Simple New Keynesian Model without Capital

Lawrence J. Christiano

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What’s It Good For?

• Conveying basic principles of macroeconomics -
  – Concept and measurement of output gap:
    • ‘difference between the actual economy and where would be if policy was managed as well as possible’.
  – Importance of aggregate demand.
    • problems when it goes awry.
  – Important policy objective: assuring the right level of aggregate demand.

• Thinking through the operating characteristics of policy rules:
  – Inflation targeting, Tax/spending rules, Leverage restrictions on banks.

• Can even use it to learn econometrics
  – how well do standard econometric estimators work?
  – how good is HP filter at estimating output gap?
Our Approach to NK Model

- We will derive the familiar ‘three equation NK model’, but they will not be our starting point.
  - Start with households, firms, technology, etc.

- Necessary to build the model from scratch -
  - need this to uncover the principles hiding inside it
  - needed to know how to ‘go back to the drawing board’ and modify the model so it can address interesting questions:
    - how should macro prudential policy be conducted?
    - how might currency mismatch problems affect the usual transmission of exchange rate depreciation to the economy?
    - what should the role of inflation, labor markets, credit growth, stock markets, etc., be in monetary policy?
    - how does an expansion of unemployment benefits in a recession affect the business cycle?
Households

- Problem:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \exp (\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^t
\]

s.t. \( P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t \)

- First order conditions:

\[
\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)
\]

\[
\exp (\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.
\]
Goods Production

• A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

\[ Y_t = \left[ \int_0^1 Y_{i,t} \frac{\epsilon-1}{\epsilon} \, dj \right]^{\frac{\epsilon}{\epsilon-1}}. \]

• Each intermediate good, \( Y_{i,t} \), is produced as follows:

\[ Y_{i,t} = \exp(a_t) \overbrace{A_t}^{\text{operator}} N_{i,t}, \quad a_t = \rho a_{t-1} + \epsilon_t^a \]

• Before discussing the firms that operate these production functions, we briefly investigate the socially efficient (‘First Best’) allocation of labor across \( i \), for given \( N_t \):

\[ N_t = \int_0^1 N_{it} \, di \]
Efficient Sectoral Allocation of Labor

• With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  – It is optimal to run them all at the same rate, i.e., $Y_{i,t} = Y_{j,t}$ for all $i,j \in [0,1]$.
• For given $N_t$, it is optimal to set $N_{i,t} = N_{j,t}$ for all $i,j \in [0,1]$.
• In this case, final output is given by $Y_t = e^{at} N_t$.

• Best way to see this is to suppose that labor is not allocated equally to all activities.
  – But, this can happen in a million different ways when there is a continuum of inputs!
  – Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i,j \in [0,1]$.
Suppose Labor Not Allocated Equally

• Example:

\[ N_{it} = \begin{cases} 
2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\
2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right], \quad 0 \leq \alpha \leq 1.
\]  

• Note that this is a particular distribution of labor across activities:

\[ \int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha)N_t = N_t \]
Labor Not Allocated Equally, cnt’ed

\[ Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{e-1}{e}} di \right]^{\frac{e}{e-1}} \]

\[ = \left[ \int_0^{\frac{1}{2}} Y_{i,t}^{\frac{e-1}{e}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{e-1}{e}} di \right]^{\frac{e}{e-1}} \]

\[ = e^{a_t} \left[ \int_0^{\frac{1}{2}} N_{i,t}^{\frac{e-1}{e}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{e-1}{e}} di \right]^{\frac{e}{e-1}} \]

\[ = e^{a_t} \left[ \int_0^{\frac{1}{2}} (2\alpha N_t)\left(\frac{e-1}{e}\right) di + \int_{\frac{1}{2}}^1 (2(1 - \alpha)N_t)\left(\frac{e-1}{e}\right) di \right]^{\frac{e}{e-1}} \]

