Application of DSGE Model:

Fixing the Interest Rate for a While, and then Returning to Taylor Rule

Standard monetary policy briefing question

- ‘What Happens if We Set the Interest Rate to Fixed Level for *y* Periods?’

Model

- Model in linearized form:

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0,$$

  – Here,
    - z_t denotes the list of endogenous variables whose values are determined at time t.
    - s_t denotes the list of exogenous variables whose values are determined at time t.

$$s_t = P s_{t-1} + \varepsilon_t.$$

- Solution: A and B *in*

$$z_t = A z_{t-1} + B s_t,$$

  – where

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0, \qquad (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

Policy Experiment

- The n^{th} equation in the system is a monetary policy rule (Taylor rule). One of the variables in z_t is the policy interest rate, R_t, in deviation from its non-stochastic steady state value:

$$R_t = \tau' z_t.$$

- τ is composed of *0*'s and a single *1*
- Policy:
  - it is now time $t=T$ and policy is $R_t = \tilde{d}$ from $t=T+1$ to $t=T+y$.
  - For $t>T+y$, policy follows the Taylor rule again.

Convenient to 'Stack' the System to be Conformable with Dynare Notation

- First set of equations is the equilibrium conditions and second set is the exogenous shock process:

$$E_t \left\{ \overbrace{\begin{bmatrix} \alpha_0 & \beta_0 \\ 0 & 0 \end{bmatrix}}^{A_0} \overbrace{\begin{pmatrix} z_{t+1} \\ s_{t+1} \end{pmatrix}}^{Z_{t+1}} + \overbrace{\begin{bmatrix} \alpha_1 & \beta_1 \\ 0 & I \end{bmatrix}}^{A_1} \overbrace{\begin{pmatrix} z_t \\ s_t \end{pmatrix}}^{Z_t} + \overbrace{\begin{bmatrix} \alpha_2 & 0 \\ 0 & -P \end{bmatrix}}^{A_2} \overbrace{\begin{pmatrix} z_{t-1} \\ s_{t-1} \end{pmatrix}}^{Z_{t-1}} + \overbrace{\begin{pmatrix} 0 \\ -\varepsilon_t \end{pmatrix}}^{\epsilon_t} \right\} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

  – or

$$E_t\{A_0 Z_{t+1} + A_1 Z_t + A_2 Z_{t-1} + \epsilon_t\} = 0.$$

  – where ϵ_t is independent over time and in time t information set.

- The n^{th} row of the above system corresponds to monetary policy rule.

Fixed *R* Equilibrium Conditions

- Delete monetary policy rule (i.e., n^{th} equation) from system and replace it by $R_t = \tilde{d}$:
  - Let $\hat{A}_0$ and $\hat{A}_2$ denote A_0 and A_2 with their n^{th} rows replaced by *0*'s.
  - Let

$$\hat{A}_1 = \begin{bmatrix} \hat{\alpha}_1 & \hat{\beta}_1 \\ 0 & I \end{bmatrix}$$

    - Where $\hat{\alpha}_1$ is α_1 with its n^{th} row replaced by τ' and $\hat{\beta}_1$ is β_1 with its n^{th} row replaced by *0*'s.
- Equilibrium conditions:

$$E_t\{\hat{A}_0 Z_{t+1} + \hat{A}_1 Z_t + \hat{A}_2 Z_{t-1} + \epsilon_t\} = d.$$

  - d is a column vector zero in all but one location and $\tau' d = \tilde{d}$

Problem

- Equilibrium conditions for *t=T+1,…,T+y*:

$$E_t\{\hat{A}_0 Z_{t+1} + \hat{A}_1 Z_t + \hat{A}_2 Z_{t-1} + \epsilon_t\} = d.$$

- Equilibrium conditions for *t>T+y:*

$$E_t\{A_0 Z_{t+1} + A_1 Z_t + A_2 Z_{t-1} + \epsilon_t\} = 0.$$

  – Solution for *t>T+y:*

$$\overbrace{\begin{pmatrix} z_t \\ s_t \end{pmatrix}}^{Z_t} = \overbrace{\begin{bmatrix} A & BP \\ 0 & P \end{bmatrix}}^{a} \overbrace{\begin{pmatrix} z_{t-1} \\ s_{t-1} \end{pmatrix}}^{Z_{t-1}} + \overbrace{\begin{pmatrix} 0 & -B \\ 0 & -I \end{pmatrix}}^{b} \overbrace{\begin{pmatrix} 0 \\ -\varepsilon_t \end{pmatrix}}^{\epsilon_t}$$

  – or,

$$Z_t = aZ_{t-1} + b\epsilon_t$$

- How is the solution for *t=T+1,…,T+y?*

Solve the Model 'Backward'

- In period *t=T+y*:

$$E_{T+y}\left\{\hat{A}_0 \overbrace{Z_{T+y+1}}^{=aZ_{T+y}+b\epsilon_{T+y+1}} + \hat{A}_1 Z_{T+y} + \hat{A}_2 Z_{T+y-1} + \epsilon_{T+y}\right\} = d$$

