Question Related to First Financial Friction Model in Handout, http://faculty.wcas.northwestern.edu/~lchrist/d11/d1111/financial_frictions.pdf(75 points) Consider an economy in which there are two periods, $t = 1, 2$.

The economy is populated by a large number of identical households. The representative household has a unit measure of agents, and each is either a 'banker' or a 'worker'. Inside the household there is perfect consumption insurance so that each agent consumes the same amount. Periods 1 and 2 consumption by each agent are denoted $c \geq 0$ and $C \geq 0$, respectively. The representative household has preferences,

$$u(c) + \beta u(C), \ 0 < \beta < 1,$$

where $u(\cdot)$ is strictly increasing, continuously differentiable, strictly concave and $u'(\cdot)$ converges to ∞ as its argument goes to zero. Workers and bankers receive endowments of $y > 0$ and $N > 0$ goods, respectively (all quantities are expressed in household per capita terms). In period 1 households split y between c and bank deposits, $d \geq 0$. Denote the gross return on deposits by R. In period 2, households allocate income earned from deposits and income earned from the profits, π, of bankers to C.

In period 1 bankers accept deposits, d, and combine these with their net worth, N, to purchase securities, $s \geq 0$, from firms. Bankers are instructed by their household to maximize profits by choice of d. Households are able to monitor the actions of their own bankers and they are assumed to be able to enforce their instructions.

Firms possess a linear production technology that can be operated at any scale. For each good put into a firm's technology in period 1, R^k units of goods appear in period 2 (R^k is a technologically given constant). There are no other costs of production and competition implies that a firm which issues s securities to a bank in period 1 returns sR^k goods to the bank in period 2. Participants in all markets are anonymous and competitive.

a) Provide a formal statement of the problems of the agents in the model and define an interior equilibrium. (An interior equilibrium is one in which $d, c, C > 0$.)

b) Prove: $R = R^k$ in an interior equilibrium. What role does interiority play in your proof?

c) Derive equations that characterize the 'first-best efficient' allocations, in the sense that the allocations solve the planning problem, $\max_{c,C} [u(c) + \beta u(C)]$ subject to the period 1 and period 2 resource constraints, $c + k \leq y + N$, $C \leq R^k k$. Here, k denotes goods put into the production technology by the planner in period 1. Show that under our assumptions, the first-best allocations satisfy $c, C > 0$.

d) Prove that the allocations in an interior equilibrium coincide with the 'first-best' allocations. Now drop the adjective, 'interior', on equilibrium. Do the allocations in an *equilibrium* coincide with the 'first-best' allocations? Explain.

e) Consider a version of the model analyzed above, in which the equilibrium is interior. Now, modify that model so that the government raises lump sum taxes, T, in period 1 and uses the proceeds to finance the purchase of securities from firms. In period 2, the government rebates TR^k to households in the form of a lump sum transfer. Indicate how the household problem must be modified to accommodate this tax policy. What is the impact on equilibrium allocations, C, c, d of the presence of T? Explain the role played by interiority of equilibrium in the analysis.

f) Now, suppose that each banker has two options after it has selected a value for d. Under the first option it can do as assumed above: earn a return on its assets, pay interest to depositors and send the difference home in the form of profits. Under the second option ('default') the bank seizes a fraction, $\theta \in (0, 1)$, of its securities, $(N + d)$, and declines to pay anything to its depositors in period 2. A defaulting bank earns profits, $\pi = R^k \theta (N + d)$, which it sends home in period 2. Depositors in a defaulting bank earn $(1 - \theta)(N + d) R^k$, on the assets not taken by the banker. Here, θ is an exogenous parameter, not chosen by the banker. At the beginning of period 1, before households have made their deposit decision, banks are required to reveal how many deposits, d, they are willing to accept.

   i) Explain why the following condition is necessary and sufficient for a bank to not default (in the case of indifference, assume a bank does not default):

$$R^k \theta (N + d) \leq R^k (N + d) - Rd. \tag{1}$$

ii) Consider an equilibrium in which banks do not default. Explain why it is optimal for an individual bank in such an equilibrium to not default. Explain why it is that when a bank in that equilibrium selects a value for d, it only considers d's that satisfy (1).

iii) Let an *interior, no-default equilibrium* be a set of numbers, c, C, d, R, such that: $c, C, d > 0$; banks do not default; d, c, C solves the household problem given R; and the value of d optimizes banks' profits, subject to (1) and given R.

   i. Show that if (1) is binding (in the sense that its Lagrange multiplier is positive), then $R^k > R$.

   ii. Provide intuition into why the interior, no-default equilibrium is not first-best efficient when (1) is binding.