\[ = e^{a_t} N_t \left[ \int_0^{\frac{1}{2}} (2\alpha)^{\left(\frac{e-1}{e}\right)} di + \int_{\frac{1}{2}}^1 (2(1 - \alpha))^{\left(\frac{e-1}{e}\right)} di \right]^{\frac{e}{e-1}} \]

\[ = e^{a_t} N_t \left[ \frac{1}{2} (2\alpha)^{\left(\frac{e-1}{e}\right)} + \frac{1}{2} (2(1 - \alpha))^{\left(\frac{e-1}{e}\right)} \right]^{\frac{e}{e-1}} \]

\[ = e^{a_t} N_t f(\alpha) \]
\[ f(\alpha) = \left[ \frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1 - \alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \]

Efficient Resource Allocation Means Equal Labor Across All Sectors

\[ \varepsilon = 6 \]

\[ \varepsilon = 10 \]
Final Goods Production

• Final good firms:
  – maximize profits:

\[
P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,
\]

subject to:

\[
Y_t = \left[ \int_0^1 Y_{i,t}^\frac{\varepsilon - 1}{\varepsilon} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}.
\]

– Foncs:

\[
Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon \rightarrow P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}
\]

"cross price restrictions"
Intermediate Goods Production

• Demand curve for $i^{th}$ monopolist:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon.$$  

• Production function:

$$Y_{i,t} = \exp(a_t) N_{i,t}, \quad \Delta a_t = \rho \Delta a_{t-1} + \varepsilon^a_t$$

• Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ \frac{P_{i,t-1}}{P_{i,t}} & \text{with probability } \theta \end{cases}.$$  

• Real marginal cost:

$$S_t = \frac{d\text{Cost}}{d\text{output}} = \frac{\left(1 - \nu\right) W_t}{\exp(a_t) P_t}$$
Optimal Price Setting by Intermediate Goods Producers

- Let

\[ \tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \quad \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}. \]

- First order condition implied by optimal price setting:

\[ \tilde{p}_t = \frac{K_t}{F_t}, \]

where

\[
K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \tag{1}
\]

\[
F_t = 1 + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \tag{2}
\]

- Note:

\[
K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1}
+ (\beta \theta)^2 E_t \bar{\pi}_{t+2}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+2} + \ldots
\]
Price Equilibrium Conditions

- Cross-price restrictions imply, given the Calvo price-stickiness:

\[ P_t = \left[ (1 - \theta) \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \]

- Dividing latter by \( P_t \) and solving for \( \tilde{p}_t \):

\[ \tilde{p}_t = \left[ \frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \]

- Combining with the first order condition for \( \tilde{p}_t \):

\[ \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3) \]
Aggregate Inputs and Aggregate Output

- Tack Yun argument:

\[ Y_t^* \equiv \int_0^1 Y_{i,t} di \quad \left( = \int_0^1 e^{a_i} N_{i,t} di = e^{a_i} N_t \right) \]

\[ \text{demand curve} \]

\[ \Rightarrow \int_0^1 Y_t P_t^\varepsilon P_{i,t}^{-\varepsilon} di = Y_t P_t^\varepsilon \int_0^1 P_{i,t}^{-\varepsilon} di \]

\[ \equiv p_t^* \]

\[ \rightarrow Y_t = \left( \frac{P_t^*}{P_t} \right)^\varepsilon e^{a_t} N_t \]

\[ P_t^* = \left[ (1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta P_{t-1}^{-\varepsilon} \right]^{\frac{1}{-\varepsilon}} \]

\[ \rightarrow p_t^* = \left[ (1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\tilde{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \]
Goods Equilibrium Conditions

- Relationship between aggregate output and aggregate inputs:

\[ C_t = p_t^* A_t N_t, \quad (6) \]

where (‘Tack Yun distortion’)

\[
p_t^* = p^* (\bar{\pi}_t, p_{t-1}^*) \equiv \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}
\]
Tack Yun Distortion: a Closer Look

- Distortion:

\[
p_t^* = p^* (\bar{\pi}_t, p_{t-1}^*) \equiv \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\varepsilon \frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}
\]

- Distortion, \( p_t^* \), increasing function of lagged distortion, \( p_{t-1}^* \).
- Current shocks affect current distortion via \( \bar{\pi}_t \) only.