  – or

$$(\hat{A}_0 a + \hat{A}_1) Z_{T+y} + \hat{A}_2 Z_{T+y-1} + \epsilon_{T+y} = d$$

$$\rightarrow Z_{T+y} = a_1 Z_{T+y-1} + b_1 \epsilon_{T+y} + d_1$$

$$a_1 \equiv -(\hat{A}_0 a + \hat{A}_1)^{-1} \hat{A}_2$$

$$b_1 \equiv -(\hat{A}_0 a + \hat{A}_1)^{-1}$$

$$d_1 \equiv (\hat{A}_0 a + \hat{A}_1)^{-1} d$$

Backward, cnt'd

- Period *t=T+y-1*:

$$E_{T+y-1}\left\{\hat{A}_0 \overbrace{Z_{T+y}}^{=a_1 Z_{T+y-1}+b_1\epsilon_{T+y}+d_1} + \hat{A}_1 Z_{T+y-1} + \hat{A}_2 Z_{T+y-2} + \epsilon_{T+y-1}\right\} = d.$$

  – or

$$(\hat{A}_0 a_1 + \hat{A}_1) Z_{T+y-1} + \hat{A}_0 d_1 + \hat{A}_2 Z_{T+y-2} + \epsilon_{T+y-1} = d.$$

$$\rightarrow Z_{T+y-1} = a_2 Z_{T+y-2} + b_2 \epsilon_{T+y-1} + d_2$$

$$a_2 = -(\hat{A}_0 a_1 + \hat{A}_1)^{-1} \hat{A}_2$$

$$b_2 = -(\hat{A}_0 a_1 + \hat{A}_1)^{-1}$$

$$d_2 = (\hat{A}_0 a_1 + \hat{A}_1)^{-1} (d - \hat{A}_0 d_1)$$

  – and so on…..

Backwards, cnt'd

- Solution for *t=T+1,…,T+y*.

$$Z_{T+y-j} = a_{j+1}Z_{T+y-j-1} + b_{j+1}\epsilon_{T+y-j},$$

  – for *j=0,1,2,…,y-1,* where

$$a_{j+1} = -(\hat{A}_0 a_j + \hat{A}_1)^{-1}\hat{A}_2,$$

$$b_{j+1} = -(\hat{A}_0 a_j + \hat{A}_1)^{-1}$$

$$d_{j+1} = (\hat{A}_0 a_j + \hat{A}_1)^{-1}(d - \hat{A}_0 d_j),$$

$$a_0 \equiv a,\ b_0 \equiv b,\ d_0 = 0.$$

In Sum

- Future stochastic realization of length, *x*, with interest rate fixed at some specified value for y<x periods….Three steps:
- Backward step:

$$a_0, a_1, \ldots, a_y; b_0, b_1, \ldots, b_y; d_0, d_1, \ldots, d_y$$

- Two forward steps: draw shocks, and simulate

$$\overbrace{\epsilon_{T+1}, \ldots, \epsilon_{T+y}}^{\text{realization of future shocks during fixed interest rate regime}}, \overbrace{\epsilon_{T+y+1}, \ldots, \epsilon_{T+x}}^{\text{realization of shocks after fixed interest rate regime}}$$

$$\begin{aligned}
Z_{T+1} &= a_y Z_T + b_y \epsilon_{T+1} + d_y \\
Z_{T+2} &= a_{y-1} Z_{T+1} + b_{y-1} \epsilon_{T+2} + d_{y-1} \\
&\ldots \\
Z_{T+y} &= a_1 Z_{T+y-1} + b_1 \epsilon_{T+y} + d_1 \\
Z_{T+y+1} &= a Z_{T+y} + b \epsilon_{T+y+1} \\
&\ldots \\
Z_{T+x} &= a Z_{T+x-1} + b \epsilon_{T+x}
\end{aligned}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \text{ (Calvo pricing equation)}$$

Net rate of inflation
(deviated from natural inflation, which is zero)

log deviation of actual and natural output ('output gap')

$$x_t = -[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} \text{ (intertemporal equation)}$$

$$r_t = \alpha r_{t-1} + (1-\alpha)[\phi_\pi \pi_t + \phi_x x_t] + u_t \text{ (policy rule)}$$

$$r_t^* = \rho \Delta a_t + \frac{1}{1+\varphi}(1-\lambda)\tau_t \text{ (natural rate)}$$

Natural rate of interest

$$y_t^* = a_t - \frac{1}{1+\varphi}\tau_t \text{ (natural output)}$$

$$x_t = y_t - y_t^* \text{ (output gap)}$$

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,\ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau,\ u_t \sim iid$$

Solving the Model

s_t should also include u_t, monetary shock

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^\tau \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -\frac{1}{\sigma} & \frac{1}{\sigma} \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ rr^*_t \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\sigma\psi\rho & -\frac{1}{\sigma+\varphi}(1-\lambda) \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Model Solution

