrium,  $r_t^*$  steers the economy so that natural equilibrium paths for  $C_t$  and  $N_t$  are realized.



# Conclusion

- Described NK model and derived equilibrium conditions.
  - The usual version of model represents monetary policy by a Taylor rule.
- When  $\theta = 0$ , so that prices are flexible, then monetary policy is (essentially) neutral.
  - Changes in money growth move prices and wages in such a way that real wages do not change and employment and output don't change.
- When prices are sticky, then a policy-induced reduction in the interest rate encourages more nominal spending.
  - The increased spending raises  $W_t$  more than  $P_t$  because of the sticky prices, thereby inducing the increased labor supply that firms need to meet the extra demand.
  - Firms are willing to produce more goods because:
    - The model assumes they *must* meet all demand at posted prices.
    - Firms make positive profits, so as long as the expansion is not too big they still make positive profits, even if not optimal.