- Derivatives:

\[
p_1^* (\bar{\pi}_t, p_{t-1}^*) = - (p_t^*)^2 \varepsilon \theta \bar{\pi}_t^{\varepsilon-2} \left[ \frac{\bar{\pi}_t}{p_{t-1}^*} - \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{1}{\varepsilon-1}} \right]
\]

\[
p_2^* (\bar{\pi}_t, p_{t-1}^*) = \left( \frac{p_t^*}{p_{t-1}^*} \right)^2 \theta \bar{\pi}_t^\varepsilon.
\]
Linear Expansion of Tack Yun Distortion in Undistorted Steady State

- Linearizing about $\bar{\pi}_t = \bar{\pi}, p_{t-1}^* = p^*$:
  \[
dp^*_t = p_1^* (\bar{\pi}, p^*) \, d\bar{\pi}_t + p_2^* (\bar{\pi}, p^*) \, dp_{t-1}^* ,
\]
  where $dx_t \equiv x_t - x$, for $x_t = p_t^*, p_{t-1}^*, \bar{\pi}_t$.

- In an undistorted steady state (i.e., $\bar{\pi}_t = p_t^* = p_{t-1}^* = 1$):
  \[
p_1^* (1, 1) = 0, \quad p_2^* (1, 1) = \theta.
\]
  so that
  \[
dp_t^* = 0 \times d\bar{\pi}_t + \theta dp_{t-1}^* \\
  \rightarrow \quad p_t^* = 1 - \theta + \theta p_{t-1}^*
\]

- Often, people that linearize NK model ignore $p_t^*$.
  - Reflects that they linearize the model around a price-undistorted steady state.
Current Period Tack Yun Distortion as a Function of Current Inflation

Graph conditioned on two alternative values for $p_{t-1}^*$ and $\theta = 0.75, \varepsilon = 6.00$

lagged Tack Yun distortion = 1 (i.e., no distortion)
lagged Tack Yun distortion = .9
Ignoring Tack Yun Distortion, a Mistake?

Tack Yun Distortion and Quarterly US CPI Inflation (Gross)
‘First Best Consumption and Employment’ useful concepts in simple NK model.

- Ramsey is the appropriate benchmark, but Ramsey and first best coincide in simple NK model.

Explained above that with socially efficient sectoral allocation of labor,

\[ Y_t = \exp (a_t) N_t. \]

First best level of employment and consumption is solution to

\[ N_t^{\text{best}} = \arg \max_N \left\{ \log [\exp (a_t) N] - \exp (\tau_t) \frac{N^{1+\varphi}}{1 + \varphi} \right\} \]

so,

\[ N_t^{\text{best}} = \exp \left( -\frac{\tau_t}{1 + \varphi} \right), \quad C_t^{\text{best}} = \exp \left( a_t - \frac{\tau_t}{1 + \varphi} \right) \]
Linearizing around Efficient Steady State

- In steady state (assuming $\bar{\pi} = 1, 1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$)

$$p^* = 1, \quad K = F = \frac{1}{1 - \beta \theta}, \quad s = \frac{\varepsilon - 1}{\varepsilon}, \quad \Delta a = \tau = 0, \quad N = 1$$

- Linearizing the Tack Yun distortion, (4):

$$p_t^* = 1, \quad t \text{ large enough}$$

- Denote the output gap in ratio form by $X_t$:

$$X_t \equiv \frac{C_t}{\exp \left( a_t - \frac{\tau_t}{1 + \varphi} \right)} = p_t^* N_t \exp \left( \frac{\tau_t}{1 + \varphi} \right),$$

where the denominator is the socially efficient (‘First Best’) level of consumption.