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

$$z_t = A z_{t-1} + B s_t$$

- where:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

Simulation

- Parameter values:

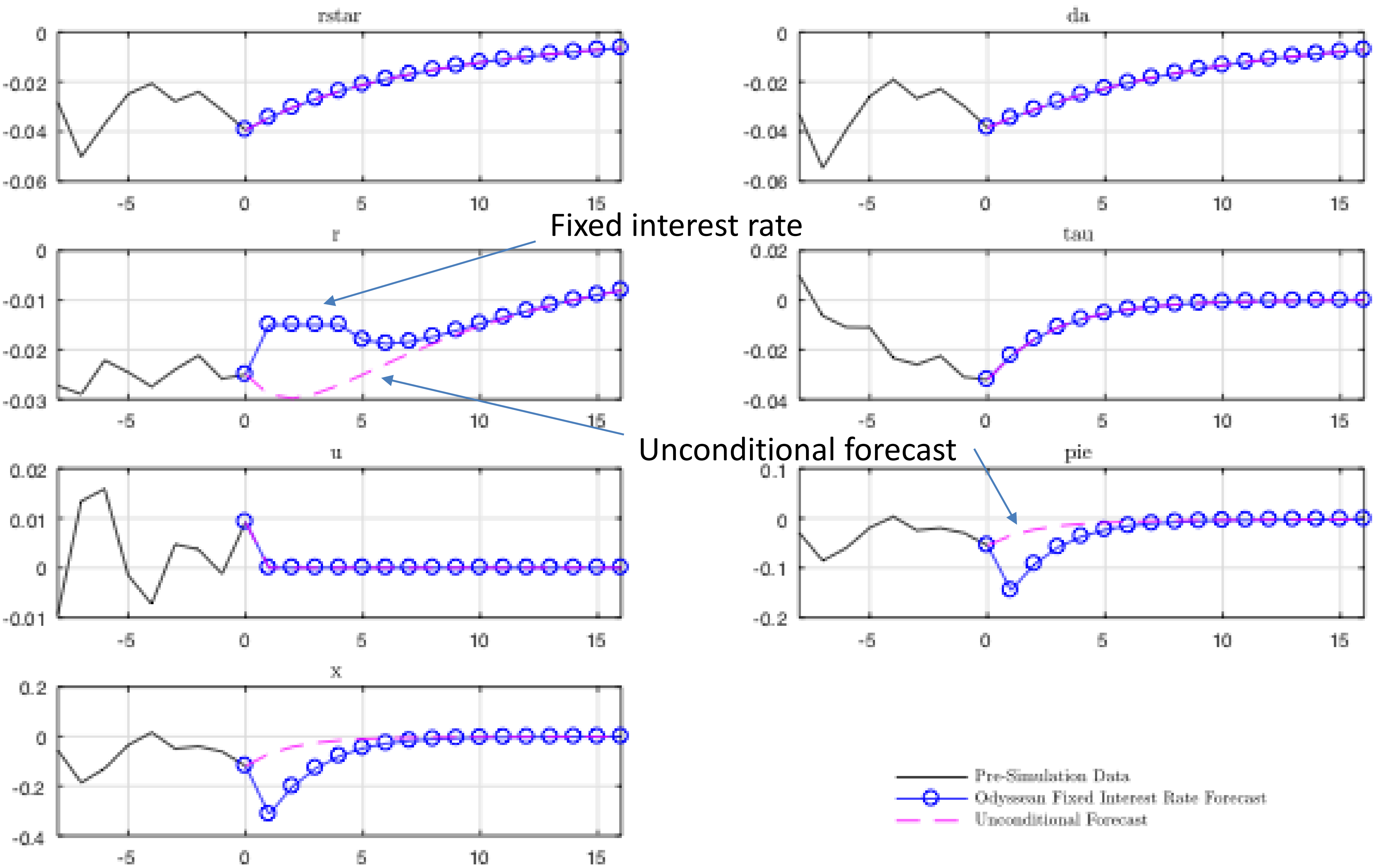
$$\alpha = 0.85, \rho = 0.90, \lambda{=}0.7$$

$\beta = 0.99,\ \phi_x = 0,\ \phi_\pi = 1.5,$ $,\ \varphi = 1,\ \theta = 0.75$ (Calvo sticky price parameter)

variance, innovation in preference shock $= 0.01^2$, variance, innovation in technology growth $= 0.01^2$

$$\kappa = \frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\theta} = 0.1717.$$

- Experiment:
  - From periods *-8, -7,…,0,* economy is stochastically fluctuating with Taylor rule in place.
  - At period *t=0,* monetary authority commits to keeping interest rate fixed in *t=1,2,3,4,* at a higher value than its value in *t=0*. Afterward, return to Taylor rule.



rstar
da
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tau
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pie
x
Fixed interest rate
Unconditional forecast
Pre-Simulation Data
Odyssean Fixed Interest Rate Forecast
Unconditional Forecast