- Then, with $x_t \equiv \hat{X}_t$ and $\hat{p}_t^* = 0$:

$$x_t = \hat{N}_t + \frac{d\tau_t}{1 + \varphi}$$
The intertemporal Euler equation, (5), after substituting for $C_t$ in terms of $X_t$:

$$
\frac{1}{X_t \exp \left( a_t - \frac{\tau_t}{1+\varphi} \right)} = \beta E_t \frac{1}{X_{t+1} \exp \left( a_{t+1} - \frac{\tau_{t+1}}{1+\varphi} \right)} \frac{R_t}{\bar{\pi}_{t+1}}
$$

$$
\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R^*_t} \frac{R_t}{\bar{\pi}_{t+1}},
$$

where

value of $R_t$ in ‘first best’

$$
\widehat{R}^*_t = \frac{1}{\beta} \exp \left( a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi} \right)
$$

then, (using $\widehat{z}_t u_t = \hat{z}_t + \hat{u}_t$, $\left( \frac{u_t}{z_t} \right) = \hat{u}_t - \hat{z}_t$):

$$
\hat{X}_t = E_t \left[ \hat{X}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}^*_t) \right]
$$
NK IS Curve, Baseline Model

- Note:

\[ Z_t = \exp(z_t), \text{ where } z_t \equiv \log Z_t \]
\[ \hat{Z}_t \equiv \frac{dZ_t}{Z} = \frac{d \exp(z_t)}{Z} = \frac{Zdz_t}{Z} = dz_t = \log Z_t - \log Z. \]

- Use this to establish, when the steady state is efficient:

\[ E_t (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}^*_{t+1}) = \log R_t - E_t \pi_{t+1} - E_t \log R^*_{t+1} = r_t - E_t \pi_{t+1} - r^*. \]

where

\[ r_t \equiv \log R_t, \quad r^*_t \equiv E_t \log R^*_{t+1}, \quad \pi_{t+1} \equiv \log \bar{\pi}_{t+1}, \]

- and, in efficient steady state:

\[ \log R^* = \log R, \quad \log \bar{\pi} = 0. \]
NK IS Curve, Baseline Model

• Substituting

\[ \hat{X}_t = E_t \left[ \hat{X}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}^*_t) \right], \ x_t \equiv \hat{X}_t, \]

we obtain NK IS curve:

\[ x_t = E_t x_{t+1} - E_t \left[ r_t - \pi_{t+1} - r^*_t \right] \]

• Also,

\[ r^*_t = -\log(\beta) + E_t \left[ a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1 + \phi} \right]. \]
Linearized Marginal Cost in Baseline Model

- Marginal cost (using $da_t = a_t$, $d\tau_t = \tau_t$ because $a = \tau = 0$):

$$s_t = (1 - \nu) \frac{\bar{w}_t}{A_t}, \quad \bar{w}_t = \exp(\tau_t) N_t^\varphi C_t$$

$$\rightarrow \hat{w}_t = \tau_t + a_t + (1 + \varphi) \hat{N}_t$$

- Then,

$$\hat{s}_t = \hat{w}_t - a_t = (\varphi + 1) \left[ \frac{\tau_t}{\varphi + 1} + \hat{N}_t \right] = (\varphi + 1) x_t$$
Linearized Phillips Curve in Baseline Model

- Log-linearize equilibrium conditions, (1)-(3), around steady state:

\[
\begin{align*}
\hat{K}_t &= (1 - \beta \theta) \hat{s}_t + \beta \theta (\varepsilon \hat{\pi}_{t+1} + \hat{K}_{t+1}) \quad (1) \\
\hat{F}_t &= \beta \theta (\varepsilon - 1) \hat{\pi}_{t+1} + \beta \theta \hat{F}_{t+1} \quad (2) \\
\hat{K}_t &= \hat{F}_t + \frac{\theta}{1 - \theta} \hat{\pi}_t \quad (3)
\end{align*}
\]

- Substitute (3) into (1)

\[
\hat{F}_t + \frac{\theta}{1 - \theta} \hat{\pi}_t = (1 - \beta \theta) \hat{s}_t + \beta \theta \left( \varepsilon \hat{\pi}_{t+1} + \hat{F}_{t+1} + \frac{\theta}{1 - \theta} \hat{\pi}_{t+1} \right)
\]

- Simplify the latter using (2), to obtain the NK Phillips curve:

\[
\pi_t = \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \hat{s}_t + \beta \pi_{t+1}
\]
Nonlinear Private Sector Equilibrium Conditions

\( K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_t^{\varepsilon} K_{t+1} \) \( (1) \)

\( F_t = 1 + \beta \theta E_t \bar{\pi}_t^{\varepsilon-1} F_{t+1} \) \( (2) \)

\[ \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \] \( (3) \)

\[ p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \] \( (4) \)

\[ \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1} \bar{\pi}_{t+1}} R_t \] \( (5) \)

\[ C_t = p_t^* A_t N_t \] \( (6) \)
The Linearized Private Sector Equilibrium Conditions

\[
\begin{align*}
    x_t &= E_t x_{t+1} - \left[ r_t - E_t \pi_{t+1} - r^*_t \right] \\
    \pi_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t + \beta E_t \pi_{t+1} \\
    \hat{s}_t &= (\phi + 1) x_t \\
    r^*_t &= -\log (\beta) + E_t \left[ a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\phi} \right]
\end{align*}
\]

Monetary policy rule:

\[
    r_t = \alpha r_{t-1} + (1 - \alpha) \left[ r + \phi r \pi_t + \phi x x_t \right]
\]
Solving the Model

• Vision about evolution of actual data:
  – Nature draws the exogenous shocks.
  – The economy transforms exogenous shocks into realization of endogenous variables, inflation, output, unemployment, etc.

• ‘Solving the model’:
  – Determining the model’s implications for the mapping from exogenous shocks to endogenous variables.
  – Potentially massive problem: current value of endogenous variables a function of past data and expected future value of endogenous variables.

• Primary strategy for solving a model:
  – Find a linear representation (‘policy rule’) of the endogenous variables, $z_t$, in terms of current and past data only:

    $$z_t = Az_{t-1} + Bs_t$$

  such that the equilibrium conditions (after linearization) are satisfied.
• Exogenous shocks:

\[ s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^a_t \\ \varepsilon^\tau_t \end{pmatrix} \]

\[ s_t = P s_{t-1} + \epsilon_t \]

• Equilibrium Conditions:

\[
\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ (1-\alpha)\phi_x \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{(1-\theta)(1-\beta\theta)}{\theta} (1+\phi) & 0 & 0 \\ 0 & -1 & -1 & 1 \\ (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{pmatrix}
\]

\[
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} s_{t+1} \\ s_t \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\]

\[ E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0 \]
Collecting:

\[ E_t \left[ \alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0 \]

\[ s_t - Ps_{t-1} - \epsilon_t = 0. \]

Policy rule:

\[ z_t = Az_{t-1} + Bs_t \]

As before, want \( A \) such that

\[ \alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0, \]

Want \( B \) such that:

\[ (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0 \]

Note: if \( \alpha = 0 \), then \( A = 0 \) is one solution (there is another one!).
\( \phi_x = 0, \phi_\pi = 1.5, \beta = 0.99, \varphi = 1, \rho = 0.2, \theta = 0.75, \alpha = 0, \delta = 0.2, \lambda = 0.5. \)
Dynamic Response to a Preference Shock

- **Inflation**
- **Output Gap**
- **Nominal Rate**
- **Natural Real Rate**
- **Preference Shock**
- **Employment**
- **Output**

Graphs show the dynamic response of various economic indicators to a preference shock.

- **Inflation** decreases over time.
- **Output Gap** decreases over time.
- **Nominal Rate** decreases over time.
- **Natural Real Rate** decreases over time.
- **Preference Shock** shows a decrease over time.
- **Employment** increases over time.
- **Output** increases over time.